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Graphing Ordered Pairs

- Points in the coordinate plane are named by ordered pairs of the form \((x, y)\). The first number, or \(x\)-coordinate, corresponds to a number on the \(x\)-axis. The second number, or \(y\)-coordinate, corresponds to a number on the \(y\)-axis.

**Example 1** Write the ordered pair for each point.

a. **A**
   - The \(x\)-coordinate is 4.
   - The \(y\)-coordinate is \(-1\).
   - The ordered pair is \((4, -1)\).

b. **B**
   - The \(x\)-coordinate is \(-2\).
   - The point lies on the \(x\)-axis, so its \(y\)-coordinate is 0.
   - The ordered pair is \((-2, 0)\).

- The \(x\)-axis and \(y\)-axis separate the coordinate plane into four regions, called quadrants. The point at which the axes intersect is called the origin. The axes and points on the axes are not located in any of the quadrants.

**Example 2** Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.

a. **G(2, 1)**
   - Start at the origin. Move 2 units right, since the \(x\)-coordinate is 2. Then move 1 unit up, since the \(y\)-coordinate is 1.
   - Draw a dot, and label it \(G\). Point \(G(2, 1)\) is in Quadrant I.

b. **H(-4, 3)**
   - Start at the origin. Move 4 units left, since the \(x\)-coordinate is \(-4\). Then move 3 units up, since the \(y\)-coordinate is 3. Draw a dot, and label it \(H\). Point \(H(-4, 3)\) is in Quadrant II.

c. **J(0, -3)**
   - Start at the origin. Since the \(x\)-coordinate is 0, the point lies on the \(y\)-axis. Move 3 units down, since the \(y\)-coordinate is \(-3\). Draw a dot, and label it \(J\). Because it is on one of the axes, point \(J(0, -3)\) is not in any quadrant.
**Example 3**

Graph a polygon with vertices \(A(-3, 3), B(1, 3), C(0, 1),\) and \(D(-4, 1)\).

Graph the ordered pairs on a coordinate plane. Connect each pair of consecutive points. The polygon is a parallelogram.

**Example 4**

Graph four points that satisfy the equation \(y = 4 - x\).

Make a table. Choose four values for \(x\). Evaluate each value of \(x\) for \(4 - x\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(4 - x)</th>
<th>(y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>1</td>
<td>4 - 1</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>4 - 2</td>
<td>2</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>3</td>
<td>4 - 3</td>
<td>1</td>
<td>(3, 1)</td>
</tr>
</tbody>
</table>

**Exercises**

Write the ordered pair for each point shown at the right.

1. \(B(-2, 3)\)
2. \(C(1, -1)\)
3. \(D(2, 2)\)
4. \(E(-3, -3)\)
5. \(F(-3, 1)\)
6. \(G(0, -3)\)
7. \(H(4, 1)\)
8. \(I(3, -2)\)
9. \(J(-1, -1)\)
10. \(K(1, 4)\)
11. \(W(3, 0)\)
12. \(M(-2, -4)\)
13. \(N(2, -4)\)
14. \(P(3, 3)\)
15. \(Q(-4, 2)\)

Graph and label each point on a coordinate plane. Name the quadrant in which each point is located. **16–31. See margin for graph.**

16. \(M(-1, 3)\)
17. \(S(2, 0)\)
18. \(R(-3, -2)\)
19. \(P(1, -4)\)
20. \(B(5, -1)\)
21. \(D(3, 4)\)
22. \(T(2, 5)\)
23. \(L(-4, -3)\)
24. \(A(-2, 2)\)
25. \(N(4, 1)\)
26. \(H(-3, -1)\)
27. \(F(0, -2)\)
28. \(C(-3, 1)\)
29. \(E(1, 3)\)
30. \(G(3, 2)\)
31. \(I(3, -2)\)

Graph the following geometric figures. **32–35. See margin.**

32. a square with vertices \(W(-3, 3), X(-3, -1), Y(1, 3),\) and \(Z(1, -1)\)
33. a polygon with vertices \(J(4, 2), K(1, -1), L(-2, 2),\) and \(M(1, 5)\)
34. a triangle with vertices \(F(2, 4), G(-3, 2),\) and \(H(-1, -3)\)
35. a rectangle with vertices \(P(-2, -1), Q(4, -1), R(-2, 1),\) and \(S(4, 1)\)

Graph four points that satisfy each equation. **36–39. See margin for sample answers.**

36. \(y = 2x\)
37. \(y = 1 + x\)
38. \(y = 3x - 1\)
39. \(y = 2 - x\)
Prerequisite Skills

Changing Units of Measure within Systems

<table>
<thead>
<tr>
<th>Metric Units of Length</th>
<th>Customary Units of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer (km) = 1000 meters (m)</td>
<td>1 foot (ft) = 12 inches (in.)</td>
</tr>
<tr>
<td>1 m = 100 centimeters (cm)</td>
<td>1 yard (yd) = 3 ft</td>
</tr>
<tr>
<td>1 cm = 10 millimeters (mm)</td>
<td>1 mile (mi) = 5280 ft</td>
</tr>
</tbody>
</table>

- To convert from larger units to smaller units, multiply.
- To convert from smaller units to larger units, divide.

**Example 1**

State which metric unit you would use to measure the length of your pen.
Since a pen has a small length, the centimeter is the appropriate unit of measure.

**Example 2**

Complete each sentence.

a. 4.2 km = __ m
   There are 1000 meters in a kilometer.
   \[4.2 \text{ km} \times 1000 = 4200 \text{ m}\]

b. 125 mm = __ cm
   There are 10 millimeters in a centimeter.
   \[125 \text{ mm} \div 10 = 12.5 \text{ cm}\]

c. 16 ft = __ in.
   There are 12 inches in a foot.
   \[16 \text{ ft} \times 12 = 192 \text{ in.}\]

d. 39 ft = __ yd
   There are 3 feet in a yard.
   \[39 \text{ ft} \div 3 = 13 \text{ yd}\]

**Example 3**

Complete each sentence.

a. 17 mm = __ m
   There are 100 centimeters in a meter. First change millimeters to centimeters.
   \[17 \text{ mm} = \frac{17}{10} \text{ cm}\]
   \[17 \text{ mm} \div 10 = 1.7 \text{ cm}\]
   Then change centimeters to meters.
   \[1.7 \text{ cm} = \frac{1.7}{100} \text{ m}\]
   \[1.7 \text{ cm} \div 100 = 0.017 \text{ m}\]
   Since 10 mm = 1 cm, divide by 10.
   Since 100 cm = 1 m, divide by 100.

b. 6600 yd = __ mi
   There are 5280 feet in one mile. First change yards to feet.
   \[6600 \text{ yd} = 3 \text{ mi}\]
   \[6600 \text{ yd} \times 3 = 19,800 \text{ ft}\]
   Then change feet to miles.
   \[19,800 \text{ ft} = \frac{19,800}{5280} \text{ mi}\]
   \[19,800 \text{ ft} \div 5280 = 3\frac{3}{4} \text{ mi}\]
   Since 5280 ft = 1 mi, divide by 5280.

**Example 4**

Complete each sentence.

a. 3.7 L = __ mL
   There are 1000 milliliters in a liter.
   \[3.7 \text{ L} \times 1000 = 3700 \text{ mL}\]

b. 16 qt = __ gal
   There are 4 quarts in a gallon.
   \[16 \text{ qt} \div 4 = 4 \text{ gal}\]
Examples c and d involve two-step conversions.

- **Examples c and d involve two-step conversions.**
  
  **c.** $7 \text{ pt} = \ ? \text{ fl oz}$
  
  There are 8 fluid ounces in a cup.
  
  First change **pints** to **cups**.
  
  $7 \text{ pt} = \ ? \text{ c}$
  
  $7 \text{ pt} \times 2 = 14 \text{ c}$
  
  Then change **cups** to **fluid ounces**.
  
  $14 \text{ c} = \ ? \text{ fl oz}$
  
  $14 \text{ c} \times 8 = 112 \text{ fl oz}$
  
  **d.** $4 \text{ gal} = \ ? \text{ pt}$
  
  There are 4 quarts in a gallon.
  
  First change **gallons** to **quarts**.
  
  $4 \text{ gal} = \ ? \text{ qt}$
  
  $4 \text{ gal} \times 4 = 16 \text{ qt}$
  
  Then change **quarts** to **pints**.
  
  $16 \text{ qt} = \ ? \text{ pt}$
  
  $16 \text{ qt} \times 2 = 32 \text{ pt}$

- The mass of an object is the amount of matter that it contains.

<table>
<thead>
<tr>
<th>Metric Units of Mass</th>
<th>Customary Units of Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilogram (kg) = 1000 grams (g)</td>
<td>1 pound (lb) = 16 ounces (oz)</td>
</tr>
<tr>
<td>1 g = 1000 milligrams (mg)</td>
<td>1 ton (T) = 2000 lb</td>
</tr>
</tbody>
</table>

**Example 5** Complete each sentence.

- **a.** $2300 \text{ mg} = \ ? \text{ g}$
  
  There are 1000 milligrams in a gram.
  
  $2300 \text{ mg} \div 1000 = 2.3 \text{ g}$

- **b.** $120 \text{ oz} = \ ? \text{ lb}$
  
  There are 16 ounces in a pound.
  
  $120 \text{ oz} \div 16 = 7.5 \text{ lb}$

- **c.** $5.47 \text{ kg} = \ ? \text{ mg}$
  
  There are 1000 milligrams in a gram.
  
  Change **kilograms** to **grams**.
  
  $5.47 \text{ kg} = \ ? \text{ g}$
  
  $5.47 \text{ kg} \times 1000 = 5470 \text{ g}$
  
  Then change **grams** to **milligrams**.
  
  $5470 \text{ g} = \ ? \text{ mg}$
  
  $5470 \text{ g} \times 1000 = 5,470,000 \text{ mg}$

- **d.** $5 \text{ T} = \ ? \text{ oz}$
  
  There are 16 ounces in a pound.
  
  Change **tons** to **pounds**.
  
  $5 \text{ T} = \ ? \text{ lb}$
  
  $5 \text{ T} \times 2000 = 10,000 \text{ lb}$
  
  Then change **pounds** to **ounces**.
  
  $10,000 \text{ lb} = \ ? \text{ oz}$
  
  $10,000 \text{ lb} \times 16 = 160,000 \text{ oz}$

**Exercises** State which metric unit you would probably use to measure each item.

1. radius of a tennis ball **cm**
2. length of a notebook **cm**
3. mass of a textbook **kg**
4. mass of a beach ball **g**
5. width of a football field **m**
6. thickness of a penny **mm**
7. amount of liquid in a cup **mL**
8. amount of water in a bath tub **L**

Complete each sentence.

- **9.** $120 \text{ in.} = \ ? \text{ ft} 10$
- **12.** $210 \text{ mm} = \ ? \text{ cm} 21$
- **15.** $90 \text{ in.} = \ ? \text{ yd} 2.5$
- **18.** $0.62 \text{ km} = \ ? \text{ m} 620$
- **21.** $32 \text{ fl oz} = \ ? \text{ c} 4$
- **24.** $48 \text{ c} = \ ? \text{ gal} 3$
- **27.** $13 \text{ lb} = \ ? \text{ oz} 208$
- **10.** $18 \text{ ft} = \ ? \text{ yd} 6$
- **13.** $180 \text{ mm} = \ ? \text{ m} 0.18$
- **16.** $5280 \text{ yd} = \ ? \text{ mi} 3$
- **19.** $370 \text{ mL} = \ ? \text{ L} 0.370$
- **22.** $5 \text{ qt} = \ ? \text{ c} 20$
- **25.** $4 \text{ gal} = \ ? \text{ pt} 16$
- **28.** $130 \text{ g} = \ ? \text{ kg} 0.130$
- **11.** $10 \text{ km} = \ ? \text{ m} 10,000$
- **14.** $3100 \text{ m} = \ ? \text{ km} 3.1$
- **17.** $8 \text{ yd} = \ ? \text{ ft} 24$
- **20.** $12 \text{ L} = \ ? \text{ mL} 12,000$
- **23.** $10 \text{ pt} = \ ? \text{ qt} 5$
- **26.** $36 \text{ mg} = \ ? \text{ g} 0.036$
- **29.** $9.05 \text{ kg} = \ ? \text{ g} 9050$
**Perimeter and Area of Rectangles and Squares**

**Perimeter** is the distance around a figure whose sides are segments. Perimeter is measured in linear units.

<table>
<thead>
<tr>
<th>Perimeter of a Rectangle</th>
<th>Perimeter of a Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Words</td>
<td>Words</td>
</tr>
<tr>
<td>Multiply two times the</td>
<td>Multiply 4 times the</td>
</tr>
<tr>
<td>sum of the length and</td>
<td>length of a side.</td>
</tr>
<tr>
<td>width.</td>
<td>Formula</td>
</tr>
<tr>
<td>$P = 2(l + w)$</td>
<td>$P = 4s$</td>
</tr>
</tbody>
</table>

**Example 1** Find the perimeter and area of each rectangle.

a. 

\[
P = 2(l + w) \quad \text{Perimeter formula}
\]

\[
= 2(4 + 9) \quad \text{Replace } l \text{ with 4 and } w \text{ with 9.}
\]

\[
= 26 \quad \text{Simplify.}
\]

\[
A = lw \quad \text{Area formula}
\]

\[
= 4 \cdot 9 \quad \text{Replace } l \text{ with 4 and } w \text{ with 9.}
\]

\[
= 36 \quad \text{Multiply.}
\]

The perimeter is 26 units, and the area is 36 square units.
b. a rectangle with length 8 units and width 3 units.

\[ P = 2(\ell + w) \]  
Perimeter formula

\[ = 2(8 + 3) \]  
Replace \( \ell \) with 8 and \( w \) with 3.

\[ = 22 \]  
Simplify.

\[ A = \ell \cdot w \]  
Area formula

\[ = 8 \cdot 3 \]  
Replace \( \ell \) with 8 and \( w \) with 3.

\[ = 24 \]  
Multiply

The perimeter is 22 units, and the area is 24 square units.

**Example 2** Find the perimeter and area of a square that has a side of length 14 feet.

\[ P = 4s \]  
Perimeter formula

\[ = 4(14) \]  
\( s = 14 \)

\[ = 56 \]  
Multiply.

\[ A = s^2 \]  
Area formula

\[ = 14^2 \]  
\( s = 14 \)

\[ = 196 \]  
Multiply.

The perimeter is 56 feet, and the area is 196 square feet.

**Exercises** Find the perimeter and area of each figure.

1.  
   \[ P = 44 \text{ in.}, \ A = 121 \text{ in}^2 \]

2.  
   \[ P = 21 \text{ km}, \ A = 22.5 \text{ km}^2 \]

3.  
   \[ P = 14 \text{ yd}, \ A = 12.25 \text{ yd}^2 \]

4.  
   \[ P = 13 \text{ ft}, \ A = 10 \text{ ft}^2 \]

5.  
   \[ P = 15 \text{ cm}, \ A = 10.26 \text{ cm}^2 \]

7. a rectangle with length 7 meters and width 11 meters \( P = 36 \text{ m}, \ A = 77 \text{ m}^2 \)

8. a square with length 4.5 inches \( P = 18 \text{ in.}, \ A = 20.25 \text{ in}^2 \)

9. a rectangular sandbox with length 2.4 meters and width 1.6 meters \( P = 8 \text{ m}, \ A = 3.84 \text{ m}^2 \)

10. a square with length 6.5 yards \( P = 26 \text{ yd}, \ A = 42.25 \text{ yd}^2 \)

11. a square office with length 12 feet \( P = 48 \text{ ft}, \ A = 144 \text{ ft}^2 \)

12. a rectangle with length 4.2 inches and width 15.7 inches \( P = 39.8 \text{ in.}, \ A = 65.94 \text{ in}^2 \)

13. a square with length 18 centimeters \( P = 72 \text{ cm}, \ A = 324 \text{ cm}^2 \)

14. a rectangle with length 5.3 feet and width 7 feet \( P = 24.6 \text{ ft}, \ A = 37.1 \text{ ft}^2 \)

15. **FENCING** Jansen purchased a lot that was 121 feet in width and 360 feet in length. If he wants to build a fence around the entire lot, how many feet of fence does he need? 962 ft

16. **CARPETING** Leonardo’s bedroom is 10 feet wide and 11 feet long. If the carpet store has a remnant whose area is 105 square feet, could it be used to cover his bedroom floor? Explain. No, \( 10(11) = 110 \text{ and } 110 > 105 \).
4 Operations with Integers

- The absolute value of any number \( n \) is its distance from zero on a number line and is written as \(|n|\). Since distance cannot be less than zero, the absolute value of a number is always greater than or equal to zero.

**Example 1** Evaluate each expression.

a. \( |3| \)
   \[ |3| = 3 \quad \text{Definition of absolute value} \]

b. \( |-7| \)
   \[ |-7| = 7 \quad \text{Definition of absolute value} \]

c. \( |-4 + 2| \)
   \[ |-4 + 2| = |-2| = -4 + 2 = -2 \]
   \[ = 2 \quad \text{Simplify} \]

- To add integers with the same sign, add their absolute values. Give the result the same sign as the integers. To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

**Example 2** Find each sum.

a. \(-3 + (-5)\)
   \[ -3 + (-5) = -8 \quad \text{Both numbers are negative, so the sum is negative.} \]
   \[ \text{Add } | -3 | \text{ and } | -5 | . \]

b. \(-4 + 2\)
   \[ -4 + 2 = -2 \quad \text{The sum is negative because } | -4 | > | 2 | . \]
   \[ \text{Subtract } | 2 | \text{ from } | -4 | . \]

c. \(6 + (-3)\)
   \[ 6 + (-3) = 3 \quad \text{The sum is positive because } | 6 | > | -3 | . \]
   \[ \text{Subtract } | -3 | \text{ from } | 6 | . \]

d. \(1 + 8\)
   \[ 1 + 8 = 9 \quad \text{Both numbers are positive, so the sum is positive} \]
   \[ \text{Add } | 1 | \text{ and } | 8 | . \]

- To subtract an integer, add its additive inverse.

**Example 3** Find each difference.

a. \(4 - 7\)
   \[ 4 - 7 = 4 + (-7) \quad \text{To subtract 7, add } -7 . \]
   \[ = -3 \]

b. \(2 - (-4)\)
   \[ 2 - (-4) = 2 + 4 \quad \text{To subtract } -4, \text{ add } 4 . \]
   \[ = 6 \]

- The product of two integers with different signs is negative. The product of two integers with the same sign is positive. Similarly, the quotient of two integers with different signs is negative, and the quotient of two integers with the same sign is positive.
Find each product or quotient.

a. \(4(-7)\)  
\[
4(-7) = -28 \quad \text{The factors have different signs.}
\]

b. \(-64 \div (-8)\)  
\[
-64 \div (-8) = 8 \quad \text{The dividend and divisor have the same sign.}
\]

c. \(-9(-6)\)  
\[
-9(-6) = 54 \quad \text{The factors have the same sign.}
\]

d. \(-55 \div 5\)  
\[
-55 \div 5 = -11 \quad \text{The dividend and divisor have different signs.}
\]

e. \[
\frac{24}{-3} = -8 \quad \text{The dividend and divisor have different signs.}
\]

*To evaluate expressions with absolute value, evaluate the absolute values first and then perform the operation.*

Evaluate each absolute value.

1. \(|-3|\) \hspace{1cm} 2. \(|4|\) \hspace{1cm} 3. \(|0|\) \hspace{1cm} 4. \(|-5|\)

Find each sum or difference.

5. \(-4 - 5\) \hspace{1cm} 6. \(3 + 4\) \hspace{1cm} 7. \(9 - 5\) \hspace{1cm} 8. \(-2 - 5\)
9. \(3 - 5\) \hspace{1cm} 10. \(-6 + 11\) \hspace{1cm} 11. \(-4 + (-4)\) \hspace{1cm} 12. \(-5 - 9\)
13. \(-3 + 1\) \hspace{1cm} 14. \(-4 + (-2)\) \hspace{1cm} 15. \(2 - (-8)\) \hspace{1cm} 16. \(7 + (-3)\)
17. \(-4 - (-2)\) \hspace{1cm} 18. \(3 - (-3)\) \hspace{1cm} 19. \(3 + (-4)\) \hspace{1cm} 20. \(-3 - (-9)\)

Evaluate each expression.

21. \(|-4| - |6| - 2\) \hspace{1cm} 22. \(|-7| + |1|\) \hspace{1cm} 23. \(|1| + |2|\) \hspace{1cm} 24. \(|2| - |5| - 3\)
25. \(|-5 + 2|\) \hspace{1cm} 26. \(|6 + 4|\) \hspace{1cm} 27. \(|3 - 7|\) \hspace{1cm} 28. \(|-3 - 3|\)

Find each product or quotient.

29. \(-36 + 9\) \hspace{1cm} 30. \(-3(-7)\) \hspace{1cm} 31. \(6(-4)\) \hspace{1cm} 32. \(-25 + 5\)
33. \(-6(-3)\) \hspace{1cm} 34. \(7(-8)\) \hspace{1cm} 35. \(-40 \div (-5)\) \hspace{1cm} 36. \(11(3)\)
37. \(44 \div (-4)\) \hspace{1cm} 38. \(-63 \div (-7)\) \hspace{1cm} 39. \(6(5)\) \hspace{1cm} 40. \(-7(-12)\)
41. \(-10(4)\) \hspace{1cm} 42. \(80 \div (-16)\) \hspace{1cm} 43. \(72 \div 9\) \hspace{1cm} 44. \(39 \div 3\)
5 Evaluating Algebraic Expressions

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

### Order of Operations

1. Evaluate expressions inside grouping symbols.
2. Evaluate all powers.
3. Do all multiplications and/or divisions from left to right.
4. Do all additions and/or subtractions from left to right.

### Example 1

Evaluate each expression.

a. \( x - 5 + y \) if \( x = 15 \) and \( y = -7 \)

\[
\begin{align*}
\text{Step 1} & \quad \text{Substitute for } x, y \rightarrow 15 - 5 + (-7) \\
\text{Step 2} & \quad \text{Add} \rightarrow 3 \\
\end{align*}
\]

b. \( 6ab^2 \) if \( a = -3 \) and \( b = 3 \)

\[
\begin{align*}
\text{Step 1} & \quad \text{Substitute for } a, b \rightarrow 6(-3)(3)^2 \\
\text{Step 2} & \quad \text{Multiply} \rightarrow 6(-3)(9) = -162 \\
\end{align*}
\]

### Example 2

Evaluate each expression if \( m = -2, n = -4, \) and \( p = 5. \)

a. \( \frac{2m + n}{p - 3} \)

\[
\begin{align*}
\text{Step 1} & \quad \text{Substitute for } m, n, p \rightarrow \frac{2(-2) + (-4)}{5 - 3} \\
\text{Step 2} & \quad \text{Multiply} \rightarrow \frac{-4}{2} = -2 \\
\text{Step 3} & \quad \text{Subtract} \rightarrow -4 \\
\end{align*}
\]

b. \( -3(m^2 + 2n) \)

\[
\begin{align*}
\text{Step 1} & \quad \text{Substitute for } m, n \rightarrow -3((-2)^2 + 2(-4)) \\
\text{Step 2} & \quad \text{Multiply} \rightarrow -3[4 + (-8)] = -3(-4) \\
\text{Step 3} & \quad \text{Add} \rightarrow -12 \\
\end{align*}
\]

### Example 3

Evaluate \( 3 \left| a - b \right| + 2 \left| c - 5 \right| \) if \( a = -2, b = -4, \) and \( c = 3. \)

\[
\begin{align*}
\text{Step 1} & \quad \text{Substitute for } a, b, c \rightarrow 3 \left| -2 - (-4) \right| + 2 \left| 3 - 5 \right| \\
\text{Step 2} & \quad \text{Simplify} \rightarrow 3 \left| 2 \right| + 2 \left| -2 \right| \\
\text{Step 3} & \quad \text{Find absolute values} \rightarrow 3(2) + 2(2) \\
\text{Step 4} & \quad \text{Simplify} \rightarrow 10 \\
\end{align*}
\]

### Exercises

Evaluate each expression if \( a = 2, b = -3, c = -1, \) and \( d = 4. \)

1. \( 2a + c \)
2. \( \frac{2b}{2c} \)
3. \( \frac{2a - n}{b} \)
4. \( 3d - c \)
5. \( \frac{3b}{5a + c} \)
6. \( 5bc \)
7. \( 2cd + 3ab \)
8. \( \frac{c - 2d}{a} \)

Evaluate each expression if \( x = 2, y = -3, \) and \( z = 1. \)

9. \( 24 + \left| x - 4 \right| 
10. \( 13 + \left| 8 + y \right| 
11. \( \left| 5 - z \right| + 11 
12. \( \left| 2y - 15 \right| + 7 
13. \( \left| y \right| - 7 
14. \( 11 - 7 + \left| -x \right| 
15. \( \left| x \right| - 2z 
16. \( \left| z - y \right| + 6 

736 Prerequisite Skills
**Solving Linear Equations**

- If the same number is added to or subtracted from each side of an equation, the resulting equation is true.

**Example 1** Solve each equation.

a. \(x - 7 = 16\)
   \[
   \begin{align*}
   x - 7 &= 16 & \text{Original equation} \\
   x - 7 + 7 &= 16 + 7 & \text{Add 7 to each side.} \\
   x &= 23 & \text{Simplify.}
   \end{align*}
   \]

b. \(m + 12 = -5\)
   \[
   \begin{align*}
   m + 12 &= -5 & \text{Original equation} \\
   m + 12 + (-12) &= -5 + (-12) & \text{Add -12 to each side.} \\
   m &= -17 & \text{Simplify.}
   \end{align*}
   \]

c. \(k + 31 = 10\)
   \[
   \begin{align*}
   k + 31 &= 10 & \text{Original equation} \\
   k + 31 - 31 &= 10 - 31 & \text{Subtract 31 from each side.} \\
   k &= -21 & \text{Simplify.}
   \end{align*}
   \]

- If each side of an equation is multiplied or divided by the same number, the resulting equation is true.

**Example 2** Solve each equation.

a. \(4d = 36\)
   \[
   \begin{align*}
   4d &= 36 & \text{Original equation} \\
   \frac{4d}{4} &= \frac{36}{4} & \text{Divide each side by 4.} \\
   x &= 9 & \text{Simplify.}
   \end{align*}
   \]

b. \(-\frac{t}{8} = -7\)
   \[
   \begin{align*}
   -\frac{t}{8} &= -7 & \text{Original equation.} \\
   -8 \left(-\frac{t}{8}\right) &= -8(-7) & \text{Multiply each side by -8.} \\
   t &= 56 & \text{Simplify.}
   \end{align*}
   \]

c. \(\frac{3}{5}x = -8\)
   \[
   \begin{align*}
   \frac{3}{5}x &= -8 & \text{Original equation.} \\
   \frac{5}{3} \left(\frac{3}{5}x\right) &= \frac{5}{3}(-8) & \text{Multiply each side by} \ \frac{5}{3}. \\
   x &= -\frac{40}{3} & \text{Simplify.}
   \end{align*}
   \]

- To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.

**Example 3** Solve each equation.

a. \(12 - m = 20\)
   \[
   \begin{align*}
   12 - m &= 20 & \text{Original equation} \\
   12 - m - 12 &= 20 - 12 & \text{Subtract 12 from each side.} \\
   -m &= 8 & \text{Simplify.} \\
   m &= -8 & \text{Divide each side by} \ -1.
   \end{align*}
   \]
b. \(8q - 15 = 49\)

\[
8q - 15 = 49 \\
8q = 49 + 15 \\
8q = 64 \\
\frac{8q}{8} = \frac{64}{8} \\
q = 8
\]

Prerequisite Skills

Add 15 to each side.
Simplify.
Divide each side by 8.
Simplify.

When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

**Example 4** Solve \(3(x - 5) = 13\).

\[
3(x - 5) = 13 \quad \text{Original equation} \\
3x - 15 = 13 \quad \text{Distributive Property} \\
3x - 15 + 15 = 13 + 15 \quad \text{Add 15 to each side.} \\
x = \frac{28}{3} \quad \text{Divide each side by 3.}
\]

**Exercises** Solve each equation.

1. \(r + 11 = 3\) \(-8\)
2. \(n + 7 = 13\) \(6\)
3. \(d - 7 = 8\) \(15\)
4. \(\frac{8}{3}n = -6\) \(-\frac{15}{4}\)
5. \(-\frac{p}{12} = 6\) \(-72\)
6. \(\frac{x}{4} = 8\) \(32\)
7. \(\frac{12}{5}f = -18\) \(-\frac{15}{2}\)
8. \(\frac{y}{7} = -11\) \(-77\)
9. \(\frac{6}{7}y = 3\) \(\frac{7}{2}\)
10. \(c - 14 = -11\) \(3\)
11. \(t - 14 = -29\) \(-15\)
12. \(p - 21 = 52\) \(73\)
13. \(b + 2 = -5\) \(-7\)
14. \(q + 10 = 22\) \(12\)
15. \(-12q = 84\) \(-7\)
16. \(5a = 30\) \(6\)
17. \(5c - 7 = 8c - 4\) \(-1\)
18. \(2c + 6 = 6c - 10\) \(4\)
19. \(\frac{m}{10} + 15 = 21\) \(60\)
20. \(-\frac{m}{8} + 7 = 5\) \(16\)
21. \(8t + 1 = 3t - 19\) \(-4\)
22. \(9n + 4 = 5n + 18\) \(\frac{7}{2}\)
23. \(5c - 24 = -4\) \(4\)
24. \(3n + 7 = 28\) \(7\)
25. \(-2y + 17 = -13\) \(15\)
26. \(-\frac{t}{13} - 2 = 3\) \(-65\)
27. \(\frac{2}{9}x - 4 = \frac{2}{3}\) \(21\)
28. \(9 - 4g = -15\) \(6\)
29. \(-4 - p = -2\) \(-2\)
30. \(21 - b = 11\) \(10\)
31. \(-2(n + 7) = 15\) \(-\frac{29}{2}\)
32. \(5(n - 1) = -25\) \(-4\)
33. \(-8a - 11 = 37\) \(-6\)
34. \(\frac{7}{4}q - 2 = -5\) \(-\frac{12}{7}\)
35. \(2(5 - n) = 8\) \(1\)
36. \(-3(d - 7) = 6\) \(5\)
Solving Inequalities in One Variable

Statements with greater than (>), less than (<), greater than or equal to (≥), or less than or equal to (≤) are inequalities.

- If any number is added or subtracted to each side of an inequality, the resulting inequality is true.

**Example 1** Solve each inequality.

a. \( x - 17 > 12 \)
   \[ x - 17 > 12 \quad \text{Original inequality} \]
   \[ x - 17 + 17 > 12 + 17 \quad \text{Add 17 to each side.} \]
   \[ x > 29 \quad \text{Simplify.} \]
   The solution set is \( \{x \mid x > 29\} \).

b. \( y + 11 \leq 5 \)
   \[ y + 11 \leq 5 \quad \text{Original inequality} \]
   \[ y + 11 - 11 \leq 5 - 11 \quad \text{Subtract 11 from each side.} \]
   \[ y \leq -6 \quad \text{Simplify.} \]
   The solution set is \( \{y \mid y \leq -6\} \).

- If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.

**Example 2** Solve each inequality.

a. \( \frac{t}{6} \geq 11 \)
   \[ \frac{t}{6} \geq 11 \quad \text{Original inequality} \]
   \[ \frac{t}{6} \geq 11 \quad \text{Multiply each side by 6.} \]
   \[ t \geq 66 \quad \text{Simplify.} \]
   The solution set is \( \{t \mid t \geq 66\} \).

b. \( 8p < 72 \)
   \[ 8p < 72 \quad \text{Original inequality} \]
   \[ \frac{8p}{8} < \frac{72}{8} \quad \text{Divide each side by 8.} \]
   \[ p < 9 \quad \text{Simplify.} \]
   The solution set is \( \{p \mid p < 9\} \).

- If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be reversed so that the resulting inequality is true.

**Example 3** Solve each inequality.

a. \( -5c > 30 \)
   \[ -5c > 30 \quad \text{Original inequality} \]
   \[ \frac{-5c}{-5} < \frac{30}{-5} \quad \text{Divide each side by } -5. \text{ Change } > \text{ to } <. \]
   \[ c < -6 \quad \text{Simplify.} \]
   The solution set is \( \{c \mid c < -6\} \).
Prerequisite Skills

6. Example 4

Solve each inequality.

a. \(-6a + 13 < -7\)

Original inequality.

\(-6a + 13 < -7\)

Subtract 13 from each side.

\(-6a < -20\)

Simplify.

\(a > \frac{10}{3}\)

Simplify.

The solution set is \([a \mid a > \frac{10}{3}]\).

b. \(4z + 7 \geq 8z - 1\)

Original inequality.

\(4z + 7 \geq 8z - 1\)

Subtract 7 from each side.

\(4z \geq 8z - 8\)

Simplify.

\(4z - 8z \geq 8z - 8 - 8z\)

Subtract 8z from each side.

\(-4z \geq -8\)

Simplify.

\(\frac{-4z}{-4} \leq \frac{-8}{-4}\)

Divide each side by \(-4\). Change \(\geq\) to \(\leq\).

\(z \leq 2\)

Simplify.

The solution set is \([z \mid z \leq 2]\).

Exercises

Solve each inequality.

1. \(x - 7 < 6 \quad \{x \mid x < 13\}\)

2. \(4c + 23 \leq -13 \quad \{c \mid c \leq -9\}\)

3. \(-\frac{p}{5} \leq 14 \quad \{p \mid p \leq -70\}\)

4. \(-\frac{a}{8} < 5 \quad \{a \mid a > -40\}\)

5. \(\frac{t}{6} > -7 \quad \{t \mid t > -42\}\)

6. \(\frac{d}{15} \leq 8 \quad \{a \mid a \leq 88\}\)

7. \(d + 8 \leq 12 \quad \{d \mid d \leq 4\}\)

8. \(m + 14 > 10 \quad \{m \mid m > 4\}\)

9. \(2z - 9 < 7z + 1 \quad \{z \mid z > -2\}\)

10. \(6t - 10 \geq 4t \quad \{t \mid t \geq 5\}\)

11. \(3z + 8 < 2 \quad \{z \mid z < -2\}\)

12. \(a + 7 \geq -5 \quad \{a \mid a \geq -12\}\)

13. \(m - 21 < 8 \quad \{m \mid m < 29\}\)

14. \(x - 6 \geq 3 \quad \{x \mid x \geq 9\}\)

15. \(-3b \leq 48 \quad \{b \mid b \geq -16\}\)

16. \(4y < 20 \quad \{y \mid y < 5\}\)

17. \(12k \leq -36 \quad \{k \mid k \leq -3\}\)

18. \(-4h > 36 \quad \{h \mid h < -9\}\)

19. \(\frac{2b}{5} - 6 \leq -2 \quad \{b \mid b \leq 10\}\)

20. \(\frac{8}{3} \cdot 1 + 1 > -5 \quad \{t \mid t > \frac{-9}{4}\}\)

21. \(7q + 3 = -4q + 25 \quad \{q \mid q \geq 2\}\)

22. \(-3n - 8 > 2n + 7 \quad \{n \mid n < -3\}\)

23. \(-3w + 1 \leq 8 \quad \{w \mid w \leq \frac{7}{3}\}\)

24. \(-\frac{3k}{2} - 17 > 11 \quad \{k \mid k < -35\}\)
Graphing Using Intercepts and Slope

- The $x$-coordinate of the point at which a line crosses the $x$-axis is called the **$x$-intercept**. The $y$-coordinate of the point at which a line crosses the $y$-axis is called the **$y$-intercept**. Since two points determine a line, one method of graphing a linear equation is to find these intercepts.

**Example 1**

Determine the $x$-intercept and $y$-intercept of $4x - 3y = 12$.

Then graph the equation.

To find the $x$-intercept, let $y = 0$.

$4x - 3y = 12$  
$4x - 3(0) = 12$  
$4x = 12$  
$x = 3$  

To find the $y$-intercept, let $x = 0$.

$4x - 3y = 12$  
$4(0) - 3y = 12$  
$-3y = 12$  
$y = -4$

Put a point on the $x$-axis at 3 and a point on the $y$-axis at $-4$. Draw the line through the two points.

- A linear equation of the form $y = mx + b$ is in slope-intercept form, where $m$ is the slope and $b$ is the $y$-intercept. When an equation is written in this form, you can graph the equation quickly.

**Example 2**

Graph $y = \frac{3}{4}x - 2$.

**Step 1**

The $y$-intercept is $-2$. So, plot a point at $(0, -2)$.

**Step 2**

The slope is $\frac{3}{4}$ rise

From $(0, -2)$, move up 3 units and right 4 units. Plot a point.

**Step 3**

Draw a line connecting the points.

**Exercises**  
Graph each equation using both intercepts. 1–6. See margin.

1. $-2x + 3y = 6$
2. $2x + 5y = 10$
3. $3x - y = 3$
4. $-x + 2y = 2$
5. $3x + 4y = 12$
6. $4y + x = 4$

Graph each equation using the slope and $y$-intercept. 7–12. See margin.

7. $y = -x + 2$
8. $y = x - 2$
9. $y = x + 1$
10. $y = 3x - 1$
11. $y = -2x + 3$
12. $y = -3x - 1$

Graph each equation using either method. 13–21. See margin.

13. $y = \frac{2}{3}x - 3$
14. $y = \frac{1}{2}x - 1$
15. $y = 2x - 2$
16. $-6x + y = 2$
17. $2y - x = -2$
18. $3x + 4y = -12$
19. $4x - 3y = 6$
20. $4x + y = 4$
21. $y = 2x - \frac{3}{2}$

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Answers continued on the following page.
Prerequisite Skills

Solving Systems of Linear Equations

- Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

**Example 1** Solve each system of equations by graphing. Then determine whether each system has **no solution**, **one solution**, or **infinitely many solutions**.

a. $y = -x + 3$
   $y = 2x - 3$
   The graphs appear to intersect at $(2, 1)$. Check this estimate by replacing $x$ with 2 and $y$ with 1 in each equation.
   **Check:**
   
   $y = -x + 3$
   $y = 2x - 3$
   $1 \neq -2 + 3$
   $1 \neq 2(2) - 3$

   The system has one solution at $(2, 1)$.

b. $y - 2x = 6$
   $3y - 6x = 9$
   The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different $y$-intercepts. Equations with the same slope and the same $y$-intercepts have an infinite number of solutions.

- It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

**Example 2** Use substitution to solve the system of equations.

$y = -4x$

$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for $y$ in the second equation.

$2y + 3x = 8$ **Second equation**

$2(-4x) + 3x = 3$  $y = -4x$

$-8x + 3x = 8$  **Simplify.**

$-5x = 8$  **Combine like terms.**

$x = -\frac{8}{5}$  **Divide each side by $-5$.**

Use $y = -4x$ to find the value of $y$.

$y = -4 \left(-\frac{8}{5}\right)$  **First equation**

$y = \frac{32}{5}$  **Simplify.**

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$. 

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Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called elimination.

**Example 3** Use elimination to solve the system of equations.

\[ \begin{align*}
3x + 5y &= 7 \\
4x + 2y &= 0
\end{align*} \]

Either \( x \) or \( y \) can be eliminated. In this example, we will eliminate \( x \).

\[ \begin{align*}
3x + 5y &= 7 & \text{Multiply by 4.} \\
12x + 20y &= 28 \quad & \text{Add the equations.} \\
4x + 2y &= 0 & \text{Multiply by } -3. \\
\frac{-12x - 6y}{14y} &= 0 & \text{Subtract 4 from each side.} \\
\frac{14y}{14} &= 28 & \text{Divide each side by } 14. \\
y &= 2 & \text{Simplify.}
\end{align*} \]

Now substitute 2 for \( y \) in either equation to find the value of \( x \).

\[ \begin{align*}
4x + 2y &= 0 & \text{Second equation} \\
4x + 2(2) &= 0 & \text{Simplify.} \\
4x + 4 &= 0 & \text{Subtract 4 from each side.} \\
x &= -4 & \text{Divide each side by } 4. \\
x &= -1 & \text{Simplify.}
\end{align*} \]

The solution is \((-1, 2)\).

**Exercises** Solve by graphing.

1. \( y = -x + 2 \)
   \( y = \frac{1}{2}x + 1 \) \((2, 0)\)

2. \( y = 3x - 3 \)
   \( y = x + 1 \) \((2, 3)\)

3. \( y = 2x - 1 \)
   \( 2y - 4x = 1 \) \text{no solution}

4. \( 2x - 4y = -2 \)
   \(-6x + 12y = 6 \) \text{infinitely many solutions}

5. \( 4x + 3y = 12 \)
   \( 3x - y = 9 \) \((3, 0)\)

6. \( 3y + x = -3 \)
   \( y - 3x = -1 \) \((0, -1)\)

7. \( -5x + 3y = 12 \)
   \( x + 2y = 8 \) \((0, 4)\)

8. \( x - 4y = 22 \)
   \( 2x + 5y = -21 \) \((2, -5)\)

9. \( y + 5x = -3 \)
   \( 3y - 2x = 8 \) \((-1, 2)\)

10. \( y - 2x = 2 \)
    \( 7y + 4x = 23 \) \((\frac{1}{2}, 3)\)

11. \( 2x - 3y = -8 \)
    \(-x + 2y = 5 \) \((-1, 2)\)

12. \( 4x + 2y = 5 \)
    \(-x + y = 10 \) \((\frac{5}{2}, \frac{5}{2})\)

**Solve by substitution.**

13. \( -3x + y = 7 \)
    \( 3x + 2y = 2 \) \((-\frac{4}{3}, \frac{3}{2})\)

14. \( 3x + 4y = -1 \)
    \(-9x - 4y = 13 \) \((-2, \frac{5}{4})\)

15. \( -4x + 5y = -11 \)
    \( 2x + 3y = 11 \) \((4, 1)\)

16. \( 6x - 5y = 1 \)
    \(-2x + 9y = 7 \) \((1, 1)\)

17. \( 3x - 2y = 8 \)
    \( 5x - 3y = 16 \) \((8, 8)\)

18. \( 4x + 7y = -17 \)
    \( 3x + 2y = -3 \) \((1, -3)\)

**Solve by elimination.**

19. \( 4x - y = 11 \) \text{elimination or substitution, } \((3, 1)\)
   \( 4x + 6y = 3 \)
   \( 2x - 3y = 3 \)

20. \( 4x - 7y = 8 \) \text{elimination, } \((\frac{11}{2}, \frac{7}{2})\)
   \( -2y + 5x = 15 \) \text{substitution, } \((3, 0)\)
   \( -2x + 5y = -1 \)

21. \( 3x - 2y = 6 \)
    \( 5x - 5y = 11 \) \text{no solution}

22. \( 3x + 3y = 6 \)
    \( 4x - 2y = -32 \) \text{elimination or substitution, } \((1, -3)\)

23. \( x + 3y = 6 \) \text{substitution, } \((-6, 4)\)
10 Square Roots and Simplifying Radicals

- A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.
- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.
- The Product Property states that for two numbers \( a \) and \( b \geq 0 \), \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \).

### Example 1

Simplify.

a. \( \sqrt{45} \)

\[
\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5} \quad \text{Prime factorization of 45}
\]

\[
= \sqrt{3^2} \cdot \sqrt{5} \quad \text{Product Property of Square Roots}
\]

\[
= 3 \sqrt{5} \quad \text{Simplify}
\]

b. \( \sqrt{3} \cdot \sqrt{3} \)

\[
\sqrt{3} \cdot \sqrt{3} = \sqrt{3^2} \quad \text{Product Property}
\]

\[
= \sqrt{9} \quad \text{Simplify}
\]

\[
= 3
\]

b. \( \sqrt{6} \cdot \sqrt{15} \)

\[
\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15} \quad \text{Product Property}
\]

\[
= \sqrt{3 \cdot 2 \cdot 3 \cdot 5} \quad \text{Prime factorization}
\]

\[
= \sqrt{3^2 \cdot 2 \cdot 5} \quad \text{Product Property}
\]

\[
= 3 \sqrt{2 \cdot 5} \quad \text{Simplify}
\]

- For radical expressions in which the exponent of the variable inside the radical is even and the resulting simplified exponent is odd, you must use absolute value to ensure nonnegative results.

### Example 2

\( \sqrt{20x^3y^2z^6} \)

\[
\sqrt{20x^3y^2z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^2 \cdot z^6} \quad \text{Prime factorization}
\]

\[
= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^2} \cdot \sqrt{z^6} \quad \text{Product Property}
\]

\[
= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x^2} \cdot \sqrt{y^2} \cdot \sqrt{z^6} \quad \text{Simplify}
\]

\[
= 2xy \sqrt{z^3} \quad \text{Simplify}
\]

- The Quotient Property states that for any numbers \( a \) and \( b \), where \( a \geq 0 \) and \( b \geq 0 \),

\[
\sqrt{a \div b} = \frac{\sqrt{a}}{\sqrt{b}}.
\]

### Example 3

Simplify \( \frac{25}{16} \)

\[
\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} \quad \text{Quotient Property}
\]

\[
= \frac{5}{4} \quad \text{Simplify}
\]
Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

**Example 4**
Simplify.

a. \[ \frac{2}{\sqrt{3}} \]
   \[ \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \]
   \[ = \frac{2\sqrt{3}}{3} \]
   Multiply by \( \frac{\sqrt{3}}{\sqrt{3}} \) and simplify.

b. \[ \frac{\sqrt{13y}}{\sqrt{18}} \]
   \[ \frac{\sqrt{13y}}{\sqrt{18}} = \frac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}} \]
   Prime factorization
   \[ = \frac{\sqrt{13y}}{3\sqrt{2}} \]
   Product Property
   \[ = \frac{\sqrt{26y}}{6} \]
   Multiply by \( \frac{\sqrt{2}}{\sqrt{2}} \) and simplify.

Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form \( p\sqrt{q} + r\sqrt{s} \) and \( p\sqrt{q} - r\sqrt{s} \).

**Example 5**
Simplify \( \frac{3}{5 - \sqrt{2}} \).

\[ \frac{3}{5 - \sqrt{2}} = \frac{3}{5 - \sqrt{2}} \cdot \frac{5 + \sqrt{2}}{5 + \sqrt{2}} \]
\[ = \frac{3(5 + \sqrt{2})}{(5 - \sqrt{2})(5 + \sqrt{2})} \]
\[ = \frac{15 + 3\sqrt{2}}{25 - 2} \]
\[ = \frac{15 + 3\sqrt{2}}{23} \]
Multiply by \( \frac{5 + \sqrt{2}}{5 + \sqrt{2}} \) and simplify.

**Exercises**

Simplify.

8. \( \frac{2}{\sqrt{3}} \)
19. \( \frac{6\sqrt{5} + 3\sqrt{10}}{2} \)
10. \( \frac{5\sqrt{3}}{5} \)
4. \( \sqrt{12} \cdot \sqrt{20} \)
6. \( \frac{5\sqrt{3}}{5} \)
15. \( \frac{10\sqrt{5}}{10} \)
8. \( \sqrt{98x^3y^6} \)
11. \( \frac{3\sqrt{14}}{4} \)
12. \( \frac{128}{147} \)
13. \( \frac{\sqrt{81}}{\sqrt{16}} \)
14. \( \frac{\sqrt{108}}{\sqrt{2p^3}} \)
15. \( \frac{4}{\sqrt{5 - 2\sqrt{5}}} \)
16. \( \frac{7\sqrt{3}}{5 - 2\sqrt{3}} \)
17. \( \frac{\sqrt{3}}{\sqrt{48}} \)
18. \( \frac{\sqrt{24}}{\sqrt{125}} \)
19. \( \frac{3\sqrt{5}}{2 - \sqrt{2}} \)
20. \( \frac{3}{-2 + \sqrt{13}} \)

Prerequisite Skills 745
Prerequisite Skills

Multiplying Polynomials

- The **Product of Powers** rule states that for any number \( a \) and all integers \( m \) and \( n \), \( a^m \cdot a^n = a^{m+n} \).

**Example 1**  Simplify each expression.

a. \((4p^3)(p^4)\)
   \[
   (4p^3)(p^4) = 4(1)(p^{3+4}) \quad \text{Product of powers} \\
   = 4p^9 \quad \text{Simplify.}
   
   (3y^5)(-9y^2z^2) \\
   (3y^5)(-9y^2z^2) = (3)(-9)(y^5)(y^2)(z^2) \quad \text{Product of powers} \\
   = -27(y^5 + 2)(z^2 + 2) \quad \text{Simplify.}
   
- The Distributive Property can be used to multiply a monomial by a polynomial.

**Example 2**  Simplify \(3x^3(-4x^2 + x - 5)\).

\[
3x^3(-4x^2 + x - 5) = 3x^3(-4x^2) + 3x^3(x) - 3x^3(5) \quad \text{Distributive Property} \\
= -12x^5 + 3x^4 - 15x^3 \quad \text{Multiply.}
\]

- To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

**Example 3**  Simplify each expression.

a. \((-3xy^4)^3\)
   \[
   (-3xy^4)^3 = (-3)^3(x^3)(y^{12}) \quad \text{Power of a product} \\
   = -27x^3y^{12} \quad \text{Power of a power}
   
   (xy)^2(-2x^4)^2 = x^3y^2(-2)^2(x^4)^2 \quad \text{Power of a product} \\
   = x^3y^2(4)x^8 \quad \text{Power of a power} \\
   = 4x^3 \cdot x^4 \cdot y^3 \quad \text{Commutative Property} \\
   = 4x^{11}y^3 \quad \text{Product of powers}
   
- To multiply two binomials, find the sum of the products of

  F  the **First** terms,  
  O  the **Outer** terms,  
  I  the **Inner** terms, and  
  L  the **Last** terms.

**Example 4**  Find each product.

a. \((2x - 3)(x + 1)\)
   \[
   (2x - 3)(x + 1) = (2x)(x) + (2x)(1) + (-3)(x) + (-3)(1) \quad \text{FOIL method} \\
   = 2x^2 + 2x - 3x - 3 \quad \text{Multiply.} \\
   = 2x^2 - x - 3 \quad \text{Combine like terms.}
   
   b. \((x + 6)(x + 5)\)
   \[
   (x + 6)(x + 5) = (x)(x) + (x)(5) + (6)(x) + (6)(5) \quad \text{FOIL method} \\
   = x^2 + 5x + 6x + 30 \quad \text{Multiply.} \\
   = x^2 + 11x + 30 \quad \text{Combine like terms.}
   
746  Prerequisite Skills
• The Distributive Property can be used to multiply any two polynomials.

**Example 5** Find \((3x - 2)(2x^2 + 7x - 4)\).

\[
(3x - 2)(2x^2 + 7x - 4) = 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4)
\]

\[
= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8
\]

\[
= 6x^3 + 17x^2 - 26x + 8
\]

**Example 6** Find each product.

a. \((2x - 3)^2\)

\[
(2x - 3)^2 = 4x^2 - 12x + 9
\]

b. \((3x + 7)(3x - 7)\)

\[
(3x + 7)(3x - 7) = 9x^2 - 49
\]

**Exercises** Find each product.

1. \((3q^2)(q^3)\) \(3q^7\)
2. \((5m)(4m^3)\) \(20m^4\)
3. \((\frac{9}{2}c)(8c^5)\) \(36c^6\)
4. \((n^6)(10n^2)\) \(10n^8\)
5. \((2a^5)(4a^2b^2)\) \(8a^7b^5\)
6. \((6a^4b^2)(2a^3b)\) \(6a^7b^5\)
7. \((\frac{8}{3}x^3y)(4x^2y^2)\) \(\frac{32}{3}x^5y^3\)
8. \((5y^2)(2y + 1)\) \(10y^3 + 4y^2\)
9. \((-2q^7)(q^3)\) \(-2q^{10}\)
10. \((5p^2 - 90p)\) \(5p^2 - 90p\)
11. \(-45e^3 + 30e^2 + 75e\)
12. \(-32y^3 - 8x^2 + 88x\)
13. \(-8m^4 + 28m^3 - 20m^2\)
14. \(-2c^2f^2\) \(4c^6d^4\)
15. \(-50x^5\) \(-125w^3x^{15}\)
16. \(169k^{10}c^3\)
17. \(-1792y^{11}z^2\)
18. \((s - 7)(s + 2)\) \(s^2 - 9s + 14\)
19. \((m - 5)(m - 4)\) \(m^2 - 9m + 4\)
20. \((s - 7)(s - 2)\) \(s^2 - 9s + 14\)
21. \((x - 3)(x + 4)\) \(x^2 + x - 12\)
22. \((x - 3)(3x + 5)\) \(3q^2 + 11q + 10\)
23. \((d + 1)(d - 1)\) \(d^2 - 1\)
24. \((a + 3)(a - 6)\) \(a^2 - 3a - 18\)
25. \((x + 4)(x - 5)\) \(x^2 - x^2 - 22x - 8\)
26. \((s - 5)^2\) \(s^2 - 10s + 25\)
27. \((2x - 5)^2\) \(4r^2 - 20r + 25\)
28. \((2y^2 - 5y - 2)\) \(x^2 + x^2 - 22x - 8\)
29. \((3y^2 - 2)(3y^2 + b + 1)\) \(9b^3 - 3b^2 + b - 2\)
30. \((3a - 3)(4a + 3)\) \(16a^2 - 9\)
31. \((3b - 2)(3b^2 + b + 1)\) \(9b^3 - 3b^2 + b - 2\)
32. \((3f - g)^2\) \(9f^2 - 6fg + g^2\)
33. \((t + 8)^2\) \(t^2 + \frac{16}{3}t + \frac{64}{9}\)
34. \((x - 2)(x^2 + 3x - 7)\) \(x^3 + x^2 - 13x + 14\)
35. \((2y + 7)(y^2 - 2y + 4)\) \(2y^3 + 3y^2 - 6y + 28\)
**Prerequisite Skills**

**Example 1** Simplify.

a. \( \frac{x^5 y^8}{-x y^3} \)

\[ \frac{x^5 y^8}{-x y^3} = \left( \frac{x^5}{-x} \right) \left( \frac{y^8}{y^3} \right) \]

Group powers that have the same base.

\[ = -x^4 y^5 \quad \text{Quotient of powers} \]

\[ = -x^4 y^5 \quad \text{Simplify} \]

b. \( \left( \frac{4x^3}{3} \right)^3 \)

\[ \left( \frac{4x^3}{3} \right)^3 = \left( \frac{4^3}{3} \right) \]

Power of a quotient

\[ = \frac{4^3}{3^3} \]

Power of a product

\[ = \frac{64}{27} \]

Power of a product

c. \( \frac{w^{-2} x^4}{2w^{-5}} \)

\[ \frac{w^{-2} x^4}{2w^{-5}} = \frac{1}{2} \left( \frac{w^{-2}}{w^{-5}} \right) x^4 \]

Group powers that have the same base.

\[ = \frac{1}{2} (w^{-2} - (-5)x^4) \quad \text{Quotient of powers} \]

\[ = \frac{1}{2} w^{3} x^4 \quad \text{Simplify} \]

**Example 2** Simplify \( \frac{15x^3 - 3x^2 + 12x}{3x} \).

\[ \frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x} \quad \text{Divide each term by } 3x. \]

\[ = 5x^2 - x + 4 \quad \text{Simplify.} \]

**Example 3** Find \( \frac{(n^2 - 8n - 9)}{(n - 9)} \).

\[ \frac{(n^2 - 8n - 9)}{(n - 9)} = \frac{n^2 - 8n - 9}{(n - 9)} \quad \text{Write as a rational expression.} \]

\[ = \frac{(n - 9)(n + 1)}{(n - 9)} \quad \text{Factor the numerator.} \]

\[ = \frac{(n - 9)(n + 1)}{(n - 9)} \quad \text{Divide by the GCF.} \]

\[ = n + 1 \quad \text{Simplify.} \]
When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

**Example 4** Find \((n^3 - 4n^2 - 9) ÷ (n - 3)\).

In this case, there is no \(n\) term, so you must rename the dividend using 0 as the coefficient of the missing term.

\[
(n^3 - 4n^2 + 9) ÷ (n - 3) = (n^3 - 4n^2 + 0n + 9) ÷ (n - 3)
\]

Divide the first term of the dividend, \(n^3\), by the first term of the divisor, \(n\).

\[
\begin{align*}
\frac{n^3}{n} &= n^2 \quad \text{Multiply } n^2 \text{ and } n - 3. \\
-n^2 + 0n &= -n^2 \quad \text{Subtract and bring down 0n.} \\
\frac{-n^2 + 3n}{n} &= \frac{-n^2 + 3n}{n} \quad \text{Multiply } -n \text{ and } n - 3. \\
-3n + 12 &= -3n + 12 \quad \text{Subtract and bring down 12.} \\
\frac{-3n + 12}{n} &= \frac{-3n + 9}{n} \quad \text{Multiply } -3 \text{ and } n - 3. \\
3 &= \quad \text{Subtract.}
\end{align*}
\]

Therefore, \((n^3 - 4n^2 + 9) ÷ (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}\). Since the quotient has a nonzero remainder, \(n - 3\) is not a factor of \(n^3 - 4n^2 + 9\).

