Student Handbook

Skills

Reference

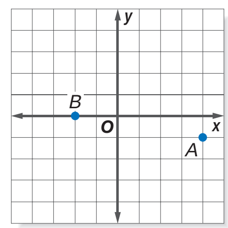
Prerequisite Skills

Prerequisite Skills

1 Graphing Ordered Pairs

- Points in the coordinate plane are named by **ordered pairs** of the form (x, y). The first number, or **x-coordinate**, corresponds to a number on the x-axis. The second number, or **y-coordinate**, corresponds to a number on the y-axis.

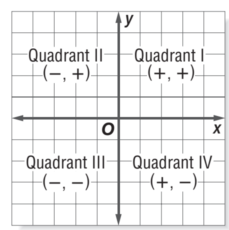
Example 1 **Write the ordered pair for each point.**



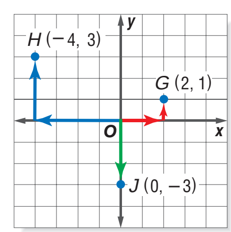
a. A
The x-coordinate is 4.
The y-coordinate is -1.
The ordered pair is $(4, -1)$.

b. B
The x-coordinate is -2.
The point lies on the x-axis, so its y-coordinate is 0.
The ordered pair is $(-2, 0)$.

- The x-axis and y-axis separate the coordinate plane into four regions, called **quadrants**. The point at which the axes intersect is called the **origin**. The axes and points on the axes are not located in any of the quadrants.



Example 2 **Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**



a. **$G(2, 1)$**
Start at the origin. Move 2 units right, since the x-coordinate is 2. Then move 1 unit up, since the y-coordinate is 1. Draw a dot, and label it G. Point $G(2, 1)$ is in Quadrant I.

b. **$H(-4, 3)$**
Start at the origin. Move 4 units left, since the x-coordinate is -4. Then move 3 units up, since the y-coordinate is 3. Draw a dot, and label it H. Point $H(-4, 3)$ is in Quadrant II.

c. **$J(0, -3)$**
Start at the origin. Since the x-coordinate is 0, the point lies on the y-axis. Move 3 units down, since the y-coordinate is -3. Draw a dot, and label it J. Because it is on one of the axes, point $J(0, -3)$ is not in any quadrant.

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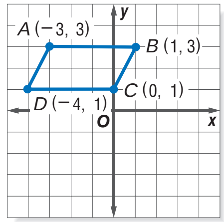
Example 3 **Graph a polygon with vertices $A(-3, 3)$, $B(1, 3)$, $C(0, 1)$, and $D(-4, 1)$.**

Graph the ordered pairs on a coordinate plane.
Connect each pair of consecutive points.
The polygon is a parallelogram.

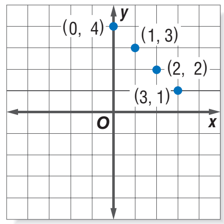


Example 4 **Graph four points that satisfy the equation $y = 4 - x$.**

Make a table.
Choose four values for x.
Evaluate each value of x for $4 - x$.

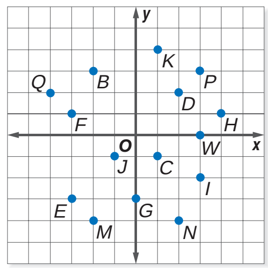
x	$4 - x$	y	(x, y)
0	4 − 0	4	(0, 4)
1	4 − 1	3	(1, 3)
2	4 − 2	2	(2, 2)
3	4 − 3	1	(3, 1)

Plot the points.