**Exercises** Find each quotient.

1. \(\frac{a^2b^2}{2a} ÷ \frac{b^2}{2}\)
2. \(\frac{5q^3}{q^2} ÷ \frac{5q^3}{r}\)
3. \(\frac{b^2 + d^2}{b^2} ÷ \frac{b^4}{d^2}\)
4. \(\frac{5y^3}{2y^2} ÷ \frac{5y^3}{2y^2}\)
5. \(\frac{3x^3 - 25x^2}{2x^3} ÷ \frac{3x^2z^3}{yz^2}\)
6. \(\frac{\frac{3x^3}{y^3}}{\frac{5}{y^2}} \div \frac{27y^6}{125}\)
7. \(\frac{3x^3 + 3x^2}{2x^3} ÷ \frac{3x^2 + 2b + 5}{b}\)
8. \(\frac{3x^3}{2x^2} ÷ \frac{3x^2}{2x^2} ÷ \frac{9}{3x}\)
9. \(\frac{a^2 - 2x^2}{2a} ÷ \frac{a^2}{a^2}\)
10. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
11. \((n^3 + 3n^2 - 5n + 1) ÷ m = m^2 + 4m - 1\)
12. \((5x^2 + 2x + 3) ÷ (x - 3) = 5x + 18 + \frac{18}{x - 3}\)
13. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
14. \((n^3 + 3n^2 - 5n + 1) ÷ m = m^2 + 4m - 1\)
15. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
16. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
17. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
18. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
19. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
20. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
21. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
22. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
23. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
24. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
25. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
26. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
27. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
28. \((n^3 + 3n^2 + 5n + 1) ÷ m = m^2 + 4m - 1\)
13 Factoring to Solve Equations

- Some polynomials can be factored using the Distributive Property.

Example 1 Factor \(5t^2 + 15t\).

Find the greatest common factor (GCF) of \(5t^2\) and \(15t\).

\[
5t^2 = 5 \cdot t \cdot t, \quad 15t = 3 \cdot 5 \cdot t \quad \text{GCF: } 5 \cdot t \text{ or } 5t
\]

\[
5t^2 + 15t = 5t(t) + 5t(3) \quad \text{Rewrite each term using the GCF.}
\]

\[
= 5t(t + 3) \quad \text{Distributive Property}
\]

- To factor polynomials of the form \(x^2 + bx + c\), find two integers \(m\) and \(n\) so that \(mn = c\) and \(m + n = b\). Then write \(x^2 + bx + c\) using the pattern \((x + m)(x + n)\).

Example 2 Factor each polynomial.

a. \(x^2 + 7x + 10\)

In this equation, \(b = 7\) and \(c = 10\).

Find two numbers with a product of 10 and with a sum of 7.

\[
x^2 + 7x + 10 = (x + m)(x + n) = (x + 2)(x + 5)
\]

b. \(x^2 - 8x + 15\)

In this equation, \(b = -8\) and \(c = 15\).

This means that \(m + n\) is negative and \(mn\) is positive. So \(m\) and \(n\) must both be negative.

\[
x^2 - 8x + 15 = (x + m)(x + n) = (x - 3)(x - 5)
\]

c. \(5x^2 - 19x - 4\)

In this equation, \(a = 5\), \(b = -19\), and \(c = -4\).

Find two numbers with a product of \(-20\) and with a sum of \(-19\).

<table>
<thead>
<tr>
<th>Factors of (-20)</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2, 10</td>
<td>8</td>
</tr>
<tr>
<td>2, -10</td>
<td>-8</td>
</tr>
<tr>
<td>-1, 20</td>
<td>19</td>
</tr>
<tr>
<td>1, -20</td>
<td>-19</td>
</tr>
</tbody>
</table>

\[
5x^2 - 19x - 4 = 5x^2 + mx + nx - 4
\]

\[
= 5x^2 + x + (-20)x - 4
\]

\[
= 5(x^2 + x) - 20(x - 1)
\]

\[
= (x(5x + 1) - 4(5x + 1))
\]

\[
= (x - 4)(5x + 1)
\]

Both \(b\) and \(c\) are positive.

\[
\begin{array}{c|c}
\text{Factors of 10} & \text{Sum of Factors} \\
\hline
1, 10 & 11 \\
2, 5 & 7 \\
\end{array}
\]

The correct factors are 2 and 5.

Write the pattern; \(m = 2\) and \(n = 5\).

\[
\begin{array}{c|c}
\text{Factors of 15} & \text{Sum of Factors} \\
\hline
-1, -15 & -16 \\
-3, -5 & -8 \\
\end{array}
\]

The correct factors are \(-3\) and \(-5\).

Write the pattern; \(m = -3\) and \(n = -5\).

\[
\begin{array}{c|c}
\text{Factors of \(-20\)} & \text{Sum of Factors} \\
\hline
-2, 10 & 8 \\
2, -10 & -8 \\
-1, 20 & 19 \\
1, -20 & -19 \\
\end{array}
\]

The correct factors are 1 and \(-20\).

Write the pattern.

\(m = 1\) and \(n = -20\)

Group terms with common factors.

Factor the GCF from each group.

Distributive Property

750 Prerequisite Skills
• Here are some special products.

**Perfect Square Trinomials**

\[ a^2 + 2ab + b^2 = (a + b)^2 \]

\[ a^2 - 2ab + b^2 = (a - b)^2 \]

**Difference of Squares**

\[ a^2 - b^2 = (a + b)(a - b) \]

**Example 3**

Factor each polynomial.

a. \(9x^2 + 6x + 1\)

\[ 9x^2 + 6x + 1 = (3x)^2 + 2(3x)(1) + 1^2 \]

Write as \(a^2 + 2ab + b^2\).  

Factor using the pattern.

b. \(x^2 - 9 = 0\)

\[ x^2 - 9 = x^2 - (3)^2 \]

Write in the form \(a^2 - b^2\) .  

Factor the difference of squares.

**Example 4**

Solve \(x^2 - 5x + 4 = 0\) by factoring.

Factor the polynomial. This expression is of the form \(x^2 + bx + c\).

\[ x^2 - 5x + 4 = 0 \quad \text{Original equation} \]

\[(x - 1)(x - 4) = 0 \quad \text{Factor the polynomial.} \]

If \(ab = 0\), then \(a = 0\), \(b = 0\), or both equal 0. Let each factor equal 0.

\[ x - 1 = 0 \quad \text{or} \quad x - 4 = 0 \]

\[ x = 1 \quad \text{or} \quad x = 4 \]

**Exercises**

Factor each polynomial.

1. \(u^2 - 12u + 36\)
2. \(w^2 + 4w + 4\)
3. \(7j^2 - 28j + 24\)
4. \(2x^2 + 24x + 72\)
5. \(2x^2 + 2x + 3x + 1\)
6. \(5t^2 - 30t + 5\)
7. \(z^2 + 10z + 21\)
8. \((z + 7)(z + 3)\)
9. \(81 + 88\)
10. \(x^2 + 14x + 48\)
11. \((x + 6)(x + 8)\)
12. \((x + 4)(x + 1)\)
13. \(q^2 - 9q + 18\)
14. \((q - 3)(q - 6)\)
15. \(p^2 - 5p + 6\)
16. \((p - 2)(p - 3)\)
17. \(n^2 - 7n - 44\)
18. \((n - 11)(n + 4)\)
19. \(3x^2 + 42x - 4\)
20. \((3x - 2)(x + 2)\)
21. \((2x - 1)(y + 5)\)
22. \(3x^2 + 11x - 4\)
23. \((3x - 1)(x + 4)\)
24. \(8a^2 + 15a - 2\)
25. \(
\frac{w^2 - 9}{4} \left( w + \frac{3}{2}\right) \left( w - \frac{3}{2}\right) \)
26. \(c^2 - 64\)
27. \((c - 8)(c + 8)\)
28. \(b^2 + 18b + 81\)
29. \(j^2 - 12j + 36\)
30. \(4t^2 - 25\)

Solve each equation by factoring.

31. \(10x^2 - 35x = 0\)
32. \(3x^2 + 15x = 0\)
33. \(k^2 + 13k + 36 = 0\)
34. \(w^2 - 8w + 12 = 0\)
35. \(c^2 - 5c - 14 = 0\)
36. \(z^2 - z - 42 = 0\)
37. \(2y^2 - 5y - 12 = 0\)
38. \(3b^2 - 4b - 15 = 0\)
39. \(t^2 + 12t + 36 = 0\)
40. \(x^2 + 5x + \frac{25}{4} = 0\)
41. \(q^2 - 8q + 16 = 0\)
42. \(a^2 - 6a + 9 = 0\)
**Operations with Matrices**

- A **matrix** is a rectangular arrangement of numbers in rows and columns. Each entry in a matrix is called an **element**. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first.

- For example, matrix $A$ has dimensions $3 \times 2$ and matrix $B$ has dimensions $2 \times 4$.

$$
\text{matrix } A = \begin{bmatrix}
6 & -2 \\
0 & 5 \\
-4 & 10
\end{bmatrix}
\text{ matrix } B = \begin{bmatrix}
7 & -1 & -2 & 0 \\
3 & 6 & -5 & 2
\end{bmatrix}
$$

- If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

**Example 1** If $A = \begin{bmatrix}12 & 7 & -3 \\ 0 & -1 & -6\end{bmatrix}$, $B = \begin{bmatrix}-3 & 0 & 5 \\ 2 & 7 & -7\end{bmatrix}$, and $C = \begin{bmatrix}9 & 1 & -5 \\ 0 & -1 & 15\end{bmatrix}$, find the sum and difference.

a. $A + B$

$A + B = \begin{bmatrix}12 & 7 & -3 \\ 0 & -1 & -6\end{bmatrix} + \begin{bmatrix}-3 & 0 & 5 \\ 2 & 7 & -7\end{bmatrix}$

Substitution

$= \begin{bmatrix}12 + (-3) & 7 + 0 & -3 + 5 \\ 0 + 2 & -1 + 7 & -6 + (-7)\end{bmatrix}$

Definition of matrix addition

$= \begin{bmatrix}9 & 7 & 2 \\ 2 & 6 & -13\end{bmatrix}$

Simplify.

b. $B - C$

$B - C = \begin{bmatrix}-3 & 0 & 5 \\ 2 & 7 & -7\end{bmatrix} - \begin{bmatrix}9 & 1 & -5 \\ 0 & -1 & 15\end{bmatrix}$

Substitution

$= \begin{bmatrix}-3 - 9 & 0 - 1 & 5 - (-5) \\ 2 - 0 & 7 - (-1) & -7 - 15\end{bmatrix}$

Definition of matrix subtraction

$= \begin{bmatrix}-12 & -1 & 10 \\ 2 & 8 & -22\end{bmatrix}$

Simplify.

- You can multiply any matrix by a constant called a **scalar**. This is called **scalar multiplication**. To perform scalar multiplication, multiply each element by the scalar.

**Example 2** If $D = \begin{bmatrix}-4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4\end{bmatrix}$, find $2D$.

$2D = 2 \begin{bmatrix}-4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4\end{bmatrix}$

Substitution

$= \begin{bmatrix}2(-4) & 2(6) & 2(-1) \\ 2(0) & 2(7) & 2(2) \\ 2(-3) & 2(-8) & 2(-4)\end{bmatrix}$

Definition of scalar multiplication

$= \begin{bmatrix}-8 & 12 & -2 \\ 0 & 14 & 4 \\ -6 & -16 & -8\end{bmatrix}$

Simplify.
You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix. The product of two matrices is found by multiplying columns and rows. The entry in the first row and first column of \(AB\), the resulting product, is found by multiplying corresponding elements in the first row of \(A\) and the first column of \(B\) and then adding.

**Example 3**

Find \(EF\) if \(E = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix}\) and \(F = \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}\).

\[
EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}
= \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix}
= \begin{bmatrix} -15 & 21 \\ 36 & -18 \end{bmatrix}
\]

**Exercises**

If \(A = \begin{bmatrix} 10 & -9 \\ 4 & -3 \\ -1 & 11 \end{bmatrix}\), \(B = \begin{bmatrix} -1 & -3 \\ 2 & 8 \\ 7 & 6 \end{bmatrix}\), and \(C = \begin{bmatrix} 8 & 0 \\ -2 & 2 \\ -10 & 6 \end{bmatrix}\), find each sum, difference, or product.

1. \(A + B\)
2. \(B + C\)
3. \(A - C\)
4. \(C - B\)
5. \(3A\)
6. \(5B\)
7. \(-4C\)
8. \(\frac{1}{2}C\)
9. \(2A + C\)
10. \(A - 5C\)
11. \(\frac{1}{2}C + B\)
12. \(3A - 3B\)

If \(X = \begin{bmatrix} 2 & -8 \\ 10 & 4 \end{bmatrix}\), \(Y = \begin{bmatrix} -1 & 0 \\ 6 & 5 \end{bmatrix}\), and \(Z = \begin{bmatrix} 4 & -8 \\ -7 & 0 \end{bmatrix}\), find each sum, difference, or product.

13. \(X + Z\)
14. \(Y + Z\)
15. \(X - Y\)
16. \(3Y\)
17. \(-6X\)
18. \(\frac{1}{2}X + Z\)
19. \(5Z - 2Y\)
20. \(XY\)
21. \(YZ\)
22. \(XZ\)
23. \(\frac{1}{3}(XZ)\)
24. \(XY + 2Z\)
Lesson 1-1

For Exercises 1–7, refer to the figure.

1. How many planes are shown in the figure? 8
2. Name three collinear points. B, O, C or D, M, J
3. Name all planes that contain point G. planes AFG, ABG, and GLK
4. Name the intersection of plane ABD and plane DIK.
5. Name two planes that do not intersect. Sample answer: planes ABD and GHJ
6. Name a plane that contains FK and EL. plane FEK
7. Is the intersection of plane ACD and plane EDJ a point or a line? Explain. A line; two planes intersect in a line, not a point.

Draw and label a figure for each relationship. 8–9. See margin.
8. Line a intersects planes A, B, and C at three distinct points.
9. Planes X and Z intersect in line m. Line b intersects the two planes in two distinct points.

Lesson 1-2

Find the precision for each measurement. Explain its meaning.
2. 0.5 mm; 85.5 to 86.5 mm
3. 251 cm; 250.5 to 251.5 cm
5. 33.45 to 33.55 in.
6. 89 m; 88.5 to 89.5 m

Find the value of the variable and BC if B is between A and C.
7. AB = 4x, BC = 5x; AB = 16 x = 4; BC = 20
8. AB = 17, BC = 3m, AC = 32 m = 5; BC = 15
9. AB = 9n, BC = 12n, AC = 42 a = 2; BC = 24
10. AB = 25, BC = 3b; AC = 7b + 13 b = 3; BC = 9
11. AB = 5n + 5, BC = 2n; AC = 54 n = 7; BC = 14
12. AB = 6c – 8, BC = 3c + 1, AC = 65 c = 8; BC = 25

Lesson 1-3

Use the Pythagorean Theorem to find the distance between each pair of points.
1. A(0, 0), B(–3, 4) 5
2. C(–1, 2), N(5, 10) 10
3. X(–6, –2), Z(6, 3) 13
4. M(–5, –8), O(3, 7) 17
5. T(–10, 2), R(6, –10) 20
6. F(5, –6), N(–5, 6) \(\sqrt{244} \approx 15.6\)

Use the Distance Formula to find the distance between each pair of points.
7. D(0, 0), M(8, –7) \(\sqrt{113} \approx 10.6\)
8. X(–1, 1), Y(1, –1) \(\sqrt{8} \approx 2.8\)
9. Z(–4, 0), A(–3, 7) \(\sqrt{50} \approx 7.1\)
10. K(6, 6), D(–3, –3) \(\sqrt{162} \approx 12.7\)
11. T(–1, 3), N(0, 2) \(\sqrt{2} \approx 1.4\)
12. S(7, 2), E(–6, 7) \(\sqrt{194} \approx 13.9\)

Find the coordinates of the midpoint of a segment having the given endpoints.
13. A(0, 0), D(–2, –8) (–1, –4)
14. D(–4, –3), E(2, 2) (–1, –0.5)
15. K(–4, –5), M(5, 4) (0.5, –0.5)
16. R(–10, 5), S(8, 4) (–1, 4.5)
17. B(2.8, –3.4), Z(1.2, 5.6) (2, 1.1)
18. D(–6.2, 7), K(3.4, –4.8) (–1.4, 1.1)

Find the coordinates of the missing endpoint given that B is the midpoint of AC.
19. C(0, 0), B(5, –6) (10, –12)
20. C(–7, –4), B(3, 5) (13, 14)
21. C(8, –4), B(–10, 2) (–28, 8)
22. C(6, 8), B(–3, 5) (–12, 2)
23. C(6, –8), B(3, –4) (0, 0)
24. C(–2, –4), B(0, 5) (2, 14)
For Exercises 1–14, use the figure at the right.

Name the vertex of each angle.
1. \( \angle 1 \) \( B \)
2. \( \angle 4 \) \( E \)
3. \( \angle 6 \) \( G \)
4. \( \angle 7 \) \( I \)

Name the sides of each angle.
5. \( \angle AIE \) \( IA, IE \)
6. \( \angle 4 \) \( ED, EF \)
7. \( \angle 6 \) \( GC, GH \)
8. \( \angle AHF \) \( HA, HF \)

Write another name for each angle.
9. \( \angle 3 \) \( DCG \)
10. \( \angle DEF \) \( \angle 4 \)
11. \( \angle 2 \) \( BCG \)

Measure each angle and classify it as right, acute, or obtuse.
12. \( \angle ABC \) 120°, obtuse
13. \( \angle CGF \) 90°, right
14. \( \angle HIF \) 60°, acute

For Exercises 1–7, refer to the figure.
1. Name two acute vertical angles. Sample answer: \( \angle BGC \), \( \angle FGE \)
2. Name two obtuse vertical angles. Sample answer: \( \angle BGF \), \( \angle CGE \)
3. Name a pair of complementary adjacent angles. Sample: \( \angle BEC \), \( \angle CED \)
4. Name a pair of supplementary adjacent angles. Sample: \( \angle CEF \), \( \angle CED \)
5. Name a pair of congruent supplementary adjacent angles. Sample: \( \angle ABE \), \( \angle CBE \)
6. If \( m \angle BGC = 4x + 5 \) and \( m \angle FGE = 6x - 15 \), find \( m \angle BGE \). 135
7. If \( m \angle BCG = 5a + 5 \), \( m \angle GCE = 3a - 4 \), and \( m \angle ECD = 4a - 7 \), find the value of \( a \) so that \( \overline{AC} \perp \overline{CD} \). 8

8. The measure of \( \angle A \) is nine less than the measure of \( \angle B \). If \( \angle A \) and \( \angle B \) form a linear pair, what are their measures? 85.5, 94.5

9. The measure of an angle’s complement is 17 more than the measure of the angle. Find the measure of the angle and its complement. 36.5, 53.5

Name each polygon by its number of sides. Classify it as convex or concave and regular or irregular. Then find the perimeter.

1. quadrilateral; convex; 90 m
2. 25 cm
3. hexagon; concave; irregular; 156 cm
4. 16-gon; concave; irregular; 264 in.

Find the perimeter of each polygon.
4. triangle with vertices at \( X(3, 3), Y(-2, 1), \) and \( Z(1, -3) \) ≈16.7 units
5. pentagon with vertices at \( P(-2, 3), E(-5, 0), N(-2, -4), T(2, -1), \) and \( A(2, 2) \) ≈21.4 units
6. hexagon with vertices at \( H(0, 4), E(-3, 2), X(-3, -2), G(0, -5), O(5, -2), \) and \( N(5, 2) \) ≈27.1 units
Lesson 2-1

1. \((-3)^2 = 9\) and a robin is a fish; false
2. \((-3)^2 = 9\) or a robin is a fish; true
3. \((-3)^2 = 9\) and an acute angle measures less than 90°; true
4. \((-3)^2 = 9\) or an acute angle measures less than 90°; true
5. \((-3)^2 \neq 9\) or a robin is a fish; false
6. \((-3)^2 = 9\) or a robin is a fish, or an acute angle measures less than 90°; false
7. A robin is a fish and an acute angle measures less than 90°; false
8. \((-3)^2 = 9\) and a robin is a fish, or an acute angle measures 90° or more; true

\(_{\text{Extra Practice}}\)

Lesson 2-2

1. \((−3)^2 = 9\) and a robin is a fish; false
2. \((−3)^2 = 9\) or a robin is a fish; true
3. \((−3)^2 = 9\) and an acute angle measures less than 90°; true
4. \((−3)^2 = 9\) or an acute angle measures less than 90°; true
5. \((−3)^2 \neq 9\) or a robin is a fish; false
6. \((−3)^2 = 9\) or a robin is a fish, or an acute angle measures less than 90°; false
7. A robin is a fish and an acute angle measures less than 90°; false
8. \((−3)^2 = 9\) and a robin is a fish, or an acute angle measures 90° or more; true

\(_{\text{Extra Practice}}\)

Lesson 2-3

1. If no sides of a triangle are equal, then it is a scalene triangle.
2. If it rains today, you will be wearing your raincoat.
3. If \(6 - x = 11\), then \(x = -5\).
4. If you are in college, you are at least 18 years old.
5. If two angles are supplementary, then the sum of their measures is 180.
6. If a triangle has two congruent sides, then it is an isosceles triangle.
7. If two lines do not intersect, then they are parallel lines.
8. If an animal is a Saint Bernard, then it is a dog.
Lesson 2-4
(pages 82–87)

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write no conclusion.

1. (1) If it rains, then the field will be muddy. See margin.
   (2) If the field is muddy, then the game will be cancelled.

2. (1) If you read a book, then you enjoy reading.
   (2) If you are in the 10th grade, then you passed the 9th grade. no conclusion

Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write invalid.

3. (1) If it snows outside, you will wear your winter coat.
   (2) It is snowing outside.
   (3) You will wear your winter coat. yes; Law of Detachment

4. (1) Two complementary angles are both acute angles.
   (2) \( \angle 1 \) and \( \angle 2 \) are acute angles.
   (3) \( \angle 1 \) and \( \angle 2 \) are complementary angles. invalid

Lesson 2-5
(pages 89–93)

Determine whether the following statements are always, sometimes, or never true. Explain.

1. \( \overline{RS} \) is perpendicular to \( \overline{PS} \). Sometimes; \( \overline{RS} \) and \( \overline{PS} \) could intersect to form a 45° angle.

2. Three points will lie on one line. Sometimes; if they are collinear, then they lie on one line.

3. Points \( B \) and \( C \) are in plane \( \mathcal{K} \). A line perpendicular to line \( BC \) is in plane \( \mathcal{K} \).
   Sometimes; the line could lie in a plane perpendicular to plane \( \mathcal{K} \).

For Exercises 4–7, use the figure at the right. In the figure, \( \overline{EC} \) and \( \overline{CD} \) are in plane \( \mathcal{R} \), and \( F \) is on \( \overline{CD} \). State the postulate that can be used to show each statement is true. 4–7. See margin.

4. \( \overline{DF} \) lies in plane \( \mathcal{R} \).
5. \( E \) and \( C \) are collinear.
6. \( D, F, \) and \( E \) are coplanar.
7. \( E \) and \( F \) are collinear.

Lesson 2-6
(pages 94–100)

State the property that justifies each statement.

1. If \( x - 5 = 6 \), then \( x = 11 \). Addition Property
2. If \( AB = CD \) and \( CD = EF \), then \( AB = EF \). Transitive Property
3. If \( a - b = r \), then \( r = a - b \). Symmetric Property

4. Copy and complete the following proof.

   Given: \( \frac{5x - 1}{8} = 3 \)
   Prove: \( x = 5 \)
   Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( \frac{5x - 1}{8} = 3 )</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( 5x - 1 = 24 )</td>
<td>b. ( \frac{5x - 1}{8} ) ( = 3 )</td>
</tr>
<tr>
<td>c. ( 5x = 25 )</td>
<td>c. ( \text{Dist. Prop. and Substitution} )</td>
</tr>
<tr>
<td>d. ( x = 5 )</td>
<td>d. ( \text{Addition Prop.} )</td>
</tr>
<tr>
<td>e. ( \text{Division Property} )</td>
<td>e. Division Property</td>
</tr>
</tbody>
</table>

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9. Converse: If a figure is a polygon, then it is a triangle; false; pentagons are polygons but are not triangles.
   Inverse: If a figure is not a triangle, then it is not a polygon; false; a hexagon is not a triangle, but it is a polygon.
   Contrapositive: If a figure is not a polygon, then it is not a triangle; true

10. Converse: If two angles have the same measure, then they are congruent angles; true
   Inverse: If two angles are not congruent angles, then they do not have the same measure; true
   Contrapositive: If two angles do not have the same measure, then they are not congruent angles; true
Lesson 2-7

9. Given: $AB \cong AF$, $AF \cong ED$, $ED \cong CD$

Prove: $AB \cong CD$

Proof:

Statements (Reasons)

1. $AB \cong AF$, $AF \cong ED$ (Given)
2. $AB \cong ED$ (Transitive)
3. $ED \cong CD$ (Given)
4. $AB \cong CD$ (Transitive)

Lesson 2-8

Find the measure of each numbered angle.

1. $\angle 9 = 141 + x$
2. $\angle 11 = x + 40$
3. $\angle 14 = x + 25$
4. $\angle 10 = 25 + x$
5. $\angle 12 = x + 10$
6. $\angle 15 = 4x + 50$
7. $\angle 13 = 3x + 30$
8. $\angle 16 = x + 45$

Determine whether the following statements are always, sometimes, or never true.

4. Two angles that are complementary are congruent. Sometimes
5. Two angles that form a linear pair are complementary. Never
6. Two congruent angles are supplementary. Sometimes
7. Perpendicular lines form four right angles. Always
8. Two right angles are supplementary. Always
9. Two lines intersect to form four right angles. Sometimes

Lesson 3-1

For Exercises 1–3, refer to the figure at the right.

1. Name all segments parallel to $\overline{AE}$. $\overline{LP}$
2. Name all planes intersecting plane $BCN$.
3. Name all segments skew to $\overline{DC}$.

2. $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ alternate interior
3. $\angle 6, \angle 7, \angle 8$ alternate exterior
4. $\angle 9$, $\angle 10$ corresponding
5. $\angle 11$ and $\angle 12$ corresponding
6. $\angle 13$ and $\angle 14$ alternate interior
7. $\angle 15$ and $\angle 16$ alternate exterior
**Lesson 3-2**

(pages 133–138)

In the figure, \( m \angle 5 = 72 \) and \( m \angle 9 = 102 \).

Find the measure of each angle.

1. \( m \angle 1 \) \( 102 \)
2. \( m \angle 13 \) \( 72 \)
3. \( m \angle 4 \) \( 102 \)
4. \( m \angle 10 \) \( 78 \)
5. \( m \angle 7 \) \( 108 \)
6. \( m \angle 16 \) \( 72 \)

Find \( x \) and \( y \) in each figure.

7. \( 8x - 5 \)° \( 75 \)° \( 9y - 3 \)°  
   \( x = 10; y = 12 \)

8. \( 4x + 7 \)°  
   \( x = 12; y = 65 \)

**Lesson 3-3**

(pages 139–144)

Find the slope of each line.

1. \( RS \) \( \frac{4}{3} \)
2. \( TU \) \( \frac{1}{5} \)
3. \( WV \) undefined
4. \( WR \) \( \frac{1}{5} \)
5. a line parallel to \( TU \) \( \frac{1}{6} \)
6. a line perpendicular to \( WR \) \( 0 \)
7. a line perpendicular to \( WV \) \( 0 \)

Determine whether \( RS \) and \( TU \) are parallel, perpendicular, or neither.

8. \( R(3, 5), S(5, 6), T(2, 0), U(4, 3) \) parallel  
9. \( R(5, 11), S(2, 2), T(1, 0), U(2, 1) \) neither  
10. \( R(1, 4), S(3, 7), T(5, 1), U(8, 1) \) parallel  
11. \( R(2, 5), S(4, 1), T(3, 3), U(1, 5) \) neither

**Lesson 3-4**

(pages 145–150)

Write an equation in slope-intercept form of the line having the given slope and \( y \)-intercept.

1. \( m = 1, y \)-intercept: \(-5\)  
   \( y = x - 5 \)
2. \( m = -\frac{1}{2}, y \)-intercept: \( \frac{1}{2} \)  
   \( y = -\frac{1}{2}x + \frac{1}{2} \)
3. \( m = 3, b = -\frac{1}{4} \)  
   \( y = 3x - \frac{1}{4} \)

Write an equation in point-slope form of the line having the given slope that contains the given point.

4. \( m = 3, (-2, 4) \)  
   \( y - 4 = 3(x + 2) \)
5. \( m = -4, (0, 3) \)  
   \( y - 3 = -4x \)
6. \( m = \frac{2}{3}, (5, 7) \)  
   \( y + 7 = \frac{2}{3}(x - 5) \)

For Exercises 7–14, use the graph at the right.

Write an equation in slope-intercept form for each line.

7. \( p \)  
   \( y = -2x + 1 \)
8. \( q \)  
   \( y = x - 3 \)
9. \( r \)  
   \( y = \frac{2}{3}x - 2 \)
10. \( s \)  
    \( y = -\frac{1}{3}x \)
11. parallel to line \( q \), contains \((2, -5)\)  
    \( y = x - 7 \)
12. perpendicular to line \( r \), contains \((0, 1)\)  
    \( y = \frac{3}{2}x + 1 \)
13. parallel to line \( s \), contains \((-2, -2)\)  
    \( y = -\frac{3}{2}x - \frac{5}{3} \)
14. perpendicular to line \( p \), contains \((0, 0)\)  
    \( y = \frac{1}{2}x \)
Lesson 3-5

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. \( \angle 9 \cong \angle 16 \)
2. \( \angle 10 \cong \angle 16 \)
3. \( \angle 12 \cong \angle 13 \)
4. \( m \angle 12 + m \angle 14 = 180 \)

1–4. See margin.

Lesson 3-6

7. \( d = \frac{7 \sqrt{2}}{2} \);

8. \( d = 1.4 \);

Lesson 3-6

Copy each figure. Draw the segment that represents the distance indicated.

1. \( P \) to \( RS \)
2. \( J \) to \( KE \)
3. \( B \) to \( F \)

Lesson 4-1

Use a protractor to classify each triangle as acute, equiangular, obtuse, or right.

1. equiangular
2. right
3. obtuse

Lesson 4-3

5. Given: \( \triangle ANG \cong \triangle NGA \)

Prove: \( \triangle ANG \) is equilateral and equiangular.

Proof: Statements (Reasons)

1. \( \triangle ANG \cong \triangle NGA \) (Given)
2. \( AN = NG \), \( \angle A = \angle N \) (CPCTC)
3. \( \triangle NGA \cong \triangle GAN \) (Given)
4. \( NG = GA \), \( \angle N = \angle G \) (CPCTC)
5. \( AN = NG = GA \) (Transitive Property of \( \cong \))
6. \( \triangle AGN \) is equilateral. (Def. of equilateral \( \triangle \))
7. \( \angle A = \angle N = \angle G \) (Transitive Property of \( \cong \))
8. \( \triangle AGN \) is equiangular. (Def. of equiangular \( \triangle \))
Lesson 4-2
Find the measure of each angle.
1. \( \angle 1 60 \)
2. \( \angle 2 60 \)
3. \( \angle 3 55 \)
4. \( \angle 4 120 \)
5. \( \angle 5 94 \)
6. \( \angle 6 86 \)
7. \( \angle 7 94 \)
8. \( \angle 8 86 \)
9. \( \angle 9 52 \)
10. \( \angle 10 24 \)

Lesson 4-3
Identify the congruent triangles in each figure.
1. \( \triangle ABC \cong \triangle FDE \)
2. \( \triangle JKH \cong \triangle JIH \)
3. \( \triangle RTS \cong \triangle UVW \)
4. \( \triangle LMN \cong \triangle NOP \)

5. Write a two-column proof. See margin.
   Given: \( \triangle ANG \cong \triangle NGA \)
   \( \triangle NGA \cong \triangle ANG \)
   Prove: \( \triangle ANG \) is equilateral and equiangular.

Lesson 4-4
Determine whether \( \triangle RST \cong \triangle JKL \) given the coordinates of the vertices. Explain.
1. \( R(-6, 2), S(-4, 4), T(-2, 2), J(6, -2), K(4, -4), L(2, -2) \) Yes; see margin for explanation.
2. \( R(-6, 3), S(-4, 7), T(-2, 3), J(2, 3), K(5, 7), L(6, 3) \) No; see margin for explanation.

Write a two-column proof. 3–4. See margin.
3. Given: \( \triangle GWN \) is equilateral.
   \( \overline{WS} \cong \overline{WI} \)
   \( \angle SWG \cong \angle IWN \)
   Prove: \( \triangle SWG \cong \triangle IWN \)

4. Given: \( \triangle ANM \cong \triangle ANI \)
   \( DI \cong OM \)
   \( ND \cong NO \)
   Prove: \( \triangle DIN \cong \triangle OMN \)

Additional Information:
- \( RS = JK, ST = KL, \text{ and } RT = JL \)
- By definition of congruent segments, all corresponding segments are congruent. Therefore, \( \triangle RST \cong \triangle JKL \).
- \( RS = \sqrt{(-6 - (-4))^2 + (4 - 2)^2} = \sqrt{4 + 4} = \sqrt{8} \)
- \( ST = \sqrt{(-4 - (-2))^2 + (4 - 2)^2} = \sqrt{4 + 4} = \sqrt{8} \)
- \( JK = \sqrt{(6 - 4)^2 + (-2 - (-4))^2} = \sqrt{4 + 4} = \sqrt{8} \)
- \( KL = \sqrt{(4 - 2)^2 + (-4 - (-2))^2} = \sqrt{4 + 4} = \sqrt{8} \)
- \( JL = \sqrt{(6 - 2)^2 + (-2 - (-2))^2} = \sqrt{16} = 4 \)

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Lesson 4-5

Write a paragraph proof. 1–2. See margin.
1. Given: \( \triangle TEN \) is isosceles with base \( TN \), \( \angle 1 \equiv \angle 4 \), \( \angle T \equiv \angle N \)
   Prove: \( \triangle TEC \equiv \triangle NEA \)

   Proof: If \( \triangle TEN \) is isosceles with base \( TN \), then \( TE \equiv NE \). Since \( \angle 1 \equiv \angle 4 \) and \( \angle T \equiv \angle N \) are given, then \( \triangle TEC \equiv \triangle NEA \) by AAS.

2. Given: \( \angle S \equiv \angle W \), \( SY \equiv YW \)
   Prove: \( ST \equiv WV \)

   Proof: \( \angle S \equiv \angle W \) and \( SY \equiv YW \)
   are given and \( \angle SYT \equiv \angle WYV \) since vertical angles are congruent. Then \( \triangle SYT \equiv \triangle WYV \) by ASA and \( ST \equiv WV \) by CPCTC.

3. Given: \( \angle 1 \equiv \angle 2 \), \( \angle 3 \equiv \angle 4 \)
   Prove: \( PT \equiv LX \)

   Proof:

\[
\angle 1 \equiv \angle 2 \quad \angle 3 \equiv \angle 4 \quad TX \equiv TX
\]

SAS

\[
\triangle PXT \equiv \triangle LTX
\]

CPCTC

Lesson 4-6

Refer to the figure for Exercises 1–6.
1. If \( AD \parallel BD \), name two congruent angles. \( \angle DAB \equiv \angle DBA \)
2. If \( BF \parallel FC \), name two congruent angles. \( \angle FBC \equiv \angle FGB \)
3. If \( BE \parallel BC \), name two congruent angles. \( \angle BEF \equiv \angle BGF \)
4. If \( \angle FBE \equiv \angle FEB \), name two congruent segments. \( FB \equiv FE \)
5. If \( \angle BCA \equiv \angle BAC \), name two congruent segments. \( BA \equiv BC \)
6. If \( \angle DBC \equiv \angle BCD \), name two congruent segments. \( BD \equiv CD \)

Lesson 4-7

Position and label each triangle on the coordinate plane. 1–4. See margin for sample answers.
1. isosceles \( \triangle ABC \) with base \( BC \) that is \( r \) units long
2. equilateral \( \triangle XYZ \) with sides \( 4b \) units long
3. isosceles right \( \triangle RST \) with hypotenuse \( ST \) and legs \( (3 + a) \) units long
4. equilateral \( \triangle CDE \) with base \( DE \) \( \frac{3}{4} b \) units long.

Name the missing coordinates of each triangle.
5. \( A(0, b), B(-a, 0) \)
6. \( F(-b, b), G(-a - 2, 0) \)
7. \( E(0, ?), O(-2, 0) \)

Extra Practice

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Lesson 5-1

For Exercises 1–4, refer to the figures at the right.

1. Suppose \( CP = 7x - 1 \) and \( PB = 6x + 3 \). If \( S \) is the circumcenter of \( \triangle ABC \), find \( x \) and \( CP \). 4; 27
2. Suppose \( m \angle ACT = 15a - 8 \) and \( m \angle ACB = 74 \). If \( S \) is the incenter of \( \triangle ABC \), find \( a \) and \( m \angle ACT \). 3; 37
3. Suppose \( TO = 7b + 5 \), \( OR = 13b - 10 \), and \( TR = 18b \). If \( Z \) is the centroid of \( \triangle TRS \), find \( b \) and \( TR \). 2.5; 45
4. Suppose \( XR = 19n - 14 \) and \( ZR = 10n + 4 \). If \( Z \) is the centroid of \( \triangle TRS \), find \( n \) and \( ZR \). 5; 64

State whether each sentence is always, sometimes, or never true.

5. The circumcenter and incenter of a triangle are the same point. sometimes
6. The three altitudes of a triangle intersect at a point inside the triangle. sometimes
7. In an equilateral triangle, the circumcenter, incenter, and centroid are the same point. always
8. The incenter is inside of a triangle. always

Lesson 5-2

Determine the relationship between the measures of the given angles.

1. \( \angle TPS, \angle TSP \) \( m \angle TPS > m \angle TSP \)
2. \( \angle PRZ, \angle ZPR \) \( m \angle PRZ > m \angle ZPR \)
3. \( \angle SPZ, \angle SZP \) \( m \angle SPZ < m \angle SZP \)
4. \( \angle SPR, \angle SRP \) \( m \angle SPR = m \angle SRP \)

5. Given: \( FH > FG \) See margin.
   Prove: \( m \angle 1 > m \angle 2 \)

6. Given: \( \overline{RQ} \) bisects \( \angle SRT \). See margin.
   Prove: \( m \angle SQR > m \angle SRQ \)

Lesson 5-3

State the assumption you would make to start an indirect proof of each statement.

1. \( \triangle ABC \equiv \triangle XYZ \) \( \triangle ABC \neq \triangle XYZ \)
2. An angle bisector of an equilateral triangle is also a median.
3. \( RS \) bisects \( \angle ABC \) \( RS \) does not bisect \( \angle ABC \).
4. An angle bisector of an equilateral triangle is not a median.

Write an indirect proof. 4–5. See margin.

4. Given: \( \angle A0Y \equiv \angle AOX \) \( \angle A0X \neq \angle Y0 \)
   Prove: \( AO \) is not the angle bisector of \( \angle XAY \).

5. Given: \( \triangle RUN \)
   Prove: There can be no more than one right angle in \( \triangle RUN \).

Lesson 5-2

5. Given: \( FH > FG \)
   Prove: \( m \angle 1 > m \angle 2 \)

Proof:

Statements (Reasons)

1. \( FH > FG \) (Given)
2. \( m \angle FGH > m \angle 2 \) (If one side of a \( \triangle \) is longer than another, the \( \angle \) opp. the longer side > than the \( \angle \) opp. the shorter side.)
3. \( m \angle 1 > m \angle FGH \) (Exterior Angle Inequality Theorem)
4. \( m \angle 1 > m \angle 2 \) (Transitive Prop. of Inequality)
Lesson 5-4

17. Given: \( RS = RT \)
   Prove: \( UV + VS > UT \)

\[ \begin{align*}
    R & \quad \quad S \\
    U & \quad \quad V \\
    T & \\
\end{align*} \]

Proof:
Statements (Reasons)
1. \( RS = RT \) (Given)
2. \( UV + VS > US \) (Triangle Inequality Theorem)
3. \( US = UR + RS \) (Segment Addition Postulate)
4. \( UV + VS > UR + RS \) (Substitution)
5. \( UV + VS > UR + RT \) (Substitution)
6. \( UR + RT > UT \) (Triangle Inequality Theorem)
7. \( UV + VS > UT \) (Transitive Property of Inequality)

18. Given: quadrilateral \( ABCD \)
   Prove: \( AD + CD + AB > BC \)

\[ \begin{align*}
    A & \quad \quad B \\
    D & \quad \quad C \\
\end{align*} \]

Proof:
Statements (Reasons)
1. quadrilateral \( ABCD \) (Given)
2. Draw \( \overline{AC} \). (Through any 2 pts. there is 1 line.)
3. \( AD + CD > AC; AB + AC > BC \) (Triangle Inequality Theorem)
4. \( AC > BC - AB \) (Subtraction Prop. of Inequality)
5. \( AD + CD > BC - AB \) (Substitution Prop. of Inequality)
6. \( AD + CD + AB > BC \) (Addition Prop. of Inequality)

Lesson 5-5

Write an inequality relating the given pair of angle or segment measures.

1. \( XZ, OZ \) \( XZ > OZ \)
2. \( \angle ZIO, \angle ZUX \) \( \angle ZIO < \angle ZUX \)
3. \( \angle AEZ, \angle AZE \) \( \angle AEZ = \angle AZE \)
4. \( IO, AE \) \( IO < AE \)
5. \( \angle AIZ, \angle IZO \) \( \angle AIZ > \angle IZO \)

Write an inequality to describe the possible values of \( x \).

6. \( 6.78 < x < 15.22 \)
7. \( 4.2 < x < 10 \)

Lesson 6-1

1. ARCHITECTURE The ratio of the height of a model of a house to the actual house is 1:63. If the width of the model is 16 inches, find the width of the actual house in feet. 84 ft

2. CONSTRUCTION A 64-inch long board is divided into lengths in the ratio 2:3. What are the two lengths into which the board is divided? 25.6 in., 38.4 in.

ALGEBRA Solve each proportion.

3. \( \frac{x + 4}{26} = \frac{38}{3} \)
4. \( \frac{3x + 1}{14} = \frac{5}{7} \)
5. \( \frac{x - 3}{4} = \frac{x + 1}{5} \)
6. \( \frac{2x + 2}{2x - 1} = \frac{7}{4} \)

7. Find the measures of the sides of a triangle if the ratio of the measures of three sides of a triangle is 9:6:5, and its perimeter is 100 inches. 45 in., 30 in., 25 in.
8. Find the measures of the angles in a triangle if the ratio of the measures of the three angles is 13:16:21. 46.8°, 57.6°, 75.6°
Lesson 6-2

Determine whether each pair of figures is similar. Justify your answer. 1–2. See margin.

1. \( \triangle ABC \) and \( \triangle DBC \)

2. \( \triangle PQR \) and \( \triangle STU \)

For Exercises 3 and 4, use \( \triangle RST \) with vertices \( R(3, 6), S(1, 2), \) and \( T(3, -1) \). Explain.

3. If the coordinates of each vertex are decreased by 3, describe the new figure.

4. If the coordinates of each vertex are multiplied by 0.5, describe the new figure.

Lesson 6-3

Determine whether each pair of triangles is similar. Justify your answer. 1–2. See margin.

1. \( \triangle MNP \) and \( \triangle XYZ \)

2. \( \triangle BRT \) and \( \triangle CTS \)

ALGEBRA Identify the similar triangles. Find \( x \) and the measures of the indicated sides.

3. \( RT \) and \( SV \)

4. \( PN \) and \( MN \)

3–4. See margin.

Lesson 6-4

1. If \( HI = 28 \), \( LH = 21 \), and \( LK = 8 \), find \( IJ \).

2. Find \( x \), \( AD \), \( DR \), and \( QR \) if \( AU = 15 \), \( QU = 25 \), \( AD = 3x + 6 \), \( DR = 8x - 2 \), and \( UD = 15 \).

Find \( x \) so that \( \overline{XY} \parallel \overline{LM} \).

3. \( XL = 3 \), \( YM = 5 \), \( LD = 9 \), \( MD = x + 3 \)

4. \( YM = 3 \), \( LD = 3x + 1 \), \( XL = 4 \), \( MD = x + 7 \)

5. \( MD = 5x - 6 \), \( YM = 3 \), \( LD = 5x + 1 \), \( XL = 5 \)

Lesson 6-2

1. \( m\angle A = 180 - 21.8 - 38.2 = 120 \), so \( m\angle A = m\angle X \). Therefore \( \angle A \equiv \angle X \).

2. \( m\angle Y = 180 - 120 - 38.2 = 21.8 \), so \( m\angle Y = m\angle B \). Therefore \( \angle Y \equiv \angle B \).

3. \( m\angle C = m\angle Z \), therefore \( \angle C \equiv \angle Z \).

All of the corresponding angles are congruent. Now determine whether corresponding sides are proportional.

\[
\frac{AB}{XY} = \frac{12.5}{5} = 2.5, \quad \frac{BC}{YZ} = \frac{17.5}{7} = 2.5, \quad \frac{AC}{XZ} = \frac{7.5}{3} = 2.5
\]

The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so \( \triangle ABC \sim \triangle XYZ \).
Lesson 6-5

Find the perimeter of each triangle.
1. \( \triangle ABC \) if \( \triangle DBC, AB = 17.5, BC = 15, BE = 6, \) and \( DE = 5 \) 45

2. \( \triangle RST \) if \( \triangle XYZ, RT = 12, XZ = 8, \) and the perimeter of \( \triangle XYZ = 22 \) 33

3. \( \triangle LMN \) if \( \triangle NXY, NX = 14, YX = 11, \)
\( YN = 9, \) and \( LN = 27 \) 102

4. \( \triangle GHI \) if \( \triangle ABC, AB = 6, GH = 10, \) and \( YN = 9, \) and \( LN = 27 \) 41 2/3

Lesson 6-6

Stage 1 of a fractal is shown drawn on grid paper.
Stage 1 is made by dividing a square into 4 congruent squares and shading the top left-hand square.

1. Draw Stage 2 by repeating the Stage 1 process in each of the 3 remaining unshaded squares. How many shaded squares are at this stage? 4

2. Draw Stage 3 by repeating the Stage 1 process in each of the unshaded squares in Stage 2. How many shaded squares are at this stage? 13

1–2. See margin for fractals.

Find the value of each expression. Then, use that value as the next \( x \) in the expression. Repeat the process and describe your observations. 3–6. See margin.

3. \( x^3 \), where \( x \) initially equals 6

4. \( 4^x \), where \( x \) initially equals 0.4

5. \( x^3 \), where \( x \) initially equals 0.5

6. \( 3^x \), where \( x \) initially equals 10

Lesson 7-1

Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.

1. 8 and 12 \( 4\sqrt{6} \approx 9.8 \)

2. 15 and 20 \( 10\sqrt{3} \approx 17.3 \)

3. 1 and 2 \( \sqrt{2} \approx 1.4 \)

4. 4 and 16 \( 8 \)

5. \( 3\sqrt{2} \) and \( 6\sqrt{2} \) \( 6 \)

6. \( \frac{1}{2} \) and 10 \( 5 \approx 2.2 \)

7. \( \frac{3}{8} \) and \( \frac{3}{2} \) \( \frac{\sqrt{3}}{4} = 0.4 \)

8. \( \frac{\sqrt{2}}{2} \) and \( \frac{3\sqrt{2}}{2} \) \( \frac{\sqrt{6}}{2} = 1.2 \)

9. \( \frac{1}{10} \) and \( \frac{7}{10} \) \( \frac{\sqrt{7}}{10} \approx 0.3 \)

Find the altitude of each triangle.

10. \( 8\sqrt{6} \approx 19.6 \)

11. \( 4\sqrt{2} \approx 5.7 \)

12. \( 2\sqrt{42} \approx 13.0 \)
Lesson 7-2

Determine whether \( \triangle DEF \) is a right triangle for the given vertices. Explain. 1–4. See margin.
1. \( D(0, 1), E(3, 2), F(2, 3) \)
2. \( D(-2, 2), E(3, -1), F(-4, -3) \)
3. \( D(2, -1), E(-2, -4), F(-4, -1) \)
4. \( D(1, 2), E(5, -2), F(-2, -1) \)

Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean triple. 5–13. See margin.
5. 1, 1, 2
6. 21, 28, 35
7. 3, 5, 7
8. 2, 5, 7
9. 24, 45, 51
10. \( \frac{5}{3}, 3', \sqrt{\frac{26}{3}} \)
11. \( \frac{6}{11}, 8, \frac{10}{11} \)
12. \( \frac{1}{2}, \frac{1}{2}, 1 \)

Lesson 7-3

Find the measures of \( x \) and \( y \).
1. \( x = 45, y = 13 \)
2. \( x = 12.5, y = 12.5\sqrt{3} \)
3. \( x = 15, y = 15\sqrt{2} \)
4. \( x = 16\sqrt{3}, y = 24 \)
5. \( x = 50\sqrt{2}, y = 100 \)
6. \( x = 18, y = 6\sqrt{3} \)

Lesson 7-4

Use \( \triangle MAN \) with right angle \( N \) to find \( \sin M, \cos M, \tan M, \sin A, \cos A, \) and \( \tan A \). Express each ratio as a fraction, and as a decimal to the nearest hundredth. 1–4. See margin.
1. \( m = 21, a = 28, n = 35 \)
2. \( m = \sqrt{2}, a = \sqrt{3}, n = \sqrt{5} \)
3. \( m = \frac{\sqrt{2}}{2}, a = \frac{\sqrt{2}}{2}, n = 1 \)
4. \( m = 3\sqrt{5}, a = 5\sqrt{3}, n = 2\sqrt{30} \)

Find the measure of each angle to the nearest tenth of a degree.
5. \( \cos A = 0.6293 \)
6. \( \sin B = 0.5664 \)
7. \( \tan C = 0.2665 \)
8. \( \sin D = 0.9352 \)
9. \( \tan M = 0.0808 \)
10. \( \cos R = 0.1097 \)

Find \( x \). Round to the nearest tenth.
11. \( x = 77.2 \)
12. \( x = 38.7 \)
13. \( x = 6.6 \)
Lesson 7-5
1. **COMMUNICATIONS** A house is located below a hill that has a satellite dish. If \( MN = 450 \) feet and \( RN = 120 \) feet, what is the measure of the angle of elevation to the top of the hill?约 14.9°

2. **AMUSEMENT PARKS** Mandy is at the top of the Mighty Screamer roller coaster. Her friend Bryn is at the bottom of the coaster waiting for the next ride. If the angle of depression from Mandy to Bryn is 26° and \( OL \) is 75 feet, what is the distance from \( L \) to \( C \)?约 153.8 ft

3. **SKIING** Mitchell is at the top of the Bridger Peak ski run. His brother Scott is looking up from the ski lodge at \( I \). If the angle of elevation from Scott to Mitchell is 13° and the distance from \( K \) to \( I \) is 2000 ft, what is the length of the ski run \( SI \)?约 2052.6 ft

Lesson 7-6

Find each measure using the given measures from \( \triangle ANG \). Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If \( m \angle N = 32, m \angle A = 47 \), and \( n = 15 \), find \( a \). 20.7
2. If \( a = 10.5, m \angle N = 26, m \angle A = 75 \), find \( n \). 4.8
3. If \( n = 18.6, a = 20.5, m \angle A = 65 \), find \( m \angle N \). 55°
4. If \( a = 57.8, n = 43.2, m \angle A = 33 \), find \( m \angle N \). 24°

Solve each \( \triangle AKX \) described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. \( m \angle X = 62, a = 28.5, m \angle K = 33 \)  \( m \angle A = 85, x \approx 25.3, k \approx 15.6 \)
6. \( k = 3.6, x = 3.7, m \angle X = 55 \)  \( m \angle K = 53, m \angle A = 72, a \approx 4.3 \)
7. \( m \angle K = 35, m \angle A = 65, x = 50 \)  \( m \angle X = 80, a \approx 46.0, k \approx 29.1 \)
8. \( m \angle A = 122, m \angle X = 15, a = 33.2 \)  \( m \angle K = 43, k \approx 26.7, x \approx 10.1 \)

Lesson 7-7

In \( \triangle CDE \), given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

1. \( c = 100, d = 125, e = 150 \); \( m \angle E \) 82.8°
2. \( c = 5, d = 6, e = 9 \); \( m \angle C \) 31.6°
3. \( c = 1.2, d = 3.5, e = 4 \); \( m \angle D \) 57.3°
4. \( c = 42.5, d = 50, e = 81.3 \); \( m \angle E \) 122.8°

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

5. \[ \begin{align*} c &= 29.1, \quad m \angle A \approx 80 \, \text{and} \quad m \angle B \approx 45 \end{align*} \]
6. \[ \begin{align*} m \angle O &= 29, \quad m \angle P \approx 71 \quad \text{and} \quad p \approx 3.4 \end{align*} \]
7. \[ \begin{align*} m \angle B &= 50, \quad m \angle X \approx 108 \quad \text{and} \quad m \angle Y \approx 22 \end{align*} \]
Lesson 8-1
(pages 404–409)

Find the sum of the measures of the interior angles of each convex polygon.

1. 25-gon 4140
2. 30-gon 5040
3. 22-gon 3600
4. 17-gon 2700
5. 5-gon 180(5a – 2)
6. h-gon 180(h – 2)

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

7. 156 15
8. 168 30
9. 162 20

Find the measures of an interior angle and an exterior angle given the number of sides of a regular polygon. Round to the nearest tenth.

10. 15 156, 24
11. 13 152.3, 27.7
12. 42 171.4, 8.6

Lesson 8-2
(pages 411–416)

Complete each statement about \(\triangle RSTU\). Justify your answer.

1. \(\angle SRU = ?\) \(\angle UTS\)
2. \(\angle UTS\) is supplementary to \(\angle\) ________.
3. \(RU \parallel ?\) \(TU\)
4. \(RU \parallel ?\) \(SU\)
5. \(\triangle RST = ?\) \(\triangle TUR\)
6. \(SV = ?\) \(UV\)

1–6. See margin for justification.

ALGEBRA Use \(\square ABCD\) to find each measure or value.

7. \(m\angle BAE = ?\) 28
8. \(m\angle BCE = ?\)
9. \(m\angle BEC = ?\) 89
10. \(m\angle CED = ?\) 91
11. \(m\angle ABE = ?\) 61
12. \(m\angle EBC = ?\) 63
13. \(a = ?\) 6
14. \(b = ?\) 4
15. \(c = ?\) 6
16. \(d = ?\) 11

Lesson 8-3
(pages 417–423)

Determine whether each quadrilateral is a parallelogram. Justify your answer. 1–3. See margin for justification.

1. \(\) no
2. \(\) yes
3. \(\) yes

ALGEBRA Find \(x\) and \(y\) so that each quadrilateral is a parallelogram.

4. \(x = 9, \ y = 13\)
5. \(x + 5y\)
6. \(x = 3, \ y = 6\)

Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

7. \(L(-3, 2), M(5, 2), N(3, -6), O(-5, -6)\); Slope Formula \(\text{yes}\)
8. \(W(-5, 6), X(2, 5), Y(-3, -4), Z(-8, -2)\); Distance Formula \(\text{no}\)
9. \(Q(-5, 4), R(0, 6), S(3, -1), T(-2, -3)\); Midpoint Formula \(\text{yes}\)
10. \(G(-5, 0), H(-13, 5), I(-10, 9), J(-2, 4)\); Distance and Slope Formulas \(\text{yes}\)
Lesson 8-6

Extra Practice

Lesson 8-7

3. Given: \( ABCD \) is a square.
Prove: \( AC \parallel BD \)

Proof:
\[
AC = \sqrt{(a - 0)^2 + (0 - a)^2} = \sqrt{2a^2} = \sqrt{2}a
\]
\[
BD = \sqrt{(a - 0)^2 + (a - 0)^2} = \sqrt{2a^2} = \sqrt{2}a
\]
\[
AC = BD = \sqrt{2}a
\]

Lesson 8-8

5. In rhombus \( QRST \), \( m \angle QRS = m \angle TSR - 40 \) and \( TS = 15 \).

6. Find \( m \angle ACD \), \( m \angle ABC \), and \( BY = 10r - 4 \).

7. Find \( m \angle ABC \), and \( m \angle AC \).

Lesson 8-9

COORDINATE GEOMETRY
For each quadrilateral with the given vertices,
a. verify that the quadrilateral is a trapezoid, and
b. determine whether the figure is an isosceles trapezoid. 1–4. See margin.

1. \( A(0, 9), B(3, 4), C(-5, 4), D(-2, 9) \)

2. \( Q(1, 4), R(4, 6), S(10, 7), T(1, 1) \)

3. \( L(1, 2), M(4, -1), N(3, -5), O(-3, 1) \)

4. \( W(1, -2), X(3, -1), Y(7, -2), Z(1, -5) \)

5. For trapezoid \( ABDC \), \( E \) and \( F \) are midpoints of the legs. Find \( CD \).

6. For trapezoid \( LMNO \), \( P \) and \( Q \) are midpoints of the legs. Find \( PQ, m \angle P, m \angle O \).

7. For isosceles trapezoid \( QRST \), find the length of the median, \( m \angle S \), and \( m \angle R \).

8. For trapezoid \( XYZW \), \( A \) and \( B \) are midpoints of the legs. For trapezoid \( XYBA \), \( C \) and \( D \) are midpoints of the legs. Find \( CD \).

EF = FG = GH = EH

Since all four sides are congruent, \( EFGH \) is a rhombus.
Lesson 8-7
(pages 447–451)

Name the missing coordinates for each quadrilateral.

1. isosceles trapezoid ABCD
2. rectangle QRST

Lesson 9-1
(pages 463–469)

COORDINATE GEOMETRY  Graph each figure and its image under the given reflection.

1. $\triangle ABC$ with vertices $A(2, 2), B(3, -2),$ and $C(-3, -1)$ in the $x$-axis  
2. rectangle $BARN$ with vertices $B(3, 3), A(3, -4), R(-1, -4),$ and $N(-1, 3)$ in the line $y = x$
3. trapezoid $ZOID$ with vertices $Z(2, 3), O(2, -4), I(-3, -3),$ and $D(-3, 1)$ in the origin
4. $\triangle PQR$ with vertices $P(-2, 1), Q(2, -2),$ and $R(-3, -4)$ in the $y$-axis
5. square $BDFH$ with vertices $B(-4, 4), D(-1, 4), F(-1, 1),$ and $H(-4, 1)$ in the origin
6. quadrilateral $QUAD$ with vertices $Q(1, 3), U(3, 1), A(-1, 0),$ and $D(-3, 4)$ in the line $y = -1$
7. $\triangle CAB$ with vertices $C(0, 4), A(1, -3),$ and $B(-4, 0)$ in the line $x = 2$

Lesson 9-2
(pages 470–475)

In each figure, $c \parallel d$. Determine whether the red figure is a translation image of the blue figure. Write yes or no. Explain your answer. 1–3. See margin.

1. 
2. 
3. 

COORDINATE GEOMETRY  Graph each figure and its image under the given translation. 4–8. See p. 781A.

4. $\triangle DEF$ with vertices $D(1, 2), E(-2, 1), and F(-3, -1)$ under the translation $(x, y) \rightarrow (x + 2, y + 1)$
5. $\triangle DEF$ with vertices $D(1, 2), E(-2, 1), and F(-3, -1)$ under the translation $(x, y) \rightarrow (x - 1, y - 3)$
6. quadrilateral $WXYZ$ with vertices $W(1, 1), X(-2, 3), Y(-3, -2),$ and $Z(2, -2)$ under the translation $(x, y) \rightarrow (x + 1, y - 1)$
7. pentagon $ABCDE$ with vertices $A(1, 3), B(-1, 1), C(-1, -2), D(3, -2),$ and $E(3, 1)$ under the translation $(x, y) \rightarrow (x - 2, y + 3)$
8. $\triangle RST$ with vertices $R(-4, 3), S(-2, -3),$ and $T(2, -1)$ under the translation $(x, y) \rightarrow (x + 3, y - 2)$

Lesson 9-2

1. Yes; it is one reflection after another with respect to the two parallel lines.
2. No; the figure has a different orientation.
3. No; it is not one reflection after another with respect to the two parallel lines.
Lesson 9-3

COORDINATE GEOMETRY  Draw the rotation image of each figure 90° in the given direction about the center point and label the vertices with coordinates. 1–2. See margin.

1. \( \triangle KLM \) with vertices \( K(4, 2), L(1, 3), \) and \( M(2, 1) \) counterclockwise about the point \( P(1, -1) \)

2. \( \triangle FGH \) with vertices \( F(-3, -3), G(2, -4), \) and \( H(-1, -1) \) clockwise about the point \( P(0, 0) \)

3. \( \triangle DEF \) with vertices \( D(1, 2), E(1, 0), \) and \( F(1, 2) \) about the point \( P(0, 0) \)

4. \( \triangle GHI \) with vertices \( G(2, 2), H(2, 1), \) and \( I(1, 1) \) clockwise about the point \( P(0, 0) \)

5. \( \triangle JKL \) with vertices \( J(-3, -3), K(-1, -1), \) and \( L(-1, 0) \) about the point \( P(0, 0) \)

6. \( \triangle MNO \) with vertices \( M(-3, -3), N(-3, -1), \) and \( O(-1, -1) \) about the point \( P(0, 0) \)

Lesson 9-4

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

1. regular hexagons and squares no

2. squares and regular pentagons no

3. regular hexagons and regular octagons no

Determine whether each statement is always, sometimes, or never true.

4. Any right isosceles triangle forms a uniform tessellation. sometimes

5. A semi-regular tessellation is uniform. always

6. A polygon that is not regular can tessellate the plane. sometimes

7. If the measure of one interior angle of a regular polygon is greater than 120, it cannot tessellate the plane. always

Lesson 9-5

Find the measure of the dilation image or the preimage of \( OM \) with the given scale factor.

1. \( OM = 1, r = -2 \) \( O'M' = 2 \)

2. \( OM = 3, r = \frac{1}{3} \) \( O'M' = 1 \)

3. \( O'M' = \frac{3}{4}, r = 3 \) \( OM = \frac{1}{4} \)

4. \( OM = \frac{7}{8}, r = \frac{5}{7} \) \( O'M' = \frac{5}{8} \)

5. \( O'M' = 4, r = -\frac{2}{3} \) \( OM = 6 \)

6. \( O'M' = 4.5, r = -1.5 \) \( OM = 3 \)

COORDINATE GEOMETRY  Find the image of each polygon, given the vertices, after a dilation centered at the origin with scale factor \( r = 3 \). Then graph a dilation with \( r = \frac{1}{3} \). 7–10. See p. 781A.

7. \( T(1, 1), R(1, -2), I(-2, 0) \)

8. \( E(2, 1), H(3, -3), O(-1, -2) \)

9. \( A(0, -1), B(-1, 1), C(0, 2), D(1, 1) \)

10. \( B(1, 0), D(2, 0), F(3, -2), H(0, -2) \)
Lesson 9-7

Find the magnitude and direction of each vector. 1–4. See margin.
1. \( \overrightarrow{AB} \) with endpoints \( A(2, 3) \) and \( B(5, 1) \)
2. \( \overrightarrow{DE} \) with endpoints \( D(-3, 2) \) and \( E(1, -4) \)
3. \( \overrightarrow{FG} \) with endpoints \( F(-1, 5) \) and \( G(3, -2) \)
4. \( \overrightarrow{HI} \) with endpoints \( H(-4, 2) \) and \( I(5, -3) \)

Find the coordinates of the image under the translation. 5–10. See margin.
5. \( AB \) with endpoints \( A(-2, 3) \) and \( B(4, 1) \), translated by \( (2, -1) \)
6. \( CD \) with endpoints \( C(-5, 2) \) and \( D(3, -4) \), translated by \( (3, -2) \)
7. \( EF \) with endpoints \( E(-2, 5) \) and \( F(1, -3) \), translated by \( (-1, 2) \)
8. \( GH \) with endpoints \( G(-3, 1) \) and \( H(2, -4) \), translated by \( (4, -3) \)
9. \( JK \) with endpoints \( J(-1, 3) \) and \( K(5, -2) \), translated by \( (-1, 4) \)
10. \( LM \) with endpoints \( L(-3, 5) \) and \( M(7, -1) \), translated by \( (2, -2) \)

Lesson 10-1

The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.
1. \( r = 18 \text{ in.} \), \( d = ? \text{ in.}, C = ? \text{ in.} \)
2. \( r = 34.2 \text{ ft} \), \( d = ? \text{ ft}, C = ? \text{ ft} \)
3. \( C = 12\pi \text{ m} \), \( r = ? \text{ m}, d = ? \text{ m} \)
4. \( C = 84.8 \text{ mi} \), \( r = ? \text{ mi}, d = ? \text{ mi} \)
5. \( d = 8.7 \text{ cm} \), \( r = ? \text{ cm}, C = ? \text{ cm} \)

Find the exact circumference of each circle.
6. \( \sqrt{10} \text{ m} \)
7. \( \sqrt{11} \text{ cm} \)
8. \( \sqrt{13} \text{ in.} \)
9. \( \sqrt{17} \text{ ft} \)
10. \( \sqrt{20} \text{ yd} \)
11. \( \sqrt{22} \text{ cm} \)
12. \( \sqrt{24} \text{ m} \)

Lesson 9-6

Find the magnitude and direction of each resultant for the given vectors. 7–9. See margin.
1. \( \overrightarrow{AB} \) with endpoints \( A(-2, 3) \) and \( B(1, 2) \)
2. \( \overrightarrow{CD} \) with endpoints \( C(-4, 5) \) and \( D(2, -1) \)
3. \( \overrightarrow{EF} \) with endpoints \( E(-1, 6) \) and \( F(3, -2) \)
4. \( \overrightarrow{GH} \) with endpoints \( G(-3, 4) \) and \( H(1, -2) \)
5. \( \overrightarrow{IJ} \) with endpoints \( I(-5, 3) \) and \( J(7, -1) \)

Graph the image of each figure under a translation by the given vector. 7–9. See margin.
1. \( \triangle ABC \) with vertices \( A(-2, 3), B(1, 2), C(4, -1) \), translated by \( (2, -1) \)
2. \( \square DEFG \) with vertices \( D(-3, 2), E(2, -1), F(-1, 4), G(4, 3) \), translated by \( (-1, 2) \)
3. \( \odot HIJ \) with center \( H(-2, 0) \) and radius \( 5 \text{ in.} \), translated by \( (3, -4) \)

Find the magnitude and direction of each resultant for the given vectors. 10–12. See margin.
10. \( \overrightarrow{AB} \) with endpoints \( A(2, 3), B(5, 1) \)
11. \( \overrightarrow{CD} \) with endpoints \( C(-4, 5), D(2, -1) \)
12. \( \overrightarrow{EF} \) with endpoints \( E(-1, 6), F(3, -2) \)

Find the coordinates of the image under the stated transformation. 1–4. See margin.
1. reflection in the \( x \)-axis
2. rotation \( 90^\circ \) clockwise about the origin
3. translation \( (x, y) \rightarrow (x - 4, y + 3) \)
4. dilation by scale factor \(-4\)
Lesson 10-2

Find each measure.
1. \( m \angle GKI \) \( 90 \)
2. \( m \angle LKJ \) \( 23 \)
3. \( m \angle LKI \) \( 113 \)
4. \( m \angle LKG \) \( 157 \)
5. \( m \angle HKJ \) \( 67 \)
6. \( m \angle HKJ \) \( 157 \)

In \( \odot X, WS, VR, \) and \( QT \) are diameters, \( m \angle WXV = 25 \) and \( m \angle XWI = 45 \).

Find each measure.
7. \( m \angle QR \) \( 90 \)
8. \( m \angle QW \) \( 65 \)
9. \( m \angle TU \) \( 45 \)
10. \( m \angle WRV \) \( 335 \)
11. \( m \angle SV \) \( 155 \)
12. \( m \angle TRW \) \( 245 \)

Lesson 10-3

In \( \odot S, HJ = 22, LG = 18, m \angle J = 35, \) and \( m \angle LM = 30 \). Find each measure.
1. \( HR \) \( 11 \)
2. \( RJ \) \( 11 \)
3. \( LT \) \( 9 \)
4. \( TG \) \( 9 \)
5. \( m \angle Hj \) \( 70 \)
6. \( m \angle LG \) \( 60 \)
7. \( m \angle MG \) \( 30 \)
8. \( m \angle HI \) \( 35 \)

In \( \odot R, CR = RE, \) and \( ED = 30 \). Find each measure.
9. \( AB \) \( 30 \)
10. \( EF \) \( 15 \)
11. \( DF \) \( 15 \)

Lesson 10-4

Find the measure of each numbered angle for each figure. 1–6. See margin.
1. \( m \angle AB = 176, \) and \( m \angle BC = 42 \)
2. \( WX = ZY, \) and \( m \angle WZ = 120 \)
3. \( m \angle QR = 40, \) and \( m \angle TS = 110 \)
4. \( \square ABCD \) is a rectangle, and \( m \angle BC = 70 \)
5. \( m \angle TR = 100, \) and \( SR \perp QT \)
6. \( m \angle Y = m \angle ZX = 56 \) and \( m \angle YX = m \angle XW = 56 \)

7. Rhombus \( ABCD \) is inscribed in a circle. What can you conclude about \( BD \)? \( \text{It is a diameter of the circle.} \)
8. Triangle \( RST \) is inscribed in a circle. If the measure of \( RS \) is 170, what is the measure of \( \angle T \)? \( 85 \)
**Lesson 10-5**
(pages 552–558)

Determine whether each segment is tangent to the given circle.

1. yes
2. no

Find \( x \). Assume that segments that appear to be tangent are tangent.

3. 3
4. \( 5\sqrt{10} \)
5. \( 8 \)

**Lesson 10-6**
(pages 561–568)

Find each measure.

1. \( m\angle 75 \)
2. \( m\angle 142.5 \)
3. \( m\angle 110 \)

Find \( x \). Assume that any segment that appears to be tangent is tangent.

4. 20
5. 25
6. 10

**Lesson 10-7**
(pages 569–574)

Find \( x \). Assume that segments that appear to be tangent are tangent.

1. 5
2. 6
3. 3

Find each variable to the nearest tenth.

4. 3.0
5. 5.7
6. 2.2
Lesson 10-8  
1. 
   \((x - 1)^2 + (y + 2)^2 = 4\)  
2. \(x^2 + y^2 = 16\)  
3. 
   \((x + 3)^2 + (y + 4)^2 = 11\)  
4. 
   \((x - 3)^2 + (y + 1)^2 = 9\)  
5. 
   \((x - 6)^2 + (y - 12)^2 = 49\)  
6. 
   \((x - 4)^2 + y^2 = 16\)  
7. 
   \((x - 6)^2 + (y + 6)^2 = 121\)  
8. 
   \((x + 5)^2 + (y - 1)^2 = 1\)

9. \(x^2 + y^2 = 25\)  
10. 
    \((x - 3)^2 + (y + 1)^2 = 9\)  
11. 
    \((x - 3)^2 + (y - 1)^2 = 2\)  
12. 
    \((x - 1)^2 + (y - 4)^2 = 1\)

Graph each equation. 9–12. See margin.

13. Find the radius of a circle whose equation is \((x + 3)^2 + (y - 1)^2 = r^2\) and contains \((-2, 1)\).
14. Find the radius of a circle whose equation is \((x - 4)^2 + (y - 3)^2 = r^2\) and contains \((8, 3)\).

Lesson 11-1  
Find the area and perimeter of each parallelogram. Round to the nearest tenth if necessary.

1. 
   \(259.8 \text{ in}^2, 70 \text{ in.}\)  
2. 
   \(178.2 \text{ ft}^2, 74 \text{ ft}\)  
3. 
   \(113.5 \text{ m}^2, 49 \text{ m}\)

COORDINATE GEOMETRY  Given the coordinates of the vertices of a quadrilateral, determine whether it is a square, a rectangle, or a parallelogram. Then find the area of the quadrilateral.

4. \(Q(-3, 3), R(-1, 3), S(-1, 1), T(-3, 1)\)  
5. \(A(-7, -6), B(-2, -6), C(-2, -3), D(-7, -3)\)  
6. \(L(5, 3), M(8, 3), N(9, 7), O(6, 7)\)  
7. \(W(-1, -2), X(-1, 1), Y(2, 1), Z(2, -2)\)

Lesson 11-2  
Find the area of each quadrilateral.

1. 
   \(432 \text{ units}^2\)  
2. 
   \(296.2 \text{ units}^2\)  
3. 
   \(561.2 \text{ units}^2\)

COORDINATE GEOMETRY  Find the area of trapezoid \(ABCD\) given the coordinates of the vertices.

4. \(A(1, 1), B(2, 3), C(4, 3), D(7, 1)\)  
5. \(A(-2, -2), B(2, 2), C(7, -3), D(-4, -3)\)  
6. \(A(1, -1), B(4, -1), C(8, 5), D(1, 5)\)  
7. \(A(-2, -2), B(4, 2), C(3, -2), D(1, -2)\)

COORDINATE GEOMETRY  Find the area of rhombus \(LMNO\) given the coordinates of the vertices.

8. \(L(-3, 0), M(1, -2), N(-3, -4), O(-7, -2)\)  
9. \(L(-3, -2), M(-4, 2), N(-3, 6), O(-2, 2)\)  
10. \(L(-1, -4), M(3, 4), N(-1, 12), O(-5, 4)\)  
11. \(L(-2, -2), M(4, 4), N(10, -2), O(4, -8)\)

Extra Practice
Lesson 11-3
(pages 610–616)
Find the area of each regular polygon. Round to the nearest tenth.
1. a square with perimeter 54 feet 182.3 ft²
2. a triangle with side length 9 inches 35.1 inches²
3. an octagon with side length 6 feet 173.8 ft²
4. a decagon with apothem length of 22 centimeters 1572.6 cm²

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.
5. 66.3 cm²
6. 61.7 ft²
7. 37.4 in²

Lesson 11-4
(pages 617–621)
Find the area of each figure. Round to the nearest tenth if necessary.
1. 187.2 units²
2. 420 units²
3. 88.3 units²

COORDINATE GEOMETRY The vertices of an irregular figure are given. Find the area of each figure.
4. R(0, 5), S(3, 3), T(3, 0) 4.5 units²
5. A(−5, −3), B(−3, 0), C(2, −1), D(2, −3) 15.5 units²
6. L(−1, 4), M(3, 2), N(3, −1), O(−1, −2), P(−3, 1) 24 units²

Lesson 11-5
(pages 622–627)
Find the total area of the sectors of the indicated color. Then find the probability of spinning the color indicated if the diameter of each spinner is 20 inches.
1. orange ≈62.8 in²; 0.20
2. blue ≈87.3 in²; 0.28
3. green ≈165.8 in²; 0.53

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region.
4. 23,561.9 units², ≈0.18
5. 54.5 units², ≈0.09
6. ≈47.6 units²; ≈0.31
Lesson 12-1

1. rectangular prism 2 units high, 3 units long, and 2 units wide
2. rectangular prism 1 unit high, 2 units long, and 3 units wide
3. pentagonal pyramid; base: OXNEP; faces: OXNEP, △PET, △ETN, △NTX, △XTO, △OTP; edges: PE, EN, NX, XO, OP, TP, TE, TN, TX, TO; vertices: T, P, E, N, X, O
4. cone; base: circle L; vertex: Z

Lesson 12-2

1. rectangular prism 2 units high, 3 units long, and 2 units wide
2. rectangular prism 1 unit high, 2 units long, and 3 units wide
3. triangular prism 3 units high with bases that are right triangles with legs 3 units and 4 units long
4. triangular prism 5 units high with bases that are right triangles with legs 4 units and 6 units long

Lesson 12-3

1. 96 units²; 166 units²
2. 180 units²; 216 units²
3. 216 units²; 264 units²
4. 1872 units²; 2304 units²
5. 3411.0 units²; 4086.0 units²
6. 7. The surface area of a right triangular prism is 228 square inches. The base is a right triangle with legs measuring 6 inches and 8 inches. Find the height of the prism. 7.5 in.
8. The surface area of a right triangular prism with height 18 inches is 1380 square inches. The base is a right triangle with a leg measuring 15 inches and a hypotenuse of length 25 inches. Find the length of the other leg of the base. 20 in.
Lesson 12-4
(pages 655–659)

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.
1. \( r = 2 \text{ ft}, h = 3.5 \text{ ft} \) \( 69.1 \text{ ft}^2 \)
2. \( d = 15 \text{ in.}, h = 20 \text{ in.} \) \( 1295.9 \text{ in}^2 \)
3. \( r = 3.7 \text{ m}, h = 6.2 \text{ m} \) \( 230.2 \text{ m}^2 \)
4. \( d = 19 \text{ mm}, h = 32 \text{ mm} \) \( 2477.1 \text{ mm}^2 \)

Find the surface area of each cylinder. Round to the nearest tenth.
5. \( r = 1.5 \text{ m}, h = 4 \text{ m} \) \( 51.8 \text{ m}^2 \)
6. \( r = 14 \text{ ft}, h = 32.5 \text{ ft} \) \( 1737.3 \text{ ft}^2 \)
7. \( r = 10.5 \text{ in.}, h = 1 \text{ in.} \) \( 34.6 \text{ in}^2 \)
8. \( r = 16.5 \text{ m}, h = 16.5 \text{ m} \) \( 3421.2 \text{ m}^2 \)

Lesson 12-5
(pages 660–665)

Find the surface area of each regular pyramid. Round to the nearest tenth.
1. \( 175 \text{ cm}^2 \)
2. \( 853.4 \text{ in}^2 \)
3. \( 3032.7 \text{ m}^2 \)
4. \( 255.4 \text{ cm}^2 \)
5. \( 736 \text{ ft}^2 \)
6. \( 15.6 \text{ cm}^2 \)

Lesson 12-6
(pages 666–670)

Find the surface area of each cone. Round to the nearest tenth.
1. \( 332.9 \text{ in}^2 \)
2. \( 2513.3 \text{ ft}^2 \)
3. \( 2191.9 \text{ cm}^2 \)
4. \( 89.4 \text{ in}^2 \)
5. \( 260.2 \text{ cm}^2 \)
6. \( 5753.7 \text{ ft}^2 \)

7. Find the surface area of a cone if the height is 28 inches and the slant height is 40 inches. \( 6153.2 \text{ in}^2 \)
8. Find the surface area of a cone if the height is 7.5 centimeters and the radius is 2.5 centimeters. \( 81.7 \text{ cm}^2 \)
Lesson 12-7
Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1. $180,955.7$ ft$^2$
2. $5674.5$ m$^2$
3. $19,950,370.0$ mi$^2$
4. $13,684.8$ cm$^2$

5. a hemisphere with the circumference of a great circle $14.1$ cm $47.5$ cm$^2$
6. a sphere with the circumference of a great circle $50.3$ in. $805.4$ in$^2$
7. a sphere with the area of a great circle $98.5$ m$^2$ $394$ m$^2$
8. a hemisphere with the circumference of a great circle $3.1$ in. $2.3$ in$^2$
9. a hemisphere with the area of a great circle $31,415.9$ ft$^2$ $94,247.7$ ft$^2$

Lesson 13-1
Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1. $5102.4$ ft$^3$
2. $3.8$ ft$^3$
3. $2160$ in$^3$
4. $426,437.6$ m$^3$

5. $2160$ in$^3$
6. $2160$ in$^3$

Find the volume of each solid to the nearest tenth.
4. $750$ in$^3$
5. $970.9$ cm$^3$
6. $225$ in$^3$

Lesson 13-2
Find the volume of each cone or pyramid. Round to the nearest tenth if necessary.

1. $62.5$ ft$^3$
2. $4188.8$ mm$^3$
3. $240$ in$^3$
4. $78.5$ m$^3$
5. $207.8$ m$^3$
6. $0.4$ in$^3$
Lesson 13-3

Find the volume of each sphere or hemisphere. Round to the nearest tenth.

1. \( 356,817.9 \text{ ft}^3 \) 2. \( 1.1 \text{ m}^3 \) 3. \( 10,289.8 \text{ mm}^3 \)

4. The diameter of the sphere is 3 cm. \( 14.1 \text{ cm}^3 \)
5. The radius of the hemisphere is \( 7\sqrt{2} \text{ m} \). \( 2031.9 \text{ m}^3 \)
6. The diameter of the hemisphere is 90 ft. \( 190,851.8 \text{ ft}^3 \)
7. The radius of the sphere is 0.5 in. \( 0.5 \text{ in}^3 \)

Lesson 13-4

Determine whether each pair of solids are similar, congruent, or neither.

1. \( \sim \) 2. \( \sim \)
3. \( \neq \) 4. \( \cong \)
5. \( \cong \) 6. \( \sim \)

Lesson 13-5

Graph the rectangular solid that contains the given point and the origin. Label the coordinates of each vertex. 1–6. See margin.

1. \((0, 0, 2)\) 2. \((3, 0, 2)\) 3. \((0, -1, 2)\)
4. \((0, -1, 3)\) 5. \((0, -1, 0)\)
6. \((0, 0, 0)\) 7. \((0, 0, 1)\) 8. \((0, 1, -1)\) 9. \((0, 0, 0)\) 10. \((1, 1, 1)\) 11. \((1, -2, 1)\) 12. \((1, -1, -1)\)
Mixed Problem Solving and Proof

Chapter 1 Points, Lines, Planes, and Angles

ARCHITECTURE For Exercises 1–4, use the following information.
The Burj Al Arab in Dubai, United Arab Emirates, is one of the world’s tallest hotels. [Lesson 1-1]
1. Trace the outline of the building on your paper.
2. Label three different planes suggested by the outline.
3. Highlight three lines in your drawing that, when extended, do not intersect.
4. Label three points on your sketch. Determine if they are coplanar and collinear.
1–4. See margin.

SKYSCRAPERS For Exercises 5–7, use the following information. [Lesson 1-2]

<table>
<thead>
<tr>
<th>Tallest Buildings in San Antonio, TX</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tower of the Americas</td>
<td>622</td>
</tr>
<tr>
<td>Marriott Rivercenter</td>
<td>546</td>
</tr>
<tr>
<td>Weston Centre</td>
<td>444</td>
</tr>
<tr>
<td>Tower Life</td>
<td>404</td>
</tr>
</tbody>
</table>

Source: www.skyscrapers.com

5. What is the precision for the measures of the heights of the buildings? 0.5 ft
6. What does the precision mean for the measure of the Tower of the Americas?
7. What is the difference in height between Weston Centre and Tower Life? 39–41 ft
8. The height is between 621.5 and 622.5 ft.

PERIMETER For Exercises 8–11, use the following information. [Lesson 1-3] 10. 18.5 units
The coordinates of the vertices of \( \triangle ABC \) are \( A(0, 6) \), \( B(-6, -2) \), and \( C(8, -4) \). Round to the nearest tenth.
8. Find the perimeter of \( \triangle ABC \). 36.9 units
9. Find the coordinates of the midpoints of each side of \( \triangle ABC \). \(-3, 2\), \(1, -3\), \(4, 1\)
10. Suppose the midpoints are connected to form a triangle. Find the perimeter of this triangle.
11. Compare the perimeters of the two triangles. See margin.

CONSTRUCTION For Exercises 14–15, use the following information.
A framer is installing a cathedral ceiling in a newly built home. A protractor and a plumb bob are used to check the angle at the joint between the ceiling and wall. The wall is vertical, so the angle between the vertical plumb line and the ceiling is the same as the angle between the wall and the ceiling. [Lesson 1-5]
14. How are \( \angle ABC \) and \( \angle CBD \) related?
15. If \( m \angle ABC = 110 \), what is \( m \angle CBD \)? 70

14. They form a linear pair and are supplementary.

STRUCTURES For Exercises 16–17, use the following information. [Lesson 1-6]
The picture shows the Hongkong and Shanghai Bank located in Hong Kong, China. 16. Name five different polygons suggested by the picture.
17. Classify each polygon you identified as convex or concave and regular or irregular.
16–17. See margin.

Chapter 2

2. Sample answer: In 2010, California will have about 245 people per square mile.
   In 2010, Michigan will have about 185 people per square mile.
6. The Hatter is correct; Alice exchanged the hypothesis and conclusion.
8. Then she should not accept it and should notify airline personnel immediately.
Chapter 2  Reasoning and Proof

POPULATION  For Exercises 1–2, use the table showing the population density for various states in 1960, 1980, and 2000. The figures represent the number of people per square mile.  (Lesson 2–1)

<table>
<thead>
<tr>
<th>State</th>
<th>1960</th>
<th>1980</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>100.4</td>
<td>151.4</td>
<td>217.2</td>
</tr>
<tr>
<td>CT</td>
<td>520.6</td>
<td>637.8</td>
<td>702.9</td>
</tr>
<tr>
<td>DE</td>
<td>225.2</td>
<td>307.6</td>
<td>401.0</td>
</tr>
<tr>
<td>HI</td>
<td>98.5</td>
<td>150.1</td>
<td>188.6</td>
</tr>
<tr>
<td>MI</td>
<td>137.7</td>
<td>162.6</td>
<td>175.0</td>
</tr>
</tbody>
</table>

Source: U.S. Census Bureau

1. Find a counterexample for the following statement. The population density for each state in the table increased by at least 30 during each 20-year period.  **Mi for both periods**

2. Write two conjectures for the year 2010.  **See margin.**

STATES  For Exercises 3–5, refer to the Venn diagram.  (Lesson 2–2)

3. How many states have less than 2,000,000 people?

4. How many states have less than 34,000 square miles in area?

5. How many states have less than 2,000,000 people and are less than 34,000 square miles in area?

6. **7 states**

LITERATURE  For Exercises 6–7, use the following quote from Lewis Carroll’s Alice’s Adventures in Wonderland.  (Lesson 2–3)

Then you should say what you mean,” the March Hare went on.

“I do,” Alice hastily replied; “at least—at least I mean—what I say—that’s the same thing, you know.”

“Not the same thing a bit!” said the Hatter.

6. Who is correct? Explain.  **See margin.**

7. How are the phrases say what you mean and mean what you say related?  **They are converses of each other.**

AIRLINE SAFETY  Airports in the United States post a sign stating if any unknown person attempts to give you any items including luggage to transport on your flight, do not accept it and notify airline personnel immediately. Write a valid conclusion to the hypothesis. If a person Candace does not know attempts to give her an item to take on her flight, do not accept it and notify airline personnel immediately, then write a paragraph proof to show that  **AIRLINES® AIRLINES®**

10. Given: \( k = \frac{\Delta l}{\ell(T - t)} \)

Prove: \( T = \frac{\Delta l}{k} + t \)

Proof:

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( k = \frac{\Delta l}{\ell(T - t)} ) (Given)</td>
</tr>
<tr>
<td>2. ( k(T - t) = \Delta l ) (Mult. Prop.)</td>
</tr>
<tr>
<td>3. ( T - t = \frac{\Delta l}{k} ) (Division Prop.)</td>
</tr>
<tr>
<td>4. ( T = \frac{\Delta l}{k} + t ) (Addition Prop.)</td>
</tr>
</tbody>
</table>

LITERATURE  For Exercises 6–7, use the following quote from Lewis Carroll’s Alice’s Adventures in Wonderland.  (Lesson 2–3)

“I do,” Alice hastily replied; “at least—at least I mean—what I say—that’s the same thing, you know.”

“Not the same thing a bit!” said the Hatter.

6. Who is correct? Explain.  **See margin.**

7. How are the phrases say what you mean and mean what you say related?  **They are converses of each other.**

ILLUSIONS  This drawing was created by German psychologist Wilhelm Wundt.  (Lesson 2–8)

12. Describe the relationship between each pair of vertical lines.  **12–13. See margin.**

13. Given: \( \angle 4 \cong \angle 2 \)

Prove: \( \angle 3 \cong \angle 1 \)

Proof:

<table>
<thead>
<tr>
<th>Statements (Reasons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 4 \cong \angle 2 ) (Given)</td>
</tr>
<tr>
<td>2. ( \angle 4 ) and ( \angle 3 ) form a linear pair; ( \angle 2 ) and ( \angle 1 ) form a linear pair. (Def. of linear pair)</td>
</tr>
<tr>
<td>3. ( \angle 4 ) and ( \angle 3 ) are supplementary; ( \angle 2 ) and ( \angle 1 ) are supplementary. (Supplement Theorem)</td>
</tr>
<tr>
<td>4. ( \angle 3 \cong \angle 1 ) (( \angle ) suppl. to ( \angle ) are ( \cong ))</td>
</tr>
</tbody>
</table>
Chapter 3

1. The triangles appear to be scalene. One leg looks longer than the other leg.

2. The triangles appear to be isosceles. Two of the sides appear to be the same length.

Chapter 4

1. Alternate interior angles are congruent, so $\angle 1 \cong \angle 2$.

11. Given: $MQ \parallel NP$

Prove: $\angle 4 \cong \angle 3$

Proof:

Statements (Reasons)

1. $MQ \parallel NP$; $\angle 4 \cong \angle 3$ (Given)
2. $\angle 3 \cong \angle 5$ (Alt. Int. $\angle$ Theorem)
3. $\angle 4 \cong \angle 5$ (Transitive Prop.)
4. $\angle 1 \cong \angle 6$ (Corres. $\angle$ Post.)
5. $\angle 1 \cong \angle 5$ (Transitive Prop.)

15. If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

16. Given: $\angle 1 \cong \angle 3$, $AB \parallel DC$

Prove: $BC \parallel AD$

Proof:

Statements (Reasons)

1. $AB \parallel DC$ (Given)
2. $\angle 1 \cong \angle 4$ (Alt. Int. $\angle$ Theorem)
3. $\angle 1 \cong \angle 3$ (Given)
4. $\angle 4 \cong \angle 3$ (Transitive Prop.)
5. $BC \parallel AD$ (If corr. $\angle$s are $\cong$, then lines are $\parallel$.)

17. The shortest distance is a perpendicular segment. You cannot walk this route because there are no streets that exactly follow this route and you cannot walk through or over buildings.

Chapter 3 Parallel and Perpendicular Lines

1. Optical Illusions

Lines $l$ and $m$ are parallel, but appear to be bowed due to the transversals drawn through $l$ and $m$. Make a conjecture about the relationship between $\angle 1$ and $\angle 2$. (Lesson 3-1)

See margin.

ARCHITECTURE

For Exercises 2–10, use the following information.

The picture shows one of two towers of the Puerta de Europa in Madrid, Spain. Lines $a$, $b$, $c$, and $d$ are parallel. The lines are cut by transversals $e$ and $f$. If $m\angle 1 = m\angle 2 = 75$, find the measure of each angle. (Lesson 3-2)

2. $\angle 3 \ 105$
3. $\angle 4 \ 105$
4. $\angle 5 \ 75$
5. $\angle 6 \ 75$
6. $\angle 7 \ 75$
7. $\angle 8 \ 30$
8. $\angle 9 \ 30$
9. $\angle 10 \ 75$
10. $\angle 11 \ 75$

11. Given: $MQ \parallel NP$

Prove: $\angle 1 \cong \angle 5$

Proof:

Statements (Reasons)

1. $MQ \parallel NP$; $\angle 4 \cong \angle 3$ (Given)
2. $\angle 3 \cong \angle 5$ (Alt. Int. $\angle$ Theorem)
3. $\angle 4 \cong \angle 3$ (Given)
4. $\angle 4 \cong \angle 3$ (Transitive Prop.)
5. $BC \parallel AD$ (If corr. $\angle$s are $\cong$, then lines are $\parallel$.)

12. Education

Between 1995 and 2000, the average cost for tuition and fees for American universities increased by an average rate of $84.20 per year. In 2000, the average cost was $2600. If costs increase at the same rate, what will the total average cost be in 2010? (Lesson 3-3)

13. The Three Forks community swimming pool holds 74,800 gallons of water. At the end of the summer, the pool is drained and winterized.

14. If the pool drains at the rate of 1200 gallons per hour, write an equation to describe the number of gallons left after $x$ hours. $y = 74,800 - 1200x$

15. Construction

An engineer and carpenter square is used to draw parallel line segments. Martin makes two cuts at an angle of 120° with the edge of the wood through points $D$ and $P$. Explain why these cuts will be parallel. (Lesson 3-5)

16. Prove: Write a two-column proof. (Lesson 3-5)

Given: $\angle 1 \cong \angle 3$

Prove: $BC \parallel AD$

17. Cities

The map shows a portion of Seattle, Washington. Describe a segment that represents the shortest distance from the Bus Station to Denny Way. Can you walk the route indicated by your segment? Explain. (Lesson 3-6)
1. Shade all right triangles red. Do these triangles appear to be scalene or isosceles? Explain.
2. Shade all acute triangles blue. Do these triangles appear to be scalene, isosceles, or equilateral? Explain. 1–2. See margin.

3. ASTROLOGY Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form ΔLEO. If the angles have measures as shown in the figure, find m∠OLE. (Lesson 4-2) 66

4. ARCHITECTURE The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent. (Lesson 4-3) See margin.

RECREATION Tapatan is a game played in the Philippines on a square board, like the one shown at the top right. Players take turns placing each of their three pieces on a different point of intersection. After all the pieces have been placed, the players take turns moving a piece along a line to another intersection. A piece cannot jump over another piece. A player who gets all their pieces in a straight line wins. Point E bisects all four line segments that pass through it. All sides are congruent, and the diagonals are congruent. Suppose a letter is assigned to each intersection. (Lesson 4-4) See margin.

8. Yes, the method is valid. Thales sighted ∠SPQ and ∠SQP. He then constructed ∠QPA congruent to ∠SPQ and ∠PQA congruent to ∠SQP. ΔSPQ and ΔAPQ share the side PQ. Since ∠QPA ≅ ∠SPQ, ∠PQA ≅ ∠SQP, and PQ ≅ PQ, ΔSPQ ≅ ΔAPQ by the ASA Postulate.

9. Given: PH bisects ∠YHX, PH ⊥ YX
Prove: ΔYHX is an isosceles triangle.

Proof:
Statements (Reasons)
1. PH bisects ∠YHX. (Given)
2. ∠YHP ≅ ∠XHP (Def. of ∠ bisector)
3. PH ⊥ YX (Given)
4. ∠YPH and ∠XPH are rt. ∠s (Def. of ⊥ lines)
5. ∠YPH ≅ ∠XPH (All rt. ∠s are ≅.)
6. ∠Y ≅ ∠X (Third ∠ Th.)
7. HY ≅ HY (Conv. of Isos. ∠ Th.)
8. ΔYHX is an isosceles triangle. (Def. of isos. ∠)
10. Given: ΔABC is a right isosceles triangle. M is the midpoint of AB.
Prove: CM ⊥ AB

Proof: Place the triangle so that the vertices are A(a, 0), B(0, a), and C(0, 0).
By the Midpoint Formula, the coordinates of M are
\( M = \left( \frac{0 + a}{2}, \frac{a + 0}{2} \right) = \left( \frac{a}{2}, \frac{a}{2} \right) \).
Find the slopes of AB and CM.
Slope of AB = \( \frac{0 - a}{a - 0} = -\frac{a}{a} = -1 \)
Slope of CM = \( \frac{\frac{a}{2} - 0}{\frac{a}{2} - 0} = \frac{\frac{2}{2}}{\frac{2}{2}} = 1 \)
The product of the slopes is \(-1\), so CM \perp AB.
1. Given: \( x + y > 634 \)
Prove: \( x > 317 \) or \( y > 317 \)

Proof:
Step 1: Assume \( x < 317 \) and \( y < 317 \).
Step 2: \( x + y < 634 \)
Step 3: This contradicts the fact that \( 2x + y > 634 \).
Therefore, at least one of the legs was longer than 317 miles.

11. Given: \( \angle ZST \cong \angle ZTS \)
\( \angle XRA \cong \angle XAR \)
\( TA = 2AX \)

Prove: \( 2XR + AZ > SZ \)

Proof:
Statements (Reasons)
1. \( \angle ZST \cong \angle ZTS \) (Given)
2. \( SZ \cong TZ \) (Isos. \( \triangle \) Th.)
3. \( SZ = TZ \) (Def. of \( \cong \))
4. \( TA + AZ > TZ \) (\( \triangle \) Ineq. Th.)
5. \( TA = 2AX \) (Given)
6. \( 2AX + AZ > TZ \) (Substitution)
7. \( \angle XRA \cong \angle XAR \) (Given)
8. \( XR = XA \) (Isos. \( \triangle \) Th.)
9. \( XR = XA \) (Def. of \( \cong \))
10. \( 2XR + AZ > TZ \) (Substitution)
11. \( 2XR + AZ > SZ \) (Substitution)

12. GEOGRAPHY The map shows a portion of Nevada. The distance from Tonopah to Round Mountain is the same as the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Use the distance measures to determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty. (Lesson 5-5) Warm Springs to Beatty

13. PROOF Write a two-column proof. (Lesson 5-5)
Given: \( DB \) is a median of \( \triangle ABC \).
\( m\angle 1 > m\angle 2 \)

Prove: \( m\angle C > m\angle A \)

Proof:
Statements (Reasons)
1. \( DB \) is a median of \( \triangle ABC \); \( m\angle 1 > m\angle 2 \) (Given)
2. \( D \) is the midpoint of \( AC \). (Def. of median)
3. \( \overline{AD} \parallel \overline{DC} \) (Midpoint Theorem)
4. \( \overline{DB} \parallel \overline{DB} \) ( Reflexive Property)
5. \( AB > BC \) (SAS Inequality)
6. \( m\angle C > m\angle A \) (If one side of a \( \triangle \) is longer than another, the \( \angle \) opp. the longer side > the \( \angle \) opp. the shorter side.)
1. **TOYS** In 2000, $34,554,900,000 was spent on toys in the U.S. The U.S. population in 2000 was 281,421,906, with 21.4% of the population 14 years and under. If all of the toys purchased in 2000 were for children 14 years and under, what was the average amount spent per child? (Lesson 6-1) $573.77

2. **QUILTING** For Exercises 2–4, use the following information. (Lesson 6-2)
   Felicia found a pattern for a quilt square. The pattern measures three-quarters of an inch on a side. Felicia wants to make a quilt that is 77 inches by 110 inches when finished.
   1. If Felicia wants to use only whole quilt squares, what is the greatest side length she can use for each square? 71 in.
   2. How many quilt squares will she need for the quilt? 70 squares
   3. By what scale factor will she need to increase the pattern for the quilt square? \( \frac{11}{3} \)

3. **PROOF** For Exercises 5 and 6, write a paragraph proof. (Lesson 6-3) 5–6. See margin.
   5. Given: \( \triangle WYZ \sim \triangle QYS \)
   Prove: \( \triangle WYZ \sim \triangle QYS \)
   6. Given: \( WX \parallel QR \), \( ZX \parallel SR \)
   Prove: \( WZ \parallel QS \)

4. **HISTORY** For Exercises 7 and 8, use the following information. (Lesson 6-4)
   In the fifteenth century, mathematicians and artists tried to construct the perfect letter. Damiano da Moile used a square as a frame to design the letter “A” as shown in the diagram. The thickness of the major stroke of the letter was to be \( \frac{1}{12} \) of the height of the letter.

5. **TRACK** A triangular track is laid out as shown. \( \triangle RST \sim \triangle WVL \). If \( UV = 500 \) feet, \( VW = 400 \) feet, \( UW = 300 \) feet, and \( ST = 1000 \) feet, find the perimeter of \( \triangle RST \). (Lesson 6-5) 2400 ft

6. Given: \( WX \parallel QR \), \( ZX \parallel SR \)
   Prove: \( WZ \parallel QS \)
   Proof: We are given that \( WX \parallel QR \), \( ZX \parallel SR \). By the Corresponding Angles Postulate, \( \angle XYW = \angle QYR \) and \( \angle YXZ = \angle YRS \). By the Reflexive Property, \( \angle QYS = \angle QYS \), \( \angle QRY = \angle QRY \) and \( \angle YRS = \angle YRS \). By AA Similarity. By the definition of similar triangles, \( \frac{WY}{SY} = \frac{YX}{YR} \) and \( \frac{ZY}{SY} = \frac{YX}{YR} \). By the Transitive Property, \( \triangle WYZ \sim \triangle QYS \) by SAS Similarity. By the definition of similar triangles \( \angle YWZ \cong \angle QYS \). \( WZ \parallel QS \) by the Corresponding Angles Postulate.

7. The bar connects the midpoints of each leg of the letter and is parallel to the base. Therefore, the length of the bar is one-half the length of the base because a midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

8. Given: \( \triangle WYZ \sim \triangle QYS \)
   Prove: \( WZ \parallel QS \)
   Proof: It is given that \( \triangle WYZ \sim \triangle QYS \) and \( \triangle YXZ \sim \triangle SYR \). By definition of similar polygons we know that \( \frac{WY}{QY} = \frac{YX}{YR} \) and \( \frac{ZX}{SY} = \frac{YR}{YR} \). Then \( \frac{WY}{QY} = \frac{ZX}{SY} \) by the Transitive Property. \( \angle WYZ = \angle QYS \) because congruence of angles is reflexive. Therefore, \( \triangle WYZ \sim \triangle QYS \) by SAS Similarity.

9. **BANKING** Ashante has $5000 in a savings account with a yearly interest rate of 2.5%. The interest is compounded twice per year. What will be the amount in the savings account after 5 years? (Lesson 6-6) $5661.35

10. **Mixed Problem Solving and Proof** 787

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**Chapter 6 Proportions and Similarity**

(pages 280–339)

1. **TOYS** In 2000, $34,554,900,000 was spent on toys in the U.S. The U.S. population in 2000 was 281,421,906, with 21.4% of the population 14 years and under. If all of the toys purchased in 2000 were for children 14 years and under, what was the average amount spent per child? (Lesson 6-1) $573.77

2. **QUILTING** For Exercises 2–4, use the following information. (Lesson 6-2)
   Felicia found a pattern for a quilt square. The pattern measures three-quarters of an inch on a side. Felicia wants to make a quilt that is 77 inches by 110 inches when finished.
   1. If Felicia wants to use only whole quilt squares, what is the greatest side length she can use for each square? 71 in.
   2. How many quilt squares will she need for the quilt? 70 squares
   3. By what scale factor will she need to increase the pattern for the quilt square? \( \frac{11}{3} \)

3. **PROOF** For Exercises 5 and 6, write a paragraph proof. (Lesson 6-3) 5–6. See margin.
   5. Given: \( \triangle WYZ \sim \triangle QYS \)
   Prove: \( \triangle WYZ \sim \triangle QYS \)
   6. Given: \( WX \parallel QR \), \( ZX \parallel SR \)
   Prove: \( WZ \parallel QS \)

4. **HISTORY** For Exercises 7 and 8, use the following information. (Lesson 6-4)
   In the fifteenth century, mathematicians and artists tried to construct the perfect letter. Damiano da Moile used a square as a frame to design the letter “A” as shown in the diagram. The thickness of the major stroke of the letter was to be \( \frac{1}{12} \) of the height of the letter.

5. **TRACK** A triangular track is laid out as shown. \( \triangle RST \sim \triangle WVL \). If \( UV = 500 \) feet, \( VW = 400 \) feet, \( UW = 300 \) feet, and \( ST = 1000 \) feet, find the perimeter of \( \triangle RST \). (Lesson 6-5) 2400 ft

6. Given: \( WX \parallel QR \), \( ZX \parallel SR \)
   Prove: \( WZ \parallel QS \)
   Proof: We are given that \( WX \parallel QR \), \( ZX \parallel SR \). By the Corresponding Angles Postulate, \( \angle XYW = \angle QYR \) and \( \angle YXZ = \angle YRS \). By the Reflexive Property, \( \angle QYS = \angle QYS \), \( \angle QRY = \angle QRY \) and \( \angle YRS = \angle YRS \). By the Transitive Property, \( \triangle WYZ \sim \triangle QYS \) by SAS Similarity. By the definition of similar triangles \( \angle YWZ = \angle QYS \). \( WZ \parallel QS \) by the Corresponding Angles Postulate.

7. The bar connects the midpoints of each leg of the letter and is parallel to the base. Therefore, the length of the bar is one-half the length of the base because a midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

8. Given: \( \triangle WYZ \sim \triangle QYS \)
   Prove: \( WZ \parallel QS \)
   Proof: It is given that \( \triangle WYZ \sim \triangle QYS \) and \( \triangle YXZ \sim \triangle SYR \). By definition of similar polygons we know that \( \frac{WY}{QY} = \frac{YX}{YR} \) and \( \frac{ZX}{SY} = \frac{YR}{YR} \). Then \( \frac{WY}{QY} = \frac{ZX}{SY} \) by the Transitive Property. \( \angle WYZ = \angle QYS \) because congruence of angles is reflexive. Therefore, \( \triangle WYZ \sim \triangle QYS \) by SAS Similarity.

9. **BANKING** Ashante has $5000 in a savings account with a yearly interest rate of 2.5%. The interest is compounded twice per year. What will be the amount in the savings account after 5 years? (Lesson 6-6) $5661.35

10. **Mixed Problem Solving and Proof** 787
Chapter 7

1. Given: \( D \) is the midpoint of \( BE \). \( BD \) is an altitude of right triangle \( ABC \).

   Prove: \( \frac{AD}{DE} = \frac{DB}{DC} \)

   Proof: Statements (Reasons)

   1. \( BD \) is an altitude of right triangle \( ABC \). (Given)
   2. \( \frac{AD}{DB} = \frac{DE}{DC} \) (The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.)
   3. \( D \) is the midpoint of \( BE \). (Given)
   4. \( DB = DE \) (Def. of midpoint)
   5. \( \frac{AD}{DE} = \frac{DB}{DC} \) (Substitution)

3. No: the measures do not satisfy the Pythagorean Theorem since \((2.7)^2 + (3.0)^2 \neq (5.3)^2\).

8. \( AE \approx 339.4 \text{ ft} \), \( EB = 300 \text{ ft} \), \( CF = 134.2 \text{ ft} \), \( DF \approx 84.9 \text{ ft} \)

Chapter 8

3. Sample answer: Make sure that opposite sides are congruent or make sure that opposite angles are congruent.

4. Given: \( \square ABCD, \ AE \cong CF \)

   Prove: Quadrilateral \( EBFD \) is a \( \square \)

   Proof: Statements (Reasons)

   1. \( \square ABCD, \ AE \cong CF \) (Given)
   2. \( AB \cong DC \) (Opp. sides of a \( \square \) are \( \cong \).)
   3. \( \angle A \cong \angle C \) (Opp. \( \angle \) of a \( \square \) are \( \cong \).)
   4. \( \triangle BAE \cong \triangle DCF \) (SAS)
   5. \( EB \cong DF \), \( \angle BEA \cong \angle DFC \) (CPCTC)
   6. \( BC \parallel AD \) (Def. of \( \square \))
   7. \( \angle DFC \cong \angle FDE \) (Alt. Int. \( \angle \) Th.)
   8. \( \angle BEA \cong \angle DFE \) (Trans. Prop.)
   9. \( EB \parallel DF \) (Corres. \( \angle \) Post.)
   10. Quadrilateral \( EBFD \) is a \( \square \).

   (If one pair of opp. sides is \( \parallel \) and \( \cong \), then the quad. is a \( \square \).)

6. Given: \( \square WXZY \), \( \angle 1 \) and \( \angle 2 \) are complementary.

   Prove: \( WXZY \) is a rectangle.

   Proof: Statements (Reasons)

   1. \( \square WXZY, \angle 1 \) and \( \angle 2 \) are complementary (Given)
   2. \( m\angle 1 + m\angle 2 = 90 \) (Def. of complementary \( \angle \))
   3. \( m\angle 1 + m\angle 2 + m\angle X = 180 \) (Angle Sum Th.)
   4. \( 90 + m\angle X = 180 \) (Substitution)
   5. \( m\angle X = 90 \) (Subtraction)
   6. \( m\angle 2 \cong m\angle Y \) (Opp. \( \angle \) of a \( \square \) are \( \cong \).
   7. \( m\angle Y = 90 \) (Substitution)
Chapter 8  Quadrilaterals  (pages 402–459)

ENGINEERING  For Exercises 1–2, use the following information.
The London Eye in London, England, is the world’s
largest observation wheel. The ride has 32 enclosed
capsules for riders. (Lesson 8-1)

1. Suppose each capsule is connected with a straight
piece of metal forming a 32-gon. Find the sum of
the measures of the interior angles. 5400
2. What is the measure of one interior angle of the
32-gon? 168.75

3. QUILTING  The quilt square shown is called the
Lone Star pattern. Describe two ways that
the quilter could ensure
the Lone Star pattern.
See margin. (Lesson 8-2)

4. PROOF  Write a two-column proof. (Lesson 8-3)
Given: □ABCD, AE = CE
Prove: Quadrilateral EBFD is a parallelogram.
See margin.

5. MUSIC  Why will the keyboard stand shown
always remain parallel to the floor? (Lesson 8-3)
See margin.

6. PROOF  Write a two-column proof. (Lesson 8-4)
Given: □WXYZ, ∠1 and ∠2 are complementary.
Prove: WXYZ is a rectangle. See margin.

7. PROOF  Write a paragraph proof. (Lesson 8-4)
Given: □KLMN
Prove: PQRS is a rectangle. See margin.

8. CONSTRUCTION  Mr. Redwing is building a
sandbox. He placed stakes at what he believes
will be the four vertices of a square with a
distance of 5 feet between each stake. How can he
be sure that the sandbox will be a square?
See margin. (Lesson 8-5)

8. DESIGN  For Exercises 9 and 10, use the square
floor tile design shown below. (Lesson 8-6)

9. Explain how you know that the trapezoids in the
design are isosceles. See margin.

10. The perimeter of the floor tile is 48 inches,
and the perimeter of the interior red square is
16 inches. Find the perimeter of one trapezoid.
16 + 8√2 in. ≈ 27.3 in.

11. PROOF  Position a quadrilateral on the
coordinate plane with vertices Q(-a, 0), R(a, 0),
S(b, c), and T(-b, c). Prove that the quadrilateral
is an isosceles trapezoid. (Lesson 8-7)
See margin.

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8. ∠X and ∠XWY are suppl., ∠X and ∠XZY are
suppl. (Cons. ∠s in □ are suppl.)
9. m∠X + m∠XWY = 180, m∠X + m∠XZY = 180
(Substitution)
10. 90 + m∠XWY = 180, 90 + m∠XZY = 180
(Substitution)
11. m∠XWY = 90, m∠XZY = 90 (Subtraction)
12. ∠X, ∠Y, ∠XWY, and ∠XZY are rt. ∠s (Def. rt. ∠)
13. WXYZ is a rect. (Def. of rect.)

9. A given: □KLMN
Prove: PQRS is a rectangle.
Proof: The diagram indicates that
∠KNS ≅ ∠SNM ≅ ∠MLQ ≅ ∠QLK
and ∠KNS ≅ ∠SKL ≅ ∠LMQ ≅ ∠OMN in □KLMN. Since ∠KLR,
∠KNS, ∠MLQ, and ∠MNP all have
two angles congruent, the third
angles are congruent by the Third
Angle Theorem. So ∠QRS ≅
∠KSN ≅ ∠MQL ≅ ∠SPQ. Since
they are vertical angles, ∠KSN ≅
∠PSR and ∠MQL ≅ ∠POR.
Therefore, ∠QRS ≅ ∠PSR ≅
∠POR ≅ ∠SPQ. PQRS is a
parallelogram since both pairs
of opposite angles are congruent,
the quadrilateral is a
parallellogram. ∠KSN and ∠KSP
form a linear pair and are
therefore supplementary angles.
∠KSP and ∠PSR form a linear
pair and are supplementary angles.
Therefore, ∠KSN and ∠PSR are
supplementary. Since they are also
congruent, each is a right angle.
If a parallelogram has one right
angle, it has four right angles.
Therefore, PQRS is a rectangle.

8. Sample answer: He should measure
the angles at the vertices to see if
they are 90 or he can check to see
if the diagonals are congruent.

9. The legs of the trapezoids are
part of the diagonals of the
square. The diagonals of a
square bisect opposite angles,
so each base angle of a
trapezoid measures 45°. One
pair of sides is parallel and the
base angles are congruent.

11. Given: Quadrilateral QRST
Prove: QRST is an isosceles
trapezoid

\[
T(-b, c) \quad S(b, c) \\
Q(-a, 0) \quad R(a, 0)
\]

Proof:
\[
TQ = \sqrt{(-b - (-a))^2 + (c - 0)^2} = \sqrt{b^2 - 2ab + a^2 + c^2}
\]
\[
SR = \sqrt{(b - a)^2 + (c - 0)^2} = \sqrt{b^2 - 2ab + a^2 + c^2}
\]
Slope of TS = \frac{c - c}{b - (-b)} = 0 or \frac{0}{2b}.
Slope of QR = \frac{c - 0}{a - (-a)} = \frac{c}{2a} or 0.
Slope of TO = \frac{c - 0}{-b - (-a)} = \frac{c}{b - a}.
Slope of SR = \frac{c - 0}{b - a} or \frac{c}{b - a}.

Exactly one pair of opposite sides
are parallel. The legs are
congruent. QRST is an isosceles
trapezoid.
Chapter 9

2. Sample answer: Look at the upper right-hand square containing two squares and four triangles. The blue triangles are reflections over a line representing the diagonal of the square. The purple pentagon is formed by reflecting a trapezoid over a line through the center of the square surrounding the pentagon. Any small pink square is a reflection of a small yellow square reflected over a diagonal of the larger square.

3. 50 mi;

4. either 45° clockwise or 45° counterclockwise

5. either 45° clockwise or 45° counterclockwise

7. Yes; the measure of one interior angle is 90, which is a factor of 360. So, a square can tessellate the plane.

9. See margin.

11. Sample answer: The matrix
\[
\begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix}
\]
will produce the vertices for a reflection of the figure in the y-axis. Then the matrix
\[
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]
will produce the vertices for a reflection of the second figure in the x-axis. This figure will be upside down.

12. The matrix
\[
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\]
will produce the vertices for a 180° rotation about the origin. The figure will be upside down and in Quadrant III.

13. The matrix for Exercise 12 has the first row entries for the first matrix used in Exercise 11 and the second row entries for the second matrix used in 11.

Chapter 9 Transformations

QUILTING For Exercises 1 and 2, use the diagram of a quilt square. (Lesson 9-1)

1. How many lines of symmetry are there for the entire quilt square? 4

2. Consider different sections of the quilt square. Describe at least three different lines of reflection and the figures reflected in those lines. See margin.

3. ENVIRONMENT A cloud of dense gas and dust pours out of Surtsey, a volcanic island off the south coast of Iceland. If the cloud blows 40 miles north and then 30 miles east, make a sketch to show the translation of the smoke particles. Then find the distance of the shortest path that would take the particles to the same position. (Lesson 9-2) See margin.

4. ART For Exercises 4–7, use the mosaic tile. See margin.

4. Identify the order and magnitude of rotation that takes a yellow triangle to a blue triangle. (Lesson 9-3)

5. Identify the order and magnitude of rotation that takes a blue triangle to a yellow triangle. (Lesson 9-3) 4–5. See margin.

6. ENVIRONMENT A cloud of dense gas and dust pours out of Surtsey, a volcanic island off the south coast of Iceland. If the cloud blows 40 miles north and then 30 miles east, make a sketch to show the translation of the smoke particles. Then find the distance of the shortest path that would take the particles to the same position. (Lesson 9-2) See margin.

9. Draw a diagram using vectors to represent this situation. See margin.

10. Find the resultant velocity and direction of the plane. about 132.4 mph; about 9.0° west of due south

AVIATION For Exercises 9 and 10, use the following information. (Lesson 9-6)

A small aircraft flies due south at an average speed of 190 miles per hour. The wind is blowing due west at 30 miles per hour.

11. Suppose you want the figure to move to Quadrant III but be upside down. Write two matrices that make this transformation, if they are applied consecutively.

12. What type of transformation is this?

13. Compare the two matrices in Exercise 11 to the matrix in Exercise 12. What do you notice?

14. Write the vertex matrix for the figure in Quadrant III and graph it on the coordinate plane.

11–14. See margin.