Exercises **Write the ordered pair for each point shown at the right.**

1. B **(−2, 3)**
2. C **(1, −1)**
3. D **(2, 2)**
4. E **(−3, −3)**
5. F **(−3, 1)**
6. G **(0, −3)**
7. H **(4, 1)**
8. I **(3, −2)**
9. J **(−1, −1)**
10. K **(1, 4)**
11. W **(3, 0)**
12. M **(−2, −4)**
13. N **(2, −4)**
14. P **(3, 3)**
15. Q **(−4, 2)**



Graph and label each point on a coordinate plane. Name the quadrant in which each point is located. **16–31. See margin for graph.**

16. $M(-1, 3)$ **II**
17. $S(2, 0)$ **none**
18. $R(-3, -2)$ **III**
19. $P(1, -4)$ **IV**
20. $B(5, -1)$ **IV**
21. $D(3, 4)$ **I**
22. $T(2, 5)$ **I**
23. $L(-4, -3)$ **III**
24. $A(-2, 2)$ **II**
25. $N(4, 1)$ **I**
26. $H(-3, -1)$ **III**
27. $F(0, -2)$ **none**
28. $C(-3, 1)$ **II**
29. $E(1, 3)$ **I**
30. $G(3, 2)$ **I**
31. $I(3, -2)$ **IV**

Graph the following geometric figures. **32–35. See margin.**

32. a square with vertices $W(-3, 3)$, $X(-3, -1)$, $Y(1, 3)$, and $Z(1, -1)$
33. a polygon with vertices $J(4, 2)$, $K(1, -1)$, $L(-2, 2)$, and $M(1, 5)$
34. a triangle with vertices $F(2, 4)$, $G(-3, 2)$, and $H(-1, -3)$
35. a rectangle with vertices $P(-2, -1)$, $Q(4, -1)$, $R(-2, 1)$, and $S(4, 1)$

Graph four points that satisfy each equation. **36–39. See margin for sample answers.**

36. $y = 2x$
37. $y = 1 + x$
38. $y = 3x - 1$
39. $y = 2 - x$

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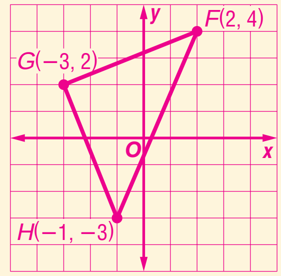
34.



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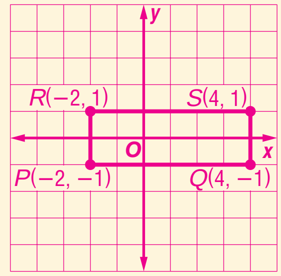

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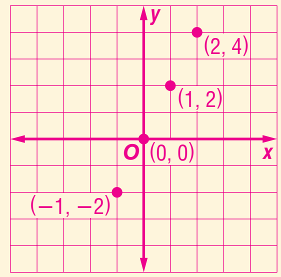

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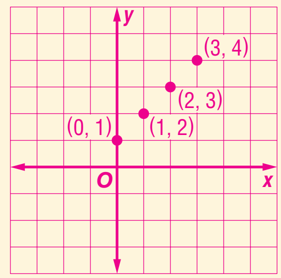

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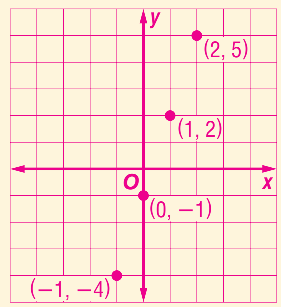

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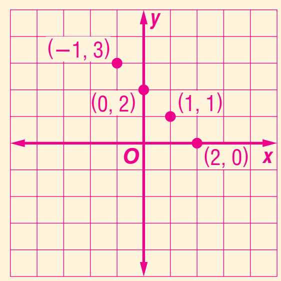
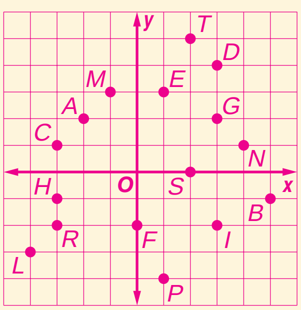


16–31.