8. CRAFTS Eduardo found a pattern for cross-stitch on the Internet. The pattern measures 2 inches by 3 inches. He would like to enlarge the piece to 4 inches by 6 inches. The copy machine available to him enlarges 150% or less by increments of whole number percents. Find two whole number percents by which he can consecutively enlarge the piece and get as close to the desired dimensions as possible without exceeding them. (Lesson 9-5)

Sample answer: 150% followed by 133%

13. The matrix for Exercise 12 has the first row entries for the first matrix used in Exercise 11 and the second row entries for the second matrix used in 11.
Chapter 10  Circles

(pages 520–589)

1. **CYCLING** A bicycle tire travels about 50.27 inches during one rotation of the wheel. What is the diameter of the tire?  
   **(Lesson 10-1)** about 16 in.

2. **SPACE** For Exercises 2–4, use the following information.  
   **(Lesson 10-2)** School children were recently surveyed about what they believe to be the most important reason to explore Mars. They were given five choices and the table below shows the results.

<table>
<thead>
<tr>
<th>Reason to Visit Mars</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learn about life beyond Earth</td>
<td>910</td>
</tr>
<tr>
<td>Learn more about Earth</td>
<td>234</td>
</tr>
<tr>
<td>Seek potential for human inhabitance</td>
<td>624</td>
</tr>
<tr>
<td>Use as a base for further exploration</td>
<td>364</td>
</tr>
<tr>
<td>Increase human knowledge</td>
<td>468</td>
</tr>
</tbody>
</table>

   **Source:** USA TODAY

3. If you were to construct a circle graph of this data, how many degrees would be allotted to each category? 2–4. **See margin.**

4. Describe the type of arc associated with each category.

5. **CRAFTS** Yvonne uses wooden spheres to make paperweights to sell at craft shows. She cuts off a flat surface for each base. If the original sphere has a radius of 4 centimeters and the diameter of the flat surface is 6 centimeters, what is the height of the paperweight?  
   **(Lesson 10-3)** about 6.6 cm

6. **PROOF** Write a two-column proof.  
   **(Lesson 10-4)**

   **Given:**  
   MHT is a semicircle.  
   TR \perp RH  
   TH = HM

   **Prove:** \(RD\) is a radius. \(DH = DA\). Since \(RG = RD\) and \(DA\), \(\triangle GDA\) is isosceles. Therefore, each angle has a measure of 60. Since \(GR\) is tangent to \(\odot D\), \(m\angle RGD = 90\). Since \(m\angle AGD = 60\), then by the Angle Addition Postulate, \(m\angle RAG = 30\). If \(m\angle DAG = 60\), then \(m\angle RAG = 120\). Then \(m\angle R = 30\). Then, \(\triangle RAG\) is isosceles, and \(RA = AG\). By the Transitive Property, \(RA = DA\). Therefore, \(AG\) bisects \(RD\).

7. **PROOF** Write a paragraph proof.  
   **(Lesson 10-5)**

   **Given:** GR is tangent to \(\odot D\) at G.  
   \(AG = DG\)

   **Prove:** \(AG\) bisects \(RD\).

8. **METEOROLOGY** A rainbow is really a full circle with a center at a point in the sky directly opposite the Sun. The position of a rainbow varies according to the viewer’s position, but its angular size, \(\angle ABC\), is always 42°. If \(m\angle D = 160\), find the measure of the visible part of the rainbow, \(m\angle AC\).  
   **(Lesson 10-6)** 76

9. **CONSTRUCTION** An arch over an entrance is 100 centimeters wide and 30 centimeters high. Find the radius of the circle that contains the arch.  
   **(Lesson 10-7)** about 56.7 cm

10. **PROOF** Mixed Problem Solving and Proof

Chapter 10

2. **Learn about life beyond Earth:** 126°;  
   **Learn more about Earth:** 32.4°; **Seek potential for human inhabitance:** 86.4°;  
   **Use as a base for further exploration:** 50.4°; **Increase human knowledge:** 64.8°

3. All of the categories are represented by minor arcs.
Chapter 11

9. The total for the black tiles is greater. For the red tiles, there are 4 hexagons and 5 squares for a perimeter of \(2(4 + 2\sqrt{2}) + 5 \cdot 4) = (72 + 16\sqrt{2})\) feet. For the black tiles, there are 8 squares and 8 triangles for a perimeter of \(2(8 + 8(2 + \sqrt{2})) = (96 + 16\sqrt{2})\) feet.

Chapter 11  Polygons and Area

REMODELING  For Exercises 1–3, use the following information.
The diagram shows the floor plan of the home that the Summers are buying. They want to replace the patio with a larger sunroom to increase their living space by one-third.  \[\text{(Lesson 11-1)}\]

HOME REPAIR  For Exercises 4 and 5, use the following information.
Scott needs to replace the shingles on the roof of his house. The roof is composed of two large isosceles trapezoids, two smaller isosceles trapezoids, and a rectangle. Each trapezoid has the same height.  \[\text{(Lesson 11-2)}\]

SPORTS  The Moore High School basketball team wants to paint their basketball court as shown. They want the center circle and the free throw areas painted blue. What is the area of the court that they will paint blue?  \[\text{(Lesson 11-3)}\]

MUSEUMS  For Exercises 7–9, use the following information.
The Hyalite Hills Museum plans to install the square mosaic pattern shown below in the entry hall. It is 10 feet on each side with each small black or red square tile measuring 2 feet on each side.  \[\text{(Lesson 11-4)}\]

1. Excluding the patio and storage area, how many square feet of living area are in the current house?  \[840 \text{ ft}^2\]
2. What area should be added to the house to increase the living area by one-third?  \[280 \text{ ft}^2\]
3. The Summers want to connect the bedroom and storage area with the sunroom. What will be the dimensions of the sunroom?  \[12 \text{ ft by 23.3 ft}\]

4. Find the height of the trapezoids.  \[16 \text{ ft}\]
5. Find the area of the roof covered by shingles.  \[2528 \text{ ft}^2\]

6. The Moore High School basketball team wants to paint their basketball court as shown. They want the center circle and the free throw areas painted blue. What is the area of the court that they will paint blue?  \[682.19 \text{ ft}^2\]

ACCOMMODATIONS  The convention center in Washington, D.C., lies in the northwest sector of the city between New York and Massachusetts Avenues, which intersect at a 130° angle. If the amount of hotel space is evenly distributed over an area with that intersection as the center and a radius of 1.5 miles, what is the probability that a visitor, randomly assigned to a hotel, will be housed in the sector containing the convention center?  \[\text{(Lesson 11-5)}\]

7. Find the area of black tiles.  \[48 \text{ ft}^2\]
8. Find the area of red tiles.  \[52 \text{ ft}^2\]
9. Which is greater, the total perimeter of the red tiles or the total perimeter of the black tiles? Explain.  \[\text{See margin.}\]

10. If the dart lands on the target, find the probability that it lands in the blue region.  \[\approx 0.378\]

11. The convention center in Washington, D.C., lies in the northwest sector of the city between New York and Massachusetts Avenues, which intersect at a 130° angle. If the amount of hotel space is evenly distributed over an area with that intersection as the center and a radius of 1.5 miles, what is the probability that a visitor, randomly assigned to a hotel, will be housed in the sector containing the convention center?  \[\frac{13}{36} \text{ or } 36.1\%\]
1. **ARCHITECTURE**
   Sketch an orthogonal drawing of the Eiffel Tower. *(Lesson 12-1)*

2. **CONSTRUCTION**
   The roof shown below is a hip-and-valley style. Use the dimensions given to find the area of the roof that would need to be shingled. *(Lesson 12-2)*

3. **AERONAUTICAL ENGINEERING**
   The surface area of the wing on an aircraft is used to determine a design factor known as wing loading. If the total weight of the aircraft and its load is \( w \) and the total surface area of its wings is \( s \), then the formula for the wing loading factor, \( \frac{w}{s} \), is \( \frac{w}{s} \). If the wing loading factor is exceeded, the pilot must either reduce the fuel load or remove passengers or cargo. Find the wing loading factor for a plane if it had a take-off weight of 750 pounds and the surface area of the wings was 532 square feet. *(Lesson 12-2)*

4. **MANUFACTURING**
   Many baking pans are given a special nonstick coating. A rectangular cake pan is 9 inches by 13 inches by 2 inches deep. What is the area of the inside of the pan that needs to be coated? *(Lesson 12-3)*

5. **COMMUNICATIONS**
   Coaxial cable is used to transmit long-distance telephone calls, cable television programming, and other communications. A typical coaxial cable contains 22 copper tubes and has a diameter of 3 inches. What is the lateral area of a coaxial cable that is 500 feet long? *(Lesson 12-4)*

6. **COLLECTIONS**
   For Exercises 6 and 7, use the following information.
   Soledad collects unique salt-and-pepper shakers. She inherited a pair of tetrahedral shakers from her mother. *(Lesson 12-5)*
   - Each edge of a shaker measures 3 centimeters. Make a sketch of one shaker. *See margin.*
   - Find the total surface area of one shaker. *about 15.6 cm\(^2\)*

7. **FARMING**
   The picture below shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions shown in the diagram to find the entire surface area of the bin. Write the exact answer and the answer rounded to the nearest square foot. *(Lesson 12-6)*

8. **GEOGRAPHY**
   For Exercises 9–11, use the following information.
   Joaquin is buying Dennis a globe for his birthday. The globe has a diameter of 16 inches. *(Lesson 12-7)*
   - What is the surface area of the globe? *804.2 in\(^2\)*
   - If the diameter of Earth is 7926 miles, find the surface area of Earth. *197,359,487.5 mi\(^2\)*
   - The continent of Africa occupies about 11,700,000 square miles. How many square inches will be used to represent Africa on the globe? *about 47.7 in\(^2\)*
1. **METEOROLOGY**  The TIROS weather satellites were a series of weather satellites, the first being launched on April 1, 1960. These satellites carried television and infrared cameras and were covered by solar cells. If the cylinder-shaped body of a TIROS had a diameter of 42 inches and a height of 19 inches, what was the volume available for carrying instruments and cameras? Round to the nearest tenth. (Lesson 13-1) 26,323.4 in³

2. **SPACECRAFT**  The smallest manned spacecraft, used by astronauts for jobs outside the Space Shuttle, is the Manned Maneuvering Unit. It is 4 feet tall, 2 feet 8 inches wide, and 3 feet 8 inches deep. Find the volume of this spacecraft in cubic feet. Round to the nearest tenth. (Lesson 13-1) 39.1 ft³

3. **MUSIC**  To play a concertina, you push and pull the end plates and press the keys. The air pressure causes vibrations of the metal reeds that make the notes. When fully expanded, the concertina is 36 inches from end to end. If the concertina is compressed, it is 7 inches from end to end. Find the volume of air in the instrument when it is fully expanded and when it is compressed. (Hint: Each endplate is a regular hexagonal prism and contains no air.) (Lesson 13-1) 2993.0 in³; 280.6 in³

4. **ENGINEERING**  The base of an oil drilling platform is made up of 24 concrete cylindrical cells. Twenty of the cells are used for oil storage. The pillars that support the platform deck rest on the four other cells. Find the total volume of the storage cells. (Lesson 13-1) 18,555,031.6 ft³

5. **HOME BUSINESS**  Jodi has a home-based business selling homemade candies. She is designing a pyramid-shaped box for the candy. The base is a square measuring 14.5 centimeters on a side. The slant height of the pyramid is 16 centimeters. Find the volume of the box. Round to the nearest cubic centimeter. (Lesson 13-2) 1000 cm³

6. Find the volume of Spaceship Earth. Round to the nearest cubic foot. (Lesson 13-3) 2,352,071 ft³

7. Find the volume of a golf ball. Round to the nearest tenth. (Lesson 13-3) 1.8 in³

8. What is the scale factor that compares Spaceship Earth to a golf ball? (Lesson 13-4) 1320 to 1

9. What is the ratio of the volume of Spaceship Earth to the volume of a golf ball? (Lesson 13-4)

10. Suppose a six-foot-tall golfer plays golf with a 1.5 inch diameter golf ball. If the ratio between golfer and ball remains the same, how tall would a golfer need to be to use Spaceship Earth as a golf ball? (Lesson 13-4) 7920 ft tall

11. Find the distance between each pair of stars. (Lesson 13-5)

12. Which star is farthest from the center of the room? the star located at S

11. \[ ST = \sqrt{354\,\text{ft}}, \quad TR = 3\sqrt{13}\,\text{ft}, \quad SR = \sqrt{115}\,\text{ft} \]
Becoming a Better Test-Taker

At some time in your life, you will have to take a standardized test. Sometimes this test may determine if you go on to the next grade or course, or even if you will graduate from high school. This section of your textbook is dedicated to making you a better test-taker.

TYPES OF TEST QUESTIONS  In the following pages, you will see examples of four types of questions commonly seen on standardized tests. A description of each type of question is shown in the table below.

<table>
<thead>
<tr>
<th>Type of Question</th>
<th>Description</th>
<th>See Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiple choice</td>
<td>Four or five possible answer choices are given from which you choose the best answer.</td>
<td>796–797</td>
</tr>
<tr>
<td>gridded response</td>
<td>You solve the problem. Then you enter the answer in a special grid and color in the corresponding circles.</td>
<td>798–801</td>
</tr>
<tr>
<td>short response</td>
<td>You solve the problem, showing your work and/or explaining your reasoning.</td>
<td>802–805</td>
</tr>
<tr>
<td>extended response</td>
<td>You solve a multi-part problem, showing your work and/or explaining your reasoning.</td>
<td>806–810</td>
</tr>
</tbody>
</table>

PRACTICE  After being introduced to each type of question, you can practice that type of question. Each set of practice questions is divided into five sections that represent the categories most commonly assessed on standardized tests.

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

USING A CALCULATOR  On some tests, you are permitted to use a calculator. You should check with your teacher to determine if calculator use is permitted on the test you will be taking, and, if so, what type of calculator can be used.

TEST-TAKING TIPS  In addition to the Test-Taking Tips like the one shown at the right, here are some additional thoughts that might help you.

- Get a good night’s rest before the test. Cramming the night before does not improve your results.
- Budget your time when taking a test. Don’t dwell on problems that you cannot solve. Just make sure to leave that question blank on your answer sheet.
- Watch for key words like NOT and EXCEPT. Also look for order words like LEAST, GREATEST, FIRST, and LAST.

Test-Taking Tip

If you are allowed to use a calculator, make sure you are familiar with how it works so that you won’t waste time trying to figure out the calculator when taking the test.
Multiple-Choice Questions

Multiple-choice questions are the most common type of question on standardized tests. These questions are sometimes called selected-response questions. You are asked to choose the best answer from four or five possible answers.

To record a multiple-choice answer, you may be asked to shade in a bubble that is a circle or an oval or just to write the letter of your choice. Always make sure that your shading is dark enough and completely covers the bubble.

Sometimes a question does not provide you with a figure that represents the problem. Drawing a diagram may help you to solve the problem. Once you draw the diagram, you may be able to eliminate some of the possibilities by using your knowledge of mathematics. Another answer choice might be that the correct answer is not given.

Example

A coordinate plane is superimposed on a map of a playground. Each side of each square represents 1 meter. The slide is located at (5, –7), and the climbing pole is located at (–1, 2). What is the distance between the slide and the pole?

\[ \text{A} \quad \sqrt{15} \text{ m} \quad \text{B} \quad 6 \text{ m} \quad \text{C} \quad 9 \text{ m} \quad \text{D} \quad 9\sqrt{13} \text{ m} \quad \text{E} \quad \text{none of these} \]

Draw a diagram of the playground on a coordinate plane. Notice that the difference in the \( x \)-coordinates is 6 meters and the difference in the \( y \)-coordinates is 9 meters.

Since the two points are two vertices of a right triangle, the distance between the two points must be greater than either of these values. So we can eliminate Choices B and C.

Use the Distance Formula or the Pythagorean Theorem to find the distance between the slide and the climbing pole. Let’s use the Pythagorean Theorem.

\[
\begin{align*}
\text{Pythagorean Theorem} \\
6^2 + 9^2 &= c^2 \\
36 + 81 &= c^2 \\
117 &= c^2 \\
3\sqrt{13} &= c
\end{align*}
\]

So, the distance between the slide and pole is \(3\sqrt{13} \) meters. Since this is not listed as choice A, B, C, or D, the answer is Choice E.

If you are short on time, you can test each answer choice to find the correct answer. Sometimes you can make an educated guess about which answer choice to try first.
1. Carmen designed a rectangular banner that was 5 feet by 8 feet for a local business. The owner of the business asked her to make a larger banner measuring 10 feet by 20 feet. What was the percent increase in size from the first banner to the second banner?  
   A) 4%  
   B) 20%  
   C) 80%  
   D) 400%  

2. A roller coaster casts a shadow 57 yards long. Next to the roller coaster is a 35-foot tree with a shadow that is 20 feet long at the same time of day. What is the height of the roller coaster to the nearest whole foot?  
   A) 98 ft  
   B) 100 ft  
   C) 299 ft  
   D) 388 ft  

3. At Speedy Car Rental, it costs $32 per day to rent a car and then $0.08 per mile. If \( y \) is the total cost of renting the car and \( x \) is the number of miles, which equation describes the relation between \( x \) and \( y \)?  
   A) \( y = 32x + 0.08 \)  
   B) \( y = 32x - 0.08 \)  
   C) \( y = 0.08x + 32 \)  
   D) \( y = 0.08x - 32 \)  

4. Eric plotted his house, school, and the library on a coordinate plane. Each side of each square represents one mile. What is the distance from his house to the library?  
   A) \( \sqrt{24} \) mi  
   B) 5 mi  
   C) \( \sqrt{26} \) mi  
   D) \( \sqrt{29} \) mi  

5. The grounds outside of the Custer County Museum contain a garden shaped like a right triangle. One leg of the triangle measures 8 feet, and the area of the garden is 18 square feet. What is the length of the other leg?  
   A) 2.25 in.  
   B) 4.5 in.  
   C) 13.5 in.  
   D) 27 in.  
   E) 54 in.  

6. The circumference of a circle is equal to the perimeter of a regular hexagon with sides that measure 22 inches. What is the length of the radius of the circle to the nearest inch? Use 3.14 for \( \pi \).  
   A) 7 in.  
   B) 14 in.  
   C) 21 in.  
   D) 24 in.  
   E) 28 in.  

7. Eduardo is planning to install carpeting in a rectangular room that measures 12 feet 6 inches by 18 feet. How many square yards of carpet does he need for the project?  
   A) 25 yd\(^2\)  
   B) 50 yd\(^2\)  
   C) 225 yd\(^2\)  
   D) 300 yd\(^2\)  

8. Marva is comparing two containers. One is a cylinder with diameter 14 centimeters and height 30 centimeters. The other is a cone with radius 15 centimeters and height 14 centimeters. What is the ratio of the volume of the cylinder to the volume of the cone?  
   A) 3 to 1  
   B) 2 to 1  
   C) 7 to 5  
   D) 7 to 10  

9. Refer to the table. Which statement is true about this set of data?  
   A) The median is less than the mean.  
   B) The mean is less than the median.  
   C) The range is 2844.  
   D) A and C are true.  
   E) B and C are true.  

<table>
<thead>
<tr>
<th>Country</th>
<th>Spending per Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>$8622</td>
</tr>
<tr>
<td>United States</td>
<td>$8098</td>
</tr>
<tr>
<td>Switzerland</td>
<td>$6827</td>
</tr>
<tr>
<td>Norway</td>
<td>$6563</td>
</tr>
<tr>
<td>Germany</td>
<td>$5841</td>
</tr>
<tr>
<td>Denmark</td>
<td>$5778</td>
</tr>
</tbody>
</table>

Source: Top 10 of Everything 2003
Gridded-Response Questions

Gridded-response questions are another type of question on standardized tests. These questions are sometimes called student-produced response or grid-in, because you must create the answer yourself, not just choose from four or five possible answers.

For gridded response, you must mark your answer on a grid printed on an answer sheet. The grid contains a row of four or five boxes at the top, two rows of ovals or circles with decimal and fraction symbols, and four or five columns of ovals, numbered 0–9. Since there is no negative symbol on the grid, answers are never negative. An example of a grid from an answer sheet is shown at the right.

How do you correctly fill in the grid?

Example 1

In the diagram, \( \triangle MPT \sim \triangle RPN \). Find \( PR \).

What do you need to find?

You need to find the value of \( x \) so that you can substitute it into the expression \( 3x + 3 \) to find \( PR \). Since the triangles are similar, write a proportion to solve for \( x \).

\[
\frac{MT}{RN} = \frac{PM}{PR} \\
\frac{4}{10} = \frac{x + 2}{3x + 3} \\
4(3x + 3) = 10(x + 2) \\
12x + 12 = 10x + 20 \\
2x = 8 \\
x = 4
\]

Now find \( PR \).

\[
PR = 3x + 3 \\
= 3(4) + 3 = 15
\]

How do you fill in the grid for the answer?

- Write your answer in the answer boxes.
- Write only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.
- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.

Many gridded-response questions result in an answer that is a fraction or a decimal. These values can also be filled in on the grid.
How do you grid decimals and fractions?

**Example 2** A triangle has a base of length 1 inch and a height of 1 inch. What is the area of the triangle in square inches?

Use the formula \( A = \frac{1}{2}bh \) to find the area of the triangle.

\[
A = \frac{1}{2}bh \\
= \frac{1}{2}(1)(1) \quad \text{Substitution} \\
= \frac{1}{2} \quad \text{or 0.5} \quad \text{Simplify.}
\]

**How do you grid the answer?**

You can either grid the fraction or the decimal. Be sure to write the decimal point or fraction bar in the answer box. The following are acceptable answer responses.

![Answer Grid Examples]

Sometimes an answer is an improper fraction. Never change the improper fraction to a mixed number. Instead, grid either the improper fraction or the equivalent decimal.

**How do you grid mixed numbers?**

**Example 3** The shaded region of the rectangular garden will contain roses. What is the ratio of the area of the garden to the area of the shaded region?

First, find the area of the garden.

\[
A = tw \\
= 25(20) \text{ or } 500
\]

Then find the area of the shaded region.

\[
A = tw \\
= 15(10) \text{ or } 150
\]

Write the ratio of the areas as a fraction.

\[
\frac{\text{area of garden}}{\text{area of shaded region}} = \frac{500}{150} \text{ or } \frac{10}{3}
\]

Leave the answer as the improper fraction \( \frac{10}{3} \), as there is no way to correctly grid \( 3\frac{1}{3} \).
1. A large rectangular meeting room is being planned for a community center. Before building the center, the planning board decides to increase the area of the original room by 40%. When the room is finally built, budget cuts force the second plan to be reduced in area by 25%. What is the ratio of the area of the room that is built to the area of the original room? \[ \frac{1.05}{1} \]

2. Greenville has a spherical tank for the city’s water supply. Due to increasing population, they plan to build another spherical water tank with a radius twice that of the current tank. How many times as great will the volume of the new tank be as the volume of the current tank? \[ 8 \]

3. In Earth’s history, the Precambrian period was about 4600 million years ago. If this number of years is written in scientific notation, what is the exponent for the power of 10? \[ 9 \]

4. A virus is a type of microorganism so small it must be viewed with an electron microscope. The largest shape of virus has a length of about 0.0003 millimeter. To the nearest whole number, how many viruses would fit end to end on the head of a pin measuring 1 millimeter? \[ 3333 \]

5. Kaia has a painting that measures 10 inches by 14 inches. She wants to make her own frame that has an equal width on all sides. She wants the total area of the painting and frame to be 285 square inches. What will be the width of the frame in inches? \[ 2.5 \text{ or } \frac{5}{2} \]

6. The diagram shows a triangle graphed on a coordinate plane. If \( AB \) is extended, what is the value of the \( y \)-intercept? \[ 13 \]

7. Tyree networks computers in homes and offices. In many cases, he needs to connect each computer to every other computer with a wire. The table shows the number of wires he needs to connect various numbers of computers. Use the table to determine how many wires are needed to connect 20 computers. \[ 190 \]

8. A line perpendicular to \( 9x - 10y = -10 \) passes through \((-1, 4)\). Find the \( x \)-intercept of the line. \[ \frac{13}{5} \text{ or } 2.6 \]

9. Find the positive solution of \( 6x^2 - 7x = 5 \). \[ \frac{5}{3} \]

10. The diagram shows \( \triangle RST \) on the coordinate plane. The triangle is first rotated \( 90^\circ \) counterclockwise about the origin and then reflected in the \( y \)-axis. What is the \( x \)-coordinate of the image of \( T \) after the two transformations? \[ 4 \]
11. An octahedron is a solid with eight faces that are all equilateral triangles. How many edges does the octahedron have? 12

12. Find the measure of $\angle A$ to the nearest tenth of a degree. 21.8

13. The Pep Club plans to decorate some large garbage barrels for Spirit Week. They will cover only the sides of the barrels with decorated paper. How many square feet of paper will they need to cover 8 barrels like the one in the diagram? Use 3.14 for $\pi$. Round to the nearest square foot. 176

14. Kara makes decorative paperweights. One of her favorites is a hemisphere with a diameter of 4.5 centimeters. What is the surface area of the hemisphere including the bottom on which it rests? Use 3.14 for $\pi$. Round to the nearest tenth of a square centimeter. 47.7

15. The record for the fastest land speed of a car traveling for one mile is approximately 763 miles per hour. The car was powered by two jet engines. What was the speed of the car in feet per second? Round to the nearest whole number. 1119

16. On average, a B-777 aircraft uses 5335 gallons of fuel on a 2.5-hour flight. At this rate, how much fuel will be needed for a 45-minute flight? Round to the nearest gallon. 1601

17. The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data? 6.8

<table>
<thead>
<tr>
<th>Name</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Kansas City Place</td>
<td>193</td>
</tr>
<tr>
<td>Town Pavilion</td>
<td>180</td>
</tr>
<tr>
<td>Hyatt Regency</td>
<td>154</td>
</tr>
<tr>
<td>Power and Light Building</td>
<td>147</td>
</tr>
<tr>
<td>City Hall</td>
<td>135</td>
</tr>
<tr>
<td>1201 Walnut</td>
<td>130</td>
</tr>
</tbody>
</table>

Source: skyscrapers.com

18. A long-distance telephone service charges 40 cents per call and 5 cents per minute. If a function model is written for the graph, what is the rate of change of the function? 5

19. In a dart game, the dart must land within the innermost circle on the dartboard to win a prize. If a dart hits the board, what is the probability, as a percent, that it will hit the innermost circle? 6.25
Short-Response Questions

Short-response questions require you to provide a solution to the problem, as well as any method, explanation, and/or justification you used to arrive at the solution. These are sometimes called constructed-response, open-response, open-ended, free-response, or student-produced questions. The following is a sample rubric, or scoring guide, for scoring short-response questions.

<table>
<thead>
<tr>
<th>Credit</th>
<th>Score</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>2</td>
<td>Full credit: The answer is correct and a full explanation is provided that shows each step in arriving at the final answer.</td>
</tr>
</tbody>
</table>
| Partial| 1     | Partial credit: There are two different ways to receive partial credit.  
• The answer is correct, but the explanation provided is incomplete or incorrect.  
• The answer is incorrect, but the explanation and method of solving the problem is correct. |
| None   | 0     | No credit: Either an answer is not provided or the answer does not make sense. |

Example

Mr. Solberg wants to buy all the lawn fertilizer he will need for this season. His front yard is a rectangle measuring 55 feet by 32 feet. His back yard is a rectangle measuring 75 feet by 54 feet. Two sizes of fertilizer are available—one that covers 5000 square feet and another covering 15,000 square feet. He needs to apply the fertilizer four times during the season. How many bags of each size should he buy to have the least amount of waste?

Full Credit Solution

Find the area of each part of the lawn and multiply by 4 since the fertilizer is to be applied 4 times. Each portion of the lawn is a rectangle, so \( A = lw \).

\[
4[(55 \times 32) + (75 \times 54)] = 23,240 \text{ ft}^2
\]

If Mr. Solberg buys 2 bags that cover 15,000 ft\(^2\), he will have too much fertilizer. If he buys 1 large bag, he will still need to cover 23,240 - 15,000 or 8240 ft\(^2\).

Find how many small bags it takes to cover 8240 ft\(^2\).

\[
8240 \div 5000 = 1.648
\]

Since he cannot buy a fraction of a bag, he will need to buy 2 of the bags that cover 5000 ft\(^2\) each.

Mr. Solberg needs to buy 1 bag that covers 15,000 square feet and 2 bags that cover 5000 square feet each.

Strategy

Estimation

Use estimation to check your solution.
Partial Credit Solution

In this sample solution, the answer is correct. However, there is no justification for any of the calculations.

First find the total number of square feet of lawn.
Find the area of each part of the yard.

\[
(55 \times 32) + (75 \times 54) = 5810 \text{ ft}^2
\]

The area of the lawn is greater than 5000 ft\(^2\), which is the amount covered by the smaller bag, but buying the bag that covers 15,000 ft\(^2\) would result in too much waste.

\[
5810 \div 5000 = 1.162
\]

Therefore, Mr. Solberg will need to buy 2 of the smaller bags of fertilizer.

Partial Credit Solution

In this sample solution, the answer is incorrect. However, after the first statement all of the calculations and reasoning are correct.

First find the total number of square feet of lawn.
Find the area of each part of the yard.

\[
(55 \times 32) + (75 \times 54) = 5810 \text{ ft}^2
\]

The area of the lawn is greater than 5000 ft\(^2\), which is the amount covered by the smaller bag, but buying the bag that covers 15,000 ft\(^2\) would result in too much waste.

\[
5810 \div 5000 = 1.162
\]

Therefore, Mr. Solberg will need to buy 2 of the smaller bags of fertilizer.

No Credit Solution

In this sample solution, the response is incorrect and incomplete.

\[
55 + 75 = 130
\]
\[
32 + 54 = 86
\]
\[
130 \times 86 \times 4 = 44,720
\]
\[
44,720 \div 15,000 = 2.98
\]

Mr. Solberg will need 3 bags of fertilizer.
5. Solve and graph $2x - 9 \leq 5x + 4$. $x \geq -\frac{13}{3}$, See margin for graph.

6. Vance rents rafts for trips on the Jefferson River. You have to reserve the raft and provide a $15 deposit in advance. Then the charge is $7.50 per hour. Write an equation that can be used to find the charge for any amount of time, where $y$ is the total charge in dollars and $x$ is the amount of time in hours. $y = 15 + 7.50x$

7. Hector is working on the design for the container shown below that consists of a cylinder with a hemisphere on top. He has written the expression $\pi r^2 + 2\pi rh + 2\pi r^2$ to represent the surface area of any size container of this shape. Explain the meaning of each term of the expression. See margin.

8. Find all solutions of the equation $6x^2 + 13x = 5$. $-\frac{5}{3}, \frac{1}{2}$

9. In 1999, there were 2,192,070 farms in the U.S., while in 2001, there were 2,157,780 farms. Let $x$ represent years since 1999 and $y$ represent the total number of farms in the U.S. Suppose the number of farms continues to decrease at the same rate as from 1999 to 2001. Write an equation that models the number of farms for any year after 1999. $y = 2,192,070 - 17,145x$

10. Refer to the diagram. What is the measure of $\angle 1$? 65

11. Quadrilateral $JKLM$ is to be reflected in the line $y = x$. What are the coordinates of the vertices of the image? $J'(2, 2)$, $K'(0, 4)$, $L'(-3, -1)$, $M'(1, -2)$
12. Write an equation in standard form for a circle that has a diameter with endpoints at \((-3, 2)\) and \((4, -5)\).

\[
(x - \frac{1}{2})^2 + (y + \frac{3}{2})^2 = 98
\]

13. In the Columbia Village subdivision, an unusually shaped lot, shown below, will be used for a small park. Find the exact perimeter of the lot. 

\[
(150 + 60\sqrt{3})\text{ ft}
\]

14. The Astronomical Unit (AU) is the distance from Earth to the Sun. It is usually rounded to 93,000,000 miles. The star Alpha Centauri is 25,556,250 million miles from Earth. What is this distance in AU? About 274.798 AU

15. Linesse handpaints unique designs on shirts and sells them. It takes her about 4.5 hours to create a design. At this rate, how many shirts can she design if she works 22 days per month for an average of 6.5 hours per day? Between 31 and 32 shirts

16. The world’s largest pancake was made in England in 1994. To the nearest cubic foot, what was the volume of the pancake? 159 ft³

17. Find the ratio of the volume of the cylinder to the volume of the pyramid. 3π to 2

18. The table shows the winning times for the Olympic men’s 1000-meter speed skating event. Make a scatter plot of the data and describe the pattern in the data. Times are rounded to the nearest second. See margin.

<table>
<thead>
<tr>
<th>Year</th>
<th>Country</th>
<th>Time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>U.S.</td>
<td>79</td>
</tr>
<tr>
<td>1980</td>
<td>U.S.</td>
<td>75</td>
</tr>
<tr>
<td>1984</td>
<td>Canada</td>
<td>76</td>
</tr>
<tr>
<td>1988</td>
<td>USSR</td>
<td>73</td>
</tr>
<tr>
<td>1992</td>
<td>Germany</td>
<td>75</td>
</tr>
<tr>
<td>1994</td>
<td>U.S.</td>
<td>72</td>
</tr>
<tr>
<td>1998</td>
<td>Netherlands</td>
<td>71</td>
</tr>
<tr>
<td>2002</td>
<td>Netherlands</td>
<td>67</td>
</tr>
</tbody>
</table>

19. Bradley surveyed 70 people about their favorite spectator sport. If a person is chosen at random from the people surveyed, what is the probability that the person’s favorite spectator sport is basketball? 20% or 0.2

20. The graph shows the altitude of a small airplane. Write a function to model the graph. Explain what the model means in terms of the altitude of the airplane. See margin.

\[y = 9000 - 1000x;\] 9000 is the greatest altitude reached by the plane during this flight. The rate of change is –1000, which means the altitude is decreasing steadily by 1000 feet per minute.
Extended-Response Questions

Extended-response questions are often called open-ended or constructed-response questions. Most extended-response questions have multiple parts. You must answer all parts to receive full credit.

Extended-response questions are similar to short-response questions in that you must show all of your work in solving the problem. A rubric is also used to determine whether you receive full, partial, or no credit. The following is a sample rubric for scoring extended-response questions.

<table>
<thead>
<tr>
<th>Credit</th>
<th>Score</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>4</td>
<td>Full credit: A correct solution is given that is supported by well-developed, accurate explanations.</td>
</tr>
<tr>
<td>Partial</td>
<td>3, 2, 1</td>
<td>Partial credit: A generally correct solution is given that may contain minor flaws in reasoning or computation or an incomplete solution. The more correct the solution, the greater the score.</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
<td>No credit: An incorrect solution is given indicating no mathematical understanding of the concept, or no solution is given.</td>
</tr>
</tbody>
</table>

Make sure that when the problem says to Show your work, you show every part of your solution including figures, sketches of graphing calculator screens, or the reasoning behind your computations.

Example

Polygon WXYZ with vertices W(−3, 2), X(4, 4), Y(3, −1), and Z(−2, −3) is a figure represented on a coordinate plane to be used in the graphics for a video game. Various transformations will be performed on the polygon to use for the game.

a. Graph WXYZ and its image W′X′Y′Z′ under a reflection in the y-axis. Be sure to label all of the vertices.

b. Describe how the coordinates of the vertices of WXYZ relate to the coordinates of the vertices of W′X′Y′Z′.

c. Another transformation is performed on WXYZ. This time, the vertices of the image W′X′Y′Z′ are W′(2, −3), X′(4, 4), Y′(−1, 3), and Z′(−3, −2). Graph WXYZ and its image under this transformation. What transformation produced W′X′Y′Z′?

Full Credit Solution

Part a A complete graph includes labels for the axes and origin and labels for the vertices, including letter names and coordinates.

- The vertices of the polygon should be correctly graphed and labeled.
- The vertices of the image should be located such that the transformation shows a reflection in the y-axis.
- The vertices of the polygons should be connected correctly. Optionally, the polygon and its image could be graphed in two contrasting colors.
Preparing for Standardized Tests

The wrong operations are used, so the answer is incorrect. Also, there are no units of measure given with any of the calculations.

Part b
The coordinates of W and W' are (−3, 2) and (3, 2). The x-coordinates are the opposite of each other and the y-coordinates are the same. For any point (a, b), the coordinates of the reflection in the y-axis are (−a, b).

Part c
The coordinates of Z and Z' have been switched. In other words, for any point (a, b), the coordinates of the reflection in the y-axis are (b, a). Since X and X' are the same point, the polygon has been reflected in the line y = x.

Partial Credit Solution
Part a This sample graph includes no labels for the axes and for the vertices of the polygon and its image. Two of the image points have been incorrectly graphed.
Preparing for Standardized Tests

Part b  Partial credit is given because the reasoning is correct, but the reasoning was based on the incorrect graph in Part a.

For two of the points, W and Z, the y-coordinates are the same and the x-coordinates are opposites. But, for points X and Y, there is no clear relationship.

Part c  Full credit is given for Part c. The graph supplied by the student was identical to the graph shown for the full credit solution for Part c. The explanation below is correct, but slightly different from the previous answer for Part c.

I noticed that point X and point X’ were the same. I also guessed that this was a reflection, but not in either axis. I played around with my ruler until I found a line that was the line of reflection. The transformation from WXYZ to W’X’Y’Z’ was a reflection in the line y = x.

This sample answer might have received a score of 2 or 1, depending on the judgment of the scorer. Had the student graphed all points correctly and gotten Part b correct, the score would probably have been a 3.

No Credit Solution

Part a  The sample answer below includes no labels on the axes or the coordinates of the vertices of the polygon. The polygon WXYZ has three vertices graphed incorrectly. The polygon that was graphed is not reflected correctly either.

Part b  I don’t see any way that the coordinates relate.

Part c  It is a reduction because it gets smaller.

In this sample answer, the student does not understand how to graph points on a coordinate plane and also does not understand the reflection of figures in an axis or other line.
Extended-Response Practice

Solve each problem. Show all your work.

**Number and Operations**

1. Refer to the table.

<table>
<thead>
<tr>
<th>City</th>
<th>1990</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phoenix, AZ</td>
<td>983,403</td>
<td>1,321,045</td>
</tr>
<tr>
<td>Austin, TX</td>
<td>465,622</td>
<td>656,562</td>
</tr>
<tr>
<td>Charlotte, NC</td>
<td>395,934</td>
<td>540,828</td>
</tr>
<tr>
<td>Mesa, AZ</td>
<td>288,091</td>
<td>396,375</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td>258,295</td>
<td>478,434</td>
</tr>
</tbody>
</table>

Source: [census.gov](http://www.census.gov)

a. For which city was the increase in population the greatest? What was the increase?

b. For which city was the percent of increase in population the greatest? What was the percent increase?

c. Suppose that the population increase of a city was 30%. If the population in 2000 was 346,668, find the population in 1990.

2. Molecules are the smallest units of a particular substance that still have the same properties as that substance. The diameter of a molecule is measured in angstroms (Å). Express each value that substance. The diameter of a molecule is 2 Å. How many angstroms are in one inch?

3. A molecule has a diameter of 2 angstroms. How many of these molecules placed side by side would fit on an eraser measuring \( \frac{1}{4} \) inch? 2a–c. See margin.

**Geometry**

5. The Silver City Marching Band is planning to create this formation with the members.

![Formation Diagram]

a. Find the missing side measures of \( \triangle EDF \). Explain.

b. Find the missing side measures of \( \triangle ABC \). Explain.

c. Find the total distance of the path: \( A \) to \( B \) to \( C \) to \( A \) to \( D \) to \( E \) to \( F \) to \( D \).

4. The depth of a reservoir was measured on the first day of each month. (Jan. = 1, Feb. = 2, and so on.)

<table>
<thead>
<tr>
<th>Month</th>
<th>Depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
</tr>
<tr>
<td>2</td>
<td>330</td>
</tr>
<tr>
<td>3</td>
<td>320</td>
</tr>
<tr>
<td>4</td>
<td>310</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>290</td>
</tr>
<tr>
<td>7</td>
<td>280</td>
</tr>
<tr>
<td>8</td>
<td>270</td>
</tr>
<tr>
<td>9</td>
<td>260</td>
</tr>
<tr>
<td>10</td>
<td>250</td>
</tr>
<tr>
<td>11</td>
<td>240</td>
</tr>
<tr>
<td>12</td>
<td>230</td>
</tr>
</tbody>
</table>

a. What is the slope of the line joining the points with \( x \)-coordinates 6 and 7? What does the slope represent?

b. Write an equation for the segment of the graph from 5 to 6. What is the slope of the line and what does this represent in terms of the graph?

c. What was the lowest depth of the reservoir? When was this depth first measured and recorded?

2a–c. See margin.

3. A. Las Vegas at 220,139  
B. Las Vegas at about 85.2%  
C. About 266,667

2. A. \( 10^{-8} \times 0.3937 \) is the number of inches. This can be rewritten as \( 3.937 \times 10^{-9} \) inches.  
B. \( 3.937 \times 10^{-9} \) inches \( = 1 \) Å, so 1 inch \( = 1 \div (3.937 \times 10^{-9}) \) \( \approx 2.54 \times 10^{8} \) Å.  
C. If 1 inch contains \( 2.54 \times 10^{8} \) Å, then one-quarter inch contains \( (2.54 \times 10^{8}) \div 4 \) or \( 6.35 \times 10^{7} \) Å. If each molecule measures 2 Å, then there are \((6.35 \times 10^{7}) + 2 \) or \( 3.175 \times 10^{7} \) of these molecules across the eraser.

3. A.  
B. \( 1800 = (24 + 2x)(29 + 2x) \)  
C. 8 feet

4. A. The slope is \(-20\). This means that the depth of the reservoir dropped by 20 feet in one month from the first day of June to the first day of July.  
B. \( y = 350 \); the slope is 0. The water depth did not change from the first day of May to the first day of June.  
C. 320 feet; it was measured on the first day of September.

5. A. \( ED = DF = 8 \sqrt{2} \approx 11.3 \) feet, since \( \triangle EDF \) is a \( 45^\circ - 45^\circ - 90^\circ \) triangle.  
B. \( AC = 8 \sqrt{2} \) since it is congruent to \( ED \). Then, since \( \triangle ABC \) is a \( 30^\circ - 60^\circ - 90^\circ \) triangle, \( AB = 16 \sqrt{2} \approx 22.6 \) feet, and \( BC = 8 \sqrt{6} \approx 19.6 \) feet.  
C. 22.6 + 19.6 + 11.3 + 11.3 + 11.3 + 16.1 + 11.3 \approx 103.4 \) feet

D. Sample answer: 6 at the points, 15 on \( EF \), 10 on each of \( ED \), \( DF \), \( DA \) and \( AC \), 19 on \( BC \), and 22 on \( AB \). The total will be 102 people. (Depending upon how students decide to round the number of feet and place the students, the answer could vary slightly.)
6. A. 420 cm$^3$
   B. approximately 502.7 cm$^3$
   C. Using the approximation of Part B, about 20% increase.

7. A. Sample answers with time given in days.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>230 days</td>
</tr>
<tr>
<td>Venus</td>
<td>270 days</td>
</tr>
<tr>
<td>Mars</td>
<td>415 days</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1003.3 days</td>
</tr>
</tbody>
</table>

B. Sample answer: Write the distance in scientific notation; for example, 138 million miles is $1.38 \times 10^8$. Then write 25,000 as $2.5 \times 10^4$. $1.38 \div 2.5 = 0.552$ and $10^8 \div 10^4 = 10^4$. $0.552 \times 10^4 = 5520$. This is the number of hours of the trip.

C. Neptune; sample explanation: 13.3 years is 116,508 hours. Multiply 116,508 by 25,000 to get $2.9127 \times 10^9$ miles, which is approximately the distance to Neptune.

8. A. Temperatures for Barrow

<table>
<thead>
<tr>
<th>Month</th>
<th>°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>20</td>
</tr>
<tr>
<td>Feb.</td>
<td>15</td>
</tr>
<tr>
<td>Mar.</td>
<td>10</td>
</tr>
<tr>
<td>Apr.</td>
<td>5</td>
</tr>
<tr>
<td>May</td>
<td>0</td>
</tr>
<tr>
<td>June</td>
<td>5</td>
</tr>
<tr>
<td>July</td>
<td>10</td>
</tr>
<tr>
<td>Aug.</td>
<td>15</td>
</tr>
<tr>
<td>Sep.</td>
<td>20</td>
</tr>
<tr>
<td>Oct.</td>
<td>25</td>
</tr>
<tr>
<td>Nov.</td>
<td>30</td>
</tr>
<tr>
<td>Dec.</td>
<td>35</td>
</tr>
</tbody>
</table>

B. Sample answer: The points suggest a curve that increases from February to June and July and then decreases back to December.

C. 10.25

D. Sample answer: If the line $y = 10.25$ is drawn on the same coordinate plane as the scatter plot, half of the graph lies below the line and half lies above the line.

9. A. $\frac{1}{49}$
   B. $\frac{8}{49} \div \frac{24}{49} = \frac{192}{2401}$
   C. Sample answer: The least probability for two darts is for each of them to land in the pink circle. The most expensive prize should be for $P$(pink) followed by $P$(pink).
Chapter 2 Reasoning and Proof

Postulate 2.1 Through any two points, there is exactly one line. (p. 89)
Postulate 2.2 Through any three points not on the same line, there is exactly one plane. (p. 89)
Postulate 2.3 A line contains at least two points. (p. 90)
Postulate 2.4 A plane contains at least three points not on the same line. (p. 90)
Postulate 2.5 If two points lie in a plane, then the entire line containing those points lies in that plane. (p. 90)
Postulate 2.6 If two lines intersect, then their intersection is exactly one point. (p. 90)
Postulate 2.7 If two planes intersect, then their intersection is a line. (p. 90)

Theorem 2.1 Midpoint Theorem If \( M \) is the midpoint of \( \overline{AB} \), then \( \overline{AM} \cong \overline{MB} \). (p. 91)

Postulate 2.8 Ruler Postulate The points on any line or line segment can be paired with real numbers so that, given any two points \( A \) and \( B \) on a line, \( A \) corresponds to zero, and \( B \) corresponds to a positive real number. (p. 101)

Postulate 2.9 Segment Addition Postulate If \( B \) is between \( A \) and \( C \), then \( AB + BC = AC \).
If \( AB + BC = AC \), then \( B \) is between \( A \) and \( C \). (p. 102)

Theorem 2.2 Congruence of segments is reflexive, symmetric, and transitive. (p. 102)

Postulate 2.10 Protractor Postulate Given \( \overline{AB} \) and a number \( r \) between 0 and 180, there is exactly one ray with endpoint \( A \), extending on either side of \( \overline{AB} \), such that the measure of the angle formed is \( r \). (p. 107)

Postulate 2.11 Angle Addition Postulate If \( R \) is in the interior of \( \angle PQS \), then \( m\angle PQR + m\angle RQS = m\angle PQS \).
If \( m\angle PQR + m\angle RQS = m\angle PQS \), then \( R \) is in the interior of \( \angle PQS \). (p. 107)

Theorem 2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles. (p. 108)

Theorem 2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. (p. 108)

Theorem 2.5 Congruence of angles is reflexive, symmetric, and transitive. (p. 108)

Theorem 2.6 Angles supplementary to the same angle or to congruent angles are congruent. (p. 109) Abbreviation: \( \angle \) suppl. to same \( \angle \) or \( \angle \) are \( \cong \).

Theorem 2.7 Angles complementary to the same angle or to congruent angles are congruent. (p. 109) Abbreviation: \( \angle \) compl. to same \( \angle \) or \( \angle \) are \( \cong \).

Theorem 2.8 Vertical Angle Theorem If two angles are vertical angles, then they are congruent. (p. 110)

Theorem 2.9 Perpendicular lines intersect to form four right angles. (p. 110)

Theorem 2.10 All right angles are congruent. (p. 110)
Chapter 3 Perpendicular and Parallel Lines

Postulate 3.1 Corresponding Angles Postulate If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent. (p. 133)

Theorem 3.1 Alternate Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent. (p. 134)

Theorem 3.2 Consecutive Interior Angles Theorem If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary. (p. 134)

Theorem 3.3 Alternate Exterior Angles Theorem If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent. (p. 134)

Theorem 3.4 Perpendicular Transversal Theorem In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (p. 134)

Postulate 3.2 Two nonvertical lines have the same slope if and only if they are parallel. (p. 141)

Postulate 3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$. (p. 141)

Postulate 3.4 If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel. (p. 151) Abbreviation: If corr. $\angle$ are $\cong$, lines are $\parallel$.

Postulate 3.5 Parallel Postulate If there is a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line. (p. 152)

Theorem 3.5 If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. (p. 152) Abbreviation: If alt. ext. $\angle$ are $\cong$, then lines are $\parallel$.

Theorem 3.6 If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. (p. 152) Abbreviation: If cons. int. $\angle$ are suppl., then lines are $\parallel$.

Theorem 3.7 If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. (p. 152) Abbreviation: If alt. int. $\angle$ are $\cong$, then lines are $\parallel$.

Theorem 3.8 In a plane, if two lines are perpendicular to the same line, then they are parallel. (p. 152) Abbreviation: If 2 lines are $\perp$ to the same line, then lines are $\parallel$.

Theorem 3.9 In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other. (p. 161)

Chapter 4 Congruent Triangles

Theorem 4.1 Angle Sum Theorem The sum of the measures of the angles of a triangle is 180. (p. 185)

Theorem 4.2 Third Angle Theorem If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent. (p. 186)
Theorem 4.3  **Exterior Angle Theorem**  The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.  (p. 186)

Corollary 4.1  The acute angles of a right triangle are complementary.  (p. 188)

Corollary 4.2  There can be at most one right or obtuse angle in a triangle.  (p. 188)

Theorem 4.4  Congruence of triangles is reflexive, symmetric, and transitive.  (p. 193)

Postulate 4.1  **Side-Side-Side Congruence (SSS)**  If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.  (p. 201)

Postulate 4.2  **Side-Angle-Side Congruence (SAS)**  If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.  (p. 202)

Postulate 4.3  **Angle-Side-Angle Congruence (ASA)**  If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent.  (p. 207)

Theorem 4.5  **Angle-Angle-Side Congruence (AAS)**  If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.  (p. 208)

Theorem 4.6  **Leg-Leg Congruence (LL)**  If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.  (p. 214)

Theorem 4.7  **Hypotenuse-Angle Congruence (HA)**  If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.  (p. 215)

Theorem 4.8  **Leg-Angle Congruence (LA)**  If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.  (p. 215)

Postulate 4.4  **Hypotenuse-Leg Congruence (HL)**  If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.  (p. 215)

Theorem 4.9  **Isosceles Triangle Theorem**  If two sides of a triangle are congruent, then the angles opposite those sides are congruent.  (p. 216)

Theorem 4.10  If two angles of a triangle are congruent, then the sides opposite those angles are congruent.  (p. 218)  **Abbreviation: Conv. of Isos. △Th.**

Corollary 4.3  A triangle is equilateral if and only if it is equiangular.  (p. 218)

Corollary 4.4  Each angle of an equilateral triangle measures 60°.  (p. 218)

**Chapter 5  Relationships in Triangles**

Theorem 5.1  Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.  (p. 238)

Theorem 5.2  Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.  (p. 238)
Theorem 5.3 **Circumcenter Theorem** The circumcenter of a triangle is equidistant from the vertices of the triangle. (p. 239)

Theorem 5.4 Any point on the angle bisector is equidistant from the sides of the angle. (p. 239)

Theorem 5.5 Any point equidistant from the sides of an angle lies on the angle bisector. (p. 239)

Theorem 5.6 **Incenter Theorem** The incenter of a triangle is equidistant from each side of the triangle. (p. 240)

Theorem 5.7 **Centroid Theorem** The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median. (p. 240)

Theorem 5.8 **Exterior Angle Inequality Theorem** If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles. (p. 248)

Theorem 5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side. (p. 249)

Theorem 5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle. (p. 250)

Theorem 5.11 **Triangle Inequality Theorem** The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 261)

Theorem 5.12 The perpendicular segment from a point to a line is the shortest segment from the point to the line. (p. 262)

Corollary 5.1 The perpendicular segment from a point to a plane is the shortest segment from the point to the plane. (p. 263)

Theorem 5.13 **SAS Inequality/Hinge Theorem** If two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle. (p. 267)

Theorem 5.14 **SSS Inequality** If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle. (p. 268)

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**Chapter 6 Proportions and Similarity**

Postulate 6.1 **Angle-Angle (AA) Similarity** If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (p. 298)

Theorem 6.1 **Side-Side-Side (SSS) Similarity** If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar. (p. 299)

Theorem 6.2 **Side-Angle-Side (SAS) Similarity** If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (p. 299)

Theorem 6.3 Similarity of triangles is reflexive, symmetric, and transitive. (p. 300)
Theorem 6.4  **Triangle Proportionality Theorem**  If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths. (p. 307)

Theorem 6.5  **Converse of the Triangle Proportionality Theorem**  If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side. (p. 308)

Theorem 6.6  **Triangle Midsegment Theorem**  A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side. (p. 308)

Corollary 6.1  If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. (p. 309)

Corollary 6.2  If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 309)

Theorem 6.7  **Proportional Perimeters Theorem**  If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides. (p. 316)

Theorem 6.8  If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. (p. 317)

**Abbreviation:** \(\sim \triangle s \text{ have corr. altitudes proportional to the corr. sides.}\)

Theorem 6.9  If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides. (p. 317)

**Abbreviation:** \(\sim \triangle s \text{ have corr. \(\angle\) bisectors proportional to the corr. sides.}\)

Theorem 6.10  If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides. (p. 317)

**Abbreviation:** \(\sim \triangle s \text{ have corr. medians proportional to the corr. sides.}\)

Theorem 6.11  **Angle Bisector Theorem**  An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides. (p. 319)

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**Chapter 7  Right Triangles and Trigonometry**

Theorem 7.1  If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other. (p. 343)

Theorem 7.2  The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. (p. 343)

Theorem 7.3  If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg. (p. 344)

Theorem 7.4  **Pythagorean Theorem**  In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse. (p. 350)

Theorem 7.5  **Converse of the Pythagorean Theorem**  If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle. (p. 351)

Theorem 7.6  In a 45°-45°-90° triangle, the length of the hypotenuse is \(\sqrt{2}\) times the length of a leg. (p. 357)
Theorem 7.7  In a $30^\circ$-$60^\circ$-$90^\circ$ triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. (p. 359)

Chapter 8 Quadrilaterals

Theorem 8.1  **Interior Angle Sum Theorem** If a convex polygon has $n$ sides and $S$ is the sum of the measures of its interior angles, then $S = 180(n - 2)$. (p. 404)

Theorem 8.2  **Exterior Angle Sum Theorem** If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360. (p. 406)

Theorem 8.3  Opposite sides of a parallelogram are congruent. (p. 412)

Abbreviation: Opp. sides of $\square$ are $\cong$.

Theorem 8.4  Opposite angles of a parallelogram are congruent. (p. 412)

Abbreviation: Opp. $\triangle$ of $\square$ are $\cong$.

Theorem 8.5  Consecutive angles in a parallelogram are supplementary. (p. 412)

Abbreviation: Cons. $\triangle$ in $\square$ are suppl.

Theorem 8.6  If a parallelogram has one right angle, it has four right angles. (p. 412)

Abbreviation: If $\square$ has 1 rt. $\triangle$, it has 4 rt. $\triangle$.

Theorem 8.7  The diagonals of a parallelogram bisect each other. (p. 413)

Abbreviation: Diag. of $\square$ bisect each other.

Theorem 8.8  The diagonal of a parallelogram separates the parallelogram into two congruent triangles. (p. 414)  

Abbreviation: Diag. of $\square$ separates $\square$ into $2 \cong \triangle$s.

Theorem 8.9  If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 418)  

Abbreviation: If both pairs of opp. sides are $\cong$, then quad. is $\square$.

Theorem 8.10  If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 418)  

Abbreviation: If both pairs of opp. $\triangle$ are $\cong$, then quad. is $\square$.

Theorem 8.11  If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 418)  

Abbreviation: If diag. bisect each other, then quad. is $\square$.

Theorem 8.12  If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. (p. 418)  

Abbreviation: If one pair of opp. sides is $\parallel$ and $\cong$, then the quad. is a $\square$.

Theorem 8.13  If a parallelogram is a rectangle, then the diagonals are congruent. (p. 424)  

Abbreviation: If $\square$ is rectangle, diag. are $\cong$.

Theorem 8.14  If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 426)  

Abbreviation: If diagonals of $\square$ are $\cong$, $\square$ is a rectangle.

Theorem 8.15  The diagonals of a rhombus are perpendicular. (p. 431)

Theorem 8.16  If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 431)

Theorem 8.17  Each diagonal of a rhombus bisects a pair of opposite angles. (p. 431)

Theorem 8.18  Both pairs of base angles of an isosceles trapezoid are congruent. (p. 439)
Theorem 8.19  The diagonals of an isosceles trapezoid are congruent.  (p. 439)

Theorem 8.20  The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.  (p. 441)

**Chapter 9  Transformations**

**Postulate 9.1**  In a given rotation, if \(A\) is the preimage, \(A'\) is the image, and \(P\) is the center of rotation, then the measure of the angle of rotation, \(\angle APA'\), is twice the measure of the acute or right angle formed by the intersecting lines of reflection.  (p. 477)

**Corollary 9.1**  Reflecting an image successively in two perpendicular lines results in a 180˚ rotation.  (p. 477)

**Theorem 9.1**  If a dilation with center \(C\) and a scale factor of \(r\) transforms \(A\) to \(E\) and \(B\) to \(D\), then \(ED = |r|\langle AB\rangle\).  (p. 491)

**Theorem 9.2**  If \(P(x, y)\) is the preimage of a dilation centered at the origin with a scale factor \(r\), then the image is \(P'(rx, ry)\).  (p. 492)

**Chapter 10  Circles**

**Theorem 10.1**  Two arcs are congruent if and only if their corresponding central angles are congruent.  (p. 530)

**Postulate 10.1  Arc Addition Postulate**  The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.  (p. 531)

**Theorem 10.2**  In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.  (p. 536)  
Abbreviations: In \(\bigcirc\), 2 minor arcs are \(\simeq\), iff corr. chords are \(\simeq\).  
In \(\bigcirc\), 2 chords are \(\simeq\), iff corr. minor arcs are \(\simeq\).