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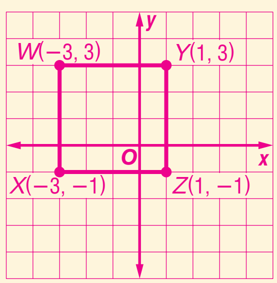

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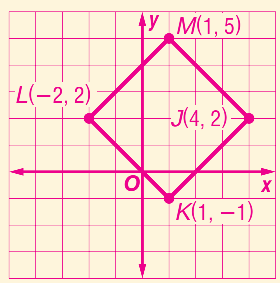

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2 Changing Units of Measure within Systems

Metric Units of Length
1 kilometer (km) = 1000 meters (m)
1 m = 100 centimeters (cm)
1 cm = 10 millimeters (mm)

Customary Units of Length
1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 ft
1 mile (mi) = 5280 ft

- To convert from larger units to smaller units, multiply.
- To convert from smaller units to larger units, divide.

Example 1 **State which metric unit you would use to measure the length of your pen.**

Since a pen has a small length, the *centimeter* is the appropriate unit of measure.

Example 2 **Complete each sentence.**

a. 4.2 km = _?_ m

There are 1000 meters in a kilometer.
4.2 km × 1000 = 4200 m

b. 125 mm = _?_ cm

There are 10 millimeters in a centimeter.
125 mm ÷ 10 = 12.5 cm

c. 16 ft = _?_ in.

There are 12 inches in a foot.
16 ft × 12 = 192 in.

d. 39 ft = _?_ yd

There are 3 feet in a yard.
39 ft ÷ 3 = 13 yd

Example 3 **Complete each sentence.**

a. 17 mm = _?_ m

There are 100 centimeters in a meter. First change *millimeters* to *centimeters*.

17 mm = _?_ cm — smaller unit → larger unit
17 mm ÷ 10 = 1.7 cm — Since 10 mm = 1 cm, divide by 10.

Then change *centimeters* to *meters*.

1.7 cm = _?_ m — smaller unit → larger unit
1.7 cm ÷ 100 = 0.017 m — Since 100 cm = 1 m, divide by 100.

b. 6600 yd = _?_ mi

There are 5280 feet in one mile. First change *yards* to *feet*.

6600 yd = _?_ ft — larger unit → smaller unit
6600 yd × 3 = 19,800 ft — Since 3 ft = 1 yd, multiply by 3.

Then change *feet* to *miles*.

19,800 ft = _?_ mi — smaller unit → larger unit
19,800 ft ÷ 5280 = $3\frac{3}{4}$ or 3.75 mi — Since 5280 ft = 1 mi, divide by 5280.

Metric Units of Capacity
1 liter (L) = 1000 millimeters (mL)

Customary Units of Capacity	
1 cup (c) = 8 fluid ounces (fl oz)	1 quart (qt) = 2 pt
1 pint (pt) = 2 c	1 gallon (gal) = 4 qt

Example 4 **Complete each sentence.**

a. 3.7 L = _?_ mL

There are 1000 milliliters in a liter.
3.7 L × 1000 = 3700 mL

b. 16 qt = _?_ gal

There are 4 quarts in a gallon.
16 qt ÷ 4 = 4 gal

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- Examples c and d involve two-step conversions.

c. 7 pt = _?_ fl oz

There are 8 fluid ounces in a cup.
First change *pints* to *cups*.

7 pt = _?_ c

7 pt × 2 = 14 c

Then change *cups* to *fluid ounces*.

14 c = _?_ fl oz

14 c × 8 = 112 fl oz

d. 4 gal = _?_ pt

There are 4 quarts in a gallon.
First change *gallons* to *quarts*.

4 gal = _?_ qt

4 gal × 4 = 16 qt

Then change *quarts* to *pints*.

16 qt = _?_ pt

16 qt × 2 = 32 pt

- The mass of an object is the amount of matter that it contains.

Metric Units of Mass
1 kilogram (kg) = 1000 grams (g)
1 g = 1000 milligrams (mg)

Customary Units of Weight
1 pound (lb) = 16 ounces (oz)
1 ton (T) = 2000 lb

Example 5 **Complete each sentence.**

a. 2300 mg = _?_ g

There are 1000 milligrams in a gram.

2300 mg ÷ 1000 = 2.3 g

b. 120 oz = _?_ lb

There are 16 ounces in a pound.