**Theorem 10.3**  In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.  (p. 537)

**Theorem 10.4**  In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.  (p. 539)

**Theorem 10.5**  If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).  (p. 544)

**Theorem 10.6**  If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.  (p. 546)  Abbreviations: Inscribed \(\triangle\) of same arc are \(\simeq\). Inscribed \(\triangle\) of \(\simeq\) arcs are \(\simeq\).

**Theorem 10.7**  If an inscribed angle intercepts a semicircle, the angle is a right angle.  (p. 547)

**Theorem 10.8**  If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.  (p. 548)

**Theorem 10.9**  If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.  (p. 553)

Postulates, Theorems, and Corollaries  R7
Theorem 10.10 If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle. (p. 553)

Theorem 10.11 If two segments from the same exterior point are tangent to a circle, then they are congruent. (p. 554)

Theorem 10.12 If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle. (p. 561)

Theorem 10.13 If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc. (p. 562)

Theorem 10.14 If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs. (p. 563)

Theorem 10.15 If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal. (p. 569)

Theorem 10.16 If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment. (p. 570)

Theorem 10.17 If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment. (p. 571)

Chapter 11 Area of Polygons And Circles

Postulate 11.1 Congruent figures have equal areas. (p. 603)

Postulate 11.2 The area of a region is the sum of the areas of all of its nonoverlapping parts. (p. 619)

Chapter 13 Volume

Theorem 13.1 If two solids are similar with a scale factor of $a : b$, then the surface areas have a ratio of $a^2 : b^2$, and the volumes have a ratio of $a^3 : b^3$. (p. 709)
Glossary/Glosario

**English**

**acute angle** (p. 30) An angle with a degree measure less than 90.

**acute triangle** (p. 178) A triangle in which all of the angles are acute angles.

**adjacent angles** (p. 37) Two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

**alternate exterior angles** (p. 128) In the figure, transversal \( t \) intersects lines \( \ell \) and \( m \). \( \angle 5 \) and \( \angle 3 \), and \( \angle 6 \) and \( \angle 4 \) are alternate exterior angles.

**alternate interior angles** (p. 128) In the figure above, transversal \( t \) intersects lines \( \ell \) and \( m \). \( \angle 1 \) and \( \angle 7 \), and \( \angle 2 \) and \( \angle 8 \) are alternate interior angles.

**altitude** 1. (p. 241) In a triangle, a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side. 2. (pp. 649, 655) In a prism or cylinder, a segment perpendicular to the bases with an endpoint in each plane. 3. (pp. 660, 666) In a pyramid or cone, the segment that has the vertex as one endpoint and is perpendicular to the base.

**ambiguous case of the Law of Sines** (p. 384) Given the measures of two sides and a nonincluded angle, there exist two possible triangles.

**angle** (p. 29) The intersection of two noncollinear rays at a common endpoint. The rays are called sides and the common endpoint is called the vertex.

**angle bisector** (p. 32) A ray that divides an angle into two congruent angles.

**Español**

**ángulo agudo** Ángulo cuya medida en grados es menos de 90.

**triángulo acutángulo** Triángulo cuyos ángulos son todos agudos.

**ángulos adyacentes** Dos ángulos que yacen sobre el mismo plano, tienen el mismo vértice y un lado en común, pero ningún punto interior.

**ángulos alternos externos** En la figura, la transversal \( t \) interseca las rectas \( \ell \) y \( m \). \( \angle 5 \) y \( \angle 3 \), y \( \angle 6 \) y \( \angle 4 \) son ángulos alternos externos.

**ángulos alternos internos** En la figura anterior, la transversal \( t \) interseca las rectas \( \ell \) y \( m \). \( \angle 1 \) y \( \angle 7 \), y \( \angle 2 \) y \( \angle 8 \) son ángulos alternos internos.

**altura** 1. En un triángulo, segmento trazado desde el vértice de un triángulo hasta el lado opuesto y que es perpendicular a dicho lado. 2. El segmento perpendicular a las bases de prismas y cilindros que tiene un extremo en cada plano. 3. El segmento que tiene un extremo en el vértice de pirámides y conos y que es perpendicular a la base.

**caso ambiguo de la ley de los senos** Dadas las medidas de dos lados y de un ángulo no incluido, existen dos triángulos posibles.

**ángulo** La intersección de dos semirrectas no colineales en un punto común. Las semirrectas se llaman lados y el punto común se llama vértice.

**bisectriz de un ángulo** Semirrecta que divide un ángulo en dos ángulos congruentes.
angle of depression (p. 372) The angle between the line of sight and the horizontal when an observer looks downward.

angle of elevation (p. 371) The angle between the line of sight and the horizontal when an observer looks upward.

angle of rotation (p. 476) The angle through which a preimage is rotated to form the image.

apothem (p. 610) A segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.

arc (p. 530) A part of a circle that is defined by two endpoints.

axis 1. (p. 655) In a cylinder, the segment with endpoints that are the centers of the bases. 2. (p. 666) In a cone, the segment with endpoints that are the vertex and the center of the base.

between (p. 14) For any two points A and B on a line, there is another point C between A and B if and only if A, B, and C are collinear and \( AC + CB = AB \).

biconditional (p. 81) The conjunction of a conditional statement and its converse.

center of rotation (p. 476) A fixed point around which shapes move in a circular motion to a new position.

central angle (p. 529) An angle that intersects a circle in two points and has its vertex at the center of the circle.

centroid (p. 240) The point of concurrency of the medians of a triangle.

chord 1. (p. 522) For a given circle, a segment with endpoints that are on the circle. 2. (p. 671) For a given sphere, a segment with endpoints that are on the sphere.

circle (p. 522) The locus of all points in a plane equidistant from a given point called the center of the circle.

circular sector (p. 524) A region bounded by two radii and the arc of a circle.

circular segment (p. 524) A region bounded by a chord and the arc of a circle.

circle of curvature (p. 524) The circle that best approximates the curve at a given point.

circular cone (p. 665) A cone whose base is a circle.

circular cylinder (p. 655) A cylinder whose base is a circle.

ángulo de depresión Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia abajo.

ángulo de elevación Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia arriba.

ángulo de rotación El ángulo a través del cual se rota una preimagen para formar la imagen.

apotema Segmento perpendicular trazado desde el centro de un polígono regular hasta uno de sus lados.

arco Parte de un círculo definida por los dos extremos de una recta.

eje 1. El segmento en un cilindro cuyos extremos forman el centro de las bases. 2. El segmento en un cono cuyos extremos forman el vértice y el centro de la base.

ubicado entre Para cualquier par de puntos A y B de una recta, existe un punto C ubicado entre A y B si y sólo si A, B y C son colineales y \( AC + CB = AB \).

bicondicional La conjunción entre un enunciado condicional y su recíproco.

centro de rotación Punto fijo alrededor del cual gira una figura hasta alcanzar una posición determinada.

ángulo central Ángulo que interseca un círculo en dos puntos y cuyo vértice se localiza en el centro del círculo.

centroide Punto de intersección de las medianas de un triángulo.

cuerda 1. Segmento cuyos extremos están en un círculo. 2. Segmento cuyos extremos están en una esfera.
círculo Lugar geométrico formado por el conjunto de puntos en un plano, equidistantes de un punto dado llamado centro.
circumcenter (p. 238) The point of concurrency of the perpendicular bisectors of a triangle.
circumference (p. 523) The distance around a circle.
circumscribed (p. 537) A circle is circumscribed about a polygon if the circle contains all the vertices of the polygon.
collinear (p. 6) Points that lie on the same line.
column matrix (p. 506) A matrix containing one column often used to represent an ordered pair or a vector, such as \([x, y]\).
complementary angles (p. 39) Two angles with measures that have a sum of 90.
component form (p. 498) A vector expressed as an ordered pair, \(\langle x, y \rangle\).
composition of reflections (p. 471) Successive reflections in parallel lines.
compound statement (p. 67) A statement formed by joining two or more statements.
concave polygon (p. 45) A polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon.
conclusion (p. 75) In a conditional statement, the statement that immediately follows the word then.
concurrent lines (p. 238) Three or more lines that intersect at a common point.
conditional statement (p. 75) A statement that can be written in if-then form.
cone (p. 666) A solid with a circular base, a vertex not contained in the same plane as the base, and a lateral surface area composed of all points in the segments connecting the vertex to the edge of the base.
**congruence transformations** (p. 194) A mapping for which a geometric figure and its image are congruent.

**congruent** (p. 15) Having the same measure.

**congruent arcs** (p. 530) Arcs of the same circle or congruent circles that have the same measure.

**congruent solids** (p. 707) Two solids are congruent if all of the following conditions are met.
1. The corresponding angles are congruent.
2. Corresponding edges are congruent.
3. Corresponding faces are congruent.
4. The volumes are congruent.

**congruent triangles** (p. 192) Triangles that have their corresponding parts congruent.

**conjecture** (p. 62) An educated guess based on known information.

**conjunction** (p. 68) A compound statement formed by joining two or more statements with the word *and*.

**consecutive interior angles** (p. 128)
In the figure, transversal $t$ intersects lines $\ell$ and $m$. There are two pairs of consecutive interior angles: $\angle 8$ and $\angle 1$, and $\angle 7$ and $\angle 2$.

**construction** (p. 15) A method of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used.

**contrapositive** (p. 77) The statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement.

**converse** (p. 77) The statement formed by exchanging the hypothesis and conclusion of a conditional statement.

**convex polygon** (p. 45) A polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon.

**coordinate proof** (p. 222) A proof that uses figures in the coordinate plane and algebra to prove geometric concepts.

**coplanar** (p. 6) Points that lie in the same plane.
corner view (p. 636) The view from a corner of a three-dimensional figure, also called the perspective view.

corollary (p. 188) A statement that can be easily proved using a theorem is called a corollary of that theorem.

corresponding angles (p. 128) In the figure, transversal $t$ intersects lines $\ell$ and $m$. There are four pairs of corresponding angles: $\angle 5$ and $\angle 1$, $\angle 8$ and $\angle 4$, $\angle 6$ and $\angle 2$, and $\angle 7$ and $\angle 3$.

cosine (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg adjacent to the acute angle to the measure of the hypotenuse.

counterexample (p. 63) An example used to show that a given statement is not always true.

cross products (p. 283) In the proportion $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, the cross products are $ad$ and $bc$. The proportion is true if and only if the cross products are equal.

cylinder (p. 638) A figure with bases that are formed by congruent circles in parallel planes.

deductive argument (p. 94) A proof formed by a group of algebraic steps used to solve a problem.

deductive reasoning (p. 82) A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

degree (p. 29) A unit of measure used in measuring angles and arcs. An arc of a circle with a measure of $1^\circ$ is $\frac{1}{360}$ of the entire circle.

diagonal (p. 404) In a polygon, a segment that connects nonconsecutive vertices of the polygon.

diameter 1. (p. 522) In a circle, a chord that passes through the center of the circle. 2. (p. 671) In a sphere, a segment that contains the center of the sphere, and has endpoints that are on the sphere.
**dilation** (p. 490) A transformation determined by a center point \( C \) and a scale factor \( k \). When \( k > 0 \), the image \( P' \) of \( P \) is the point on \( CP \) such that \( CP' = |k| \cdot CP \). When \( k < 0 \), the image \( P' \) of \( P \) is the point on the ray opposite \( CP \) such that \( CP' = k \cdot CP \).

**direct isometry** (p. 481) An isometry in which the image of a figure is found by moving the figure intact within the plane.

**direction** (p. 498) The measure of the angle that a vector forms with the positive \( x \)-axis or any other horizontal line.

**disjunction** (p. 68) A compound statement formed by joining two or more statements with the word **or**.

**equal vectors** (p. 499) Vectors that have the same magnitude and direction.

**equiangular triangle** (p. 178) A triangle with all angles congruent.

**equilateral triangle** (p. 179) A triangle with all sides congruent.

**exterior** (p. 29) A point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle.

**exterior angle** (p. 186) An angle formed by one side of a triangle and the extension of another side.

**extremes** (p. 283) In \( \frac{a}{b} = \frac{c}{d} \), the numbers \( a \) and \( d \).

**flow proof** (p. 187) A proof that organizes statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate the order of the statements.
fractal (p. 325) A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity.

geometric mean (p. 342) For any positive numbers $a$ and $b$, the positive number $x$ such that $\frac{a}{x} = \frac{x}{b}$.

geometric probability (p. 622) Using the principles of length and area to find the probability of an event.

glide reflection (p. 475) A composition of a translation and a reflection in a line parallel to the direction of the translation.

great circle (p. 671) For a given sphere, the intersection of the sphere and a plane that contains the center of the sphere.

glitch (p. 223) A momentary error in a computer program or output.

height of a parallelogram (p. 595) The length of an altitude of a parallelogram.

hemisphere (p. 672) One of the two congruent parts into which a great circle separates a sphere.

hypothesis (p. 75) In a conditional statement, the statement that immediately follows the word if.

if-then statement (p. 75) A compound statement of the form “if $A$, then $B$”, where $A$ and $B$ are statements.

incenter (p. 240) The point of concurrency of the angle bisectors of a triangle.

included angle (p. 201) In a triangle, the angle formed by two sides is the included angle for those two sides.

included side (p. 207) The side of a triangle that is a side of each of two angles.

indirect isometry (p. 481) An isometry that cannot be performed by maintaining the orientation of the points, as in a direct isometry.
indirect proof (p. 255) In an indirect proof, one assumes that the statement to be proved is false. One then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

indirect reasoning (p. 255) Reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis or some other accepted fact, like a postulate, theorem, or corollary. Then, since the assumption has been proved false, the conclusion must be true.

inductive reasoning (p. 62) Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.

inscribed (p. 537) A polygon is inscribed in a circle if each of its vertices lie on the circle.

intercepted (p. 544) An angle intercepts an arc if and only if each of the following conditions are met.
1. The endpoints of the arc lie on the angle.
2. All points of the arc except the endpoints are in the interior of the circle.
3. Each side of the angle contains an endpoint of the arc.

interior (p. 29) A point is in the interior of an angle if it does not lie on the angle itself and it lies on a segment with endpoints that are on the sides of the angle.

inverse (p. 77) The statement formed by negating both the hypothesis and conclusion of a conditional statement.

irregular figure (p. 617) A figure that cannot be classified as a single polygon.

irregular polygon (p. 618) A polygon that is not regular.
**isometry** (p. 463) A mapping for which the original figure and its image are congruent.

**isosceles trapezoid** (p. 439) A trapezoid in which the legs are congruent, both pairs of base angles are congruent, and the diagonals are congruent.

**isosceles triangle** (p. 179) A triangle with at least two sides congruent. The congruent sides are called **legs**. The angles opposite the legs are **base angles**. The angle formed by the two legs is the **vertex angle**. The side opposite the vertex angle is the **base**.

**iteration** (p. 325) A process of repeating the same procedure over and over again.

**kite** (p. 438) A quadrilateral with exactly two distinct pairs of adjacent congruent sides.

**lateral area** (p. 649) For prisms, pyramids, cylinders, and cones, the area of the figure, not including the bases.

**Law of Cosines** (p. 385) Let $\triangle ABC$ be any triangle with $a$, $b$, and $c$ representing the measures of sides opposite the angles with measures $A$, $B$, and $C$ respectively. Then the following equations are true.

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
b^2 &= a^2 + c^2 - 2ac \cos B \\
c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]

**Law of Detachment** (p. 82) If $p \rightarrow q$ is a true conditional and $p$ is true, then $q$ is also true.
**Law of Sines** (p. 377) Let \( \triangle ABC \) be any triangle with \( a, b, \) and \( c \) representing the measures of sides opposite the angles with measures \( A, B, \) and \( C \) respectively. Then, \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \).

**Law of Syllogism** (p. 83) If \( p \rightarrow q \) and \( q \rightarrow r \) are true conditionals, then \( p \rightarrow r \) is also true.

**line** (p. 6) A basic undefined term of geometry. A line is made up of points and has no thickness or width. In a figure, a line is shown with an arrowhead at each end. Lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters.

**line of reflection** (p. 463) A line through a figure that separates the figure into two mirror images.

**line of symmetry** (p. 466) A line that can be drawn through a plane figure so that the figure on one side is the reflection image of the figure on the opposite side.

**line segment** (p. 13) A measurable part of a line that consists of two points, called endpoints, and all of the points between them.

**linear pair** (p. 37) A pair of adjacent angles whose non-common sides are opposite rays.

**locus** (p. 11) The set of points that satisfy a given condition.

**logically equivalent** (p. 77) Statements that have the same truth values.

**magnitude** (p. 498) The length of a vector.

**major arc** (p. 530) An arc with a measure greater than 180. \( \overrightarrow{ACB} \) is a major arc.

**magnitude** La longitud de un vector.

**major arc** Arco que mide más de 180°. \( \overrightarrow{ACB} \) es un arco mayor.
matrix logic  (p. 88) A method of deductive reasoning that uses a table to solve problems.

means  (p. 283) In \( \frac{a}{b} = \frac{c}{d} \), the numbers \( b \) and \( c \).

median  1. (p. 240) In a triangle, a line segment with endpoints that are a vertex of a triangle and the midpoint of the side opposite the vertex. 2. (p. 440) In a trapezoid, the segment that joins the midpoints of the legs.

midpoint  (p. 22) The point halfway between the endpoints of a segment.

midsegment  (p. 308) A segment with endpoints that are the midpoints of two sides of a triangle.

minor arc  (p. 530) An arc with a measure less than 180. \( AB \) is a minor arc.

negation  (p. 67) If a statement is represented by \( p \), then \( \neg p \) is the negation of the statement.

net  (p. 644) A two-dimensional figure that when folded forms the surfaces of a three-dimensional object.

\( n \)-gon  (p. 46) A polygon with \( n \) sides.

non-Euclidean geometry  (p. 165) The study of geometrical systems that are not in accordance with the Parallel Postulate of Euclidean geometry.

oblique cone  (p. 666) A cone that is not a right cone.

oblique cylinder  (p. 655) A cylinder that is not a right cylinder.

oblique prism  (p. 649) A prism in which the lateral edges are not perpendicular to the bases.
obtuse angle (p. 30) An angle with degree measure greater than 90 and less than 180.

obtuse triangle (p. 178) A triangle with an obtuse angle.

opposite rays (p. 29) Two rays \( BA \) and \( BC \) such that \( B \) is between \( A \) and \( C \).

ordered triple (p. 714) Three numbers given in a specific order used to locate points in space.

orthocenter (p. 240) The point of concurrency of the altitudes of a triangle.

orthogonal drawing (p. 636) The two-dimensional top view, left view, front view, and right view of a three-dimensional object.

paragraph proof (p. 90) An informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true.

parallel lines (p. 126) Coplanar lines that do not intersect.

parallel planes (p. 126) Planes that do not intersect.

parallel vectors (p. 499) Vectors that have the same or opposite direction.

parallelogram (p. 411) A quadrilateral with parallel opposite sides. Any side of a parallelogram may be called a base.

perimeter (p. 46) The sum of the lengths of the sides of a polygon.

perpendicular bisector (p. 238) In a triangle, a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side.

ángulo obtuso Ángulo que mide más de 90° y menos de 180°.

triángulo obtusángulo Triángulo que tiene un ángulo obtuso.

semirrectas opuestas Dos semirrectas \( BA \) y \( BC \) tales que \( B \) se localiza entre \( A \) y \( C \).

triple ordenado Tres números dados en un orden especifico que sirven para ubicar puntos en el espacio.

ortocentro Punto de intersección de las alturas de un triángulo.

vista ortogonal Vista bidimensional desde arriba, desde la izquierda, desde el frente o desde la derecha de un cuerpo tridimensional.

demostación de párrafo Demostración informal escrita en forma de párrafo que explica por qué una conjetura acerca de una situación dada es verdadera.

rectas paralelas Rectas coplanares que no se intersecan.

planos paralelos Planos que no se intersecan.

vectores paralelos Vectores que tienen la misma dirección o la dirección opuesta.

paralelograma Cuadrilátero cuyos lados opuestos son paralelos entre sí. Cualquier lado del paralelograma puede ser la base.

perímetro La suma de la longitud de los lados de un polígono.

mediatriz Recta, segmento o semirrecta que atraviesa el punto medio del lado de un triángulo y que es perpendicular a dicho lado.
**perpendicular lines** (p. 40) Lines that form right angles.

**perspective view** (p. 636) The view of a three-dimensional figure from the corner.

**pi (π)** (p. 524) An irrational number represented by the ratio of the circumference of a circle to the diameter of the circle.

**plane** (p. 6) A basic undefined term of geometry. A plane is a flat surface made up of points that has no depth and extends indefinitely in all directions. In a figure, a plane is often represented by a shaded, slanted 4-sided figure. Planes are usually named by a capital script letter or by three noncollinear points on the plane.

**plane Euclidean geometry** (p. 165) Geometry based on Euclid’s axioms dealing with a system of points, lines, and planes.

**Platonic Solids** (p. 637) The five regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, or icosahedron.

**point** (p. 6) A basic undefined term of geometry. A point is a location. In a figure, points are represented by a dot. Points are named by capital letters.

**point of concurrency** (p. 238) The point of intersection of concurrent lines.

**point of symmetry** (p. 466) The common point of reflection for all points of a figure.

**point of tangency** (p. 552) For a line that intersects a circle in only one point, the point at which they intersect.

**point-slope form** (p. 145) An equation of the form $y - y_1 = m(x - x_1)$, where $(x_1, y_1)$ are the coordinates of any point on the line and $m$ is the slope of the line.
polygon (p. 45) A closed figure formed by a finite number of coplanar segments called sides such that the following conditions are met:  
1. The sides that have a common endpoint are noncollinear.  
2. Each side intersects exactly two other sides, but only at their endpoints, called the vertices.

polyhedrons (p. 637) Closed three-dimensional figures made up of flat polygonal regions. The flat regions formed by the polygons and their interiors are called faces. Pairs of faces intersect in segments called edges. Points where three or more edges intersect are called vertices.

postulate (p. 89) A statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true without proof.

precision (p. 14) The precision of any measurement depends on the smallest unit available on the measuring tool.

prism (p. 637) A solid with the following characteristics:  
1. Two faces, called bases, are formed by congruent polygons that lie in parallel planes.  
2. The faces that are not bases, called lateral faces, are formed by parallelograms.  
3. The intersections of two adjacent lateral faces are called lateral edges and are parallel segments.

proof (p. 90) A logical argument in which each statement you make is supported by a statement that is accepted as true.

proof by contradiction (p. 255) An indirect proof in which one assumes that the statement to be proved is false. One then uses logical reasoning to deduce a statement that contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

proportion (p. 283) An equation of the form \( \frac{a}{b} = \frac{c}{d} \) that states that two ratios are equal.

pyramid (p. 637) A solid with the following characteristics:  
1. All of the faces, except one face, intersect at a point called the vertex.  
2. The face that does not contain the vertex is called the base and is a polygonal region.  
3. The faces meeting at the vertex are called lateral faces and are triangular regions.

polígono Figura cerrada formada por un número finito de segmentos coplanares llamados lados, y que satisfacer las siguientes condiciones:  
1. Los lados que tienen un extremo común son no colineales.  
2. Cada lado interseca exactamente dos lados, pero sólo en sus extremos, formando los vértices.

poliedro Figura tridimensional cerrada formada por regiones poligonales planas. Las regiones planas definidas por un polígono y sus laterales se llaman caras. Cada intersección entre dos caras se llama arista. Los puntos donde se intersecan tres o más aristas se llaman vértices.

postulado Enunciado que describe una relación fundamental entre los términos primitivos de geometría. Los postulados se aceptan como verdaderos sin necesidad de demostración.

precisión La precisión de una medida depende de la unidad de medida más pequeña del instrumento de medición.

prisma Sólido que posee las siguientes características:  
1. Tiene dos caras llamadas bases, formadas por polígonos congruentes que yacen en planos paralelos.  
2. Las caras que no son las bases, llamadas caras laterales, son formadas por paralelogramos.  
3. Las intersecciones de dos aristas laterales adyacentes se llaman aristas laterales y son segmentos paralelos.

demostración Argumento lógico en que cada enunciado está basado en un enunciado que se acepta como verdadero.

demostración por contradicción Demostración indirecta en que se asume que el enunciado que se va a demostrar es falso. Después, se razona lógicamente para deducir un enunciado que contradiga un postulado, un teorema o una de las conjeturas. Una vez que se obtiene una contradicción, se concluye que el enunciado que se supuso falso es, en realidad, verdadero.

proporción Ecuación de la forma \( \frac{a}{b} = \frac{c}{d} \) que establece que dos razones son iguales.

pirámide Sólido con las siguientes características:  
1. Todas, excepto una de las caras, se intersecan en un punto llamado vértice.  
2. La cara que no contiene el vértice se llama base y es una región poligonal.  
3. Las caras que se encuentran en los vértices se llaman caras laterales y son regiones triangulares.
Pythagorean identity (p. 391) The identity \( \cos^2 \theta + \sin^2 \theta = 1 \).

Pythagorean triple (p. 352) A group of three whole numbers that satisfies the equation \( a^2 + b^2 = c^2 \), where \( c \) is the greatest number.

Pythagorean identity (p. 391) La identidad \( \cos^2 \theta + \sin^2 \theta = 1 \).

Pythagorean triple (p. 352) Grupo de tres números enteros que satisfacen la ecuación \( a^2 + b^2 = c^2 \), donde \( c \) es el número más grande.

radius 1. (p. 522) In a circle, any segment with endpoints that are the center of the circle and a point on the circle. 2. (p. 671) In a sphere, any segment with endpoints that are the center and a point on the sphere.

radio 1. Cualquier segmento cuyos extremos están en el centro de un círculo y en un punto cualquiera del mismo. 2. Cualquier segmento cuyos extremos forman el centro y en punto de una esfera.

to rate of change (p. 140) Describes how a quantity is changing over time.

tasa de cambio Describe cómo cambia una cantidad a través del tiempo.

ray (p. 29) \( PQ \) is a ray if it is the set of points consisting of \( PQ \) and all points \( S \) for which \( Q \) is between \( P \) and \( S \).

raya \( PQ \) es una semirrecta si consta del conjunto de puntos formado por \( PQ \) y todos los \( S \) puntos \( S \) para los que \( Q \) se localiza entre \( P \) y \( S \).

reciprocal identity (p. 391) Each of the three trigonometric ratios called cosecant, secant, and cotangent, that are the reciprocals of sine, cosine, and tangent, respectively.

identidad recíproca Cada una de las tres razones trigonométricas llamadas cosecante, secante y tangente que son los recíprocos del seno, el coseno y la tangente, respectivamente.

rectangle (p. 424) A quadrilateral with four right angles.

rectángulo Cuadrilátero que tiene cuatro ángulos rectos.

reflection (p. 463) A transformation representing a flip of the figure over a point, line, or plane.

reflexión Transformación que se obtiene cuando se “voltea” una imagen sobre un punto, una línea o un plano.

reflection matrix (p. 507) A matrix that can be multiplied by the vertex matrix of a figure to find the coordinates of the reflected image.

matriz de reflexión Matriz que al ser multiplicada por la matriz de vértices de una figura permite hallar las coordenadas de la imagen reflejada.

regular polygon (p. 46) A convex polygon in which all of the sides are congruent and all of the angles are congruent.

polígono regular Polígono convexo en el que todos los lados y todos los ángulos son congruentes entre sí.

regular polyhedron (p. 637) A polyhedron in which all of the faces are regular congruent polygons.

poliedro regular Poliedro cuyas caras son polígonos regulares congruentes.

regular prism (p. 637) A right prism with bases that are regular polygons.

prisma regular Prisma recto cuyas bases son polígonos regulares.
**regular tessellation** (p. 484) A tessellation formed by only one type of regular polygon.

**related conditionals** (p. 77) Statements such as the converse, inverse, and contrapositive that are based on a given conditional statement.

**relative error** (p. 19) The ratio of the half-unit difference in precision to the entire measure, expressed as a percent.

**remote interior angles** (p. 186) The angles of a triangle that are not adjacent to a given exterior angle.

**resultant** (p. 500) The sum of two vectors.

**rhombus** (p. 431) A quadrilateral with all four sides congruent.

**right angle** (p. 30) An angle with a degree measure of 90.

**right cone** (p. 666) A cone with an axis that is also an altitude.

**right cylinder** (p. 655) A cylinder with an axis that is also an altitude.

**right prism** (p. 649) A prism with lateral edges that are also altitudes.

**right triangle** (p. 178) A triangle with a right angle. The side opposite the right angle is called the hypotenuse. The other two sides are called legs.

**rotation** (p. 476) A transformation that turns every point of a preimage through a specified angle and direction about a fixed point, called the center of rotation.

**rotation matrix** (p. 507) A matrix that can be multiplied by the vertex matrix of a figure to find the coordinates of the rotated image.

**rotational symmetry** (p. 478) If a figure can be rotated less than 360° about a point so that the image and the preimage are indistinguishable, the figure has rotational symmetry.

**teselado regular** Teselado formado por un solo tipo de polígono regular.

**enunciados condicionales relacionados** Enunciados tales como el recíproco, la inversa y la antítesis que están basados en un enunciado condicional dado.

**error relativo** La razón entre la mitad de la unidad más precisa de la medición y la medición completa, expresada en forma de porcentaje.

**ángulos internos no adyacentes** Ángulos de un triángulo que no son adyacentes a un ángulo exterior dado.

**resultante** La suma de dos vectores.

**rombo** Cuadrilátero cuyos cuatro lados son congruentes.

**ángulo recto** Ángulo cuya medida en grados es 90.

**cono recto** Cono cuyo eje es también su altura.

**cilindro recto** Cilindro cuyo eje es también su altura.

**prisma recto** Prisma cuyas aristas laterales también son su altura.

**triángulo rectángulo** Triángulo con un ángulo recto. El lado opuesto al ángulo recto se conoce como hipotenusa. Los otros dos lados se llaman catetos.

**rotación** Transformación en que se hace girar cada punto de la preimagen a través de un ángulo y una dirección determinadas alrededor de un punto, conocido como centro de rotación.

**matriz de rotación** Matriz que al ser multiplicada por la matriz de vértices de la figura permite calcular las coordenadas de la imagen rotada.

**simetría de rotación** Si se puede rotar una imagen menos de 360° alrededor de un punto y la imagen y la preimagen son idénticas, entonces la figura presenta simetría de rotación.
**scalar** (p. 501) A constant multiplied by a vector.

**scalar multiplication** (p. 501) Multiplication of a vector by a scalar.

**scale factor** (p. 290) The ratio of the lengths of two corresponding sides of two similar polygons or two similar solids.

**scalene triangle** (p. 179) A triangle with no two sides congruent.

**secant** (p. 561) Any line that intersects a circle in exactly two points.

**sector of a circle** (p. 623) A region of a circle bounded by a central angle and its intercepted arc.

**segment** (p. 13) See line segment.

**segment bisector** (p. 24) A segment, line, or plane that intersects a segment at its midpoint.

**segment of a circle** (p. 624) The region of a circle bounded by an arc and a chord.

**self-similar** (p. 325) If any parts of a fractal image are replicas of the entire image, the image is self-similar.

**semicircle** (p. 530) An arc that measures 180.

**semi-regular tessellation** (p. 484) A uniform tessellation formed using two or more regular polygons.

**similar polygons** (p. 289) Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
similar solids (p. 707) Solids that have exactly the same shape, but not necessarily the same size.

similarity transformation (p. 491) When a figure and its transformation image are similar.

sine (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the hypotenuse.

skew lines (p. 127) Lines that do not intersect and are not coplanar.

slope (p. 139) For a (nonvertical) line containing two points \((x_1', y_1')\) and \((x_2', y_2')\), the number \(m\) given by the formula \(m = \frac{y_2 - y_1}{x_2 - x_1}\) where \(x_2 \neq x_1\).

slope-intercept form (p. 145) A linear equation of the form \(y = mx + b\). The graph of such an equation has slope \(m\) and \(y\)-intercept \(b\).

solving a triangle (p. 378) Finding the measures of all of the angles and sides of a triangle.

space (p. 8) A boundless three-dimensional set of all points.

sphere (p. 638) In space, the set of all points that are a given distance from a given point, called the center.

spherical geometry (p. 165) The branch of geometry that deals with a system of points, greatcircles (lines), and spheres (planes).

square (p. 432) A quadrilateral with four right angles and four congruent sides.

standard position (p. 498) When the initial point of a vector is at the origin.

statement (p. 67) Any sentence that is either true or false, but not both.

strictly self-similar (p. 325) A figure is strictly self-similar if any of its parts, no matter where they are located or what size is selected, contain the same figure as the whole.

similar solids Sólidos que tienen exactamente la misma forma, pero no necesariamente el mismo tamaño.

transformación de semejanza Aquélla en que la figura y su imagen transformada son semejantes.

seno Es la razón entre la medida del cateto opuesto al ángulo agudo y la medida de la hipotenusa de un triángulo rectángulo.

rectas alabeadas Rectas que no se intersecan y que no son coplanares.

pendiente Para una recta (no vertical) que contiene dos puntos \((x_1', y_1')\) y \((x_2', y_2')\), el número \(m\) dado por la fórmula \(m = \frac{y_2 - y_1}{x_2 - x_1}\) donde \(x_2 \neq x_1\).

forma pendiente-intersección Ecuación lineal de la forma \(y = mx + b\). En la gráfica de tal ecuación, la pendiente es \(m\) y la intersección \(y\) es \(b\).

resolver un triángulo Calcular las medidas de todos los ángulos y todos los lados de un triángulo.

esfera El conjunto de todos los puntos en el espacio que se encuentran a cierta distancia de un punto dado llamado centro.

géometría esférica Rama de la geometría que estudia los sistemas de puntos, círculos máximos (rectas) y esferas (planos).

cuadrado Cuadrilátero con cuatro ángulos rectos y cuatro lados congruentes.

posición estándar Ocurre cuando la posición inicial de un vector es el origen.

enunciado Una oración que puede ser falsa o verdadera, pero no ambas.

exactamente autosemejante Una figura es estrictamente autosemejante si cualquiera de sus partes, sin importar su localización o su tamaño, contiene la figura completa.
**supplementary angles** (p. 39) Two angles with measures that have a sum of 180.

**surface area** (p. 644) The sum of the areas of all faces and side surfaces of a three-dimensional figure.

**tangent** 1. (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the leg adjacent to the acute angle. 2. (p. 552) A line in the plane of a circle that intersects the circle in exactly one point. The point of intersection is called the **point of tangency**. 3. (p. 671) A line that intersects a sphere in exactly one point.

**tessellation** (p. 483) A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces.

**theorem** (p. 90) A statement or conjecture that can be proven true by undefined terms, definitions, and postulates.

**transformation** (p. 462) In a plane, a mapping for which each point has exactly one image point and each image point has exactly one preimage point.

**translation** (p. 470) A transformation that moves all points of a figure the same distance in the same direction.

**translation matrix** (p. 506) A matrix that can be added to the vertex matrix of a figure to find the coordinates of the translated image.

**transversal** (p. 127) A line that intersects two or more lines in a plane at different points.

**trapezoid** (p. 439) A quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called **bases**. The nonparallel sides are called **legs**. The pairs of angles with their vertices at the endpoints of the same base are called **base angles**.
trigonometric identity (p. 391) An equation involving a trigonometric ratio that is true for all values of the angle measure.

trigonometric ratio (p. 364) A ratio of the lengths of sides of a right triangle.

trigonometry (p. 364) The study of the properties of triangles and trigonometric functions and their applications.

truth table (p. 70) A table used as a convenient method for organizing the truth values of statements.

truth value (p. 67) The truth or falsity of a statement.

two-column proof (p. 95) A formal proof that contains statements and reasons organized in two columns. Each step is called a statement, and the properties that justify each step are called reasons.

undefined terms (p. 7) Words, usually readily understood, that are not formally explained by means of more basic words and concepts. The basic undefined terms of geometry are point, line, and plane.

uniform tessellations (p. 484) Tessellations containing the same arrangement of shapes and angles at each vertex.

vector (p. 498) A directed segment representing a quantity that has both magnitude, or length, and direction.

vertex matrix (p. 506) A matrix that represents a polygon by placing all of the column matrices of the coordinates of the vertices into one matrix.

vertical angles (p. 37) Two nonadjacent angles formed by two intersecting lines.

volume (p. 688) A measure of the amount of space enclosed by a three-dimensional figure.
**Selected Answers**

### Chapter 1 Points, Lines, Planes, and Angles

#### Page 5 Chapter Getting Started

1–4.

- \( D(-1, 2) \)
- \( B(4, 0) \)
- \( A(3, -2) \)

5. \( \frac{11}{8} \) 7. \( \frac{5}{16} \) 9. -15

11. 25 13. 20 in.

15. 24.6 m

#### Pages 9–11 Lesson 1-1

1. point, line, plane 3. Micha; the points must be noncollinear to determine a plane.

5. Sample answer:

7. 6 9. No; \( A, C, \) and \( I \) lie in plane \( ABC \), but \( D \) does not.

11. point 13. \( \pi \) 15. \( \mathcal{R} \)

17. Sample answer: \( \overline{PQ} \)

19. \((D, 9)\)

21.

23. Sample answer:

25.

27.

29. points that seem collinear; sample answer: \((0, -2), (1, -3), (2, -4), (3, -5)\)

31. 1 33. anywhere on \( \overline{AB} \) 35. \( A, B, C, D \) or \( E, F, C, B \)

37. \( \overline{AC} \) 39. lines 41. plane 43. point 45. point

47.

51. Sample answer:

53. vertical 55. Sample answer: Chairs wobble because all four legs do not touch the floor at the same time. Answers should include the following:

- The ends of the legs represent points. If all points lie in the same plane, the chair will not wobble.
- Because it only takes three points to determine a plane, a chair with three legs will never wobble.

57. B

59. part of the coordinate plane above the line \( y = -2x + 1 \).

61. =

63. =

65. <

#### Pages 16–19 Lesson 1-2

1. Align the 0 point on the ruler with the leftmost endpoint of the segment. Align the edge of the ruler along the segment. Note where the rightmost endpoint falls on the scale and read the closest eighth of an inch measurement.

3. \( \frac{13}{4} \) in. 5. 0.5 m; 14 cm could be 13.5 to 14.5 m 7. 3.7 cm

9. \( x = 3; LM = 9 \) 11. \( \overline{BC} \equiv \overline{CD}, \overline{BE} \equiv \overline{ED}, \overline{BA} \equiv \overline{DA} \)

13. 4.5 cm or 45 mm 15. \( \frac{13}{4} \) in. 17. 0.5 cm; 21.5 to 22.5 mm

19. 0.5 cm; 307.5 to 308.5 cm 21. \( \frac{11}{8} \) ft.; \( \frac{33}{8} \) ft.

23. \( \frac{11}{4} \) in. 25. 2.8 cm 27. \( \frac{13}{4} \) in. 29. \( x = 11; ST = 22 \)

31. \( x = 2; ST = 4 \) 33. \( y = 2; ST = 3 \) 35. no 37. yes

39. yes 41. \( \overline{CF} \equiv \overline{DG}, \overline{AB} \equiv \overline{HI}, \overline{CE} \equiv \overline{ED} \equiv \overline{EF} \equiv \overline{EG} \)

43. 50,000 visitors 45. No; the number of visitors to Washington state parks could be as low as 46.35 million or as high as 46.45 million. The visitors to Illinois state parks could be as low as 44.45 million or as high as 44.55 million visitors. The difference in visitors could be as high as 2.0 million.
47. 15.5 cm; Each measurement is accurate within 0.5 cm, so the greatest perimeter is 3.5 cm + 5.5 cm + 6.5 cm.

49. 

51. Sample answer: Units of measure are used to differentiate between size and distance, as well as for accuracy. Answers should include the following.
• When a measurement is stated, you do not know the precision of the instrument used to make the measure. Therefore, the actual measure could be greater or less than that stated.
• You can assume equal measures when segments are shown to be congruent.

53. 1.7% 55. 0.08% 57. D 59. Sample answer: planes ABC and BCD 61. 5 63. 22 65. 1

Page 19 Practice Quiz 1
1.  2.  PR 3. PR 5. 8.35

Pages 25–27 Lesson 1-3
1. Sample answers: (1) Use one of the Midpoint Formulas if you know the coordinates of the endpoints. (2) Draw a segment and fold the paper so that the endpoints match to locate the middle of the segment. (3) Use a compass and segment and fold the paper so that the endpoints match to locate the middle of the segment. (4) Use a compass and straightedge to construct the bisector of the segment.

3. 8 5. 10 7. −6 9. (−2.5, 4) 11. (3, 5) 13. 2
15. 3 17. 11 19. 10 21. 13 23. 15 25. 25. 90 ≈ 9.5
27. 29. 8 29. 17.3 units
31. −3 33. 25 35. 1
37. (10, 3) 39. (−10, −3) 41. (5.6, 2.85) 43. R(2, 7)
45. T(12, 11) 47. Lafayette, LA 49a. 111.8 49b. 212.0
49c. 353.4 49d. 420.3 49e. 37.4 49f. 2092.9 51. 72.1

53. Sample answer: The perimeter increases by the same factor. 55. (−1, −3) 57. B 59. 41/4 in.

61. Sample answer:

Pages 33–36 Lesson 1-4
1. Yes; they all have the same measure. 3. m∠A = m∠Z
21. ∠FED, ∠4 23. ∠AED, ∠DEA, ∠EAB, ∠BEA, ∠EAC, ∠CEA 25. 15° 27. 30°, 30° 29. 60°, acute 31. 90°, right
33. 120°, obtuse 35. 65 37. 4 39. 4 41. Sample answer: Acute can mean something that is sharp or having a very fine tip like a pen, a knife, or a needle. Obtuse means not pointed or blunt, so something that is obtuse would be wide. 43. 31; 59 45. 1, 3, 6, 10, 15 47. 21, 45 49. Sample answer: A degree is 1/360 of a circle. Answers should include the following.
• Place one side of the angle to coincide with 0 on the protractor and the vertex of the angle at the center point of the protractor. Observe the point at which the other side of the angle intersects the scale of the protractor.
• See students’ work.

51. C 53. √80 ≈ 8.9; (2, 2) 55. 92/3 in. 57. 13 59. F, L, J
61. 5 63. −45 65. 8

Page 36 Practice Quiz 2
1. (−1, 2); √65 ≈ 8.1 3. (0, 0); √2000 ≈ 44.7 5. 34; 135

Pages 41–62 Lesson 1-5
1. 

3. Sample answer: The noncommon sides of a linear pair of angles form a straight line.

5. Sample answer: ∠ABC, ∠CBE
7. x = 24, y = −20
9. Yes; they share a common side and vertex, so they are adjacent. Since PR falls between PQ and PS, m∠PQR < 90, so the two angles cannot be complementary or supplementary.
11. ∠WUT, ∠WUX
13. ∠UWT, ∠WTY
15. ∠WTY, ∠WTV 17. 53, 37 19. 148 21. 84, 96 23. always
25. sometimes 27. 3.75 29. 114 31. Yes; the symbol denotes that ∠DAB is a right angle. 33. Yes; their sum of their measures is m∠ADC, which 90. 35. No; we do not know m∠ABC.

37. Sample answer:

39. Because ∠WUT and ∠TUV are supplementary, let m∠WUT = x and m∠TUV = 180 − x. A bisector creates measures that are half of the original angle, so m∠YUT = 1/2 m∠WUT or 1/2 x and m∠TUZ = 1/2 m∠TUV or 1/2 (180 − x). Then m∠YUZ = m∠YUT + m∠TUZ or (1/2) x + (1/2) (180 − x). This sum simplifies to 180/2 = 90. Because m∠YUZ = 90, YU ⊥ UZ. 41. A 43. 41°AB, m∠AB, n∠AB 45. obtuse 47. right 49. obtuse 51. 8

53. √173 ≈ 13.2 55. √20 ≈ 4.5 57. n = 3, QR = 20
59. 24 61. 40

Pages 48–50 Lesson 1-6
1. Divide the perimeter by 10. 3. P = 3s 5. pentagon; concave; irregular 7. 33 ft 9. 16 units 11. 4605 ft
13. octagon; convex; regular 15. pentagon 17. triangle
19. 82 ft 21. 40 units 23. The perimeter is tripled.
25. 125 m 27. 30 units 29. All are 15 cm. 31. 13 units, 13 units, 5 units 33. 4 in., 4 in., 17 in., 17 in. 35. 52 units
37. Sample answer: Some toys use pieces to form polygons. Others have polygon-shaped pieces that connect together.

Answers should include the following.
• triangles, quadrilaterals, pentagons
• 

39. D 41. sometimes 43. 63

Pages 53–56 Chapter 1 Study Guide and Review
1. d 3. f 5. b 7. p or m 9. F

11. 

13. x = 6, PB = 18
15. s = 3, PB = 12
17. yes
19. not enough information
21. √101 ≈ 10.0

23. √13 ≈ 3.6 25. (3, −5) 27. (0.6, −6.35) 29. FE, FG
31. 70°, acute 33. 50°, acute 35. 36 37. 40 39. ∠TWY, ∠XWY
41. 9 43. not a polygon 45. ≈ 22.5 units
Chapter 2 Reasoning and Proof

Page 61 Chapter 2 Getting Started
1. 10 3. 0 5. 50 7. 21 9. –9 11. \( \frac{10}{5} \) 13. 16

Pages 63–66 Lesson 2-1
1. Sample answer: After the news is over, it’s time for dinner.
3. Sample answer: When it's cloudy, it rains.
Counterexample: It is often cloudy and it does not rain.
5. 7
7. A, B, C, and D are noncollinear.
9. true
11. 
13. Sample answer:
15. 11
17. 16
19. 30
21. Lines \( \ell \) and \( m \) form four right angles.
23. \( \angle 3 \) and \( \angle 4 \) are supplementary.
25. \( \triangle PQR \) is a scalene triangle.
27. \( P = SR, QR = PS \)
29. false;
31. false;
33. true
35. False; \( JKLM \) may not have a right angle.
37. trial and error, a process of inductive reasoning
39. \( C_2H_6 \)
41. false; \( n = 41 \)
43. C
45. hexagon, convex, irregular
47. heptagon, concave, irregular
49. No; we do not know anything about the angle measures.
51. Yes; they form a linear pair.
53. (2, –1)
55. (1, –12)
57. (5.5, 2.2)
59. 8; 56
61. 4; 16
63. 10; 43
65. 4, 5
67. 5, 6, 7

Pages 71–74 Lesson 2-2
1. The conjunction \( (p \text{ and } q) \) is represented by the intersection of the two circles.
3. A conjunction is a compound statement using the word and, while a disjunction is a compound statement using the word or.
5. \( 9 + 5 = 14 \) and a square has four sides; true.
7. \( 9 + 5 = 14 \) or February does not have 30 days; true.
9. \( 9 + 5 \neq 14 \) or a square does not have four sides; false.

11. Sample answer:

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15. 14
17. 3
19. \( \sqrt{64} = 8 \) or an equilateral triangle has three congruent sides; true.
21. \( 0 < 0 \) and an obtuse angle measures greater than \( 90^\circ \) and less than \( 180^\circ \); false.
23. An equilateral triangle has three congruent sides and an obtuse angle measures greater than \( 90^\circ \) and less than \( 180^\circ \); true.
25. An equilateral triangle has three congruent sides and \( 0 < 0 \); false.
27. An obtuse angle measures greater than \( 90^\circ \) and less than \( 180^\circ \) or an equilateral triangle has three congruent sides; true.
29. An obtuse angle measures greater than \( 90^\circ \) and less than \( 180^\circ \), or an equilateral triangle has three congruent sides and \( 0 < 0 \); true.

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53. Sample answer: Logic can be used to eliminate false choices on a multiple choice test. Answers should include the following:
   • Math is my favorite subject and drama club is my favorite activity.
   • See students’ work.
55. C 57. 81 59. 1 61. 405 63. 34.4 65. 29.5 67. 55º; acute 69. 222 feet 71. 44 73. 184

Pages 78–80 Lesson 2-3
1. Writing a conditional in if-then form is helpful so that the hypothesis and conclusion are easily recognizable.
3. In the inverse, you negate both the hypothesis and the conclusion of the conditional. In the contrapositive, you negate the hypothesis and the conclusion of the converse.
5. H: x – 3 = 7; C: x = 10 7. If a pitcher is a 32-ounce pitcher, then it holds a quart of liquid. 9. If an angle is formed by perpendicular lines, then it is a right angle.
11. true 13. Converse: If plants grow, then they have water; true. Inverse: If plants do not have water, then they will not grow; true. Contrapositive: If plants do not grow, then they do not have water. False; they may have been killed by overwatering. 15. Sample answer: If you are in Colorado, then aspen trees cover high areas of the mountains. If you are in Florida, then cypress trees rise from the swamps. If you are in Vermont, then maple trees are prevalent. 17. H: you are a teenager; C: you are at least 13 years old 19. H: three points lie on a line; C: the points are collinear 21. H: the measure of an is between 0 and 90; C: the angle is acute 23. If you are a math teacher, then you love to solve problems. 25. Sample answer: If two angles are adjacent, then they have a common side.
27. Sample answer: If two triangles are equiangular, then they are equilateral. 29. true 31. true 33. false 35. true 37. false 39. true 41. Converse: If you are in good shape, then you exercise regularly; true. Inverse: If you do not exercise regularly, then you are not in good shape; true. Contrapositive: If you are not in good shape, then you do not exercise regularly. False; an ill person may exercise a lot, but still not be in good shape.
43. Converse: If a figure is a quadrilateral, then it is a rectangle; false, rhombus. Inverse: If a figure is not a rectangle, then it is not a quadrilateral; false, rhombus. Contrapositive: If a figure is not a quadrilateral, then it is not a rectangle; true. 45. Converse: If an angle has measure less than 90, then it is acute; true. Inverse: If an angle is not acute, then its measure is not less than 90; true. Contrapositive: If an angle’s measure is not less than 90, then it is not acute; true. 47. Sample answer: In Alaska, if there are more hours of daylight than darkness, then it is summer. In Alaska, if there are more hours of darkness than daylight, then it is winter.
51. B
53. A hexagon has five sides or \(60 \times 3 = 18\); false 55. A hexagon doesn’t have five sides or \(60 \times 3 = 18\); true 57. George Washington was not the first president of the United States and \(60 \times 3 \neq 18\); false 59. The sum of the measures of the angles in a triangle is 180.
61. \(\angle PQR\) is a right angle.
63. \(\sqrt{41}\) or 6.4 65. \(\sqrt{125}\) or 11.2 67. Multiply each side by 2.

Page 80 Practice Quiz 1
1. false 3. Sample answer:

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5. Converse: If two angles have a common vertex, then the angles are adjacent. False; \(\angle ABD\) is not adjacent to \(\angle ABC\).

Inverse: If two angles are not adjacent, then they do not have a common vertex. False; \(\angle ABC\) and \(\angle DBE\) have a common vertex and are not adjacent.

Contrapositive: If two angles do not have a common vertex, then they are not adjacent; true.
• Doctors need to note a patient’s symptoms to determine which medication to prescribe, then determine how much to prescribe based on weight, age, severity of the illness, and so on.
• Doctors use what is known to be true about diseases and when symptoms appear, then deduce that the patient has a particular illness.

35. B 37. They are a fast, easy way to add fun to your family’s menu.

39. Sample answer:

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<td>F</td>
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</tbody>
</table>

41. Sample answer:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>q (\lor) r</th>
<th>p (\lor) (q (\lor) r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
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<td>F</td>
</tr>
</tbody>
</table>

43. \(\angle HDG\) 45. Sample answer: \(\angle JHK\) and \(\angle DHK\)

47. Yes, slashes on the segments indicate that they are congruent. 49. 10 51. \(\sqrt{130} \approx 11.4\)

53. 55.

57. Sample answer: \(\angle 1\) and \(\angle 2\) are complementary, \(m\angle 1 + m\angle 2 = 90\).

Pages 91–93 Lesson 2-5

1. Deductive reasoning is used to support claims that are made in a proof. 3. postulates, theorems, algebraic properties, definitions 5. 15 7. definition of collinear

9. Through any two points, there is exactly one line. 11. 15 ribbons 13. 10 15. 21 17. Always; if two points lie in a plane, then the entire line containing those points lies in that plane. 19. Sometimes; the three points cannot be on the same line. 21. Sometimes; \(l\) and \(m\) could be skew so they would not lie in the same plane \(P\). 23. If two points lie in a plane, then the entire line containing those points lies in that plane. 25. If two points lie in a plane, then the entire line containing those points lies in the plane. 27. Through any three points not on the same line, there is exactly one plane. 29. She will have 4 different planes and 6 lines. 31. one, ten 33. C 35. yes; Law of Detachment

37. Converse: If \(\triangle ABC\) has an angle with measure greater than 90, then \(\triangle ABC\) is a right triangle. False; the triangle would be obtuse. Inverse: If \(\triangle ABC\) is not a right triangle, none of its angle measures are greater than 90. False; it could be an obtuse triangle. Contrapositive: If \(\triangle ABC\) does not have an angle measure greater than 90, \(\triangle ABC\) is not a right triangle. False; \(m\angle ABC\) could still be 90 and \(\triangle ABC\) be a right triangle. 39. \(\sqrt{17} \approx 4.1\) 41. \(\sqrt{106} \approx 10.3\)

43. 45. 12 47. 10

Selected Answers

Pages 97–100 Lesson 2-6

1. Sample answer: If \(x = 2\) and \(x + y = 6\), then \(2 + y = 6\).

3. hypothesis; conclusion 5. Multiplication Property 7. Addition Property 9a. \(5 - \frac{1}{2}x = 1\) 9b. Mult. Prop. 9c. Dist. Prop. 9d. \(-2x = -12\) 9e. Div. Prop.

11. Given: Rectangle \(ABCD\), \(AD = 3, AB = 10\)

Prove: \(AC = BD\)

Proof:

<table>
<thead>
<tr>
<th>Statement Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rectangle (ABCD), (AD = 3, AB = 10)</td>
</tr>
<tr>
<td>2. (\triangle ABC) and (\triangle BCD) are right triangles.</td>
</tr>
<tr>
<td>3. (AC = \sqrt{3^2 + 10^2}) (BD = \sqrt{3^2 + 10^2})</td>
</tr>
<tr>
<td>4. (AC = BD)</td>
</tr>
</tbody>
</table>


25a. \(2x - 7 = \frac{1}{3}x - 2\) 25b. \(3(x - 7) = 3\left(\frac{1}{3}x - 2\right)\)

25c. Dist. Prop. 25d. \(5x - 21 = -6\) 25e. Add. Prop. 25f. \(x = 3\)

27. Given: \(-2y + \frac{3}{2} = 8\)

Prove: \(y = -\frac{15}{4}\)

Proof:

<table>
<thead>
<tr>
<th>Statement Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (-2y + \frac{3}{2} = 8)</td>
</tr>
<tr>
<td>2. ((-2y + \frac{3}{2}) = 2(8))</td>
</tr>
<tr>
<td>5. (y = -\frac{13}{4})</td>
</tr>
</tbody>
</table>

29. Given: \(5 - \frac{2}{3}z = 1\)

Prove: \(z = 6\)

Proof:

<table>
<thead>
<tr>
<th>Statement Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (5 - \frac{2}{3}z = 1)</td>
</tr>
<tr>
<td>2. (3(5 - \frac{2}{3}z) = 3(1))</td>
</tr>
<tr>
<td>3. (15 - 2x = 3)</td>
</tr>
<tr>
<td>5. (-2x = -12)</td>
</tr>
<tr>
<td>6. (-\frac{2x}{-2} = -\frac{12}{-2})</td>
</tr>
<tr>
<td>7. (x = 6)</td>
</tr>
</tbody>
</table>
31. Given: \( m\angle ACB = m\angle ABC \)
Prove: \( m\angle XCA = m\angle YBA \)

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m\angle ACB = m\angle ABC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle XCA + m\angle ACB = 180 )</td>
<td>2. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>3. ( m\angle XCA + m\angle ACB = 180 )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. ( m\angle XCA + m\angle ACB = m\angle YBA + m\angle ABC )</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( m\angle XCA = m\angle YBA )</td>
<td>5. Subt. Prop.</td>
</tr>
</tbody>
</table>

33. All of the angle measures would be equal. 35. See students’ work. 37. B 39. 6 41. Invalid; 27 ÷ 6 = 4.5, which is not an integer. 43. Sample answer: If people are happy, then they rarely correct their faults. 45. Sample answer: If a person is a champion, then the person is afraid of losing. 47. \( \frac{1}{2} \) ft 49. 0.5 in. 51. 11 53. 47

Page 100 Practice Quiz 2

1. invalid 3. If two lines intersect, then their intersection is exactly one point.

5. Given: \( 2(n - 3) + 5 = 3(n - 1) \)
Prove: \( n = 2 \)

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 2(n - 3) + 5 = 3(n - 1) )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( 2n - 6 + 5 = 3n - 3 )</td>
<td>2. Dist. Prop.</td>
</tr>
<tr>
<td>3. ( 2n - 1 = 3n - 3 )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. ( 2n - 1 - 2n = 3n - 3 - 2n )</td>
<td>4. Subt. Prop.</td>
</tr>
<tr>
<td>5. ( -1 = n - 3 )</td>
<td>5. Substitution</td>
</tr>
<tr>
<td>6. ( -1 + 3 = n - 3 + 3 )</td>
<td>6. Add. Prop.</td>
</tr>
<tr>
<td>7. ( 2 = n )</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>8. ( n = 2 )</td>
<td>8. Symmetric Prop.</td>
</tr>
</tbody>
</table>

Pages 103–106 Lesson 2-7

1. Sample answer: The distance from Cleveland to Chicago is the same as the distance from Cleveland to Chicago.
3. If \( A, B, \) and \( C \) are collinear and \( AB + BC = AC \), then \( B \) is between \( A \) and \( C \). 5. Symmetric

7. Given: \( PQ \parallel RS, QS \parallel ST \)
Prove: \( PS \parallel RT \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( PQ \parallel RS, QS \parallel ST )</td>
<td>a. Given</td>
</tr>
<tr>
<td>b. ( PQ = RS, QS = ST )</td>
<td>b. Def. of ( \parallel ) segments</td>
</tr>
<tr>
<td>c. ( PS = PQ + QS, RT = RS + ST )</td>
<td>c. Segment Addition Post.</td>
</tr>
<tr>
<td>d. ( PQ + QS = RS + ST )</td>
<td>d. Addition Property</td>
</tr>
<tr>
<td>e. ( PS = RT )</td>
<td>e. Substitution</td>
</tr>
<tr>
<td>f. ( PS \parallel RT )</td>
<td>f. Def. of ( \parallel ) segments</td>
</tr>
</tbody>
</table>

9. Given: \( HI \equiv TU, HJ \equiv TV \)
Prove: \( IJ \equiv UV \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( HI \equiv TU, HJ \equiv TV )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( HI + IJ = HJ )</td>
<td>2. Seg. Add. Post.</td>
</tr>
<tr>
<td>3. ( TU + IJ = TV )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>6. ( TU + IJ = TU + UV )</td>
<td>6. Substitution</td>
</tr>
<tr>
<td>7. ( SU = ST )</td>
<td>7. Reflexive Prop.</td>
</tr>
<tr>
<td>8. ( IJ = UV )</td>
<td>8. Subt. Prop.</td>
</tr>
<tr>
<td>9. ( IJ \equiv UV )</td>
<td>9. Def. of ( \equiv ) segments</td>
</tr>
</tbody>
</table>

11. Helena is between Missoula and Miles City.

19. Given: \( XY \equiv WZ \) and \( WZ \equiv AB \)
Prove: \( XY \equiv AB \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( XY \equiv WZ ) and ( WZ \equiv AB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( XY = WZ ) and ( WZ = AB )</td>
<td>2. Def. of ( \equiv ) segments</td>
</tr>
<tr>
<td>3. ( XY = AB )</td>
<td>3. Transitive Prop.</td>
</tr>
<tr>
<td>4. ( XY \equiv AB )</td>
<td>4. Def. of ( \equiv ) segments</td>
</tr>
</tbody>
</table>

21. Given: \( WY \equiv ZX \)
A is the midpoint of \( WY \).
A is the midpoint of \( ZX \).

Prove: \( WA \equivZA \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( WY = ZX )</td>
<td>a. Given</td>
</tr>
<tr>
<td>a. ( WA = WY )</td>
<td>b. Def. of ( \equiv ) segs.</td>
</tr>
<tr>
<td>a. ( WY = ZX )</td>
<td>c. Definition of midpoint</td>
</tr>
<tr>
<td>a. ( ZX = ZA + AX )</td>
<td>d. Segment Addition Post.</td>
</tr>
<tr>
<td>a. ( WA + ZA = ZA + AX )</td>
<td>e. Substitution</td>
</tr>
<tr>
<td>a. ( WA + WA = ZA + ZA )</td>
<td>f. Substitution</td>
</tr>
<tr>
<td>a. ( 2WA = 2ZA )</td>
<td>g. Substitution</td>
</tr>
<tr>
<td>a. ( WA = ZA )</td>
<td>h. Division Property</td>
</tr>
<tr>
<td>a. ( WA \equiv ZA )</td>
<td>i. Def. of ( \equiv ) segs.</td>
</tr>
</tbody>
</table>

23. Given: \( AB = BC \)
Prove: \( AC = 2BC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB = BC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>3. ( AC = BC + BC )</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. ( AC = 2BC )</td>
<td>4. Substitution</td>
</tr>
</tbody>
</table>
25. Given: \( AB \equiv DE \), C is the midpoint of \( BD \).
Prove: \( AC \equiv CE \)

---

27. Sample answers: \( \overline{LN} \equiv \overline{QO} \) and \( \overline{LM} \equiv \overline{MN} \equiv \overline{RS} \equiv \overline{ST} \equiv \overline{QP} \equiv \overline{PO} \)
29. B 31. Substitution 33. Addition Property 35. Never; the midpoint of a segment divides it into two congruent segments. 37. Always; if two planes intersect, they intersect in a line. 39. 3; 9 cm by 13 cm
41. 15 43. 45 45. 25

---

33. Given: \( \ell \perp m \)
Prove: \( \angle 2, \angle 3, \) and \( \angle 4 \) are rt. \( \triangle \)

---

35. Given: \( \ell \perp m \)
Prove: \( \triangle 1 \equiv \triangle 2 \)

---

37. Given: \( \angle ABD \equiv \angle CBD \),
\( \angle ABD \) and \( \angle BDC \) form a linear pair.
Prove: \( \angle ABD \) and \( \angle CBD \) are rt. \( \triangle \).
41. Because the lines are perpendicular, the angles formed are right angles. All right angles are congruent. Therefore, \( \angle 1 \) is congruent to \( \angle 2 \). 43. Two angles that are supplementary to the same angle are congruent. Answers should include the following:

- \( \angle 1 \) and \( \angle 2 \) are supplementary; \( \angle 2 \) and \( \angle 3 \) are supplementary.
- \( \angle 1 \) and \( \angle 3 \) are vertical angles, and are therefore congruent.
- If two angles are complementary to the same angle, then the angles are congruent. 45. B

47. Given: \( X \) is the midpoint of \( WY \).
Prove: \( WX + YZ = XZ \)

Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( X ) is the midpoint of ( WY ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( WX = XY )</td>
<td>2. Def. of midpoint</td>
</tr>
<tr>
<td>3. ( XY + YZ = XZ )</td>
<td>3. Segment Addition</td>
</tr>
<tr>
<td>4. ( WX + YZ = XZ )</td>
<td>4. Substitution</td>
</tr>
</tbody>
</table>

49. \( \angle O M N, \angle M N R \) 51. \( N \) or \( R \) 53. obtuse
55. \( \angle N M L, \angle N M P, \angle N M O, \angle R M N, \angle O M N \)

13. In a right triangle with right angle \( C \), \( a^2 + b^2 = c^2 \) or the sum of the measures of two supplementary angles is 180; true. 15. \(-1 > 0 \), and in a right triangle with right angle \( C \), \( a^2 + b^2 = c^2 \), or the sum of the measures of two supplementary angles is 180; false. 17. In a right triangle with right angle \( C \), \( a^2 + b^2 = c^2 \) and the sum of the measures of two supplementary angles is 180, and \(-1 > 0 \); false. 19. Converse: If a month has 31 days, then it is March. False; July has 31 days. Inverse: If a month is not March, then it does not have 31 days. False; July has 31 days. Contrapositive: If a month does not have 31 days, then it is not March; true. 21. true 23. false 25. Valid; by definition, adjacent angles have a common vertex.

27. yes; Law of Detachment 29. yes; Law of Syllogism 31. Always; if \( P \) is the midpoint of \( XY \), then \( XP = PY \). By definition of congruent segments, \( XP = PY \).

33. Sometimes; if the points are collinear. 35. Sometimes; if the right angles form a linear pair. 37. Never; adjacent angles must share a common side, and vertical angles do not. 39. Distributive Property 41. Subtraction Property

43. Given: \( 5 = 2 - \frac{1}{2} \)
Prove: \( x = -6 \)

Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 5 = 2 - \frac{1}{2} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( 5 - 2 = 2 - \frac{1}{2} )</td>
<td>2. Subt. Prop.</td>
</tr>
<tr>
<td>3. ( 3 = -\frac{1}{2} )</td>
<td>3. Substitution</td>
</tr>
</tbody>
</table>

45. Given: \( AC = AB, AC = 4x + 1 \), \( AB = 6x - 13 \)
Prove: \( x = 7 \)

Proof:
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AC = AB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( 4x + 1 = 6x - 13 )</td>
<td>2. Substitution</td>
</tr>
<tr>
<td>3. ( 4x + 1 - 1 = 6x - 13 - 1 )</td>
<td>3. Subt. Prop.</td>
</tr>
<tr>
<td>4. ( 4x = 6x - 14 )</td>
<td>4. Substitution</td>
</tr>
<tr>
<td>5. ( 4x - 6x = 6x - 14 - 6x )</td>
<td>5. Subt. Prop.</td>
</tr>
<tr>
<td>6. (-2x = -14 )</td>
<td>6. Substitution</td>
</tr>
<tr>
<td>7. ( \frac{-2x}{-2} = -\frac{14}{-2} )</td>
<td>7. Div. Prop.</td>
</tr>
<tr>
<td>8. ( x = 7 )</td>
<td>8. Substitution</td>
</tr>
</tbody>
</table>

55. 145 57. 90

Chapter 3 Parallel and Perpendicular Lines

Page 125 Chapter 3 Getting Started
1. SP 3. ST 5. \( \angle 4, \angle 6, \angle 8 \) 7. \( \angle 1, \angle 5, \angle 7 \) 9. \( \frac{3}{2} \)

Pages 128–131 Lesson 3-1
1. Sample answer: The bottom and top of a cylinder are contained in parallel planes.
3. Sample answer: looking down railroad tracks
5. \( AB, JK, LM \) 7. \( q \) and \( r \), \( q \) and \( t \), \( r \) and \( t \)
9. \( p \) and \( r \), \( p \) and \( t \), \( r \) and \( t \)
11. alternate interior 13. consecutive interior 15. \( p \), consecutive interior 17. \( q \), alternate interior 19. Sample answer: The roof and the floor are parallel planes.
21. Sample answer: The top of the memorial "cuts" the pillars. 23. \( ABC, ABQ, PQS, QDS, APQ, DET \)
25. \( AP, AQ, CR, FU, Q_1, Q_2, RS, TS, CU \) 27. \( BC, CD, DE, EF, OR, RS, ST, TU \)
29. \( a \) and \( c \), \( a \) and \( r \), \( r \) and \( c \)
31. \( a \) and \( b \), \( a \) and \( c \), \( b \) and \( c \) 33. alternate exterior
35. corresponding 37. alternate interior 39. consecutive interior 41. \( p \), alternate interior 43. \( \ell \), alternate exterior
45. \( q \), alternate interior 47. \( m \), consecutive interior
49. \( CD, DH, EI \) 51. No; plane \( ADE \) will intersect all the planes if they are extended. 53. infinite number
55. Sample answer: Parallel lines and planes are used in architecture to make structures that will be stable. Answers should include the following:
   - Opposite walls should form parallel planes; the floor may be parallel to the ceiling.
   - The plane that forms a stairway will not be parallel to some of the walls.

57. 16, 20, or 28

59. Given: \( PQ \parallel ZY, QR \parallel XY \)
Prove: \( PR \parallel XZ \)

Proof: Since \( PQ \parallel ZY \) and \( QR \parallel XY \), \( PQ = ZY \) and \( QR = XY \) by the definition of congruent segments. By the Addition Property, \( PQ + QR = ZY + XY \). Using the Segment Addition Postulate, \( PR = PQ + QR \) and \( XZ = XY + YZ \). By substitution, \( PR = XZ \). Because the measures are equal, \( PR \parallel XZ \) by the definition of congruent segments.

61. \( m \angle EFG \) is less than 90; Detachment

63. 8.25

65. 15.81

67. 10.20

69.

Pages 136–138 Lesson 3-2

1. Sometimes; if the transversal is perpendicular to the parallel lines, then \( \angle 1 \) and \( \angle 2 \) are right angles and are congruent.

3. 1

5. 110

7. 70

9. 55

11. \( x = 13, y = 6 \)

13. 67

15. 75

17. 105

19. 105

21. 43

23. 43

25. 137

27. 60

29. 70

31. 120

33. \( x = 34, y = \pm 5 \)

35. 113

37. \( x = 14, y = 11, z = 73 \)

39. (1) Given (2) Corresponding Angles Postulate (3) Vertical Angles Theorem (4) Transitive Property

41. Given: \( \ell \perp m, m \parallel n \)
Prove: \( \ell \perp n \)

Proof: Since \( \ell \perp m \), we know that \( \angle 1 \equiv \angle 2 \), because perpendicular lines form congruent right angles. Then by the Corresponding Angles Postulate, \( \angle 1 \equiv \angle 3 \) and \( \angle 2 \equiv \angle 4 \). By the definition of congruent angles, \( m \angle 1 = m \angle 2 \), \( m \angle 1 = m \angle 3 \), and \( m \angle 2 = m \angle 4 \). By substitution, \( m \angle 3 = m \angle 4 \). Because \( \angle 3 \) and \( \angle 4 \) form a congruent linear pair, they are right angles. By definition, \( \ell \perp n \).

43. \( 39^\circ \)

45. \( 2001 \)

47. \( y = \frac{1}{2}x - \frac{11}{2} \)

49. C

51. 131

53. 49

55. \( \frac{2}{3} \)

57. \( \frac{3}{8} \)

59. \( -\frac{4}{5} \)

Page 138 Practice Quiz 1

1. \( p \); alternate exterior

3. \( q \); alternate interior

5. 75

Pages 142–144 Lesson 3-3

1. horizontal; vertical

3. horizontal line, vertical line

5. \( -\frac{1}{2} \)

7. 2

9. parallel

13. (1500, –120) or (–1500, –120)

15. \( \frac{1}{7} \)

17. –5

19. \( p \); alternate exterior

21. neither

23. parallel

25. –3

27. 6

29. 6

31. undefined

53. 59

61. \( p \); alternate interior

63. \( H, I, \) and \( J \) are noncollinear.

65. \( R, S, \) and \( T \) are collinear.

67. obtuse

69. obtuse

71. \( y = -\frac{1}{2}x - \frac{5}{4} \)

Pages 147–150 Lesson 3-4

1. Sample answer: Use the point-slope form where \( (x_1, y_1) = (-2, 8) \) and \( m = -\frac{2}{5} \).
3. Sample answer: $y = x$  

5. $y = -\frac{3}{5}x - 2$

7. $y + 1 = \frac{3}{2}(x - 4)$

9. $y - 137.5 = 1.25(x - 20)$

11. $y = -x + 2$

13. $y = 39.95, y = 0.95x + 4.95$

15. $y = \frac{1}{6}x - 4$

17. $y = \frac{5}{8}x - 6$

19. $y = -x - 3$

21. $y - 1 = 2(x - 3)$  
23. $y + 5 = -\frac{4}{5}(x + 12)$

25. $y - 17.12 = 0.48(x - 5)$  
27. $y = -3x - 2$

29. $y = 2x - 4$  
31. $y = -x + 5$  
33. $y = -\frac{1}{8}x$

35. $y = -\frac{3}{5}x + 3$

37. $y = -\frac{1}{5}x - 4$  
41. no slope-intercept form, $x = -6$

43. $y = \frac{2}{5}x - \frac{24}{5}$

45. $y = 0.05x + 750$, where $x$ = total price of appliances sold  
47. $y = -750x + 10,800$

49. in 10 days  
51. $y = x - 180$

53. Sample answer: In the equation of a line, the $b$ value indicates the fixed rate, while the $mx$ value indicates charges based on usage. Answers should include the following:

- The fee for air time can be considered the slope of the equation.
- We can find where the equations intersect to see where the plans might be equal.

55. B  
57. undefined  
59. 58$

61. 75  
63. 73

65. Given: $AC = DF$, $AB = DE$  
Prove: $BC = EF$

Proof:

Statements | Reasons
---|---
1. $AC = DF, AB = DE$ | 1. Given
2. $AC = AB + BC$ | 2. Segment Addition
$DF = DE + EF$ | Postulate
3. $AB + BC = DE + EF$ | 3. Substitution Property
4. $BC = EF$ | 4. Subtraction Property

67. 26, 69  
69. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$, $\angle 3$ and $\angle 7$  
71. $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 5$

Page 150 Practice Quiz 2

1. neither  
3. $\frac{7}{2}$  
5. $\frac{5}{4}$  
7. $y = -\frac{4}{5}x + \frac{16}{5}$

9. $y + 8 = -\frac{1}{4}(x - 5)$

Pages 154–157 Lesson 3-5

1. Sample answer: Use a pair of alternate exterior $\angle$ that are $\equiv$ and cut by a transversal; show that a pair of consecutive interior $\angle$ are suppl.; show that alternate interior $\angle$ are $\equiv$; show two lines are $\perp$ to same line; show corresponding $\angle$ are $\equiv$.  
3. Sample answer: A basketball court has parallel lines, as does a newspaper. The edges should be equidistant along the entire line.  
5. $\ell \parallel m; \equiv$ alt. int. $\angle$  
7. $p \parallel q; \equiv$ alt. ext. $\angle$

9. 11.375  
11. The slope of $CD$ is $\frac{1}{8}$, and the slope of line $AB$ is $\frac{1}{7}$. The slopes are not equal, so the lines are not parallel.  
13. $a \parallel b; \equiv$ alt. int. $\angle$

15. $\ell \parallel m; \equiv$ corr. $\angle$  
17. $AE \parallel BF; \equiv$ corr. $\angle$

19. $AC \parallel EG; \equiv$ alt. int. $\angle$  
21. $HS \parallel JT; \equiv$ corr. $\angle$

23. $KN \parallel PR; \text{ suppl. cons. int. } \angle$

25. 1. Given  
2. Definition of perpendicular  
3. All rt. $\angle$ are $\equiv$.  
4. If corresponding $\angle$ are $\equiv$, then lines $\parallel$.

27. 15  
29. 8  
31. 21.6

33. Given: $\angle 4 \equiv \angle 6$  
Prove: $\ell \parallel m$

Proof: We know that $\angle 4 \equiv \angle 6$. Because $\angle 6$ and $\angle 7$ are vertical angles they are congruent. By the Transitive Property of Congruence, $\angle 4 \equiv \angle 7$. Since $\angle 4$ and $\angle 7$ are corresponding angles, they are congruent, $\ell \parallel m$.

35. Given: $\overline{AD} \perp \overline{CD}$  
$\angle 1 \equiv \angle 2$  
Prove: $\overline{BC} \perp \overline{CD}$

Proof:

Statements | Reasons
---|---
1. $\overline{AD} \perp \overline{CD}$, $\angle 1 \equiv \angle 2$ | 1. Given
2. $\overline{AD} \parallel \overline{BC}$ | 2. If alternate interior $\angle$ are $\equiv$, lines are $\parallel$.
3. $\overline{BC} \perp \overline{CD}$ | 3. Perpendicular Transversal Th.

37. Given: $\angle RSP \equiv \angle PQR$  
$\angle QRS$ and $\angle PQR$ are supplementary.  
Prove: $\overline{PS} \parallel \overline{QR}$

Proof:

Statements | Reasons
---|---
1. $\angle RSP \equiv \angle PQR$ | 1. Given
$\angle QRS$ and $\angle PQR$ are supplementary. | 2. Def. of $\equiv$ $\angle$
3. $m\angle RSP = m\angle PQR$ | 3. Def. of suppl. $\angle$
4. $m\angle QRS + m\angle PQR = 180$ | 4. Substitution
5. $m\angle QRS + m\angle RSP = 180$ | 5. Def. of suppl. $\angle$
6. $\overline{PS} \parallel \overline{QR}$ | 6. If consecutive interior $\angle$ are suppl., lines $\parallel$.

39. No, the slopes are not the same.  
41. The 10-yard lines will be parallel because they are all perpendicular to the sideline and two or more lines perpendicular to the same line are parallel.  
43. See students’ work.  
45. B

47. $y = 0.3x - 6$  
49. $y = -\frac{1}{2}x + \frac{19}{2}$  
51. $\frac{5}{4}$  
53. 1

55. undefined

<table>
<thead>
<tr>
<th>$p$</th>
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</tbody>
</table>
Selected Answers

Pages 162–164 Lesson 3-6
1. Construct a perpendicular line between them.
2. Sample answer: Measure distances at different parts; compare slopes; measure angles. Finding slopes is the most readily available method.
3. Sample answer: We want new shelves to be parallel so they will line up. Answers should include the following:
   - After marking several points, a slope can be calculated, which should be the same slope as the original brace.
   - Building walls requires parallel lines.
   - After marking several points, a slope can be calculated, which should be the same slope as the original brace.
4. $\text{OP} = 2/3$ units; $\text{OQ} = 1 - y = 1 - 2/3 = 1/3$ units
5. $\text{PQ} = \sqrt{(x - 2)^2 + (y - 2)^2}$
6. $\text{PQ} = \sqrt{(x - 2)^2 + (y - 2)^2}$
7. $\text{PQ} = \sqrt{(x - 2)^2 + (y - 2)^2}$
8. $\text{PQ} = \sqrt{(x - 2)^2 + (y - 2)^2}$

Pages 167–170 Chapter 3 Study Guide and Review
1. alternate
2. parallel
3. alternate exterior
4. consecutive
5. interior
6. alternate exterior
7. corresponding
8. consecutive interior
9. alternate interior
10. 17
11. 127
12. $\text{AB} = 5$, $\text{BC} = 5$
13. $\text{AB} = 5$, $\text{BC} = 5$
14. $\text{AB} = 5$, $\text{BC} = 5$
15. $\text{AB} = 5$, $\text{BC} = 5$
16. $\text{AB} = 5$, $\text{BC} = 5$
17. $\text{AB} = 5$, $\text{BC} = 5$
18. $\text{AB} = 5$, $\text{BC} = 5$
19. $\text{AB} = 5$, $\text{BC} = 5$
20. $\text{AB} = 5$, $\text{BC} = 5$

Pages 177 Chapter 4 Getting Started
1. $-6\frac{1}{2}$
2. 1
3. $2\frac{3}{4}$
4. $7/3$
5. $\angle 2, \angle 12, \angle 15, \angle 6, \angle 9, \angle 3, \angle 13$
6. $\angle 6, \angle 9, \angle 3, \angle 13, \angle 2, \angle 8, \angle 12, \angle 15$
7. $\angle 11.2$
8. $\angle 14.6$

Pages 180–183 Lesson 4-1
1. Triangles are classified by sides and angles. For example, a triangle can have a right angle and have no two sides congruent. Always; equiangular triangles have three acute angles.
5. obtuse
7. $\triangle MJK, \triangle KLM, \triangle KJN, \triangle LMK$
9. $x = 4, JM = 3, MN = 3, JN = 2$
11. $TW = \sqrt{125}, WZ = \sqrt{125}$
13. right
15. acute
17. obtuse
19. equilateral, equiangular
21. isosceles, acute
23. $\triangle BAC, \triangle CDB$
25. $\triangle ABD, \triangle ACD, \triangle BAC, \triangle CDB$
27. $x = 5, MN = 9, MP = 9, NP = 9$
29. $x = 8, JL = 11, JK = 11, KL = 7$
31. Scalene; it is 184 miles from Lexington to Nashville, 265 miles from Cairo to Lexington, and 144 miles from Cairo to Nashville.
33. $\text{AB} = \sqrt{106}, \text{BC} = \sqrt{233}, \text{AC} = \sqrt{65};$ scalene
35. $\text{AB} = \sqrt{29}, \text{BC} = 4, \text{AC} = \sqrt{29};$ isosceles
37. $\text{AB} = \sqrt{124}, \text{BC} = \sqrt{124}, \text{AC} = 8; \text{isosceles}$
39. Given: $m\angle NPM = 33$
Prove: $\angle RPM$ is obtuse.

Proof: $\angle NPM$ and $\angle RPM$ form a linear pair. $\angle NPM$ and $\angle RPM$ are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle NPM + m\angle RPM = 180$. It is given that $m\angle NPM = 33$. By substitution, $33 + m\angle RPM = 180$. Subtract to find that $m\angle RPM = 147$. $\angle RPM$ is obtuse by definition. $\angle RPM$ is obtuse by definition.
41. $AD = \sqrt{(0 - a)^2 + (0 - b)^2}$  
$CD = \sqrt{(a - a)^2 + (0 - b)^2}$  
$= \sqrt{\frac{a^2}{4} + b^2}$  

$AD = CD$, so $\triangle ADC$ is isosceles by definition.

43. Sample answer: Triangles are used in construction as structural support. Answers should include the following:

- Triangles can be classified by sides and angles. If the measure of each angle is less than 90, the triangle is acute. If the measure of one angle is greater than 90, the triangle is obtuse. If one angle equals 90°, the triangle is right. If each angle has the same measure, the triangle is equilateral. If no two sides are congruent, the triangle is scalene. If at least two sides are congruent, it is isosceles. If all of the sides are congruent, the triangle is equilateral.

- Isosceles triangles seem to be used more often in architecture and construction.

45. B

49. 51. 44 53. any three: $\angle 2$ and $\angle 11$, $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 7$, $\angle 3$ and $\angle 12$, $\angle 7$ and $\angle 10$, $\angle 8$ and $\angle 11$ 55. $\angle 6$, $\angle 9$, and $\angle 12$

57. $\angle 2$, $\angle 5$, and $\angle 8$

46. $\sqrt{8}$

54. $\triangle MNO$

58. $\triangle M$ is a right angle.

Prove: There can be at most one right angle in a triangle.

Proof:
In $\triangle MNO$, $\angle M$ is a right angle. $m\angle M + m\angle N + m\angle O = 180$. If $m\angle N = 90$, then $m\angle M + m\angle O = 90$. If $\angle N$ were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.

48. $\triangle AED$

51. $\triangle BEC$

53. $\sqrt{20}$ units

55. $\sqrt{112}$ units

57. $x = 112$, $y = 28$, $z = 22$

59. reflexive 61. symmetric 63. transitive

47. A

50. $\triangle AED$

52. $\triangle ABC$

54. $\triangle MNO$

56. $\sqrt{13}$

58. $x = 112$, $y = 28$, $z = 22$

59. reflexive 61. symmetric 63. transitive
31. Given: \( \triangle RST \cong \triangle XYZ \)

Prove: \( \triangle XYZ \cong \triangle RST \)

Proof:

\[
\begin{align*}
\angle R &= \angle X, \angle S &= \angle Y, \angle T &= \angle Z, \\
RS &= XY, ST &= YZ, RT &= XZ
\end{align*}
\]

CPCTC

\[
\angle X = \angle R, \angle Y = \angle S, \angle Z = \angle T, \\
XY = RS, YZ = ST, XZ = RT
\]

Congruence of \( \triangle \) and segments is symmetric.

\( \triangle XYZ \cong \triangle RST \)

Def. of \( \triangle \)

33. Given: \( \triangle RST \cong \triangle XYZ \)

35. Given: \( \triangle DEF \)

Prove: \( \triangle DEF \cong \triangle DEF \)

Proof:

\[
\begin{align*}
DE &= DE, EF &= EF, \\
DF &= DF
\end{align*}
\]

Congruence of segments is reflexive.

\( \triangle DEF \cong \triangle DEF \)

Def. of \( \triangle \)

37. Sample answer: Triangles are used in bridge design for structure and support. Answers should include the following.

- The shape of the triangle does not matter.
- Some of the triangles used in the bridge supports seem to be congruent.

39. D 41. 58 43. \( x = 3, BC = 10, CD = 10, BD = 5 \)

45. \( y = -\frac{3}{2}x + 3 \) 47. \( y = -4x - 11 \) 49. \( \sqrt{5} \) 51. \( \sqrt{13} \)

Pages 203–206  Lesson 4-4

1. Sample answer: In \( \triangle QRS \), \( \angle R \) is the included angle of the sides \( QR \) and \( RS \).

3. \( EG = 2, MP = 2, FG = 4, NP = 4, EF = \sqrt{20}, \) and \( MN = \sqrt{20} \). The corresponding sides have the same measure and are congruent. \( \triangle EFG \cong \triangle MNP \) by SSS.

5. Given: \( DE \) and \( BC \) bisect each other

Prove: \( \triangle DGB \cong \triangle EGC \)

Proof: \( DE \) and \( BC \) bisect each other.

\[
\begin{align*}
DG &= GE, BG &= GC
\end{align*}
\]

Def. of bisector of segments

\( \triangle DGB \cong \triangle EGC \)

SAS

7. SAS

9. Given: \( T \) is the midpoint of \( \overline{SQ} \).

Prove: \( \triangle SRT \cong \triangle QRT \)

Proof:

Statements | Reasons
---|---
1. \( T \) is the midpoint of \( \overline{SQ} \). | 1. Given
2. \( ST = QT \). | 2. Midpoint Theorem
3. \( SR = QR \). | 3. Given
4. \( RT = RT \). | 4. Reflexive Property
5. \( \triangle SRT \cong \triangle QRT \). | 5. SSS

11. \( JK = \sqrt{10}, KL = \sqrt{10}, JL = \sqrt{20}, FG = \sqrt{2}, GH = \sqrt{50} \), and \( FH = 6 \). The corresponding sides are not congruent so \( \triangle JKL \) is not congruent to \( \triangle FGH \).

13. \( JK = \sqrt{10}, KL = \sqrt{10}, JL = \sqrt{20}, FG = \sqrt{2}, GH = \sqrt{10} \), and \( FH = 20 \). Each pair of corresponding sides have the same measure so they are congruent. \( \triangle JKL \equiv \triangle FGH \) by SSS.

15. Given: \( R \equiv T \equiv Q \equiv W \), \( \angle RQY \equiv \angle WQT \)

Prove: \( \triangle QWT \equiv \triangle QYR \)

Proof:

\[
\begin{align*}
\angle RQY &= \angle WQT
\end{align*}
\]

Given

\( \triangle QWT \equiv \triangle QYR \)

SAS

17. Given: \( \triangle MRN \equiv \triangle QRP \), \( \angle MNP \equiv \angle QPN \)

Prove: \( \triangle MNP \equiv \triangle QPN \)

Proof:

Statement | Reason
---|---
1. \( \triangle MRN \equiv \triangle QRP \), \( \angle MNP \equiv \angle QPN \) | 1. Given
2. \( MN \equiv QP \) | 2. CPCTC
3. \( NP \equiv NP \) | 3. Reflexive Property
4. \( \triangle MNP \equiv \triangle QPN \) | 4. SAS
19. Given: $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$

\[
\begin{array}{c|c}
\text{Statement} & \text{Reason} \\
1. \triangle GHJ \cong \triangle LKJ & 1. \text{Given} \\
2. HJ \cong KL, & 2. \text{CPCTC} \\
\text{GH} \cong LK, & \text{Def. of } \cong \text{ segments} \\
3. HJ = KL, GJ = LJ & 3. \text{Addition Property} \\
4. HJ + LJ = KL + JG & 4. \text{Segment Addition} \\
5. KJ + GJ = KG; & 5. \text{Reflexive Property} \\
HJ + LJ = HL & 6. \text{SSS} \\
6. KG = HL & 7. \text{Reflexive Property} \\
7. KG \equiv HL & 8. \text{SSS} \\
8. GL \equiv GL & 9. \text{SSS} \\
9. \triangle GHL \cong \triangle LKG & 10. \text{SSS}
\end{array}
\]

21. Given: $EF \cong HI$
Prove: $\triangle EFG \cong \triangle HFG$

\[
\begin{array}{c|c}
\text{Statements} & \text{Reasons} \\
1. EF \cong HI; G is the midpoint of EH. & 1. \text{Given} \\
2. EG \cong GH & 2. \text{Midpoint Theorem} \\
3. FG \cong FG & 3. \text{Reflexive Property} \\
4. \triangle EFG \cong \triangle HFG & 4. \text{SSS}
\end{array}
\]

23. not possible 25. SSS or SAS

27. Given: $TS \cong SF \cong FH \cong HT$
Prove: $\angle TSF, \angle SFH, \angle FHT,$ and $\angle HTS$ are right angles.

\[
\begin{array}{c|c}
\text{Statements} & \text{Reasons} \\
1. TS \cong SF \cong FH \cong HT & 1. \text{Given} \\
2. \angle TSF, \angle SFH, \angle FHT, \text{ and } \angle HTS \text{ are right angles.} & 2. \text{Given} \\
3. \angle STH \cong \angle SFH & 3. \text{All rt. } \angle \text{ are } \cong. \\
4. \angle STH \cong \angle SFH & 4. \text{SAS} \\
5. \angle STH \cong \angle SFH & 5. \text{CPCTC}
\end{array}
\]

29. Sample answer: The properties of congruent triangles help land surveyors double check measurements. Answers should include the following:

- If each pair of corresponding angles and sides are congruent, the triangles are congruent by definition. If two pairs of corresponding sides and the included angle are congruent, the triangles are congruent by SAS. If each pair of corresponding sides are congruent, the triangles are congruent by SSS.

- Sample answer: Architects also use congruent triangles when designing buildings.

31. B 33. $\triangle WXZ \cong \triangle YXZ$ 35. 78 37. 68 39. 59
41. $-1$ 43. There is a steeper rate of decline from the second quarter to the third.
45. $\angle CBD$ 47. $CD$

**Pages 210–213 Lesson 4-5**

1. Two triangles can have corresponding congruent angles without corresponding congruent sides. $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$. However, $\overline{AB} \not\cong \overline{DE}$, so $\triangle ABC \not\cong \triangle DEF$.

3. AAS can be proven using the Third Angle Theorem. Postulates are accepted as true without proof.

5. Given: $XW \parallelYZ$, $\angle X \cong \angle Z$
Prove: $\triangle WXY \cong \triangle YZW$

\[
\begin{array}{c}
\text{Proof:} \\
\text{Given: } XW \parallel YZ \\
\angle ZXY \cong \angle YZW \\
\text{Alt. int. } \angle \text{ are } \cong. \\
\angle WXZ \cong \angle YZW \\
\text{Reflexive Property} \quad \text{AAS}
\end{array}
\]

7. Given: $\angle E \cong \angle K$,
$\angle DGH \cong \angle DHG$,
$\angle GJ \cong \angle HJ$.

Prove: $\triangle EGD \cong \triangle KHD$

\[
\begin{array}{c}
\text{Proof:} \\
\text{Since } \angle EGD \text{ and } \angle DGH \text{ are a linear pair, the angles are supplementary. Likewise, } \angle KHD \text{ and } \angle DGH \text{ are supplementary. We are given that } \angle DGH \cong \angle DHG. \\
\angle \text{ supplementary to congruent angles are } \cong \text{ so } \angle EGD \cong \angle KHD. \text{ Since we are given that } \angle E \cong \angle K \text{ and } \angle GJ \cong \angle HJ, \triangle EGD \cong \triangle KHD \text{ by ASA.}
\end{array}
\]

9. Given: $EF \parallel GH$, $EF \cong GH$
Prove: $EK \cong KH$

\[
\begin{array}{c}
\text{Proof:} \\
\text{Given: } EF \parallel GH \\
\angle E \cong \angle H \\
\text{Alt. int. } \angle \text{ are } \cong. \\
\triangle EKF \cong \triangle HKG \\
\text{AAS} \\
\angle GKH \cong \angle EKF \\
\text{Vert. } \angle \text{ are } \cong. \\
\triangle EKF \cong \triangle HKG \\
\text{CPCTC}
\end{array}
\]

11. Given: $\angle V \cong \angle S$, $TV \cong QS$
Prove: $\angle 1 \cong \angle 2$

\[
\begin{array}{c}
\text{Proof:} \\
\text{Given: } \angle V \cong \angle S \\
\angle V \cong \angle S \\
\text{Alt. int. } \angle \text{ are } \cong. \\
\triangle TRV \cong \triangle QRS \\
\text{AAS} \\
\angle V \cong \angle S \\
\text{CPCTC}
\end{array}
\]
13. Given: $MN \equiv PQ$, $\angle M \equiv \angle Q$
$\angle 2 \equiv \angle 3$

Prove: $\triangle MLP \equiv \triangle QLN$

Proof:

$\begin{align*}
MN & \equiv PQ \\
MN & \equiv PQ \\
MN + NP &= NP + PQ \\
MP & \equiv NP \\
MP & \equiv NP \\
\angle M & \equiv \angle Q \\
\angle 2 & \equiv \angle 3
\end{align*}$

15. Given: $\angle NOM \equiv \angle POR$, $\angle M \equiv \angle R$, $\angle N \equiv \angle Q$
$\angle 1 \equiv \angle 2$

Prove: $\triangle BEA \equiv \triangle DAE$

Proof:

$\begin{align*}
DA & \equiv BE \\
BA & \equiv DE \\
\triangle BAE & \equiv \triangle DAE \\
AE & \equiv AE
\end{align*}$

17. Given: $\angle F \equiv \angle J$, $\angle E \equiv \angle H$, $\angle G \equiv \angle H$

Prove: $\angle F \equiv \angle J$

Proof: $\angle F \equiv \angle J$, $\angle E \equiv \angle H$, $\angle G \equiv \angle H$

19. Given: $\angle MYT \equiv \angle NYT$

Prove: $\angle RYM \equiv \angle RYN$

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
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<tbody>
<tr>
<td>1. $\angle MYT \equiv \angle NYT$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle MTY \equiv \angle NTY$</td>
<td>2. Reflexive Property</td>
</tr>
<tr>
<td>3. $\angle Y \equiv \angle Y$</td>
<td>3. ASA</td>
</tr>
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<td>4. $\angle MY \equiv \angle NY$</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>5. $\angle RYM \equiv \angle RYN$</td>
<td>5. Def. of linear pair</td>
</tr>
<tr>
<td>$\angle MYT \equiv \angle NYT$</td>
<td>$\angle MTY \equiv \angle NTY$</td>
</tr>
</tbody>
</table>

33. Given: $\triangle BAE \equiv \triangle DAE$

Prove: $\angle BA \equiv \angle BE$

Proof:

$\begin{align*}
DA & \equiv BE \\
BA & \equiv DE \\
\triangle BAE & \equiv \triangle DAE \\
AE & \equiv AE
\end{align*}$

35. Turn: $RS = \sqrt{2}$, $R'S' = \sqrt{2}$, $ST = 1$, $S'T' = 1$, $RT = 1$, $R'T' = 1$. Use a protractor to confirm that the corresponding angles are congruent.

7. Given: $\angle CTE$ is isosceles with vertex $\angle C$.

Prove: $\angle CTE$ is equilateral.

Proof:

$\begin{align*}
\angle T & \equiv \angle C \\
\angle E & \equiv \angle T \\
\angle E & \equiv \angle T \\
m\angle T & = 60
\end{align*}$
5. \( m \angle T = 60 \)
6. \( m \angle E = 60 \)
7. \( m \angle C + m \angle E + m \angle T = 180 \)
8. \( m \angle C + 60 + 60 = 180 \)
9. \( m \angle C = 60 \)
10. \( \triangle CTE \) is equiangular.
11. \( \triangle CTE \) is equilateral.
9. \( \angle LTR \equiv \angle LRT \)
10. \( \angle LSQ \equiv \angle LQS \)
11. \( LS \equiv LR \)
12. \( x = 3; y = 18 \)

29. Given: \( \triangle XKF \) is equilateral. 
Prove: \( J \) is the midpoint of \( KF \).

Proof:

Statements Reasons
1. \( \triangle XKF \) is equilateral. 1. Given
2. \( KX \equiv FX \)
3. \( \angle 1 \equiv \angle 2 \)
4. \( XJ \) bisects \( \angle X \)
5. \( \angle KXF \equiv \angle FXJ \)
6. \( \angle XKF \equiv \angle FXJ \)
7. \( KJ \equiv JF \)
8. \( J \) is the midpoint of \( KF \).

31. Case I:
Given: \( \triangle ABC \) is an equilateral triangle.
Prove: \( \triangle ABC \) is an equilateral triangle.

Proof:

Statements Reasons
1. \( \triangle ABC \) is an equilateral triangle. 1. Given
2. \( AB \equiv AC \equiv BC \)
3. \( \angle A \equiv \angle B \equiv \angle C \)
4. \( \angle A \equiv \angle B \equiv \angle C \)
5. \( \triangle ABC \) is an equiangular \( \triangle \).

Case II:
Given: \( \triangle ABC \) is an equiangular \( \triangle \).
Prove: \( \triangle ABC \) is an equilateral \( \triangle \).

Proof:

Statements Reasons
1. \( \triangle ABC \) is an equiangular triangle. 1. Given
2. \( \angle A \equiv \angle B \equiv \angle C \)
3. \( AB \equiv AC \equiv BC \)
4. \( AB \equiv AC \equiv BC \)
5. \( \triangle ABC \) is an equilateral \( \triangle \).
Given: \(BC(0, 0)\) is the midpoint of \(AH\).  
Prove: \(\triangle ABC\) is isosceles.

Proof: Use the Distance Formula to find \(AB\) and \(BC\).
\[
AB = \sqrt{(2 - 0)^2 + (8 - 0)^2} = \sqrt{4 + 64} = \sqrt{68} \\
BC = \sqrt{(4 - 2)^2 + (0 - 8)^2} = \sqrt{4 + 64} = \sqrt{68}
\]
Since \(AB = BC\), \(AB \equiv BC\). Since the legs are congruent, \(\triangle ABC\) is isosceles.

Selected Answers

29. Given: \(\triangle ABD, \triangle FBD\)  
Prove: \(\triangle ABD \equiv \triangle FBD\)

Proof: \(BD \equiv BD\) by the Reflexive Property. \(AD = \sqrt{(3 - 0)^2 + (1 - 1)^2} = \sqrt{9 + 0} = 3\)  
\(DF = \sqrt{(6 - 3)^2 + (1 - 1)^2} = \sqrt{9 + 0} = 3\)  
Since \(AD = DF\), \(AD \equiv DF\).
\[
AB = \sqrt{(3 - 0)^2 + (4 - 1)^2} = \sqrt{9 + 9} = 3\sqrt{2} \\
BF = \sqrt{(6 - 3)^2 + (1 - 4)^2} = \sqrt{9 + 9} = 3\sqrt{2}
\]
Since \(AB = BF\), \(AB \equiv BF\). \(\triangle ABD \equiv \triangle FBD\) by SSS.

31. Given: \(\triangle BPR, \triangle BAR\)
Prove: \(PR = 800, BR = 800, RA = 800\)

Proof: \(PB \equiv BA\)

\[
PB = \sqrt{(800 - 0)^2 + (800 - 0)^2} = \sqrt{1,280,000} \\
BA = \sqrt{(800 - 1600)^2 + (800 - 0)^2} = \sqrt{1,280,000} \\
PB = BA\), so \(PB \equiv BA\).

33. \(\sqrt{680,000}\) or about 824.6 ft  
35. \((2a, 0)\)  
37. \(AB = 4a\)  
\[AC = \sqrt{(0 - (2a))^2 + (2a - 0)^2} = \sqrt{4a^2 + 4a^2} = \sqrt{8a^2}\]  
\[CB = \sqrt{(2a - 0)^2 + (2a - 0)^2} = \sqrt{4a^2 + 4a^2} = \sqrt{8a^2}\]  
\[\text{Slope of } AC = \frac{-2a - 0}{0 - (2a)} = 1; \text{ slope of } CB = \frac{2a - 0}{0 - 2a}\] or \(-1\).

41. Given: \(\angle 3 \equiv \angle 4\)
Prove: \(QR \equiv QS\)

Proof:

Statements | Reasons
--- | ---
1. \(\angle 3 \equiv \angle 4\) | 1. Given
2. \(\angle 2\) and \(\angle 4\) form a linear pair, \(\angle 1\) and \(\angle 3\) form a linear pair. | 2. Def. of linear pair
3. \(\angle 2\) and \(\angle 4\) are supplementary, \(\angle 1\) and \(\angle 3\) are supplementary. | 3. If \(\angle 2\) form a linear pair, then they are suppl.
4. \(\angle 2 \equiv \angle 1\) | 4. Angles that are suppl. to \(\equiv \angle\) are \(\equiv\).
5. \(\text{Conv. of Isos. } \triangle\) Th. | 5. Conv. of Isos. \(\triangle\) Th.
43. Given: \( \overline{AD} \parallel \overline{CE}, \overline{AD} \parallel \overline{CE} \)
Prove: \( \triangle ABD \equiv \triangle EBC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AD} \parallel \overline{CE} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle A = \angle E, \angle D = \angle C )</td>
<td>2. Alt. int. ( \triangle ) are ( \equiv. )</td>
</tr>
<tr>
<td>3. ( \overline{AD} \parallel \overline{CE} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \triangle ABD \equiv \triangle EBC )</td>
<td>4. ASA</td>
</tr>
</tbody>
</table>

45. BC \( \parallel \overline{AD} \); if alt. int. \( \triangle \) are \( \equiv, \) lines are \( \parallel. \)

47. \( l \parallel m; \) if 2 lines are \( \parallel \) to the same line, they are \( \parallel. \)

49. \( BC \parallel \overline{AD}; \) if alt. int. \( \triangle \) are \( \equiv, \) lines are \( \parallel. \)

51. BC \( \parallel \overline{AD}; \) if alt. int. \( \triangle \) are \( \equiv, \) lines are \( \parallel. \)

Pages 227–230  Chapter 4  Study Guide and Review

1. h, d, a, b, 9. obtuse, isosceles
11. equiangular, equilateral
13. 25 15. \( \angle E = \angle D, \angle F = \angle C, \angle G = \angle B, \overline{EF} = \overline{DC}, \overline{FG} = \overline{CB}, \overline{GE} = \overline{BD} \)
17. \( \triangle KNC \equiv \triangle KRE, \triangle NKC \equiv \triangle KER, \triangle CKN \equiv \triangle EKR, \overline{CN} \equiv \overline{KE}, \overline{CK} \equiv \overline{ER}, \overline{KN} \equiv \overline{RK} \)
19. \( MN = \sqrt{20}, NP = \sqrt{5}, MP = 5, QR = \sqrt{20}, RS = \sqrt{5}, QS = 5. \) Each pair of corresponding sides has the same measure. Therefore, \( \triangle MNP \equiv \triangle QRS \) by SSS.

21. Given: \( \triangle DGC \equiv \triangle DGE, \angle GCF \equiv \angle GEF \)
Proof: \( \triangle DFC \equiv \triangle DFE \)

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle DGC \equiv \triangle DGE, \angle GCF \equiv \angle GEF )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle CDG \equiv \angle EDG, \overline{CD} \equiv \overline{ED}, \text{ and} \angle CFD \equiv \angle EFD )</td>
<td>2. CPCTC</td>
</tr>
<tr>
<td>3. ( \angle DFC \equiv \angle DFE )</td>
<td>3. AAS</td>
</tr>
</tbody>
</table>

23. \( 40 \)
25. \( 80 \)
27. \( (3m, n) \)

Chapter 5 Relationships in Triangles

Page 235  Chapter 5  Getting Started

1. \(-4, 5 \) 3. \((-0.5, -5) \) 5. 68 7. 40 9. 26 11. 14 13. The sum of the measures of the angles is 180.

Pages 242–245  Lesson 5-1

1. Sample answer: Both pass through the midpoint of a side. A perpendicular bisector is perpendicular to the side of a triangle, and does not necessarily pass through the vertex opposite the side, while a median does pass through the vertex and is not necessarily perpendicular to the side.
3. Sample answer: An altitude and angle bisector of a triangle are the same segment in an equilateral triangle.

5. Given: \( \overline{XY} \equiv \overline{XZ} \)
Prove: \( \overline{YM} \equiv \overline{ZN} \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{XY} \equiv \overline{XZ}, \overline{YM} ) and ( \overline{ZN} ) are medians.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( M ) is the midpoint of ( \overline{XZ} ). ( N ) is the midpoint of ( \overline{XY} ).</td>
<td>2. Def. of median</td>
</tr>
<tr>
<td>3. ( \overline{XY} \equiv \overline{XZ} )</td>
<td>3. Def. of ( \equiv. ) segs.</td>
</tr>
<tr>
<td>4. ( MZ \equiv MZ, \overline{XN} \equiv \overline{NY} )</td>
<td>4. Def. of median</td>
</tr>
<tr>
<td>5. ( MZ \equiv MZ, \overline{XN} \equiv \overline{NY} )</td>
<td>5. Def. of ( \equiv. ) segs.</td>
</tr>
<tr>
<td>7. Substitution</td>
<td>7. Substitution</td>
</tr>
<tr>
<td>10. Def. of ( \equiv. ) segs.</td>
<td>10. Division Property</td>
</tr>
<tr>
<td>12. Isosceles Triangle Theorem</td>
<td>12. Isosceles Triangle Theorem</td>
</tr>
<tr>
<td>14. SAS</td>
<td>14. SAS</td>
</tr>
<tr>
<td>15. CPCTC</td>
<td>15. CPCTC</td>
</tr>
</tbody>
</table>

31. Given: \( \overline{CA} \equiv \overline{CB}, \overline{AD} \equiv \overline{BD} \)
Prove: \( C \) and \( D \) are on the perpendicular bisector of \( \overline{AB}. \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
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</thead>
<tbody>
<tr>
<td>1. ( \overline{CA} \equiv \overline{CB}, \overline{AD} \equiv \overline{BD} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{CD} \equiv \overline{CD} )</td>
<td>2. Reflexive Property</td>
</tr>
<tr>
<td>3. ( \triangle ACD \equiv \triangle BCD )</td>
<td>3. SSS</td>
</tr>
<tr>
<td>5. ( \overline{CE} \equiv \overline{CE} )</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>6. ( \triangle CEA \equiv \triangle CEB )</td>
<td>5. Reflexive Property</td>
</tr>
<tr>
<td>6. SAS</td>
<td>6. SAS</td>
</tr>
</tbody>
</table>
7. \( \overline{AE} \cong \overline{BE} \)
8. \( E \) is the midpoint of \( \overline{AB} \).
9. \( \angle CEA \cong \angle CEB \)
10. \( \angle CEA \) and \( \angle CEB \) form a linear pair.
11. \( \angle CEA \) and \( \angle CEB \) are supplementary.
12. \( m \angle CEA + m \angle CEB = 180 \)
13. \( m \angle CEA + m \angle CEB = 180 \)
14. \( 2(m \angle CEA) = 180 \)
15. \( m \angle CEA = 90 \)
16. \( \angle CEA \) and \( \angle CEB \) are rt. \( \angle \).
17. \( CD \perp \overline{AB} \)
18. \( CD \) is the perpendicular bisector of \( \overline{AB} \).
19. \( C \) and \( D \) are on the perpendicular bisector of \( \overline{AB} \).

33. Given: \( \triangle ABC, \overline{AD}, \overline{BE}, \overline{CF}, \overline{KP} \perp \overline{AB}, \overline{KQ} \perp \overline{BC}, \overline{KR} \perp \overline{AC} \)
Prove: \( KP = KQ = KR \)

Proof:

<table>
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</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC, \overline{AD}, \overline{BE}, \overline{CF}, \overline{KP} \perp \overline{AB}, \overline{KQ} \perp \overline{BC}, \overline{KR} \perp \overline{AC} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( KP = KQ, KQ = KR, KP = KR )</td>
<td>2. Any point on the perpendicular bisector is equidistant from the sides of the angle.</td>
</tr>
</tbody>
</table>

35. 4

37. \( (0, 6), 12 \)

43. Sample answer:

45. Sample answer:

47. \( \angle 5 \cong \angle 11 \)
49. \( ML \parallel MN \)
51. >
53. >

Pages 251–254 Lesson 5-2

1. never
3. Grace; she placed the shorter side with the smaller angle, and the longer side with the larger angle.
5. \( \angle 3 \)
7. \( \angle 4 \), \( \angle 5 \), \( \angle 6 \)
9. \( \angle 2 \), \( \angle 3 \), \( \angle 5 \), \( \angle 6 \)
11. \( m \angle XZY < m \angle XYZ \)
13. \( AE < EB \)
15. \( BC = EC \)
17. \( \angle 1 \)
19. \( \angle 7 \)
21. \( \angle 7 \)
23. \( \angle 2 \), \( \angle 7 \), \( \angle 8 \), \( \angle 10 \)
25. \( \angle 3 \), \( \angle 5 \)
27. \( \angle 8 \), \( \angle 7 \), \( \angle 3 \), \( \angle 1 \)
29. \( m \angle KAJ < m \angle A[\overline{K} \overline{J}] \)
31. \( m \angle SMJ > m \angle MJS \)
33. \( m \angle MYJ < m \angle [\overline{M} \overline{J}] \)
35. Given: \( \overline{JM} \parallel \overline{KL} \), \( \overline{JL} \equiv \overline{KL} \)
Prove: \( m \angle 1 > m \angle 2 \)

Proof:

<table>
<thead>
<tr>
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<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{JM} \parallel \overline{KL} ), ( \overline{JL} \equiv \overline{KL} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Isosceles ( \triangle ) Theorem</td>
<td></td>
</tr>
<tr>
<td>3. Def. of ( \angle )</td>
<td></td>
</tr>
<tr>
<td>4. Ext. ( \angle ) Inequality Theorem</td>
<td></td>
</tr>
<tr>
<td>5. Substitution</td>
<td></td>
</tr>
<tr>
<td>6. Ext. ( \angle ) Inequality Theorem</td>
<td></td>
</tr>
<tr>
<td>7. Trans. Prop. of Inequality</td>
<td></td>
</tr>
</tbody>
</table>

37. \( ZY > YR \)
39. \( RZ > SR \)
41. \( TY < ZY \)
43. \( \angle M, \angle L, \angle K \)
45. Phoenix to Atlanta, Des Moines to Phoenix, Atlanta to Des Moines
47. 5; \( PR, QR, PQ \)
49. 12; \( QR, PR, PQ \)
51. \( 2(y + 1) > \frac{x}{3}; y > \frac{x - 6}{6} \)
53. \( 3x + 15 > 4x + 7 > 0, \frac{7}{4} < x < 8 \)
55. \( A \)
57. \( (15, -6) \)
59. Yes; \( \frac{1}{3}(-3) = -1 \), and \( F \) is the midpoint of \( \overline{BD} \).
61. Label the midpoints of \( \overline{AB}, \overline{BC}, \) and \( \overline{CA} \) as \( E, F, \) and \( G \) respectively. Then the coordinates of \( E, F, \) and \( G \) are \( (\frac{a}{2}, 0), (\frac{a + b}{2}, \frac{c}{2}), \) and \( (\frac{b}{2}, \frac{c}{2}) \) respectively. The slope of \( \overline{AF} = \frac{c}{a + b} \) and the slope of \( \overline{AD} = \frac{c}{a + b} \), so \( D \) is on \( \overline{AF} \). The slope of \( \overline{BG} = \frac{c}{b - 2a} \) and the slope of \( \overline{BD} = \frac{c}{b - 2a} \), so \( D \) is on \( \overline{BG} \). The slope of \( \overline{CE} = \frac{2c}{2b - a} \) and the slope of \( \overline{CD} = \frac{2c}{2b - a} \), so \( D \) is on \( \overline{CE} \).
Since \( D \) is on \( \overline{AF}, \overline{BG}, \) and \( \overline{CE} \), it is the intersection point of the three segments.
63. \( \angle C \cong \angle R, \angle D \cong \angle S, \angle G \cong \angle W, \overline{CD} \parallel \overline{RS}, \overline{DG} \parallel \overline{SW}, \overline{CG} \parallel \overline{RW} \)
65. 9.5
67. false

Page 254 Practice Quiz 1
1. 5
3. never
5. sometimes
7. no triangle
9. \( m \angle Q = 56, m \angle R = 61, m \angle S = 63 \)

Pages 257–260 Lesson 5-3

1. If a statement is shown to be false, then its opposite must be true.
3. Sample answer: \( \triangle ABC \) is isosceles.
   Given: \( \triangle ABC; AB \neq BC; BC \neq AC; AB \neq AC \)
   Prove: \( \triangle ABC \) is scalene.
   Proof:
   Step 1: Assume \( \triangle ABC \) is not scalene.
   Case 1: \( \triangle ABC \) is isosceles.
   If \( \triangle ABC \) is isosceles, then \( AB = BC, BC = AC, \) or \( AB = AC \). This contradicts the given information, so \( \triangle ABC \) is not isosceles.
   Case 2: \( \triangle ABC \) is equilateral.
   In order for a triangle to be equilateral, it must also be isosceles, and Case 1 proved that \( \triangle ABC \) is not isosceles.
   Thus, \( \triangle ABC \) is not scalene. Therefore, \( \triangle ABC \) is scalene.
5. The lines are not parallel.
7. Given: \( a > 0 \)
Prove: \( \frac{1}{a} > 0 \)
Proof:
Step 1: Assume \( \frac{1}{a} \leq 0 \).
Step 2: \( \frac{1}{a} \leq 0; a \cdot \frac{1}{a} \leq 0 \cdot a, 1 \leq 0 \)
Step 3: The conclusion that \( 1 \leq 0 \) is false, so the assumption that \( \frac{1}{a} \leq 0 \) must be false. Therefore, \( \frac{1}{a} > 0 \).

9. Given: \( \triangle ABC \)
Prove: There can be no more than one obtuse angle in \( \triangle ABC \).
Proof:
Step 1: Assume that there can be more than one obtuse angle in \( \triangle ABC \).
Step 2: The measure of an obtuse angle is greater than 90, \( x > 90 \), so the measure of two obtuse angles is greater than 180, \( 2x > 180 \).
Step 3: The conclusion contradicts the fact that the sum of the angles of a triangle equals 180. Thus, there can be at most one obtuse angle in \( \triangle ABC \).

11. Given: \( \triangle ABC \) is a right triangle; \( \angle C \) is a right angle.
Prove: \( AB > BC \) and \( AB > AC \)
Proof:
Step 1: Assume that the hypotenuse of a right triangle is not the longest side. That is, \( AB < BC \) or \( AB < AC \).
Step 2: If \( AB < BC \), then \( m \angle C < m \angle A \). Since \( m \angle C = 90 \), \( m \angle A > 90 \).
So, \( m \angle C + m \angle A > 180 \). By the same reasoning, if \( AB < BC \), then \( m \angle C + m \angle B > 180 \).
Step 3: Both relationships contradict the fact that the sum of the measures of the angles of a triangle equals 180. Therefore, the hypotenuse must be the longest side of a right triangle.