120 oz ÷ 16 = 7.5 lb

- Examples c and d involve two-step conversions.

c. 5.47 kg = _?_ mg

There are 1000 milligrams in a gram.
Change *kilograms* to *grams*.

5.47 kg = _?_ g

5.47 kg × 1000 = 5470 g

Then change *grams* to *milligrams*.

5470 g = _?_ mg

5470 g × 1000 = 5,470,000 mg

d. 5 T = _?_ oz

There are 16 ounces in a pound.
Change *tons* to *pounds*.

5 T = _?_ lb

5 T × 2000 = 10,000 lb

Then change *pounds* to *ounces*.

10,000 lb = _?_ oz

10,000 lb × 16 = 160,000 oz

Exercises **State which metric unit you would probably use to measure each item.**

1. radius of a tennis ball **cm**
2. length of a notebook **cm**
3. mass of a textbook **kg**
4. mass of a beach ball **g**
5. width of a football field **m**
6. thickness of a penny **mm**
7. amount of liquid in a cup **mL**
8. amount of water in a bath tub **L**

Complete each sentence.

9. 120 in. = _?_ ft **10**
10. 18 ft = _?_ yd **6**
11. 10 km = _?_ m **10,000**
12. 210 mm = _?_ cm **21**
13. 180 mm = _?_ m **0.18**
14. 3100 m = _?_ km **3.1**
15. 90 in. = _?_ yd **2.5**
16. 5280 yd = _?_ mi **3**
17. 8 yd = _?_ ft **24**
18. 0.62 km = _?_ m **620**
19. 370 mL = _?_ L **0.370**
20. 12 L = _?_ mL **12,000**
21. 32 fl oz = _?_ c **4**
22. 5 qt = _?_ c **20**
23. 10 pt = _?_ qt **5**
24. 48 c = _?_ gal **3**
25. 4 gal = _?_ qt **16**
26. 36 mg = _?_ g **0.036**
27. 13 lb = _?_ oz **208**
28. 130 g = _?_ kg **0.130**
29. 9.05 kg = _?_ g **9050**

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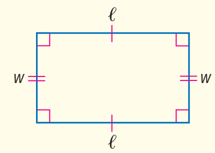
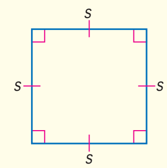
3 Perimeter and Area of Rectangles and Squares

Perimeter is the distance around a figure whose sides are segments. Perimeter is measured in linear units.

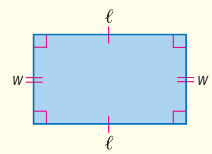
Perimeter of a Rectangle		Perimeter of a Square	
Words	Multiply two times the sum of the length and width.	**Words**	Multiply 4 times the length of a side.
Formula	$P = 2(\ell + w)$	**Formula**	$P = 4s$

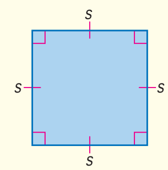




Area is the number of square units needed to cover a surface. Area is measured in square units.

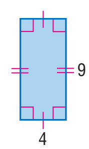
Area of a Rectangle		Area of a Square	
Words	Multiply the length and width.	**Words**	Square the length of a side.
Formula	$A = \ell w$	**Formula**	$A = s^2$





Example 1 **Find the perimeter and area of each rectangle.**

a.



$P = 2(\ell + w)$ Perimeter formula

$= 2(4 + 9)$ Replace ℓ with 4 and w with 9.

$= 26$ Simplify.

$A = \ell w$ Area formula

$= 4 \cdot 9$ Replace ℓ with 4 and w with 9.

$= 36$ Multiply.

The perimeter is 26 units, and the area is 36 square units.

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b. a rectangle with length 8 units and width 3 units.

$P = 2(\ell + w)$ Perimeter formula

$= 2(8 + 3)$ Replace ℓ with 8 and w with 3.

$= 22$ Simplify.

$A = \ell \cdot w$ Area formula

$= 8 \cdot 3$ Replace ℓ with 8 and w with 3.

$= 24$ Multiply

The perimeter is 22 units, and the area is 24 square units.

Example 2 **Find the perimeter and area of a square that has a side of length 14 feet.**

$P = 4s$ Perimeter formula

$= 4(14)$ $s = 14$

$= 56$ Multiply.

$A = s^2$ Area formula

$= 14^2$ $s = 14$

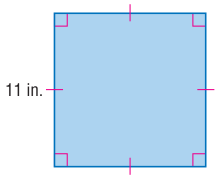
$= 196$ Multiply.

The perimeter is 56 feet, and the area is 196 square feet.

Exercises

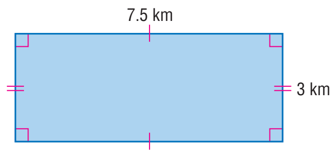
Find the perimeter and area of each figure.

1.



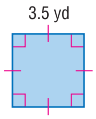
$P = 44$ in., $A = 121$ in^2

2.



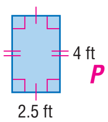
$P = 21$ km, $A = 22.5$ km^2

3.



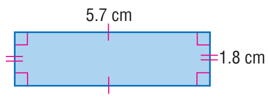
$P = 14$ yd, $A = 12.25$ yd^2

4.



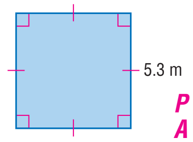
$P = 13$ ft, $A = 10$ ft^2

5.



$P = 15$ cm, $A = 10.26$ cm^2

6.



$P = 21.2$ m, $A = 28.09$ m^2

7. a rectangle with length 7 meters and width 11 meters $P = 36$ m, $A = 77$ m^2

8. a square with length 4.5 inches $P = 18$ in., $A = 20.25$ in^2

9. a rectangular sandbox with length 2.4 meters and width 1.6 meters $P = 8$ m, $A = 3.84$ m^2

10. a square with length 6.5 yards $P = 26$ yd, $A = 42.25$ yd^2

11. a square office with length 12 feet $P = 48$ ft, $A = 144$ ft^2

12. a rectangle with length 4.2 inches and width 15.7 inches $P = 39.8$ in., $A = 65.94$ in^2

13. a square with length 18 centimeters $P = 72$ cm, $A = 324$ cm^2

14. a rectangle with length 5.3 feet and width 7 feet $P = 24.6$ ft, $A = 37.1$ ft^2

15. FENCING Jansen purchased a lot that was 121 feet in width and 360 feet in length. If he wants to build a fence around the entire lot, how many feet of fence does he need? **962 ft**

16. CARPETING Leonardo's bedroom is 10 feet wide and 11 feet long. If the carpet store has a remnant whose area is 105 square feet, could it be used to cover his bedroom floor? Explain. **No, $10(11) = 110$ and $110 > 105$.**

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4 Operations with Integers

- The absolute value of any number n is its distance from zero on a number line and is written as $|n|$. Since distance cannot be less than zero, the absolute value of a number is always greater than or equal to zero.

Example 1 **Evaluate each expression.**

a. $|3|$

$|3| = 3$ — Definition of absolute value

b. $|-7|$

$|-7| = 7$ — Definition of absolute value

c. $|-4 + 2|$

$|-4 + 2| = |-2|$ — $-4 + 2 = -2$

$= 2$ — Simplify.

- To add integers with the same sign, add their absolute values. Give the result the same sign as the integers. To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

Example 2 **Find each sum.**

a. $-3 + (-5)$ — Both numbers are negative, so the sum is negative.

$-3 + (-5) = -8$ — Add $|-3|$ and $|-5|$.

b. $-4 + 2$ — The sum is negative because $|-4| > |2|$.

$-4 + 2 = -2$ — Subtract $|2|$ from $|-4|$.

c. $6 + (-3)$ — The sum is positive because $|6| > |-3|$.

$6 + (-3) = 3$ — Subtract $|-3|$ from $|6|$.

d. $1 + 8$ — Both numbers are positive, so the sum is positive

$1 + 8 = 9$ — Add $|1|$ and $|8|$.