13. \( \overline{PQ} \neq \overline{ST} \)
15. A number cannot be expressed as \( \frac{a}{b} \).
17. Points \( P, Q, \) and \( R \) are noncollinear.
19. Given: \( \frac{1}{a} < 0 \)
Prove: \( a \) is negative.
Proof:
Step 1: Assume \( a > 0 \). \( a \neq 0 \) since that would make \( \frac{1}{a} \) undefined.
Step 2: \( \frac{1}{a} < 0 \)
\( a \left( \frac{1}{a} \right) < 0 \cdot a \)
\( 1 < 0 \)
Step 3: \( 1 > 0 \), so the assumption must be false. Thus, \( a \) must be negative.

21. Given: \( \overline{PQ} \equiv \overline{PR} \)
\( \angle 1 \neq \angle 2 \)
Prove: \( \overline{PZ} \) is not a median of \( \triangle PQR \).
Proof:
Step 1: Assume \( \overline{PZ} \) is a median of \( \triangle PQR \).
Step 2: If \( \overline{PZ} \) is a median of \( \triangle PQR \), then \( Z \) is the midpoint of \( QR \), and \( \overline{QZ} \equiv \overline{RZ} \). \( \overline{PZ} \equiv \overline{PZ} \) by the Reflexive Property. \( \triangle PZQ \equiv \triangle PZR \) by SSS. \( \angle 1 \equiv \angle 2 \) by CPCTC.

Step 3: This conclusion contradicts the given fact \( \angle 1 \neq \angle 2 \). Thus, \( \overline{PZ} \) is not a median of \( \triangle PQR \).

23. Given: \( a > 0, b > 0, \) and \( a > b \)
Prove: \( \frac{a}{b} > 1 \)
Proof:
Step 1: Assume that \( \frac{a}{b} \leq 1 \).
Step 2: Case 1  Case 2
\( \frac{a}{b} < 1 \)  \( \frac{a}{b} = 1 \)
\( a < b \)  \( a = b \)
Step 3: The conclusion of both cases contradicts the given fact \( a > b \). Thus, \( \frac{a}{b} > 1 \).

25. Given: \( \triangle ABC \) and \( \triangle ABD \) are equilateral.
Prove: \( \triangle ACD \) is not equilateral.
Proof:
Step 1: Assume that \( \triangle BCD \) is an equilateral triangle.
Step 2: If \( \triangle BCD \) is an equilateral triangle, then \( BC \equiv \overline{CD} \equiv \overline{DB} \). Since \( \triangle ABC \) and \( \triangle ABD \) are equilateral triangles, \( AC \equiv \overline{AB} \equiv \overline{BC} \) and \( AD \equiv \overline{AB} \equiv \overline{DB} \). By the Transitive Property, \( AC \equiv \overline{AD} \equiv \overline{CD} \). Therefore, \( \triangle ACD \) is an equilateral triangle.
Step 3: This conclusion contradicts the given information. Thus, the assumption is false. Therefore, \( \triangle BCD \) is not an equilateral triangle.

27. Use \( r = \frac{d}{t} \), \( t = 3 \), and \( d = 175 \).
Proof:
Step 1: Assume that Ramon’s average speed was greater than or equal to 60 miles per hour, \( r \geq 60 \).
Step 2: Case 1  Case 2
\( r = 60 \)  \( r > 60 \)
\( 60 \leq \frac{175}{3} \)  \( 175 \leq \frac{3}{2} \)
\( 58.3 \leq 60 \)
\( 58.3 > 60 \)
Step 3: The conclusions are false, so the assumption must be false. Therefore, Ramon’s average speed was less than 60 miles per hour.

29. \( 1500 \cdot 15\% = 225 \)
2500 \( \cdot 0.15 = 375 \)
225 = 225
31. Yes; if you assume the client was at the scene of the crime, it is contradicted by his presence in Chicago at that time. Thus, the assumption that he was present at the crime is false.
33. Proof:
Step 1: Assume that \( \sqrt{2} \) is a rational number.
Step 2: If \( \sqrt{2} \) is a rational number, it can be written as \( \frac{a}{b} \), where \( a \) and \( b \) are integers with no common factors, and \( b \neq 0 \). If \( \sqrt{2} = \frac{a}{b} \), then \( 2 = \frac{a^2}{b^2} \), and \( 2b^2 = a^2 \). Thus \( a^2 \) is an even number, as is \( a \). Because \( a \) is even it can be written as \( 2n \).
\( 2b^2 = a^2 \)
\( 2b^2 = (2n)^2 \)
\( 4n^2 \)
\( b^2 = 2n^2 \)
Thus, \( b^2 \) is an even number. So, \( b \) is also an even number.
Step 3: Because \( b \) and \( a \) are both even numbers, they have a common factor of 2. This contradicts the definition of rational numbers. Therefore, \( \sqrt{2} \) is not rational.
35. \( D \) 37. \( \angle P \)

39. Given: \( CD \) is an angle bisector. \( CD \) is an altitude.

Prove: \( \triangle ABC \) is isosceles.

Proof:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1. ( CD ) is an angle bisector. ( CD ) is an altitude.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle ACD \equiv \angle BCD )</td>
<td>2. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>3. ( CD \perp AB )</td>
<td>3. Def. of altitude</td>
</tr>
<tr>
<td>4. ( \angle ACD ) and ( \angle BCD ) are rt. ( \angle )s.</td>
<td>4. ( \perp ) lines form 4 rt. ( \angle )s.</td>
</tr>
<tr>
<td>5. ( \angle ACD \equiv \angle BCD )</td>
<td>5. All rt. ( \angle )s are ( \equiv ).</td>
</tr>
<tr>
<td>6. ( \triangle ABC \equiv \triangle DBC )</td>
<td>6. Reflexive Prop.</td>
</tr>
<tr>
<td>7. ASA</td>
<td>7. ASA</td>
</tr>
<tr>
<td>8. ( AC \equiv BC )</td>
<td>8. CPCTC</td>
</tr>
<tr>
<td>9. ( \triangle ABC ) is isosceles.</td>
<td>9. Def. of isosceles ( \triangle )</td>
</tr>
</tbody>
</table>

41. Given: \( \triangle ABC \equiv \triangle DEF; BG \) is an angle bisector of \( \angle ABC \). \( EH \) is an angle bisector of \( \angle DEF \).

Prove: \( BG \equiv EH \)

Proof:

<table>
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</tr>
<tr>
<td>2. ( \angle A \equiv \angle D, AB \equiv DE, \angle ABC \equiv \angle DEF )</td>
<td>2. Def. of ( \equiv ) segment</td>
</tr>
<tr>
<td>3. ( BG ) is an angle bisector of ( \triangle ABC ). ( EH ) is an angle bisector of ( \triangle DEF ).</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle ABG \equiv \angle GBC, \angle DEH \equiv \angle HEF )</td>
<td>4. Def. of ( \angle ) bisector</td>
</tr>
<tr>
<td>5. ( m\angle ABC = m\angle DEF )</td>
<td>5. Def. of ( \equiv \ \angle )</td>
</tr>
<tr>
<td>6. ( m\angle ABG = m\angle GBC, m\angle DEH = m\angle HEF )</td>
<td>6. Def. of ( \equiv \ \angle )</td>
</tr>
<tr>
<td>7. Angle Addition Property</td>
<td>7. Angle Addition Property</td>
</tr>
<tr>
<td>8. Substitution</td>
<td>8. Substitution</td>
</tr>
<tr>
<td>10. ( 2m\angle ABG = 2m\angle DEH )</td>
<td>10. Addition</td>
</tr>
<tr>
<td>11. ( m\angle ABG = m\angle DEH )</td>
<td>11. Division</td>
</tr>
<tr>
<td>12. ( \angle ABG \equiv \angle DEH )</td>
<td>12. Def. of ( \equiv \ \angle )</td>
</tr>
<tr>
<td>13. ( \angle ABC \equiv \angle DEF )</td>
<td>13. ASA</td>
</tr>
<tr>
<td>14. CPCTC</td>
<td>14. CPCTC</td>
</tr>
</tbody>
</table>

43. \( y - 3 = 2(x - 4) \)

45. \( y + 9 = 11(x + 4) \)

47. False

Pages 263–266 Lesson 5-4

1. Sample answer: If the lines are not horizontal, then the segment connecting their \( y \)-intercepts is not perpendicular to either line. Since distance is measured along a perpendicular segment, this segment cannot be used.

3. Sample answer:

\[
\begin{align*}
2, 3, 4 \text{ and } 1, 2, 3; \\
5, 6 & \text{ no; } 5 + 10 > 15 \quad \text{7. yes; } 5.2 + 5.6 > 10.1 \\
9 & \text{ no; } 13 + 16 > 29 \quad \text{17. no; } 17 + 30 > 30 \\
\end{align*}
\]

5. \( n \); 8 must be false. Therefore, \( x \) must be false. Therefore, \( x > 8 \).

5. Given: \( m\angle ADC \neq m\angle ADB \)

Prove: \( \overline{AD} \) is not an altitude of \( \triangle ABC \).

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AD} ) is an altitude of ( \triangle ABC ).</td>
<td>1. Assumption</td>
</tr>
<tr>
<td>2. ( \angle ADC ) and ( \angle ADB ) are right angles.</td>
<td>2. Def. of altitude</td>
</tr>
<tr>
<td>3. ( m\angle ADC = m\angle ADB )</td>
<td>3. All rt ( \angle )s are ( \equiv ).</td>
</tr>
<tr>
<td>4. ( m\angle ADC = m\angle ADB )</td>
<td>4. Def. of ( \equiv ) angles</td>
</tr>
</tbody>
</table>

This contradicts the given information that \( m\angle ADC \neq m\angle ADB \). Thus, \( \overline{AD} \) is not an altitude of \( \triangle ABC \).

7. \( n \); 25 + 35 \neq 60 \quad \text{9. yes; } 5 + 6 > 10

Pages 270–273 Lesson 5-5

1. Sample answer: A pair of scissors illustrates the SSS inequality. As the distance between the tips of the scissors decreases, the angle between the blades decreases, allowing the blades to cut.

3. \( AB < CD \)

5. \( \frac{7}{3} < x < 6 \)

7. Given: \( \overline{PQ} \equiv \overline{SQ} \)

Prove: \( PR > SR \)
21. Given: \( PQ = RS \), \( QR < PS \)
Prove: \( m \angle 3 < m \angle 1 \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( PQ = RS )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( QS = QS )</td>
<td>2. Reflexive Property</td>
</tr>
<tr>
<td>3. ( QR &lt; PS )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( m \angle 3 &lt; m \angle 1 )</td>
<td>4. SSS Inequality</td>
</tr>
</tbody>
</table>

23. Given: \( ED \parallel DF; m \angle 1 > m \angle 2; D \) is the midpoint of \( CB; AE \parallel AF \)
Prove: \( AC > AB \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( ED \parallel DF; D ) is the midpoint of ( DB )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( CD = BD )</td>
<td>2. Def. of midpoint</td>
</tr>
<tr>
<td>3. ( CD = BD )</td>
<td>3. Def. of ( \equiv ) segments</td>
</tr>
<tr>
<td>4. ( m \angle 1 &gt; m \angle 2 )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( EC &gt; FB )</td>
<td>5. SAS Inequality</td>
</tr>
<tr>
<td>6. ( AE \parallel AF )</td>
<td>6. Given</td>
</tr>
<tr>
<td>7. ( AE = AF )</td>
<td>7. Def. of ( \equiv ) segments</td>
</tr>
<tr>
<td>8. ( AE + EC &gt; AE + FB )</td>
<td>8. Add. Prop. of Inequality</td>
</tr>
<tr>
<td>9. ( AE + EC &gt; AF + FB )</td>
<td>9. Substitution Prop. of Inequality</td>
</tr>
<tr>
<td>10. ( AE + EC = AC ), ( AF + FB = AB )</td>
<td>10. Segment Add. Post.</td>
</tr>
<tr>
<td>11. ( AC &gt; AB )</td>
<td>11. Substitution</td>
</tr>
</tbody>
</table>

25. As the door is opened wider, the angle formed increases and the distance from the end of the door to the door frame increases.

27. As the vertex angle increases, the base angles decrease. Thus, as the base angles decrease, the altitude of the triangle decreases.

29. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Stride (m)</td>
<td>Velocity (m/s)</td>
</tr>
<tr>
<td>0.25</td>
<td>0.07</td>
</tr>
<tr>
<td>0.50</td>
<td>0.22</td>
</tr>
<tr>
<td>0.75</td>
<td>0.43</td>
</tr>
<tr>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>1.25</td>
<td>1.01</td>
</tr>
<tr>
<td>1.50</td>
<td>1.37</td>
</tr>
</tbody>
</table>

31. Sample answer: A backhoe digs when the angle between the two arms decreases and the shovel moves through the dirt. Answers should include the following.
- As the operator digs, the angle between the arms decreases.
- The distance between the ends of the arms increases as the angle between the arms increases, and decreases as the angle decreases.

39. Given: \( AD \) bisects \( BE; \) \( AB \parallel DE \).

Prove: \( \triangle ABC \equiv \triangle DEC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AD ) bisects ( BE; AB \parallel DE )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Def. of seg. bisector</td>
<td>2. Def. of seg. bisector</td>
</tr>
<tr>
<td>3. Alt. int. ( \angle ) Thm.</td>
<td>3. Alt. int. ( \angle ) Thm.</td>
</tr>
<tr>
<td>4. ( \triangle BCA \equiv \triangle ECD )</td>
<td>4. Vert. ( \angle ) are ( \equiv )</td>
</tr>
<tr>
<td>5. ( \triangle ABC \equiv \triangle DEC )</td>
<td>5. ASA</td>
</tr>
</tbody>
</table>

41. \( EF = 5, FG = 50, EG = 5; \) isosceles 43. \( EF = \sqrt{145}, FG = \sqrt{544}, EG = 35; \) scalene 45. yes, by the Law of Detachment

Chapter 6 Proportions and Similarity

Page 281  Chapter 6 Getting Started

<table>
<thead>
<tr>
<th>1. 15 3. 10 5. 2 7. (-\frac{6}{5} ) 9. yes; ( \equiv ) alt. int. ( \angle )</th>
<th>11. 2, 4, 8, 16 13. 1, 7, 25, 79</th>
</tr>
</thead>
</table>

Page 284–287 Lesson 6-1

1. Cross multiply and divide by 28. 3. Suki; Madeline did not find the cross products correctly. 5. \( \frac{1}{12} \) 7. 2.1275 9. 54, 48, 42 13. 130 13. 76:89 15. 25:3:1 17. 18 ft, 24 ft 19. 43.2, 64.8, 72 21. 18 in., 24 in., 30 in. 23. \( \frac{3}{2} \) 25. 2:19 27. 16.4 lb 29. 1.295 31. 14 33. 3 35. \(-1, -\frac{2}{3} \) 37. 36%

39. Sample answer: It appears that Tiffany used rectangles with areas that were in proportion as a background for this artwork. Answers should include the following.
- The center column pieces are to the third column from the left pieces as the pieces from the third column are to the pieces in the outside column.
- The dimensions are approximately 24 inches by 34 inches.
53. Yes; 100 km and 62 mi are the same length, so \( AB = CD \). By the definition of congruent segments, \( \overline{AB} \equiv \overline{CD} \). 55. 13.0
57. 1.2

**Page 292–297 Lesson 6-2**

1. Both students are correct. One student has inverted the ratio and reversed the order of the comparison. 3. If two polygons are congruent, then they are similar. All of the corresponding angles are congruent, and the ratio of measures of the corresponding sides is 1. Two similar figures have congruent angles, and the sides are in proportion, but not always congruent. If the scale factor is 1, then the figures are congruent. 5. Yes; \( \angle A \equiv \angle E \), \( \angle B \equiv \angle F \), \( \angle C \equiv \angle G \), \( \angle D \equiv \angle H \) and \( \frac{AB}{EF} = \frac{BC}{GH} = \frac{CD}{GF} = \frac{DE}{EH} = \frac{3}{2} \). So \( \triangle ABCD \sim \triangle EFGH \). 7. Polygon \( ABCD \sim \) polygon \( EFGH \); 23; 28; 20; 32; \( \frac{1}{2} \). 9. 60 m 11. \( ABCF \) is similar to \( EDCF \) since they are congruent. 13. \( \triangle ABC \) is not similar to \( \triangle DEF \). \( \angle A \not\equiv \angle D \). 15. \( \frac{1}{3} \) 17. polygon \( ABCD \sim \) polygon \( EFGH \); \( \frac{13}{3} \); \( AB = \frac{16}{3} \), \( CD = \frac{10}{3} \). 19. \( \triangle ABE \sim \triangle ACD \); 6; \( BC = 8; ED = 5 \). \( \frac{5}{9} \). 21. about 3.9 in. by 6.25 in. 23. \( \frac{25}{16} \)

25. \( \frac{5}{2} \text{ in.} \)

27. always 29. never 31. sometimes 33. always 35. 30; 70 37. 27; 14 39. 71.05; 48.45 41. 7; 5 43. 108 45. 73.2 47. \( \frac{8}{5} \)

61. Sample answer: Artists use geometric shapes in patterns to create another scene or picture. The included objects have the same shape but are different sizes. Answers should include the following:

• The objects are enclosed within a circle. The objects seem to go on and on
• Each “ring” of figures has images that are approximately the same width, but vary in number and design.

65. D

67. \( \frac{AB}{A'B'} = \frac{AC}{AC'} = \frac{BC}{B'C} = \frac{1}{2} \)

69. The sides are proportional and the angles are congruent, so the triangles are similar.

71. \( -23 \) 73. \( OC > AO \)

75. \( m \angle ABD > m \angle ADB \)

77. 91 79. \( m \angle 1 = m \angle 2 = 111 \). 81. 62 83. 118 85. 62 87. 118

Page 301–306 Lesson 6-3

1. Sample answer: Two triangles are congruent by the SSS, SAS, and ASA Postulates and the AAS Theorem. In these triangles, corresponding parts must be congruent. Two triangles are similar by AA Similarity, SSS Similarity, and SAS Similarity. In similar triangles, the sides are proportional and the angles are congruent. Congruent triangles are always similar triangles. Similar triangles are congruent only when the scale factor for the proportional sides is 1. SSS and SAS are common relationships for both congruent and similar. 3. Alicia; while both have corresponding sides in a ratio, Alicia has them in proper order with the numerators from the same triangle.

5. \( \triangle ABC \sim \triangle DEF \); \( x = 10; AB = 10; DE = 6 \). 7. Yes: \( \triangle DEF \sim \triangle ACB \) by SSS Similarity 9. 135 ft 11. Yes: \( \triangle QRS \sim \triangle TVU \) by SSS Similarity 13. Yes; \( \triangle RST \sim \triangle JKL \) by AA Similarity 15. Yes; \( \triangle ABC \sim \triangle JKL \) by SAS Similarity 17. No; sides are not proportional.

19. \( \triangle ABE \sim \triangle ACD \); \( x = \frac{8}{5}; AB = \frac{33}{5}; AC = \frac{93}{5} \)

21. \( \triangle ABC \sim \triangle ARS \); \( x = 8; 15; \frac{3}{2} \) 25. true

27. \( \triangle EAB \sim \triangle EFC \sim \triangle AFD \) by AA Similarity

29. \( KP = 5; KM = 15; MR = 13 \frac{1}{3}; ML = 20; MN = 12 \)

31. \( m \angle TUV = 43, m \angle R = 43, m \angle RSU = 47, m \angle SUV = 47 \)

33. \( x = y \); if \( BD \parallel \overline{AE} \), then \( \triangle BCD \sim \triangle ACE \) by AA Similarity and \( \frac{BC}{AC} = \frac{DC}{EC} \). Thus, \( \frac{2}{4} = \frac{x}{x + y} \). Cross multiply and solve for \( y \), yielding \( y = x \).

35. Given: \( \overline{LP} \parallel \overline{MN} \)

Prove: \( \frac{LJ}{JN} = \frac{PI}{JM} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{LP} \parallel \overline{MN} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle PLN \equiv \angle LNM, \angle LPM \equiv \angle PMN )</td>
<td>2. Alt. Int. ( \angle ) Theorem</td>
</tr>
<tr>
<td>3. ( \triangle LPJ \sim \triangle NMJ )</td>
<td>3. AA Similarity</td>
</tr>
<tr>
<td>4. ( \frac{LJ}{JN} = \frac{PI}{JM} )</td>
<td>4. Corr. sides of ( \sim ) s are proportional.</td>
</tr>
</tbody>
</table>

37. Given: \( \triangle ABC \) and \( \triangle EDF \) are right triangles. \( \frac{AB}{AC} = \frac{DE}{DF} \)

Prove: \( \triangle ABC \sim \triangle DEF \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC ) and ( \triangle EDF ) are right triangles.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle ABC ) and ( \angle EDF ) are right angles.</td>
<td>2. Def. of rt. ( \triangle )</td>
</tr>
</tbody>
</table>
| 3. \( \triangle ABC \equiv \angle EDF \) | 3. All rt. \( \triangle \) s are \( \equiv \).
| 4. \( \frac{AB}{DE} = \frac{AC}{DF} \) | 4. Given |
| 5. \( \triangle ABC \sim \triangle DEF \) | 5. SAS Similarity |

39. 13.5 ft 41. about 420.5 m 43. 10.75 m
45. Sample answer: City planners use maps in their work. Answers should include the following:
- City planners need to know geometry facts when developing zoning laws.
- A city planner would need to know that the shortest distance between two parallel lines is the perpendicular distance.

47. $\triangle ABC \sim \triangle ACD$; $\triangle ABC \sim \triangle CBD$; $\triangle ACD \sim \triangle CBD$; they are similar by AA Similarity.

49. A

51. $PQRS \sim ABCD$; 1.6; 1.4; 1; 1.2

53. 5

55. 15

57. No; $\overline{AT}$ is not perpendicular to $\overline{BC}$.

59. (5.5, 13)

61. (3.5, −2.5)
35. Given: $\overline{IF}$ bisects $\angle EFG$.

Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$

Proof:

1. $\overline{IF}$ bisects $\angle EFG$, $EH \parallel FG$, $EF \parallel HG$
2. $\angle EFK = \angle KFG$
3. $\angle KFG = \angle JKH$
4. $\angle JKH = \angle EKF$
5. $\angle EFK = \angle EKF$
6. $\angle FJH = \angle EFK$
7. $\angle FJH = \angle EKF$
8. $\triangle EKF \sim \triangle GJF$
9. $\frac{EK}{KF} = \frac{GJ}{JF}$

37. Given: $\triangle RST \sim \triangle ABC$, $W$ and $D$ are midpoints of $\overline{TS}$ and $\overline{CB}$, respectively.

Prove: $\triangle RWS \sim \triangle ADB$

Proof:

1. $\triangle RST \sim \triangle ABC$
2. $\angle S = \angle B$
3. $W$ and $D$ are midpoints.
4. $\overline{RS} = \overline{TS}$
5. $\overline{AS} = \overline{CB}$
6. $\overline{RS} = 2\overline{WS}$
7. $\overline{AB} = 2\overline{BD}$
8. $\overline{RS} = \overline{WS}$
9. $\overline{BD}$
10. $\triangle RWS \sim \triangle ADB$

39. 12.9 41. no; sides not proportional 43. yes; $\frac{LM}{MO} = \frac{LN}{NP}$

45. $\triangle PQT \sim \triangle PRS$, $x = 7$, $PQ = 15$

47. $y = 2x + 1$

50. $320, 640$

51. $-27, -33$

Page 323 Practice Quiz 2

1. 20 3. no; sides not proportional 5. 12.75 7. 10.5 9. 5

Page 326–331 Lesson 6-6

1. Sample answer: irregular shape formed by iteration of self-similar shapes. Sample answer: icebergs, ferns, leaf veins 5. $A_n = 2(2^n - 1)$ 7. 1.4142...; 1.1892... 9. Yes, the procedure is repeated over and over again.

13. Yes, any part contains the same figure as the whole, 9 squares with the middle shaded. 15. 1, 3, 6, 10, 15...; Each difference is 1 more than the preceding difference.

17. The result is similar to a Stage 3 Sierpinski triangle. 19. 25

23. Yes; the smaller and smaller details of the shape have the same geometric characteristics as the original form.

25. $A_n = 4^n$; 65,536 27. Stage 0: 3 units, Stage 1: $3 \cdot \frac{4}{3}$ or 4 units, Stage 2: $3 \left(3 \cdot \frac{4}{3} \right) - 3 \cdot \frac{4}{3} = 3 \cdot \frac{4}{3}$ or 5 1 units, Stage 3: $3 \cdot \frac{4}{3}$ or 7 1 units 29. The original triangle and the new triangles are equilateral and thus, all of the angles are equal to 60. By AA Similarity, the triangles are similar. 31. 0.2, 5, 0.2, 5, 0.2; the numbers alternate between 0.2 and 5. 33. 1, 2, 4, 16, 65,536; the numbers approach positive infinity. 35. 0, -5, -10 37. -6, 24, -66 39. When $x = 0$: 0.64, 0.9216, 0.2890..., 0.8219..., 0.5854..., 0.9708..., 0.1133..., 0.4019..., 0.9615..., 0.1478...; when $x = 0.2$: 0.6423..., 0.9188..., 0.2981..., 0.8369..., 0.5458..., 0.9916..., 0.0333..., 0.1287..., 0.4487..., 0.9894... Yes, the initial value affected the tenth value. 41. The leaves in the tree and the branches of the trees are self-similar. These self-similar shapes are repeated throughout the painting. 43. See students’ work.

45. Sample answer: Fractal geometry can be found in the repeating patterns of nature. Answers should include the following:

- Broccoli is an example of fractal geometry because the shape of the florets is repeated throughout; one floret looks the same as the stalk.
- Sample answer: Scientists can use fractals to study the human body, rivers, and tributaries, and to model how landscapes change over time.

47. $C$ 49. $\frac{13}{5} 51. \frac{7}{3} 53. \frac{16}{14} 55$. Miami, Bermuda, San Juan 57. 10 ft, 10 ft, 17 ft, 17 ft
Chapter 7 Right Triangles and Trigonometry

Page 541 Chapter 7 Getting Started
1. $a = 16$ 3. $e = 24, f = 12$ 5. $x = 13$ 7. $21.21$ 9. $2\sqrt{2}$ 11. 15 13. 98 15. 23

Pages 345–348 Lesson 7-1
1. Sample answer: 2 and 72. 3. Ian; his proportion shows that the altitude is the geometric mean of the two segments of the hypotenuse. 5. 42 7. $2\sqrt{3} = 3.5$ 9. $4\sqrt{3} = 6.9$ 11. $x = 6; y = 4\sqrt{3}$ 13. $\sqrt{30} = 5.5$ 15. $2\sqrt{15} = 7.7$ 17. $\frac{\sqrt{15}}{5} = 0.8$ 19. $\frac{\sqrt{5}}{3} = 0.7$ 21. $3\sqrt{5} = 6.7$ 23. $8\sqrt{2} = 11.3$ 25. $\sqrt{26} = 5.1$ 27. $x = 2\sqrt{15} \approx 4.9$; $y = \sqrt{33} \approx 5.7$; $z = 2\sqrt{6} = 4.9$ 29. $x = \frac{40}{3}; y = \frac{5}{3}; z = 10\sqrt{2} = 14.1$ 31. $x = 6\sqrt{6} = 14.7$; $y = 6\sqrt{42} \approx 38.9$; $z = 36\sqrt{5} = 95.2$ 33. $\frac{17}{7}$ 35. never 37. sometimes 39. $\triangle FGH$ is a right triangle. $\overline{OG}$ is the altitude from the vertex of the right angle to the hypotenuse of that triangle. So, by Theorem 7.2, $\overline{OG}$ is the geometric mean between $\overline{OF}$ and $\overline{OH}$, and so on. 41. 2.4 yd 43. yes; Indiana and Virginia

45. Given: $\angle QPR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$.

Prove: $\triangle PSQ \sim \triangle PQR$  $	riangle PSQ \sim \triangle QSR$

Proof:

\begin{align*}
\text{Statements} & \quad \text{Reasons} \\
1. \angle QPR \text{ is a right angle.} & \quad 1. \text{Given} \\
2. \overline{QS} \perp \overline{RP} & \quad 2. \text{Definition of altitude} \\
3. \angle 1 \text{ and } \angle 2 \text{ are right angles.} & \quad 3. \text{Definition of perpendicular lines} \\
4. \angle 1 \equiv \angle PQR & \quad 4. \text{All right } \angle \text{s are } \equiv. \\
5. \angle 2 \equiv \angle PQR & \quad 5. \text{Congruence of angles is reflexive.} \\
6. \triangle PSQ \sim \triangle PQR & \quad 6. \text{AA Similarity} \\
7. \triangle PSQ \sim \triangle QSR & \quad \text{Statements 4 and 5} \\
7. \angle 2 \equiv \angle PQR & \quad 7. \text{Similarity of triangles is transitive.} \\
\end{align*}

47. Given: $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.

Prove: $\frac{AB}{AD} = \frac{AD}{AC}$  $\frac{BC}{DC} = \frac{DC}{AC}$

Proof:

\begin{align*}
\text{Statements} & \quad \text{Reasons} \\
1. \angle ADC \text{ is a right angle.} & \quad 1. \text{Given} \\
2. \overline{DB} \text{ is an altitude of } \triangle ADC. & \quad 2. \text{Definition of right triangle} \\
3. \triangle ABD \sim \triangle ADC & \quad 3. \text{If the altitude is drawn from the vertex of the } \angle \text{ to the hypotenuse of a } \triangle, \text{ then the 2 } \triangle s \text{ formed are similar to the given } \triangle \text{ and to each other.} \\
4. \triangle ABD \sim \triangle ADC & \quad 4. \text{Definition of similar polygons} \\
\end{align*}
2. \((CB)^2 + (AC)^2 = (AB)^2\)
3. \(\left| x_2 - x_1 \right| = CB\)
   \(\left| y_2 - y_1 \right| = AC\)
4. \(x_2 - x_1 = a\)
   \(y_2 - y_1 = d\)
5. \((x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2\)
6. \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d\)
7. \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

2. Pythagorean Theorem
3. Distance on a number line
4. Substitution
5. Substitution
6. Take square root of each side.
7. Reflexive Property

41. about 76.53 ft  43. about 13.4 mi  45. Sample answer: The road, the tower that is perpendicular to the road, and the cables form the right triangles. Answers should include the following.
   • Right triangles are formed by the bridge, the towers, and the cables.
   • The cable is the hypotenuse in each triangle.

47. C   49. yes  51. 6\(V\sqrt{3} \approx 10.4\)  53. 3\(\sqrt{6} \approx 7.3\)
55. \(\sqrt{10} \approx 3.2\)  57. 3; approaches positive infinity.
59. 0.25; alternates between 0.25 and 4.
61. \(\frac{7\sqrt{3}}{3}\)  63. \(\sqrt{7}\)
65. 12\(\sqrt{2}\)  67. 2\(\sqrt{2}\)  69. \(\frac{\sqrt{2}}{2}\)

Pages 360–363 Lesson 7-3
1. Sample answer: Construct two perpendicular lines. Use a ruler to measure 3 cm from the point of intersection on the one ray. Use the compass to copy the 3 cm segment.
   Connect the two endpoints to form a 45°-45°-90° triangle with sides of 3 cm and a hypotenuse of 3\(\sqrt{2}\) cm.
   3. The length of the rectangle is \(\sqrt{3}\) times the width; \(\ell = \sqrt{3}w\).
5. \(x = 5\sqrt{2}; y = 5\sqrt{2}\)
7. \(a = 4; b = 4\sqrt{3}\)

9.

11. 90\(\sqrt{2}\) or 127.28 ft  13. \(x = \frac{17\sqrt{2}}{2}; y = 45\)  15. \(x = 8\sqrt{3}\);
   \(y = 8\sqrt{3}\)
17. \(x = 5\sqrt{2}; y = \frac{5\sqrt{2}}{2}\)
19. \(a = 14\sqrt{3}; CE = 21\)
\(y = 21\sqrt{3}; b = 42\)  21. 7.5\(\sqrt{3}\) cm \(\approx 12.99\) cm
23. 14.8\(\sqrt{3}\) m \(\approx 25.63\) m  25. 8\(\sqrt{2}\) \(\approx 11.31\)  27. (4, 8)
29. \((-3 - \frac{13\sqrt{3}}{3}, -6)\) or about \((-10.51, -6)\)
31. \(a = 3\sqrt{3}, b = 9, c = 3\sqrt{3}, d = 9\)
33. 30° angle
35. Sample answer:
37. \(BH = 16\)
39. 12\(\sqrt{3}\) \(\approx 20.78\) cm
41. 52 + 4\(\sqrt{3}\) + 4\(\sqrt{6}\)
   units  43. C   45. yes, yes
47. no, no  49. yes, yes
51. 2\(\sqrt{21}\) \(\approx 9.2, 21; 25\)
53. \(\frac{40}{5}, \frac{5}{3}, 10\sqrt{2} \approx 14.1\)
55. \(\angle ALK < \angle NLO\)  57. \(\angle KLO = \angle ALN\)  59. 15

Pages 367–370 Lesson 7-4
1. The triangles are similar, so the ratios remain the same.
3. All three ratios involve two sides of a right triangle. The sine ratio is the measure of the opposite leg divided by the measure of the hypotenuse. The cosine ratio is the measure of the adjacent leg divided by the measure of the hypotenuse. The tangent ratio is the measure of the opposite leg divided by the measure of the adjacent leg.
5. \(\frac{14}{50} = 0.28, \frac{48}{50} = 0.96, \frac{14}{29} = 0.49, \frac{48}{50} = 0.96, \frac{14}{50} = 0.28; \frac{48}{14} \approx 3.43\)
7. 0.8387  9. 0.8387  11. 1.0000  13. \(\angle A \approx 54.8\)
15. \(\angle A \approx 33.7\)  17. 2997 ft  19. \(\frac{\sqrt{3}}{3} \approx 0.58; \frac{\sqrt{6}}{3} \approx 0.82; \frac{\sqrt{2}}{2} \approx 0.71; \frac{\sqrt{6}}{3} \approx 0.82; \frac{\sqrt{3}}{3} \approx 0.58; \frac{\sqrt{2}}{2} \approx 1.41\)
21. \(\frac{2}{3} \approx 0.67; \frac{2\sqrt{2}}{3} \approx 0.75; \frac{2\sqrt{3}}{5} \approx 0.89; \frac{\sqrt{6}}{3} \approx 0.75; \frac{2}{3} \approx 0.67; \frac{\sqrt{5}}{2} \approx 1.12\)
23. 0.9260  25. 0.9974  27. 0.2393
29. \(\frac{5}{8} = 0.625\)  30. 3\(\sqrt{2}/6 \approx 0.9806\)  33. \(\frac{1}{5} = 0.2000\)
35. \(\frac{26}{26} \approx 0.1961\)  37. 46.4  39. 84.0  41. 83.0
43. \(x = 8.5\)  45. \(x = 28.2\)  47. \(x = 22.6\)  49. 4.1 mi
51. about 5.18 ft  53. about 54.5  55. about 47.9 in.
57. \(x = 17.1, y = 23.4\)  59. about 272,837 astronomical units
61. \(2\sqrt{5}/3\)  63. C  65. \(\csc A = \frac{5}{3}; \sec A = \frac{5}{4}\)
67. \(\csc A = 2; \sec A = \frac{2\sqrt{3}}{3}; \csc B = \frac{2\sqrt{3}}{3}; \sec B = \frac{2\sqrt{3}}{3}\)
   \(\csc C = \frac{2\sqrt{3}}{3}; \sec C = \frac{2\sqrt{3}}{3}\)
69. \(b = 4\sqrt{3}, c = 8\)  71. \(a = 2.5, b = 2.5\sqrt{3}\)  73. yes, yes
75. no, no  77. 117  79. 150
81. 63

Pages 373–376 Lesson 7-5
1. Sample answer: \(\angle ABC\)
3. The angle of depression is \(\angle FPB\) and the angle of elevation is \(\angle TBPE\).
5. 22°  7. 706 ft  9. about 173.2 yd  11. about 5.3°
13. about 118.2 yd  15. about 4°  17. about 40.2°
19. 100 ft, 300 ft
21. about 8.3 in.  23. no  25. About 5.1 mi
27. Answers should include the following.
   • Pilots use angles of elevation when they are ascending and angles of depression when descending.
   • Angles of elevation are formed when a person looks upward and angles of depression are formed when a person looks downward.
29. \(A = 31.0, 30.8, 70.0\)  35. 19.5  37. 14\(\sqrt{3}\)  28
39. 31.2 cm  41. 5  43. 34  45. 52  47. 3.75

Pages 380–383 Lesson 7-6
1. Felipe; Makayla is using the definition of the sine ratio for a right triangle, but this is not a right triangle. 3. In one case you need the measures of two sides and the measure of an angle opposite one of the sides. In the other
case you need the measures of two angles and the measure of a side. 5. 13.1 7. 55 9. \(m \angle R = 19, m \angle Q = 56\), \(q \approx 27.5\) 11. \(m \angle R = 43, m \angle R = 17, r \approx 9.5\) 13. \(m \angle P = 37, p \approx 11.1, m \angle R = 32\) 15. about 237 feet 17. 2.7 19. 29 21. 29 23. \(m \angle X = 25.6, m \angle W = 58.4, w \approx 20.3\) 25. \(m \angle X = 19.3, m \angle W = 48.7, w \approx 45.4\) 27. \(m \angle X = 82, x \approx 5.2, y \approx 4.7\) 29. \(m \angle X = 49.6, m \angle Y = 42.4, y \approx 14.2\) 31. 56.9 units 33. about 14.9 mi, about 13.6 mi 35. about 536 ft 37. about 1000.7 m 39. about 13.6 mi 41. Sample answer: Triangles are used to determine distances in space. Answers should include the following.

- The VLA is one of the world’s premier astronomical radio observatories. It is used to make pictures from the radio waves emitted by astronomical objects.
- Triangles are used in the construction of the antennas.

43. A 45. about 5.97 ft 47. \(20, 29 \approx 0.69; 22, 29 \approx 0.72; 20, 21 \approx 0.95; \frac{21}{22}, \frac{20}{29} \approx 0.72, \frac{20}{21} \approx 0.95; \frac{\sqrt{2}}{2} \approx 0.71; \frac{\sqrt{2}}{2} \approx 1.00; \frac{\sqrt{2}}{2} \approx 0.71; \frac{\sqrt{2}}{2} \approx 1.00 \) 51. 54 53. \(\frac{13}{112}, \frac{11}{80} \) 57. \(\frac{7}{15} \)

Pages 383 Chapter 7 Practice Quiz 2 1. 58.0 3. 53.2 5. \(m \angle D = 41, m \angle E = 57, e \approx 10.2\)

Pages 387–390 Lesson 7-7

1. Sample answer: Use the Law of Cosines when you have all three sides given (SSS) or two sides and the included angle (SAS).

3. If two angles and one side are given, then the Law of Cosines cannot be used. 5. 159.7 7. 98 9. \(\ell \approx 17.9\); \(m \angle K = 55; m \angle M = 78\) 11. \(u \approx 4.9\) 13. \(t \approx 22.5\) 15. 16 17. 36 19. \(m \angle H = 31; m \angle G = 109; g \approx 14.7\) 21. \(m \angle B = 86; m \angle C = 56; m \angle D = 38\) 23. \(c \approx 6.3; m \angle A = 80; m \angle B = 63\) 25. \(m \angle B = 99; b \approx 31.3; a \approx 25.3\) 27. \(m \angle M = 18.6; m \angle N = 138.4; n \approx 91.8\) 29. \(\ell \approx 21.1; m \angle M = 42.8; m \angle N = 88.2\) 31. \(m \angle L = 101.9; m \angle M = 36.3; m \angle N = 41.8\) 33. \(m \angle L = 6.0; m \angle P \approx 22.2; m \angle N \approx 130.8\) 35. \(m \angle M \approx 18.5; m \angle L \approx 40.9; m \angle N \approx 79.1\) 37. \(m \angle N \approx 42.8; m \approx 86.2; m \approx 51.4\) 39. 561.2 units 41. 59.8, 63.4, 56.8 43a. Pythagorean Theorem 43b. Substitution 43c. Pythagorean Theorem 43d. Substitution 43e. Def. of cosine 43f. Cross products 43g. Substitution 43h. Computative Property 45. Sample answer: Triangles are used to build supports, walls, and foundations. Answers should include the following.

- The triangular building was more efficient with the cells around the edge.
- The Law of Sines requires two angles and a side or two sides and an angle opposite one of those sides.

47. C 49. 33 51. yes 53. no

55. Given: \(\triangle JFM \sim \triangle EFB\) \(\triangle LFM \sim \triangle AGB\)

Prove: \(\triangle JFL \sim \triangle EFG\)

Proof:

Since \(\triangle JFM \sim \triangle EFB\) and \(\triangle LFM \sim \triangle AGB\), then by the definition of similar triangles, \(\frac{EF}{BF} = \frac{MF}{LF}\) and \(\frac{MF}{LF} = \frac{LF}{GF}\). By the Transitive Property of Equality, \(\frac{EF}{BF} = \frac{MF}{LF}\). \(\triangle \equiv \triangle F\) by the Reflexive Property of Congruence. Then, by SAS Similarity, \(\triangle JFL \sim \triangle EFG\).

Page 403 Chapter 8 Getting Started

1. 130 3. 120 5. \(\frac{1}{6}; -6\); perpendicular 7. \(\frac{4}{3} - \frac{3}{4}\) perpendicular 9. \(\frac{a}{b}\)

Pages 407–409 Lesson 8-1

1. A concave polygon has at least one obtuse angle, which means the sum will be different from the formula.

3. Sample answer:

regular quadrilateral, 360°
quadrilateral that is not regular, 360°

47. B 49. 92.1 51. 50.1 53. \(m \angle G = 67, m \angle H = 60, h \approx 16.1\) 55. \(m \angle F = 57, f \approx 63.7, h \approx 70.0\)

57. Given: \(\parallel JK\), \(\parallel LM\)

Prove: \(\triangle JKL \equiv \triangle MLK\)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\parallel JK), (\parallel LM)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (\angle MKL \equiv \angle JLK)</td>
<td>2. Alt. int. (\angle) are (\equiv)</td>
</tr>
<tr>
<td>3. (\angle JKL \equiv \angle MLK)</td>
<td>3. Reflexive Property</td>
</tr>
<tr>
<td>4. (\angle JKL \equiv \triangle MLK)</td>
<td>4. ASA</td>
</tr>
<tr>
<td>5. (m \angle JKL \equiv \triangle MLK)</td>
<td>63. (\angle 3) and (\angle 5, \angle 2) and (\angle 6) 65. none</td>
</tr>
</tbody>
</table>
13. Given: \( \square VZRQ \) and \( \square WQST \)
Prove: \( \angle Z \equiv \angle T \)

Proof:
\[
\begin{align*}
1. & \quad \square VZRQ \text{ and } \square WQST & \text{Given} \\
2. & \quad \angle Z \equiv \angle Q, \quad \angle Q \equiv \angle T & \text{Opp. } \angle \text{ s of a } \square \equiv. \\
3. & \quad \angle Z \equiv \angle T & \text{Transitive Prop.}
\end{align*}
\]

15. C 17. \( \angle CDB \), alt. int. \( \angle \) are \( \equiv \). 19. \( \overline{CD} \), diag. of \( \square \) bisect each other. 21. \( \angle BAC \), alt. int. \( \angle \) are \( \equiv \). 23. 33

35. \( a \), \( b \) = 3, \( DB = 32 \). 37. \( EQ = 5 \), \( QG = 5 \), \( HQ = \sqrt{13} \). 39. Slope of \( EH \) is undefined, slope of \( EF = -\frac{1}{3} \); no, the slopes of the sides are not negative reciprocals of each other.

41. Given: \( \square PQRS \)
Prove: \( \overline{PQ} \equiv \overline{RS} \) \( \angle Z \equiv \angle T \)

Proof:
\[
\begin{align*}
1. & \quad \square PQRS & \text{Given} \\
2. & \quad \text{Draw an auxiliary segment } PR \text{ and label angles } 1, 2, 3, \text{ and } 4 \text{ as shown.} \\
3. & \quad \overline{PQ} \parallel \overline{SR}, \overline{PS} \parallel \overline{QR} & \text{SAS} \\
4. & \quad \angle 1 \equiv \angle 2, \quad \angle 3 \equiv \angle 4 & \text{Opp. sides of } \square \equiv. \\
5. & \quad \overline{PR} \equiv \overline{PS} & \text{Opp. } \angle \text{ s of } \square \equiv. \\
6. & \quad \angle QPR \equiv \angle SPR & \text{ASA} \\
7. & \quad \angle P \equiv \angle Q & \text{CPCTC}
\end{align*}
\]

45. Given: \( \square WXYZ \)
Prove: \( \triangle WXZ \equiv \triangle YZX \)

Proof:
\[
\begin{align*}
1. & \quad \square WXYZ & \text{Given} \\
2. & \quad \overline{WX} \equiv \overline{ZY}, \overline{WZ} \equiv \overline{XY} & \text{Opp. sides of } \square \equiv. \\
3. & \quad \angle WXZ \equiv \angle XZY & \text{Opp. } \angle \text{ s of } \square \equiv. \\
4. & \quad \triangle WXZ \equiv \triangle YZX & \text{SAS}
\end{align*}
\]

49. The graphic uses the illustration of wedges shaped like parallelograms to display the data. Answers should include the following:
- The opposite sides are parallel and congruent, the opposite angles are congruent, and the consecutive angles are supplementary.
- Sample answer:

51. B 53. 3600 55. 6120 57. Sines; \( m \angle C = 69.9 \), \( m \angle A = 53.1, a = 11.9 \). 59. 30 61. side, \( \frac{7}{3} \) 63. side, \( \frac{7}{3} \)

P445-446
Lesson 8-3
1. Both pairs of opposite sides are congruent; both pairs of opposite angles are congruent; diagonals bisect each other; one pair of opposite sides is parallel and congruent.
3. Shaniqua’s Carter’s description could result in a shape that is not a parallelogram. 5. Yes; each pair of opp. \( \angle \) is \( \equiv \). 7. \( x = 41, y = 16 \), 9. yes

11. Given: \( PT \equiv TR \)
Prove: \( \angle TSP \equiv \angle TQR \)

Proof:
\[
\begin{align*}
1. & \quad \overline{PT} \equiv \overline{TR} & \text{Given} \\
2. & \quad \angle TSP \equiv \angle RTQ & \text{Vertical } \angle \text{ s are } \equiv. \\
3. & \quad \overline{RT} \equiv \overline{RT} & \text{AAS} \\
4. & \quad \angle TSP \equiv \angle RTQ & \text{CPCTC} \\
5. & \quad \overline{PS} \equiv \overline{QR} & \text{If alt. int. } \angle \text{ s are } \equiv, \text{ lines are } \parallel. \\
6. & \quad \triangle TSP \equiv \triangle TQR & \text{If one pair of opp. sides is } \parallel \text{ and } \equiv, \text{ then the quad. is a } \square.
\end{align*}
\]
13. Yes; each pair of opposite angles is congruent. 15. Yes; opposite angles are congruent. 17. Yes; one pair of opposite sides is parallel and congruent. 19. $x = 6, y = 24$
21. $x = 1, y = 2$ 23. $x = 34, y = 44$ 25. yes 27. yes
29. no 31. yes 33. Move $M$ to $(-4, 1), N$ to $(-3, 4), P$ to $(0, -9), or R$ to $(-7, 3)$. 35. $(2, -2), (4, 10), or (10, 0)$
37. Parallelogram; $KM$ and $JL$ are diagonals that bisect each other.

39. Given: $AD \equiv BC$
   $AB \equiv DC$
Prove: $ABCD$ is a parallelogram.
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AD \equiv BC, AB \equiv DC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw $DB$.</td>
<td>2. Two points determine a line.</td>
</tr>
<tr>
<td>3. $DB \equiv DB$</td>
<td>3. Reflexive Property</td>
</tr>
<tr>
<td>4. $\triangle ABD \equiv \triangle CDB$</td>
<td>4. SSS</td>
</tr>
<tr>
<td>5. $\angle 1 \equiv \angle 2, \angle 3 \equiv \angle 4$</td>
<td>5. CPCTC</td>
</tr>
<tr>
<td>6. $AD \parallel BC, AB \parallel CD$</td>
<td>6. If alt. int. $\angle$ are $\equiv$, lines are $\parallel$.</td>
</tr>
<tr>
<td>7. $ABCD$ is a parallelogram.</td>
<td>7. Definition of parallelogram</td>
</tr>
</tbody>
</table>

41. Given: $\overline{AB} \equiv \overline{DC}$
   $\overline{AB} \parallel \overline{DC}$
Prove: $ABCD$ is a parallelogram.
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AB \equiv DC, AB \parallel DC$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw $\overline{AC}$</td>
<td>2. Two points determine a line.</td>
</tr>
<tr>
<td>3. $\angle 1 \equiv \angle 2$</td>
<td>3. Alternate Interior Angles Theorem</td>
</tr>
<tr>
<td>4. $\overline{AC} \equiv \overline{AC}$</td>
<td>4. Reflexive Property</td>
</tr>
<tr>
<td>5. $\triangle ABC \equiv \triangle CDA$</td>
<td>5. SAS</td>
</tr>
<tr>
<td>6. $AD \equiv BC$</td>
<td>6. CPCTC</td>
</tr>
<tr>
<td>7. $ABCD$ is a parallelogram.</td>
<td>7. If both pairs of opp. sides are $\equiv$, then the quad. is $\square$.</td>
</tr>
</tbody>
</table>

43. Given: $ABCDEF$ is a regular hexagon.
Prove: $FDCA$ is a parallelogram.

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $ABCDEF$ is a regular hexagon.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB \equiv DE, BC \equiv EF$</td>
<td>2. Def. of regular hexagon</td>
</tr>
<tr>
<td>$\angle E \equiv \angle B, FA \equiv CD$</td>
<td>3. SAS</td>
</tr>
<tr>
<td>3. $\triangle ABC \equiv \triangle DEF$</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>$\overline{AC} \equiv \overline{DF}$</td>
<td>5. If both pairs of opp. sides are $\equiv$, then the quad. is $\square$.</td>
</tr>
</tbody>
</table>

Pages 423–430 Lesson 8-4

1. If consecutive sides are perpendicular or diagonals are congruent, then the parallelogram is a rectangle.
3. McKenna; Consuelo’s definition is correct if one pair of opposite sides is parallel and congruent.
5. 40 72 or 10
9. Make sure that the angles measure 90° or that the diagonals are congruent. 11. 11 13. 15 14. 4 17. 60
19. 30 21. 60 23. 30
25. Measure the opposite sides and the diagonals to make sure they are congruent. 27. No; $\overline{DH}$ and $\overline{FG}$ are not parallel.
29. Yes; opp. sides are $\parallel$, diag. are $\equiv$. 31. $\frac{1}{2}, \frac{3}{2}$
33. Yes; consec. sides are $\perp$.
35. Move $L$ and $K$ until the length of the diagonals is the same.
37. See students’ work.
39. Sample answer: $\overline{AC} \equiv \overline{BD}$ but $ABCD$ is not a rectangle.

41. Given: $\square{WXYZ}$ and $\overline{WY} \equiv \overline{XZ}$
Prove: $WXYZ$ is a rectangle.
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\square{WXYZ}$ and $\overline{WY} \equiv \overline{XZ}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{XY} \equiv \overline{WZ}$</td>
<td>2. Opp. sides of $\square$ are $\equiv$.</td>
</tr>
<tr>
<td>3. $\overline{WX} \equiv \overline{WX}$</td>
<td>3. Reflexive Property</td>
</tr>
<tr>
<td>4. $\triangle WZX \equiv \triangle YXW$</td>
<td>4. SSS</td>
</tr>
<tr>
<td>5. $\angle ZXW$ and $\angle YXW$ are supplementary.</td>
<td>5. CPCTC</td>
</tr>
<tr>
<td>6. $\angle ZXW$ and $\angle YXW$ are right angles.</td>
<td>6. Consec. $\angle$ of $\square$ are suppl.</td>
</tr>
<tr>
<td>7. $\angle WZY$ and $\angle XYZ$ are right angles.</td>
<td>7. If $2$ $\angle$ are $\equiv$ and suppl, each $\angle$ is a rt. $\angle$.</td>
</tr>
<tr>
<td>8. $\square{WXYZ}$ is a rectangle.</td>
<td>8. If $\square$ has 1 rt. $\angle$, it has 4 rt. $\angle$.</td>
</tr>
</tbody>
</table>

43. Given: $DEAC$ and $FEAB$ are rectangles.
   $\angle GKH \equiv \angle JHK$; $\overline{GJ}$ and $\overline{HK}$ intersect at $L$.
Prove: $GHJK$ is a parallelogram.
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $DEAC$ and $FEAB$ are rectangles.</td>
<td>1. Given</td>
</tr>
<tr>
<td>$\angle GKH \equiv \angle JHK$</td>
<td>2. Def. of parallelogram</td>
</tr>
<tr>
<td>$\overline{GJ}$ and $\overline{HK}$ intersect at $L$.</td>
<td>3. Def. of parallel planes</td>
</tr>
<tr>
<td>$\overline{DE} \parallel \overline{AC}$ and $\overline{FE} \parallel \overline{AB}$</td>
<td>4. Def. of intersecting lines</td>
</tr>
<tr>
<td>$G, J, H, K, L$ are in the same plane.</td>
<td>5. Def. of parallel lines</td>
</tr>
<tr>
<td>$\overline{GJ}$ and $\overline{HJ}$</td>
<td>6. If alt. $\angle$ are $\equiv$, lines are $\parallel$.</td>
</tr>
<tr>
<td>$\overline{GH} \parallel \overline{KJ}$</td>
<td>7. Def. of parallelogram</td>
</tr>
</tbody>
</table>

Page 427–430 Lesson 8-4

45. B 47. 12 49. 14 units 51. 8 53. 30 55. 72 57. 45, $12\sqrt{2}$ 59. $16\sqrt{3}, 16$ 61. 5, $-\frac{3}{2}$ not $\perp$ 63. $\frac{2}{3}, -\frac{3}{2}, \perp$
45. No; there are no parallel lines in spherical geometry.
47. No; the sides are not parallel. 49. A 51. 31 53. 43
55. 49 57. 5 59. \( \sqrt{297} \approx 17.2 \) 61. 5 63. 29

**Pages 434–437 Lesson 8-5**

1. Sample answer:

<table>
<thead>
<tr>
<th>Square</th>
<th>Rectangle (( \ell ) 1 right ( \angle ) )</th>
<th>Rhombus (( \square ) with 4 ( \parallel ) sides)</th>
<th>Parallelogram (opposite sides ( \parallel ) )</th>
</tr>
</thead>
</table>

3. A square is a rectangle with all sides congruent.
5. 7. 96.8 9. None; the diagonals are not congruent or perpendicular. 11. If the measure of each angle is 90 or if the diagonals are congruent, then the floor is a square. 13. 120 15. 30

17. 53 19. 5 21. Rhombus; the diagonals are perpendicular. 23. None; the diagonals are not congruent or perpendicular.

25. Sample answer:

![Diagram](image)

27. always 29. sometimes 31. always 33. 40 cm

35. Given: \( ABCD \) is a parallelogram.
\[ AC \parallel BD \]
Prove: \( ABCD \) is a rhombus.
Proof: We are given that \( ABCD \) is a parallelogram. The diagonals of a parallelogram bisect each other, so \( AE \equiv EC \). \( BE \equiv BE \) because congruence of segments is reflexive. We are also given that \( AC \parallel BD \). Thus, \( \angle AEB \) and \( \angle BEC \) are right angles by the definition of perpendicular lines. Then \( \angle AEB \equiv \angle BEC \) because all right angles are congruent. Therefore, \( \triangle AEB \equiv \triangle CEB \) by SAS. \( AB \equiv CB \) by CPCTC. Opposite sides of parallelograms are congruent, so \( AB \equiv CD \) and \( BC \equiv AD \). Then since congruence of segments is transitive, \( AB \equiv CD \equiv CB \equiv AD \). All four sides of \( ABCD \) are congruent, so \( ABCD \) is a rhombus by definition.

37. No; it is about 11,662.9 mm. 39. The flag of Denmark contains four red rectangles. The flag of St. Vincent and the Grenadines contains a blue rectangle, a green rectangle, a yellow rectangle, a blue and yellow rectangle, a yellow and green rectangle, and three green rhombi. The flag of Trinidad and Tobago contains two white parallelograms and one black parallelogram.

41. Given: \( \triangle TPX \equiv \triangle QPX \equiv \triangle QRX \equiv \triangle TRX \)
Prove: \( TPQR \) is a rhombus.

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle TPX \equiv \triangle QPX \equiv \triangle QRX \equiv \triangle TRX )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( TP \equiv PQ \equiv QR \equiv TR )</td>
<td>2. CPCTC</td>
</tr>
<tr>
<td>3. ( TPQR ) is a rhombus</td>
<td>3. Def. of rhombus</td>
</tr>
</tbody>
</table>

43. Given: \( QRST \) and \( QRTV \) are rhombi.
Prove: \( \triangle QRT \) is equilateral.

**Proof:**

<table>
<thead>
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<tbody>
<tr>
<td>1. ( QRST ) and ( QRTV ) are rhombi</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( QT \equiv TR \equiv QR )</td>
<td>2. Def. of rhombus</td>
</tr>
<tr>
<td>3. ( QT \equiv TR \equiv QR )</td>
<td>3. Substitution Property</td>
</tr>
<tr>
<td>4. ( \triangle QRT ) is equilateral</td>
<td>4. Def. of equilateral triangle</td>
</tr>
</tbody>
</table>

45. Sample answer: You can ride a bicycle with square wheels over a curved road. Answers should include the following.
- Rhombi and squares both have all four sides congruent, but the diagonals of a square are congruent. A square has four right angles and rhombi have each pair of opposite angles congruent, but not all angles are necessarily congruent.
- Sample answer: Since the angles of a rhombus are not all congruent, riding over the same road would not be smooth.