- To subtract an integer, add its additive inverse.

Example 3 **Find each difference.**

a. $4 - 7$

$4 - 7 = 4 + (-7)$ — To subtract 7, add -7.

$= -3$

b. $2 - (-4)$

$2 - (-4) = 2 + 4$ — To subtract -4, add 4.

$= 6$

- The product of two integers with different signs is negative. The product of two integers with the same sign is positive. Similarly, the quotient of two integers with different signs is negative, and the quotient of two integers with the same sign is positive.

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Example 4 **Find each product or quotient.**

a. $4(-7)$ — The factors have different signs.

$4(-7) = -28$ — The product is negative.

b. $-64 \div (-8)$ — The dividend and divisor have the same sign.

$-64 \div (-8) = 8$ — The quotient is positive.

c. $-9(-6)$ — The factors have the same sign.

$-9(-6) = 54$ — The product is positive.

d. $-55 \div 5$ — The dividend and divisor have different signs.

$-55 \div 5 = -11$ — The quotient is negative.

e. $\frac{24}{-3}$ — The dividend and divisor have different signs.

$\frac{24}{-3} = -8$ — The quotient is negative.

- To evaluate expressions with absolute value, evaluate the absolute values first and then perform the operation.

Example 5 **Evaluate each expression.**

a. $|-3| - |5|$

$|-3| - |5| = 3 - 5$ — $|-3| = 3$, $|5| = 5$

$= -2$ — Simplify.

b. $|-5| + |-2|$

$|-5| + |-2| = 5 + 2$ — $|-5| = 5$, $|-2| = 2$

$= 7$ — Simplify.

Exercises **Evaluate each absolute value.**

1. $|-3|$ **3** **2.** $|4|$ **4** **3.** $|0|$ **0** **4.** $|-5|$ **5**

Find each sum or difference.

5. $-4 - 5$ **−9** **6.** $3 + 4$ **7** **7.** $9 - 5$ **4** **8.** $-2 - 5$ **−7**

9. $3 - 5$ **−2** **10.** $-6 + 11$ **5** **11.** $-4 + (-4)$ **−8** **12.** $5 - 9$ **−4**

13. $-3 + 1$ **−2** **14.** $-4 + (-2)$ **−6** **15.** $2 - (-8)$ **10** **16.** $7 + (-3)$ **4**

17. $-4 - (-2)$ **−2** **18.** $3 - (-3)$ **6** **19.** $3 + (-4)$ **−1** **20.** $-3 - (-9)$ **6**

Evaluate each expression.

21. $|-4| - |6|$ **−2** **22.** $|-7| + |-1|$ **8** **23.** $|1| + |-2|$ **3** **24.** $|2| - |-5|$ **−3**

25. $|-5 + 2|$ **3** **26.** $|6 + 4|$ **10** **27.** $|3 - 7|$ **4** **28.** $|-3 - 3|$ **6**

Find each product or quotient.

29. $-36 \div 9$ **−4** **30.** $-3(-7)$ **21** **31.** $6(-4)$ **−24** **32.** $-25 \div 5$ **−5**

33. $-6(-3)$ **18** **34.** $7(-8)$ **−56** **35.** $-40 \div (-5)$ **8** **36.** $11(3)$ **33**

37. $44 \div (-4)$ **−11** **38.** $-63 \div (-7)$ **9** **39.** $6(5)$ **30** **40.** $-7(12)$ **−84**

41. $-10(4)$ **−40** **42.** $80 \div (-16)$ **−5** **43.** $72 \div 9$ **8** **44.** $39 \div 3$ **13**

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5 Evaluating Algebraic Expressions

An expression is an algebraic expression if it contains sums and/or products of variables and numbers. To evaluate an algebraic expression, replace the variable or variables with known values, and then use the order of operations.

Order of Operations	
Step 1	Evaluate expressions inside grouping symbols.
Step 2	Evaluate all powers.
Step 3	Do all multiplications and/or divisions from left to right.
Step 4	Do all additions and/or subtractions from left to right.

Example 1 **Evaluate each expression.**

a. $x - 5 + y$ **if** $x = 15$ **and** $y = -7$

$x - 5 + y = 15 - 5 + (-7)$ $\quad x = 15, y = -7$

$= 10 + (-7)$ $\quad$ Subtract 5 from 15.

$= 3$ $\quad$ Add.

b. $6ab^2$ **if** $a = -3$ **and** $b = 3$

$6ab^2 = 6(-3)(3)^2$ $\quad a = -3, b = 3$

$= 6(-3)(9)$ $\quad 3^2 = 9$

$= (-18)(9)$ $\quad$ Multiply.

$= -162$ $\quad$ Multiply.

Example 2 **Evaluate each expression if** $m = -2$, $n = -4$, **and** $p = 5$.

a. $\dfrac{2m + n}{p - 3}$

The division bar is a grouping symbol. Evaluate the numerator and denominator before dividing.

$\dfrac{2m + n}{p - 3} = \dfrac{2(-2) + (-4)}{5 - 3}$ $\quad$ Replace m with -2, n with -4, and p with 5.

$= \dfrac{-4 - 4}{5 - 3}$ $\quad$ Multiply.

$= \dfrac{-8}{2}$ $\quad$ Subtract.

$= -4$ $\quad$ Simplify.

b. $-3(m^2 + 2n)$

$-3(m^2 + 2n) = -3[(-2)^2 + 2(-4)]$ $\quad$ Replace m with -2 and n with -4.

$= -3[4 + (-8)]$ $\quad$ Multiply.

$= -3(-4)$ $\quad$ Add.

$= 12$ $\quad$ Multiply.

Example 3 **Evaluate** $3|a - b| + 2|c - 5|$ **if** $a = -2$, $b = -4$, **and** $c = 3$.

$3|a - b| + 2|c - 5| = 3|-2 - (-4)| + 2|3 - 5|$ $\quad$ Substitute for a, b, and c.

$= 3|2| + 2|-2|$ $\quad$ Simplify.

$= 3(2) + 2(2)$ $\quad$ Find absolute values.

$= 10$ $\quad$ Simplify.

Exercises **Evaluate each expression if** $a = 2$, $b = -3$, $c = -1$, **and** $d = 4$.

1. $2a + c$ **3**
2. $\dfrac{bd}{2c}$ **6**
3. $\dfrac{2d - a}{b}$ **−2**
4. $3d - c$ **13**
5. $\dfrac{3b}{5a + c}$ **−1**
6. $5bc$ **15**
7. $2cd + 3ab$ **−26**
8. $\dfrac{c - 2d}{a}$ $-\dfrac{9}{2}$

Evaluate each expression if $x = 2$, $y = -3$, **and** $z = 1$.

9. $24 + |x - 4|$ **26**
10. $13 + |8 + y|$ **18**
11. $|5 - z| + 11$ **15**
12. $|2y - 15| + 7$ **28**
13. $|y| - 7$ **−4**
14. $11 - 7 + |-x|$ **6**
15. $|x| - |2z|$ **0**
16. $|z - y| + 6$ **10**

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6 Solving Linear Equations

- If the same number is added to or subtracted from each side of an equation, the resulting equation is true.

Example 1 **Solve each equation.**

a. $\mathbf{x - 7 = 16}$

$x - 7 = 16$	Original equation
$x - 7 + 7 = 16 + 7$	Add 7 to each side.
$x = 23$	Simplify.

b. $\mathbf{m + 12 = -5}$

$m + 12 = -5$	Original equation
$m + 12 + (-12) = -5 + (-12)$	Add -12 to each side.
$m = -17$	Simplify.

c. $\mathbf{k + 31 = 10}$

$k + 31 = 10$	Original equation
$k + 31 - 31 = 10 - 31$	Subtract 31 from each side.
$k = -21$	Simplify.

- If each side of an equation is multiplied or divided by the same number, the resulting equation is true.