47. C 49. 140 51. \( x = 2 \) \( y = 3 \) 53. yes 55. no

57. 13.5 59. 20 61. \( \angle A'H = \angle AH \) 63. \( AK = AB \)

65. 2.4 67. 5

**Pages 442-445 Lesson 8-6**

1. Exactly one pair of opposite sides is parallel.

3. Sample answer:

![Diagram](image)

The median of a trapezoid is parallel to both bases.

5. isosceles, \( QR = \sqrt{20} \), \( ST = \sqrt{20} \), \( Z \), 9a. \( \overline{AD} \parallel \overline{BC} \), \( CD \parallel AB \) 9b. not isosceles, \( AB = 17 \) and \( CD = 5 \)
11a. \( \overline{DC} \parallel \overline{FE} \), \( \overline{DE} \parallel \overline{FC} \) 11b. isosceles, \( DE = V \), \( CF = \sqrt{50} \)
13. 8 15. 14, 110, 110 17. 62 19. 15

21. Sample answer: triangles, quadrilaterals, trapezoids, hexagons 23. trapezoid, exactly one pair opp. sides \| 25. square, all sides \( \equiv \), consecutive sides \( \perp \) 27. \( A(-2, 3.5), B(4, -1) \) 29. \( \overline{DF} \parallel \overline{EF} \), not isosceles, \( \overline{DE} \neq \overline{GF} \), \( \overline{DE} \parallel \overline{GF} \)
31. \( WV = 6 \)

33. Given: \( \triangle TZX \equiv \triangle YXZ \), \( \overline{WX} \parallel \overline{ZY} \)
Prove: \( XYZW \) is a trapezoid.

**Proof:**

![Diagram](image)

If alt. int. \( \angle \) are \( \equiv \), then the lines are \( \parallel \).

\( \overline{WX} \parallel \overline{ZY} \)

\( XYZW \) is a trapezoid.

Selected Answers R59
35. **Given:** E and C are midpoints of $AD$ and $DB$.
**Prove:** $ABCE$ is an isosceles trapezoid.

**Proof:**

$E$ and $C$ are midpoints of $AD$ and $DB$.

Given

$EC \parallel AB$  
A segment joining the midpoints of two sides of a triangle is parallel to the third side.

$\frac{1}{2}AD = \frac{1}{2}DB$  
Def. of Midpt.

$AE = BC$  
Substitution

$ABCE$ is an isosceles trapezoid.  
Def. of isosceles trapezoid

37. Sample answer:

![Diagram](image)

39. 4

41. Sample answer: Trapezoids are used in monuments as well as other buildings. Answers should include the following:
- Trapezoids have exactly one pair of opposite sides parallel.
- Trapezoids can be used as window panes.

43. B  45. 10  47. 70  49. $RS = 7\sqrt{2}, TV = \sqrt{113}$

51. No; opposite sides are not congruent and the diagonals do not bisect each other.  53. $\frac{17}{5}$  55. $\frac{13}{2}$  57. 0  59. $\frac{2b}{a}$

---

**Page 445**  
**Chapter 8**  
**Practice Quiz 2**

1. 12  3. rhombus, opp. sides $\parallel$, diag. $\perp$, consec. sides not $\perp$  5. 18

**Pages 449–451**  
**Lesson 8-7**

1. Place one vertex at the origin and position the figure so another vertex lies on the positive $x$-axis.

3. $D(0, a + b)$

5. $(c, b)$

7. **Given:** $ABCD$ is a square.  
**Prove:** $AC \perp DB$  
**Proof:**

Slope of $DB = \frac{0 - a}{a - 0}$ or $-1$

Slope of $AC = \frac{0 - a}{0 - a}$ or 1

The slope of $AC$ is the negative reciprocal of the slope of $DB$, so they are perpendicular.

9.

11. $B(-b, c)$

13. $G(a, 0), E(-b, c)$

15. $T(-2a, c), W(-2a, -c)$

17. **Given:** $ABCD$ is a rectangle.  
**Prove:** $AC \equiv DB$  
**Proof:**

Use the Distance Formula to find $AC = \sqrt{a^2 + b^2}$ and $BD = \sqrt{a^2 + b^2}$. $AC$ and $BC$ have the same length, so they are congruent.

19. **Given:** isosceles trapezoid $ABCD$ with $\overline{AD} \equiv \overline{BC}$  
**Prove:** $BD \equiv AC$  
**Proof:**

$BD = \sqrt{(a-b)^2 + (c-0)^2} = \sqrt{(a-b)^2 + c^2}$

$AC = \sqrt{((a-b)-0)^2 + (c-0)^2} = \sqrt{(a-b)^2 + c^2}$

$BD = AC$ and $BD \equiv AC$

21. **Given:** $ABCD$ is a rectangle.  
**Prove:** $QRST$ is a rhombus.  
**Proof:**

Midpoint $Q$ is $\left(\frac{0 + 0}{2}, \frac{0 + b}{2}\right)$ or $(0, \frac{b}{2})$.

Midpoint $R$ is $\left(\frac{a + a}{2}, \frac{b + b}{2}\right)$ or $(\frac{a}{2}, \frac{b}{2})$.

Midpoint $S$ is $\left(\frac{a + a}{2}, \frac{b + b}{2}\right)$ or $(\frac{a}{2}, \frac{b}{2})$.

Midpoint $T$ is $\left(\frac{0 + 0}{2}, \frac{0 + 0}{2}\right)$ or $(0, 0)$.

$QR = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$RS = \sqrt{\left(\frac{a}{2} - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$ST = \sqrt{\left(\frac{a}{2} - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$QT = \sqrt{\left(\frac{a}{2} - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$

$QR = RS = ST = QT$ so $QR \equiv RS \equiv ST \equiv QT$. $QRST$ is a rhombus.

23. Sample answer: $C(a + c, b), D(2a + c, 0)$  25. No, there is not enough information given to prove that the sides of the tower are parallel.  27. Sample answer: The coordinate plane is used in coordinate proofs. The Distance Formula, Midpoint Formula and Slope Formula are used to prove theorems. Answers should include the following:
- Place the figure so one of the vertices is at the origin.  
- Place at least one side of the figure on the positive $x$-axis.  
- Keep the figure in the first quadrant if possible and use coordinates that will simplify calculations.
- Sample answer: Theorem 8.3 Opposite sides of a parallelogram are congruent.

29. A  31. 55  33. 160  35. $\sqrt{60} \approx 7.7$  37. $m \angle VXZ = m \angle VXZ$  39. $m \angle ZXY > m \angle ZXY$
**Pages 452–456  Chapter 8  Study Guide and Review**

1. true    3. false, rectangle    5. false, trapezoid    7. true    9. 120    11. 90    13. m∠W = 62, m∠X = 108, m∠Y = 80, m∠Z = 110    15. 52    17. 87.9    19. 6    21. no    23. yes    25. 52    27. 28    29. Yes, opp. sides are parallel and diag. are congruent    31. 7.5    33. 102

35. Given: \(ABCD\) is a square.
Prove: \(AC \perp BD\)

Proof:
Slope of \(AC\) = \(\frac{a - 0}{a - 0}\) or 1
Slope of \(BD\) = \(\frac{a - 0}{0 - a}\) or \(-1\)
The slope of \(AC\) is the negative reciprocal of the slope of \(BD\). Therefore, \(AC \perp BD\).

37. \(P(3a, c)\)

**Chapter 9  Transformations**

**Page 461  Chapter 9  Getting Started**

1. 3. 5. \(B(-1, 3)\) \(A(1, 3)\)

3. \(E(-2, 1)\) \(O\) \(F(-1, -2)\)

5. \(J(-7, 10)\) \(K(-6, 7)\)

7. 36.9    9. 41.8    11. 41.4

13. \(\begin{bmatrix} -5 & -1 \\ 10 & 5 \end{bmatrix}\)

15. \(\begin{bmatrix} -2 & -5 & 1 \\ 3 & -4 & -5 \end{bmatrix}\)

31. 33. 35. 2; yes    37. 1; no    39. same shape, but turned or rotated

41. \(A(4, 7), B(10, -3)\), and \(C(-6, -8)\)

43. Consider point \((a, b)\). Upon reflection in the origin, its image is \((-a, -b)\). Upon reflection in the \(x\)-axis and then the \(y\)-axis, its image is \((a, -b)\) and then \((-a, -b)\). The images are the same.

45. vertical line of symmetry    47. vertical, horizontal lines of symmetry; point of symmetry at the center

49. \(D\)

51. Given: Quadrilateral \(LMNP\): \(X, Y, Z,\) and \(W\) are midpoints of their respective sides.
Prove: \(YW\) and \(XZ\) bisect each other.

Proof:
Midpoint \(Y\) of \(MN\) is \(\left(\frac{2d + 2a}{2}, \frac{2e + 2c}{2}\right)\) or \(\left(a + b, c + e\right)\).
Midpoint \(Z\) of \(NP\) is \(\left(\frac{2a + 2b}{2}, \frac{2e + 0}{2}\right)\) or \((a + b, 0)\).
Midpoint \(W\) of \(PL\) is \(\left(\frac{0 + 2b}{2}, \frac{0 + 0}{2}\right)\) or \((b, 0)\).
Midpoint \(X\) of \(LM\) is \(\left(\frac{0 + 2d}{2}, \frac{2e + 2c}{2}\right)\) or \((d, e)\).
Midpoint of \(XY\) is \(\left(\frac{d + a + b + e + c + 0}{2}, \frac{a + b + d + e + c}{2}\right)\).
Midpoint of \(ZX\) is \(\left(\frac{d + a + b}{2}, \frac{e + c}{2}\right)\) or \((a + b + d, e + c)\).
The midpoints of \(XZ\) and \(XY\) are the same, so \(XZ\) and \(XY\) bisect each other.
53. \( f = 25.5, m\angle H = 76, h = 28.8 \)  
59. \( \sqrt{2} \)

61. \( \sqrt{5} \)

**Pages 470–475 Lesson 9-2**

1. Sample answer: \( A(3, 5) \) and \( B(-4, 7) \); start at 3, count to the left to -4, which is 7 units to the left or -7. Then count up 2 units from 5 to 7 or +2. The translation from \( A \) to \( B \) is \( (x, y) \to (x - 7, y + 2) \).

3. Allie; counting from the point \( (2, 1) \) to \( (1, 1) \) is right 3 and down 2 to the image. The reflections would be too far to the right. The image would be reversed as well.

5. No; quadrilateral \( WXYZ \) is oriented differently than quadrilateral \( NPQR \).

7. Yes; it is one reflection after another with respect to the two parallel lines.

11. No; it is a reflection followed a rotation.

13. Yes; it is one reflection after another with respect to the two parallel lines.

33. No; the percent per figure is different in each category.

35. Translations and reflections preserve the congruences of segments and angles. The composition of the two transformations will preserve both congruences. Therefore, a glide reflection is an isometry.

45. 23 ft

47. You did not fill out an application.

51. 5

53. \( 3\sqrt{2} \)

55. 57.

59.

**Pages 476–482 Lesson 9-3**

1. clockwise \( (x, y) \to (y, -x) \); counterclockwise \( (x, y) \to (-y, x) \)

3. Both translations and rotations are made up of two reflections. The difference is that translations reflect across parallel lines while rotations reflect across intersecting lines.

5.

7.

9. order 6; magnitude 60°

11. order 5 and magnitude 72°; order 4 and magnitude 90°; order 3 and magnitude 120°
13. \[ \triangle P \]

15. \[ \triangle T \]

17. \[ \triangle T \]

19. \[ \triangle Z \]

21. \[ \triangle Z \]

23. \[ K''(0, -5), L''(4, -2), \text{ and } M''(4, 2); 90^\circ \text{ clockwise} \]

25. \( \sqrt{3}, 1 \)

27. Yes; it is a proper successive reflection with respect to the two intersecting lines.

29. yes 31. no 33. 9

35. \((x, y) \rightarrow (y, -x)\)

37. any point on the line of reflection

39. no invariant points

41. A

43. \[ \triangle W \]

45. direct 47. Yes; it is one reflection after another with respect to the two parallel lines. 49. Yes; it is one reflection after another with respect to the two parallel lines. 51. C

53. \( \angle AGF \) 55. \( TR \); diagonals bisect each other 57. \( \angle QRS \); opp. \( \angle \equiv \)

59. yes 61. yes 63. (0, 4), (1, 2), (2, 0) 65. (0, 12), (1, 8), (2, 4), (3, 0)

67. (0, 12), (1, 6), (2, 0)

**Page 482 Chapter 9 Practice Quiz 1**

1. \[ \triangle W \]

3. \[ \triangle Y \]

5. order 36; magnitude 10°

**Pages 483–488 Lesson 9-4**

1. Semi-regular tessellations contain two or more regular polygons, but uniform tessellations can be any combination of shapes. 3. The figure used in the tessellation appears to be a trapezoid, which is not a regular polygon. Thus, the tessellation cannot be regular. 5. no; measure of interior angle = 168° 7. yes 9. yes; not uniform 11. no; measure of interior angle = 140° 13. yes; measure of interior angle = 60° 15. no; measure of interior angle = 164.3° 17. no 19. yes 21. yes; uniform 23. yes; not uniform 25. yes; not uniform 27. yes; uniform, regular 29. semi-regular, uniform 31. Never; semi-regular tessellations have the same combination of shapes and angles at each vertex like uniform tessellations. The shapes for semi-regular tessellations are just regular. 33. Always; the sum of the measures of the angles of a quadrilateral is 360°. So if each angle of the quadrilateral is rotated at the vertex, then that equals 360° and the tessellation is possible. 35. yes 37. uniform, regular 39. Sample answer: Tessellations can be used in art to create abstract art. Answers should include the following.

- The equilateral triangles are arranged to form hexagons, which are arranged adjacent to one another.
- Sample answers: kites, trapezoids, isosceles triangles

41. A

43. \[ \triangle B \]

45. \[ \triangle D \]

47. \( x = 4, y = 1 \)

49. \( x = 56, y = 12 \)

51. no 53. yes, no 55. no 57. \( AB = 7, BC = 10, AC = 9 \)

59. \( l(-1) = -1 \) and \( -1(1) = -1 \)

61. square

63. 15 65. 22.5

**Pages 490–497 Lesson 9-5**

1. Dilations only preserve length if the scale factor is 1 or \(-1\). So for any other scale factor, length is not preserved and the dilation is not an isometry. 3. Trey; Desiree found the image using a positive scale factor.
11. \( r = 2 \); enlargement
13. C

15.

\[
\text{Selected Answers}
\]

17.

19.

21. \( S'T' = \frac{3}{5} \)
23. \( ST = 4 \)
25. \( S'T' = 0.9 \)

27.

29.

31. \( \frac{1}{2} \); reduction
33. \( \frac{1}{3} \); reduction
35. –2; enlargement
37. 7.5 by 10.5
39. The perimeter is four times the original perimeter.

41. Given: dilation with center \( C \) and scale factor \( r \)
Proof: \( ED = r(AB) \)
Proof:
\[ CE = r(CA) \text{ and } CD = r(CB) \]
by the definition of a dilation. \( \frac{CE}{CA} = r \) and \( \frac{CD}{CB} = r \).
So, \( \frac{CE}{CA} = \frac{CD}{CB} \) by substitution.
\[ \angle ACB = \angle ECD, \] since congruence of angles is reflexive. Therefore, by SAS Similarity, \( \triangle ACB \) is similar to \( \triangle ECD \). The corresponding sides of similar triangles are proportional, so \( \frac{ED}{AB} = \frac{CE}{CA} \). We know that \( \frac{CE}{CA} = r \), so \( \frac{ED}{AB} = r \) by substitution. Therefore, \( ED = r(AB) \) by the Multiplication Property of Equality.

43. 2
45. \( \frac{1}{20} \)
47. 60%
49.

51.

53. Sample answer: Yes; a cut and paste produces an image congruent to the original. Answers should include the following:
- Congruent figures are similar, so cutting and pasting is a similarity transformation.
- If you scale both horizontally and vertically by the same factor, you are creating a dilation.

55. A
57. no
59. no

63. Given: \( \angle I \equiv \angle L \) \( B \) is the midpoint of \( \overline{IL} \)
Proof: \( \triangle JHB \equiv \triangle LCB \)
Proof: It is known that \( \angle J \equiv \angle L \). Since \( B \) is the midpoint of \( \overline{IL} \), \( \overline{JB} \equiv \overline{LB} \) by the Midpoint Theorem.
\( \angle JHB \equiv \angle LBC \) because vertical angles are congruent.
Thus, \( \triangle JHB \equiv \triangle LCB \) by ASA. 65. 76.0
5. $A'(-5, -1)$,
   $B'(-\frac{1}{3}, -3)$,
   $C'(2, -2)$

3. Sample answer: A vector to translate a figure is the same as using an ordered pair because a vector has horizontal and vertical components, each of which can be represented by one coordinate of an ordered pair.

7. $2\sqrt{13} \approx 7.2, 213.7^\circ$

5. $(4, -3)$

9. $306.9^\circ$

13. $6\sqrt{13} \approx 21.6, 303.7^\circ$

15. $(2, 6)$

16. $(-7, -4)$

19. $(-3, 5)$

21. $5, 0^\circ$

23. $2\sqrt{5} \approx 4.5, 296.6^\circ$

25. $7\sqrt{5} \approx 15.7, 26.6^\circ$

27. $25, \approx 73.7^\circ$

29. $5\sqrt{41} \approx 32.0, \approx 218.7^\circ$

31. $6\sqrt{2} \approx 8.5, 135.0^\circ$

33. $4\sqrt{10} \approx 12.6, 198.4^\circ$

35. $2V 122 \approx 22.1, 275.2^\circ$

59. Sample answer: Quantities such as velocity are vectors. The velocity of the wind and the velocity of the plane contribute factor into the overall flight plan. Answers should include the following:

- A wind from the west would add to the velocity contributed by the plane resulting in an overall velocity with a smaller magnitude.

- When traveling east, the prevailing winds add to the velocity of the plane. When traveling west, they detract from it.

61. D

63. $A'B' = 6$

65. $AB = 48$

67. yes; not uniform

69. 12

71. 30

73. $[-4 -3 -10 -4]$

75. $[-27 -15 -3 15]$

77. $[-4 -4 -12 -12]$

1. yes; uniform; semi-regular

3. Sample answer: Using a vector to translate a figure is the same as using an ordered pair because a vector has horizontal and vertical components, each of which can be represented by one coordinate of an ordered pair.
51. $\frac{1}{2}$; reduction
53. 60, 120
55. 36, 144

Pages 512–516 Chapter 9 Study Guide and Review
1. false, center 3. false, component form 5. false, center of rotation 7. false, scale factor
9. $245.$
11. $24, 12, 12$

13. $B'(3, -5), C'(3, -3), D'(5, -3); 180^\circ$

15. $L'(-2, 2), M'(-3, 5), N'(-6, 3); 90^\circ$ counterclockwise

17. $P'(2, -6), Q'(-4, -4), R'(-2, 2)$

19. $200^\circ$ 21. yes; not uniform 23. yes; uniform 25. Yes; the measure of an interior angle is 60, which is a factor of 360.

27. $C'D' = 24$
29. $CD = 4$

31. $C'D' = 10$
33. $P'(2, -6), Q'(-4, -4), R'(-2, 2)$
35. (3, 4) 37. (0, 8) 39. $14.8, \approx 208.3^\circ$ 41. $129.9, \approx 213.3^\circ$
43. $D'(-\frac{12}{5}, -\frac{8}{5}), E'(0, 4), F'(\frac{8}{5}, -\frac{16}{5})$
45. $D'(-2, 3), E'(5, 0), F'(-4, -2)$
47. $W'(-16, 2), X'(-4, 6), Y'(-2, 0), Z'(-12, -6)$

Chapter 10 Circles

Pages 521 Chapter 10 Getting Started
1. 162 3. 2.4 5. $r = \frac{C}{2\pi}$ 7. 15 9. 17.0
11. 1.5, -0.9 13. 2.5, -3

Pages 522–528 Lesson 10-1
1. Sample answer: The value of $\pi$ is calculated by dividing the circumference of a circle by the diameter. 3. Except for a diameter, two radii and a chord of a circle can form a triangle. The Triangle Inequality Theorem states that the sum of two sides has to be greater than the third. So, $2r$ has to be greater than the measure of any chord that is not a diameter, but $2r$ is the measure of the diameter. So the diameter has to be longer than any other chord of the circle.
5. $F A, F B, F C, \text{ or } F D$ 7. $A C$ or $B D$ 9. 10.4 in.
11. 6
29. 64 in. or 5 ft 4 in. 31. 0.6 m 33. 3 35. 12 37. 34
39. 20 41. 5 43. 2.5 45. 13.4 cm, 84.19 cm
47. 24.32 m, 12.16 m 49. $13\frac{1}{2}$ in., 42.41 in. 51. 0.33a, 1.05a
53. 5 ft 55. 8.$\pi$ cm 57. 0; The longest chord of a circle is the diameter, which contains the center.
59. 500–600 ft
61. 24\pi units 63. 27 65. 10$\pi$, 20$\pi$, 30$\pi$ 67. 9.8, $60^\circ$
69. 44.7, $27^\circ$ 71. 24

73. Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
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</thead>
<tbody>
<tr>
<td>1. $\overline{RQ}$ bisects $\angle SRT.$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle SRQ \equiv \angle QRT$</td>
<td>2. Def. of $\angle$ bisector</td>
</tr>
<tr>
<td>3. $m\angle SRQ = m\angle QRT$</td>
<td>3. Def. of $\equiv \angle$</td>
</tr>
<tr>
<td>4. $m\angle SQR = m\angle T + m\angle QRT$</td>
<td>4. Exterior Angle Theorem</td>
</tr>
<tr>
<td>5. $m\angle SQR &gt; m\angle QRT$</td>
<td>5. Def. of Inequality</td>
</tr>
<tr>
<td>6. $m\angle SQR &gt; m\angle SRQ$</td>
<td>6. Substitution</td>
</tr>
</tbody>
</table>

Pages 529–535 Lesson 10-2
1. Sample answer: $AB, BC, AC, AB, BC, CA$;
$mAB = 110$, $mBC = 160$, $mAC = 90$, $mABC = 270$,
$mBCA = 250$, $mCAB = 200$
3. Sample answer: Concentric circles have the same center, but different radius measures;
congruent circles usually have different centers but the same radius measure.
5. 137 7. 103 9. 180 11. 138
13. Sample answer: 25% = 90°, 23% = 83°, 28% = 101°,
22% = 79°, 2% = 7° 15. 60 17. 30 19. 120 21. 115
23. 65 25. 90 27. 90 29. 135 31. 270 33. 76 35. 52
37. 256 39. 308 41. 24$\pi$ = 75.40 units 43. 4$\pi$ = 12.57 units
45. The first category is a major arc, and the other three categories are minor arcs.
47. always 49. never
51. $m\angle 1 = 80$, $m\angle 2 = 120$, $m\angle 3 = 160$
53. 56.5 ft
55. No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent.
57. B 59. 20; 62.83
61. 28; 14 63. 84.9 newtons, 32° north of due east
65. 36.68 67. $\sqrt{245}$ 69. If $ABC$ has three sides, then $ABC$ is a triangle.
71. 42 73. 100 75. 36

Pages 536–543 Lesson 10-3
1. Sample answer: An inscribed polygon has all vertices on the circle. A circumscribed circle means the circle is drawn around so that the polygon lies in its interior and all vertices lie on the circle.
3. To bisect the chord, it must be a diameter and be perpendicular.
5. 30
7. 5$\sqrt{3}$ 9. 10$\sqrt{5}$ 5 22.36 11. 15 13. 15 15. 40
17. 80 19. 4 21. 5 23. $mAB = mBC = mCD = mDE = mEF = mFG = mGH = mHA = 45$
25. $mNP = mRQ = 120$; $mNR = mPQ = 60$
27. 30 29. 15 31. 16 33. 6
35. $\sqrt{2} \approx 1.41$
37. Given: \( \odot O, OS \perp RT, OV \perp UW, OS = OV \)
Prove: \( RT \equiv UW \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( OT = OW )</td>
<td>1. All radii of a ( \odot ) are ( = ).</td>
</tr>
<tr>
<td>2. ( OS \perp RT, OV \perp UW, OS = OV )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \triangle OST, \triangle OVW ) are right angles.</td>
<td>3. Definition of ( \perp ) lines</td>
</tr>
<tr>
<td>4. ( \triangle STO \equiv \triangle VWO )</td>
<td>4. HL</td>
</tr>
<tr>
<td>5. ( ST = VW )</td>
<td>5. CPCTC</td>
</tr>
<tr>
<td>6. ( ST = VW )</td>
<td>6. Definition of ( = ) segments</td>
</tr>
<tr>
<td>7. ( 2(ST) = 2(VW) )</td>
<td>7. Radius ( \perp ) to a chord bisects the chord.</td>
</tr>
<tr>
<td>8. ( OS ) bisects ( RT; OV ) bisects ( UW ).</td>
<td>8. Definition of segment bisector</td>
</tr>
<tr>
<td>9. ( RT = 2(ST), UW = 2(VW) )</td>
<td>9. Substitution</td>
</tr>
<tr>
<td>10. ( RT = UW )</td>
<td>10. Definition of ( = ) segments</td>
</tr>
<tr>
<td>11. ( RT \equiv UW )</td>
<td>11.</td>
</tr>
</tbody>
</table>

43. \( 2 \sqrt{135} \approx 23.24 \) yd

45. Let \( r \) be the radius of \( \odot P \). Draw radii to points \( D \) and \( E \) to create triangles. The length \( DE \) is \( r \sqrt{3} \) and \( AB = 2r; r \sqrt{3} \neq \frac{1}{2} (2r) \). Inscribed equilateral triangle; the six arcs making up the circle are congruent because the chords intercepting them were congruent by construction. Each of the three chords drawn intersect two of the congruent chords. Thus, the three larger arcs are congruent. So, the three chords are congruent, making this an equilateral triangle.

49. No; congruent arcs must be in the same circle, but these are in concentric circles.

51. Sample answer: The grooves of a waffle iron are chords of the circle. The ones that pass horizontally and vertically through the center are diameters. Answers should include the following:

- If you know the measure of the radius and the distance the chord is from the center, you can use the Pythagorean Theorem to find the length of half of the chord and then multiply by 2.
- There are four grooves on either side of the diameter, so each groove is about 1 in. from the center. In the figure, \( EF = 2 \) and \( EB = 4 \) because the radius is half the diameter. Using the Pythagorean Theorem, you find that \( FB = 3.464 \) in. so \( AB = 6.93 \) in. Approximate lengths for other chords are 5.29 in. and 7.75 in., but exactly 8 in. for the diameter.

53. 14,400  55. 180  57. \( \overline{SU} \)  59. \( \overline{RM, AM, DM, IM} \)  61. 50  63. 10  65. 20

Page 543 Chapter 10 Practice Quiz 1

1. \( BC, BD, BA \)  3. 95  5. 9  7. 28  9. 21

Page 544–551 Lesson 10–4

1. Sample answer: \( \angle 1 = 30, \angle 2 = 60, \angle 3 = 60, \angle 4 = 30, \angle 5 = 30, \angle 6 = 60, \angle 7 = 60, \angle 8 = 30 \)  5. \( \angle 1 = 35, \angle 2 = 55, \angle 3 = 39, \angle 4 = 39 \)  7. 1  9. \( \angle 1 \equiv \angle 2 = 30, \angle 3 = 25 \)

11. Given: \( \overline{AB \equiv DE, AC \equiv CE} \)
Prove: \( \triangle ABC \equiv \triangle EDC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( AB \equiv DE, AC \equiv CE )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( mAB = mDE, mAC = mCE )</td>
<td>2. Def. of ( \equiv ) arcs</td>
</tr>
<tr>
<td>3. ( \frac{1}{2} mAB = \frac{1}{2} mDE )</td>
<td>3. Mult. Prop.</td>
</tr>
<tr>
<td>( 4. m \angle ACB = \frac{1}{2} mAC ), ( m \angle ECD = \frac{1}{2} mCE )</td>
<td>4. Inscribed Angle Theorem</td>
</tr>
<tr>
<td>( m \angle 1 = \frac{1}{2} mAC ), ( m \angle 2 = \frac{1}{2} mCE )</td>
<td>5. Substitution</td>
</tr>
<tr>
<td>( m \angle ECD )</td>
<td>6. Def. of ( \equiv )</td>
</tr>
<tr>
<td>( \angle 1 \equiv \angle 2 )</td>
<td>7. ( \equiv ) arcs have ( \equiv ) chords.</td>
</tr>
<tr>
<td>( 8. \triangle ABC \equiv \triangle EDC )</td>
<td>8. AAS</td>
</tr>
<tr>
<td>13. ( \angle 1 = \angle 2 = 13 )  15. ( \angle 1 = 51, \angle 2 = 90, \angle 3 = 39 )  17. 45, 30, 120 \ 19. ( m \angle B = 120, m \angle C = 120, m \angle D = 60 )  21. Sample answer: ( EF ) is a diameter of the circle and a diagonal and angle bisector of ( EDFG ).  23. 72  25. 144</td>
<td></td>
</tr>
</tbody>
</table>

27. 162  29. 9  31. \( \frac{9}{9} \)  33. 1

35. Given: \( T \) lies inside \( \angle PRQ, RK \) is a diameter of \( \odot T \).
Prove: \( m \angle PRQ = \frac{1}{2} mPKQ \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m \angle PRQ = m \angle PRK + m \angle KRQ )</td>
<td>1. Angle Addition Theorem</td>
</tr>
<tr>
<td>2. ( mPKQ = m \overline{PK} + m \overline{KQ} )</td>
<td>2. Arc Addition Theorem</td>
</tr>
<tr>
<td>3. ( \frac{1}{2} mPKQ = \frac{1}{2} m \overline{PK} + \frac{1}{2} m \overline{KQ} )</td>
<td>3. Multiplication Property</td>
</tr>
</tbody>
</table>
4. \( m \angle PRK = \frac{1}{2} m \overline{PK} \), 
\( m \angle KRQ = \frac{1}{2} m \overline{KQ} \)

5. \( \frac{1}{2} m \overline{PKQ} = m \angle PRK + m \angle KRQ \)

6. \( \frac{1}{2} m \overline{PKQ} = m \angle PRQ \)

4. The measure of an inscribed angle whose side is a diameter is half the measure of the intercepted arc (Case 1).

5. Substitution (Steps 3, 4)

6. Substitution (Steps 5, 1)

37. Given: inscribed \( \angle MLN \) and \( \angle CED \), \( CD \equiv MN \)
Prove: \( \angle CED \equiv \angle MLN \)

**Proof:**

**Statements**

1. \( \angle MLN \) and \( \angle CED \) are inscribed; \( CD \equiv MN \)
2. \( m \angle MLN = \frac{1}{2} m \overline{MN} \);
   \( m \angle CED = \frac{1}{2} m \overline{CD} \)
3. \( m \overline{CD} = m \overline{MN} \)
4. \( \frac{1}{2} m \overline{CD} = \frac{1}{2} m \overline{MN} \)
5. \( m \angle CED = m \angle MLN \)
6. \( \angle CED \equiv \angle MLN \)

**Reasons**

1. Given
2. Measure of an inscribed \( \angle = \) half measure of intercepted arc.
3. Def. of \( \equiv \) arcs
5. Substitution
6. Def. of \( \equiv \)

39. Given: quadrilateral \( ABCD \)
Prove: \( \angle A \) and \( \angle C \) are supplementary.
\( \angle B \) and \( \angle D \) are supplementary.

**Proof:** By arc addition and the definitions of arc measure and the sum of central angles, \( m \overline{DCB} + m \overline{DAB} = 360 \). Since \( m \angle C = \frac{1}{2} m \overline{DAB} \) and \( m \angle A = \frac{1}{2} m \overline{DCB} \),
\( m \angle C + m \angle A = \frac{1}{2} (m \overline{DCB} + m \overline{DAB}) \), but \( m \overline{DCB} + m \overline{DAB} = 360 \), so \( m \angle C + m \angle A = 180 \) or 90. This makes \( \angle C \) and \( \angle A \) supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, \( m \angle A + m \angle C + m \angle B + m \angle D = 360 \). But
\( m \angle A + m \angle C = 180 \), so \( m \angle B + m \angle D = 180 \), making them supplementary also.

41. Isosceles right triangle because sides are congruent radii making it isosceles and \( \angle AOC \) is a central angle for an arc of 90\(^\circ\), making it a right angle.

43. Square because each angle intersects a semicircle, making them 90\(^\circ\) angles. Each side is a chord of congruent arcs, so the chords are congruent.

45. Sample answer: The socket is similar to an inscribed polygon because the vertices of the hexagon can be placed on a circle that is concentric with the outer circle of the socket. Answers should include the following:
- An inscribed polygon is one in which all of its vertices are points on a circle.
- The side of the regular hexagon inscribed in a circle \( \frac{3}{4} \) inch wide is \( \frac{3}{8} \) inch.

47. 234 49. \( \sqrt{135} \approx 11.62 \) 51. 4\( \pi \) units 53. always
55. sometimes 57. no

---

**Page 552–558 Lesson 10-5**

1a. Two; from any point outside the circle, you can draw only two tangents. 1b. None; a line containing a point inside the circle would intersect the circle in two points. A tangent can only intersect a circle in one point. 1c. One; since a tangent intersects a circle in exactly one point, there is one tangent containing a point on the circle.

3. Sample answer:
   polygon circumscribed about a circle
   polygon inscribed in a circle

5. Yes; \( 5^2 + 12^2 = 13^2 \) 7. 576 ft 9. no 10. yes 11. 13. 16
15. 12 17. 3 19. 30 21. See students’ work. 23. 60 units 25. 15\( \sqrt{3} \) units

27. Given: \( \overline{AB} \) is tangent to \( \odot O \) at \( B \). \( \overline{AC} \) is tangent to \( \odot O \) at \( C \).
Prove: \( \overline{AB} \equiv \overline{AC} \)

**Proof:**

**Statements**

1. \( \overline{AB} \) is tangent to \( \odot O \) at \( B \). \( \overline{AC} \) is tangent to \( \odot O \) at \( C \).
2. Draw \( \overline{BX} \), \( \overline{CX} \), and \( \overline{AX} \).
3. \( \overline{AB} \perp \overline{BX} \), \( \overline{AC} \perp \overline{CX} \)
4. \( \angle ABX \) and \( \angle ACX \) are right angles.
5. \( \overline{BX} \equiv \overline{CX} \)
6. \( \overline{AX} \equiv \overline{AX} \)
7. \( \triangle ABX \equiv \triangle ACX \)
8. \( \overline{AB} \equiv \overline{AC} \)

29. \( \overline{AE} \) and \( \overline{BF} \)
31. 12; Draw \( \overline{PG} \), \( \overline{NL} \), and \( \overline{PL} \). Construct \( \overline{EQ} \perp \overline{GP} \), thus \( \overline{LQGN} \) is a rectangle. \( GQ = NL = 4 \), so \( QP = 5 \). Using the Pythagorean Theorem, \( (QP)^2 + (QL)^2 = (PL)^2 \). So, \( QL = 12 \). Since \( GN = QL \), \( GN = 12 \).
33. 27 35. \( \overline{AD} \) and \( \overline{BC} \)
37. 45 39. 4

41. Sample answer:
   **Given:** \( ABCD \) is a rectangle. \( E \) is the midpoint of \( \overline{AB} \).
   **Prove:** \( \triangle CED \) is isosceles.
**Page 561–568 Lesson 10-6**

1. Sample answer: A tangent intersects the circle in only one point and no part of the tangent is in the interior of the circle. A secant intersects the circle in two points and some of its points do lie in the interior of the circle.  

3. 138  
5. 20 7. 235 9. 55 11. 110 13. 60 15. 110 17. 90  
19. 30 21. 30 23. 8 25. 4 27. 25 29. 130 31. 10  
33. 141 35. 44 37. 118 39. about 103 ft 41. 4.6 cm

43a. Given: \( \overline{AB} \) is a tangent to \( \odot O \). \( \overline{AC} \) is a secant to \( \odot O \). \( \angle CAB \) is acute.  

**Prove:** \( m \angle CAB = \frac{1}{2} m \overline{CA} \)

**Proof:** \( \angle DAB \) is a right \( \angle \) with measure 90, and \( \overline{DCA} \) is a semicircle with measure 180, since if a line is tangent to a \( \odot \), it is \( \perp \) to the radius at the point of tangency. Since \( \angle CAB \) is acute, \( C \) is in the interior of \( \angle DAB \), so by the Angle and Arc Addition Postulates, \( m \angle DAB = m \angle DAC + m \angle CAB \) and \( m \overline{DCA} = m \overline{DC} + m \overline{CA} \). By substitution, \( 90 = m \angle DAC + m \angle CAB \) and \( 180 = m \overline{DC} + m \overline{CA} \). So, \( 90 = \frac{1}{2} m \overline{DC} + \frac{1}{2} m \angle CAB \) by Division Prop., and \( m \angle DAC + m \angle CAB = \frac{1}{2} m \overline{DC} + \frac{1}{2} m \angle CAB \) by substitution. \( m \angle DAC = \frac{1}{2} m \overline{DC} \) since \( \angle DAC \) is inscribed, so substitution yields \( \frac{1}{2} m \overline{DC} + m \angle CAB = \frac{1}{2} m \overline{DC} + \frac{1}{2} m \angle CAB \). By Subtraction Prop., \( m \angle CAB = \frac{1}{2} m \angle CAB \).

43b. Given: \( \overline{AB} \) is a tangent to \( \odot O \). \( \overline{AC} \) is a secant to \( \odot O \). \( \angle CAB \) is obtuse.  

**Prove:** \( m \angle CAB = \frac{1}{2} m \overline{CA} \)

**Proof:** \( \angle CAB \) and \( \angle CAE \) form a linear pair, so \( m \angle CAB + m \angle CAE = 180 \). Since \( \angle CAB \) is obtuse, \( \angle CAE \) is acute and Case 1 applies, so \( m \angle CAE = \frac{1}{2} m \overline{CA} \). \( m \overline{CA} + m \overline{CDA} = 360 \), so \( \frac{1}{2} m \overline{CA} + \frac{1}{2} m \overline{CDA} = 180 \) by Division Prop., and \( m \angle CAE + \frac{1}{2} m \overline{CDA} = 180 \) by substitution. By the Transitive Prop., \( m \angle CAB + m \angle CAE = m \angle CAE + \frac{1}{2} m \overline{CDA} \), so by Subtraction Prop., \( m \angle CAB = \frac{1}{2} m \overline{CA} \).

45. \( \angle 3, \angle 1, \angle 2; m \angle 3 = m \overline{RQ}, m \angle 1 = \frac{1}{2} m \overline{RQ} \) so \( m \angle 3 > m \angle 1, m \angle 2 = \frac{1}{2} (m \overline{RQ} - m \overline{TP}) = \frac{1}{2} m \overline{RQ} - \frac{1}{2} m \overline{TP} \), which is less than \( \frac{1}{2} m \overline{RQ} \), so \( m \angle 2 < m \angle 1 \).  
47. A 49. 16

51. 33 53. 44.5 55. 30 in. 57. 4, -10 59. 3, 5

**Page 568 Chapter 10 Practice Quiz 2**

1. 67.5 3. 12 5. 115.5

**Page 569–574 Lesson 10-7**

1. Sample answer: The product equation for secant segments equates the product of exterior segment measure and the whole segment measure for each secant. In the case of secant-tangent, the product involving the tangent segment becomes (measure of tangent segment)² because the exterior segment and the whole segment are the same segment.

3. Sample answer:  
5. 28.1 7. \( \approx 7 \times \approx 3.54 \) 9. 4  
11. 2 13. 6 15. 3.2  
17. 4 19. 5.6

21. Given: \( \overline{WY} \) and \( \overline{XX} \) intersect at \( T \).  

**Prove:** \( \overline{WT} \cdot \overline{TY} = \overline{ZT} \cdot \overline{TX} \)

**Proof:**

**Statements**  | **Reasons**
---|---
**a.** \( \angle W \equiv \angle Z, \angle X \equiv \angle Y \)  | a. Inscribed angles that intercept the same arc are congruent.
**b.** \( \triangle WXT \sim \triangle ZYT \)  | b. AA Similarity
**c.** \( \frac{\overline{WT}}{\overline{TX}} = \frac{\overline{ZT}}{\overline{TY}} \)  | c. Definition of similar triangles
**d.** \( \overline{WT} \cdot \overline{TY} = \overline{ZT} \cdot \overline{TX} \)  | d. Cross products

31. Given: tangent \( \overline{RS} \) and secant \( \overline{US} \)  

**Prove:** \( (\overline{RS})^2 = \overline{US} \cdot \overline{TS} \)

**Proof:**

**Statements**  | **Reasons**
---|---
1. tangent \( \overline{RS} \) and secant \( \overline{US} \)  | 1. Given
2. \( m \angle RUT = \frac{1}{2} m \overline{RT} \)  | 2. The measure of an inscribed angle equals half the measure of its intercepted arc.
3. \( m \angle SRT = \frac{1}{2} m \overline{RT} \)  | 3. The measure of an angle formed by a secant and a tangent equals half the measure of its intercepted arc.
4. \( m \angle RUT = m \angle SRT \)  | 4. Substitution
33. Sample answer: The product of the parts of one intersecting chord equals the product of the parts of the other chord. Answers should include the following.

- Definition of ∝ and ∝
- Reflexive Prop.
- AA Similarity
- Definition of ∝
- Cross products

Chapter 11 Areas of Polygons and Circles

Page 593 Chapter 11 Getting Started

1. 10 3 4 6 10 18 7 54 9 13 11 36 19 15

Pages 598–600 Lesson 11-1

1. The area of a rectangle is the product of the length and the width. The area of a parallelogram is the product of the base and the height. For both quadrilaterals, the measure of the length of one side is multiplied by the length of the altitude. 3. 28 ft; 39.0 ft² 5. 12.8 m; 10.2 m² 7. rectangle, 170 units² 9. 80 in.; 259.8 in² 11. 21.6 cm; 29.2 cm²

3. 44 m; 103.9 m² 15. 45.7 mm² 17. 108.5 m 19. \( h = 40 \text{ units}, b = 50 \text{ units} \)

21. parallelogram, 56 units²

23. parallelogram, 64 units² 25. square, 13 units² 27. 150 units² 29. Yes; the dimensions are 32 in. by 18 in.

31. \( = 13.9 \text{ ft} \) 33. The perimeter is 19 m, half of 38 m. The area is 20 m². 35. 5 in., 7 in. 37. C 39. (5, 2), \( r = 7 \)

41. \( (-\frac{2}{3}, \frac{1}{9}), r = \frac{2}{3} \) 43. 32 45. 21 47. \( F'(4, 0), G'(-2, -2), H'(-2, 2); 90° \) counterclockwise 49. 13 ft

51. 16 53. 20

Pages 605–609 Lesson 11-2

1. Sample answer: 3. Sometimes; two rhombi can have different corresponding diagonal lengths and have the same area.

4. 280 in² 8. 20 units² 9. 4 units² 11. 45 m 12. 12.4 cm²

15. 95 km² 17. 1200 ft² 19. 50 m² 21. 129.9 mm²

23. 55 units² 25. 22.5 units² 27. 20 units² 29. 16 units²

31. \( = 26.8 \text{ ft} \) 33. \( = 22.6 \text{ m} \) 35. 20 cm 37. about 8.7 ft

39. 13,326 ft² 41. 120 in² 43. \( = 10.8 \text{ in}² \) 45. 21 ft²

47. False; sample answer: the area for each of these right triangles is 6 square units. The perimeter of one triangle is 12 and the perimeter of the other is \( 8 + \sqrt{40} \) or about 14.3.

49. area = 12, area = 3; perimeter = 8\( \sqrt{3} \), 51. \( \frac{2}{\pi} \) 53. The ratio is the same.

55. 4:1; The ratio of the areas is the square of the scale factor. 57. 45 ft²; The ratio of the areas is 5:9. 59. B

61. area = \( \frac{1}{2}ab \sin C \) 63. 6.02 cm² 65. 374 cm²
39. Multiply the measure of the central angle of the sector by 1.

67. 231 ft²  69. \((x + 4)^2 + \left( y - \frac{1}{2} \right)^2 = \frac{121}{4} \)  71. 275 in.

73. (172,4, 220,6)  75. 20.1

Pages 613–616 Lesson 11-3
1. Sample answer: Separate a hexagon inscribed in a circle into six congruent nonoverlapping isosceles triangles. The area of one triangle is one-half the product of one side of the hexagon and the apothem of the hexagon. The area of the hexagon is \(6\left(\frac{3a}{2}\right)\). The perimeter of the hexagon is 6, so the formula is \(\frac{1}{2}Pa\).

3. 127.3 yd²  5. 10.6 cm²  7. about 3.6 yd²  9. 882 m²  11. 1995.3 in²  13. 482.8 km²  15. 30.4 units²  17. 26.6 units²  19. 4.1 units²  21. 271.2 units²  23. 2:1  25. One 16-inch pizza; the area of the 16-inch pizza is greater than the area of two 8-inch pizzas, so you get more pizza for the same price. 27. 83.1 units²  29. 48.2 units²  31. 227.0 units²  33. 646.8 units²  35. triangles; 629 tiles  37. \(= 380.1 \text{ in}^2\)  39. 34.6 units²  41. 157.1 units²  43. 471.2 units²  45. 54,677.8 ft²; 899.8 ft  47. 225\(\pi \approx 706.9 \text{ ft}^2\)  49. 2:3  51. The ratio is the same. 53. The ratio of the areas is the square of the scale factor. 55. 3 to 4  57. \(B\)  59. 260 cm²  61. \(\approx 2829.0 \text{ yd}^2\)  63. square; 36 units²  65. rectangle; 30 units²  67. 42  69. 6  71. \(4\sqrt{2}\)

Pages 619–621 Lesson 11-4
Sample answer: \(\approx 18.3 \text{ units}^2\)  3. 53.4 units²  5. 24 units²  7. \(= 1247.4 \text{ in}^2\)  9. 70.9 units²  11. 4185 units²  13. 154.1 units²  15. \(= 2236.9 \text{ in}^2\)  17. 23.1 units²  19. 21 units²  21. 33 units²  23. Sample answer: 57,500 mi²  25. 462 27. Sample answer: Reduce the width of each rectangle.

29. Sample answer: Windsurfers use the area of the sail to catch the wind and stay afloat on the water. Answers should include the following.
- To find the area of the sail, separate it into shapes. Then find the area of each shape. The sum of areas is the area of the sail.
- Sample answer: Surfboards and sailboards are also irregular figures.

31. \(C\)  33. 154.2 units²  35. 156.3 ft²  37. \(= 384.0 \text{ m}^2\)  39. 0.63  41. 0.19

Page 621 Practice Quiz 2
1. 679.0 mm²  3. 1208.1 units²  5. 44.5 units²

Pages 625–627 Lesson 11-5
1. Multiply the measure of the central angle of the sector by the area of the circle and then divide the product by 360°.
3. Rachel; Taimi did not multiply \(\frac{62}{360}\) by the area of the circle.  5. \(= 114.2 \text{ units}^2, \approx 0.36\)  7. 0.60  9. 0.54  11. \(= 58.9 \text{ units}^2\)  13. \(= 19.6 \text{ units}^2, 0.1\)  15. 74.6 units²  0.42 17. \(= 3.3 \text{ units}^2, \approx 0.03\)  19. \(= 25.8 \text{ units}^2, \approx 0.15\)  21. 0.68  23. 0.68  25. 0.19  27. \(= 0.29\)

The chances of landing on a black or white sector are the same, so they should have the same point value. 31a. No; each colored sector has a different central angle. 31b. No; there is not an equal chance of landing on each color.

5. Hexagonal pyramid; base: \(ABCDEF\); faces: \(ABCDEF, \triangle AGF, \triangle FGE, \triangle EGD, \triangle DGC, \triangle CGB, \triangle BGA\); edges: \(AF, FE, ED, DC, CB, BA, AG, FG, EG, DG, CG, and GB\); vertices: \(A, B, C, D, E, F, and G\).

7. cylinder; bases: circles \(P\) and \(Q\)

17. rectangular pyramid; base: \(\square DEFG\); faces: \(\square DEFG, \triangle DHG, \triangle GHF, \triangle FHE, \triangle DHE\); edges: \(DG, GF, FE, ED, DH, EH, FH, and GH\); vertices: \(D, E, F, G, and H\).

19. cylinder; bases: circles \(S\) and \(T\)  21. cone; base: circle \(B\); vertex \(A\)  23. No, not enough information is provided by the top and front views to determine the shape.

25. parabola  27. circle  29. rectangle

31. intersecting three faces  33. intersecting all four faces, and parallel to base; not parallel to any face;

35. cylinder  37. rectangles, triangles, quadrilaterals
39a. triangular  
39b. cube, rectangular, or hexahedron  
39c. pentagonal  
39d. hexagonal  
39e. hexagonal  
41. No; the number of faces is not enough information to classify a polyhedron. A polyhedron with 6 faces could be a cube, rectangular prism, hexahedron, or a pentagonal pyramid. More information is needed to classify a polyhedron.  
43. Sample answer: Archaeologists use two dimensional drawings to learn more about the structure they are studying. Egyptologists can compare two-dimensional drawings to learn more about the structure they are studying. Egyptologists can compare two-dimensional drawings of the pyramids and note similarities and any differences. Answers should include the following:  
- Viewpoint drawings and corner views are types of two-dimensional drawings that show three dimensions.  
- To show three dimensions in a drawing, you need to know the views from the front, top, and each side.  
45. D  
47. infinite  
49. 0.242  
51. 0.611  
53. 21 units$^2$  
55. 11 units$^2$  
57. 90 ft, 433.0 ft$^2$  
59. 300 cm$^2$  
61. 4320 in$^2$  

**Pages 645–648 Lesson 12-2**

1. Sample answer:  

5. 188 in$^2$;  

7. 64 cm$^2$;  

9.  

11.  

13.  

15. 66 units$^2$;  

17. 56 units$^2$;  

19. 121.5 units$^2$;  

21. 116.3 units$^2$;
33. A: \( 6 \text{ units}^2 \); B: \( (9 + \frac{\sqrt{3}}{2}) = 9.87 \text{ units}^2 \); C: \( 76 \text{ units}^2 \);

37. The surface area quadruples when the dimensions are doubled. For example, the surface area of the cube is \( 6(1^2) \) or 6 square units. When the dimensions are doubled the surface area is \( 6(2^2) \) or 24 square units.

39. No; 5 and 3 are opposite faces; the sum is 8.

43. \( \sqrt{3} \)

51. 35

53. \( \frac{1}{2} \)

55. 1963.50 \( \text{in}^2 \)

57. 21,124.07 \( \text{mm}^2 \)

Pages 657–659 Lesson 12-4

1. Multiply the circumference of the base by the height and add the area of each base.

3. Jamie; since the cylinder has one base removed, the surface area will be the sum of the lateral area and one base.

5. 1520.5 m²

9. 2352.4 m²

11. 517.5 in²

13. 251.3 ft²

15. 30.0 cm²

17. 3 cm

19. 8 m

21. The lateral areas will be in the ratio 3:2:1; \( 45\pi \text{ in}^2 \), \( 30\pi \text{ in}^2 \), \( 15\pi \text{ in}^2 \).

23. The lateral area is tripled. The surface area is increased, but not tripled.

25. 1.25 m

27. Sample answer: Extreme sports participants use a semicylinder for a ramp. Answers should include the following.
• To find the lateral area of a semicylinder like the half-pipe, multiply the height by the circumference of the base and then divide by 2.
• A half-pipe ramp is half of a cylinder if the ramp is an equal distance from the axis of the cylinder.

Page 659 Practice Quiz 1
1. Sample answer:

Pages 663–665 Lesson 12-5
1. Sample answer:

Pages 668–670 Lesson 12-6
1. Sample answer:

Pages 674–676 Lesson 12-7
1. Sample answer:

Pages 678–682 Chapter 12 Study Guide and Review
Pages 704–706 Lesson 13-3
1. The volume of a sphere was generated by adding the volumes of an infinite number of small pyramids. Each pyramid has its base as the surface of the sphere and its height from the base to the center of the sphere.
3. \(9202.8 \text{ in}^3\) 5. \(268.1 \text{ in}^3\) 7. \(155.2 \text{ m}^3\) 9. \(1853.3 \text{ m}^3\)
11. \(3261.8 \text{ ft}^3\) 13. \(233.4 \text{ in}^3\) 15. \(68.6 \text{ m}^3\) 17. \(7238.2 \text{ in}^3\)
19. \(= 21,990,642,871 \text{ km}^3\) 21. No, the volume of the cone is \(41.9 \text{ cm}^3\); the volume of the ice cream is about \(33.5 \text{ cm}^3\).
23. \(= 20,579.5 \text{ mm}^3\) 25. \(= 1162.1 \text{ mm}^2\) 27. \(\frac{2}{3}\)
29. \(= 587.7 \text{ in}^3\) 31. \(= 32.7 \text{ m}^3\) 33. about \(184 \text{ mm}^3\)
35. See students’ work. 37. \(A\) 39. \(142.3 \text{ m}^3\)
41. \((x - 2)^2 + (y + 1)^2 = 64\) 43. \((x - 2)^2 + (y - 1)^2 = 34\)
45. \(27\text{m}^3\) 47. \(\frac{867.0}{125}\)

Pages 710–713 Practice Quiz 1
1. 125.7 \text{ in}^3\) 3. 935.3 \text{ cm}^3\) 5. 42.3 \text{ in}^3\)

Pages 713 Practice Quiz 2
1. \(67,834.4 \text{ ft}^3\) 3. \(\frac{7}{5}\) 5. \(\frac{343}{125}\)

Pages 717–719 Lesson 13-5
1. The coordinate plane has 4 regions or quadrants with 4 possible combinations of signs for the ordered pairs. Three-dimensional space is the intersection of 3 planes that create 8 regions with 8 possible combinations of signs for the ordered triples. 3. A dilation of a rectangular prism will provide a similar figure, but not a congruent one unless \(r = 1\) or \(r = -1\).
5. 

\[
\begin{align*}
Q &= (-1, 0, 2) \\
P &= (0, 0, 2) \\
R &= (0, 0, 2) \\
U &= (-1, 0, 0) \\
V &= (0, 0, 0) \\
S &= (1, 0, 0) \\
T &= (0, 1, 0) \\
W &= (0, 0, 0) \\
N &= (0, 0, 0)
\end{align*}
\]
7. \( \sqrt{186}; \left(1, -\frac{7}{2}, \frac{7}{2}\right) \)
9. (12, 8, 8), (12, 0, 8), (0, 8, 8), (12, 8, 0), (12, 0, 0), (0, 0, 0), and (0, 8, 0); (−36, 8, 24), (−36, 0, 24), (−48, 0, 24), (−48, 8, 24) (−36, 8, 16), (−36, 0, 16), (−48, 0, 16), and (−48, 8, 16)

11.

13.

15.

17. \( PQ = \sqrt{115}; \left(\frac{1}{2}, -\frac{7}{2}, \frac{7}{2}\right) \)
19. \( GH = \sqrt{17}; \left(\frac{3}{5}, -\frac{7}{10}, 4\right) \)
21. \( BC = \sqrt{39}; \left(-\frac{\sqrt{3}}{2}, 3, 3\sqrt{2}\right) \)

23.

25. \( P'(0, 2, -2), Q'(0, 5, -2), R'(2, 5, -2), S'(2, 2, -2), T'(0, 5, -5), U'(0, 2, -5), V'(2, 2, -5), \) and \( W'(2, 5, -5) \)

27. \( A'(4, 5, 1), B'(4, 2, 1), C'(1, 2, 1), D'(1, 5, 1), E'(4, 5, -2), F'(4, 2, -2), G'(1, 2, -2), \) and \( H'(1, 5, -2) \)

29. \( A'(6, 6, 6), B'(6, 0, 6), C'(0, 0, 6), D'(0, 6, 6), E'(6, 6, 0), F'(6, 0, 0), G'(0, 0, 0), \) and \( H'(0, 6, 0); \ V = 216 \text{ units}^3 \)

31. 8.2 mi
33. (0, −14, 14)
35. \((x, y, z) \rightarrow (x + 2, y + 3, z - 5) \)
37. Sample answer: Three-dimensional graphing is used in computer animation to render images and allow them to move realistically. Answers should include the following.
- Ordered triples are a method of locating and naming points in space. An ordered triple is unique to one point.
- Applying transformations to points in space would allow an animator to create realistic movement in animation.

39. B
41. The locus of points in space with coordinates that satisfy the equation of \( x + z = 4 \) is a plane perpendicular to the \( xz \)-plane whose intersection with the \( xz \)-plane is the graph of \( z = -x + 4 \) in the \( xz \)-plane.

43. similar
45. 1150.3 yd\(^3\)
47. 12,770.1 ft\(^3\)

Pages 720–722 Chapter 13 Study Guide and Review
1. pyramid
3. an ordered triple
5. similar
7. the Distance Formula in Space
9. Cavalieri’s Principle
11. 504 in\(^3\)
13. 749.5 ft\(^3\)
15. 1466.4 ft\(^3\)
17. 33.5 ft\(^3\)
19. 4637.6 mm\(^3\)
21. 523.6 units\(^3\)
23. similar
25. \( CD = \sqrt{58}; (-9, 5.5, 5.5) \)
27. \( FG = \sqrt{422}; (1.5, \sqrt{2}, 3\sqrt{2}, -3) \)
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Red type denotes items only in the Teacher’s Wraparound Edition.

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AAS. See Angle-Angle-Side

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