Example 2 **Solve each equation.**

a. $\mathbf{4d = 36}$

$4d = 36$	Original equation
$\frac{4d}{4} = \frac{36}{4}$	Divide each side by 4.
$x = 9$	Simplify.

b. $\mathbf{-\frac{t}{8} = -7}$

$-\frac{t}{8} = -7$	Original equation.
$-8\left(-\frac{t}{8}\right) = -8(-7)$	Multiply each side by -8.
$t = 56$	Simplify.

c. $\mathbf{\frac{3}{5}x = -8}$

$\frac{3}{5}x = -8$	Original equation.
$\frac{5}{3}\left(\frac{3}{5}x\right) = \frac{5}{3}(-8)$	Multiply each side by $\frac{5}{3}$.
$x = -\frac{40}{3}$	Simplify.

- To solve equations with more than one operation, often called *multi-step equations*, undo operations by working backward.

Example 3 **Solve each equation.**

a. $\mathbf{12 - m = 20}$

$12 - m = 20$	Original equation
$12 - m - 12 = 20 - 12$	Subtract 12 from each side.
$-m = 8$	Simplify.
$m = -8$	Divide each side by -1.

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b. $\mathbf{8q - 15 = 49}$

$8q - 15 = 49$	Original equation
$8q - 15 + 15 = 49 + 15$	Add 15 to each side.
$8q = 64$	Simplify.
$\frac{8q}{8} = \frac{64}{8}$	Divide each side by 8.
$q = 8$	Simplify.

c. $\mathbf{12y + 8 = 6y - 5}$

$12y + 8 = 6y - 5$	Original equation
$12y + 8 - 8 = 6y - 5 - 8$	Subtract 8 from each side.
$12y = 6y - 13$	Simplify.
$12y - 6y = 6y - 13 - 6y$	Subtract 6y from each side.
$6y = -13$	Simplify.
$\frac{6y}{6} = \frac{-13}{6}$	Divide each side by 6.
$y = -\frac{13}{6}$	Simplify.

- When solving equations that contain grouping symbols, first use the Distributive Property to remove the grouping symbols.

Example 4 **Solve** $\mathbf{3(x - 5) = 13}$**.**

$3(x - 5) = 13$	Original equation
$3x - 15 = 13$	Distributive Property
$3x - 15 + 15 = 13 + 15$	Add 15 to each side.
$3x = 28$	Simplify.
$x = \frac{28}{3}$	Divide each side by 3.

Exercises **Solve each equation.**

1. $r + 11 = 3$ **-8**

2. $n + 7 = 13$ **6**

3. $d - 7 = 8$ **15**

4. $\frac{8}{5}a = -6$ **$-\frac{15}{4}$**

5. $-\frac{p}{12} = 6$ **-72**

6. $\frac{x}{4} = 8$ **32**

7. $\frac{12}{5}f = -18$ **$-\frac{15}{2}$**

8. $\frac{y}{7} = -11$ **-77**

9. $\frac{6}{7}y = 3$ **$\frac{7}{2}$**

10. $c - 14 = -11$ **3**

11. $t - 14 = -29$ **-15**

12. $p - 21 = 52$ **73**

13. $b + 2 = -5$ **-7**

14. $q + 10 = 22$ **12**

15. $-12q = 84$ **-7**

16. $5s = 30$ **6**

17. $5c - 7 = 8c - 4$ **-1**

18. $2\ell + 6 = 6\ell - 10$ **4**

19. $\frac{m}{10} + 15 = 21$ **60**

20. $-\frac{m}{8} + 7 = 5$ **16**

21. $8t + 1 = 3t - 19$ **-4**

22. $9n + 4 = 5n + 18$ **$\frac{7}{2}$**

23. $5c - 24 = -4$ **4**

24. $3n + 7 = 28$ **7**

25. $-2y + 17 = -13$ **15**

26. $-\frac{t}{13} - 2 = 3$ **-65**

27. $\frac{2}{9}x - 4 = \frac{2}{3}$ **21**

28. $9 - 4g = -15$ **6**

29. $-4 - p = -2$ **-2**

30. $21 - b = 11$ **10**

31. $-2(n + 7) = 15$ **$-\frac{29}{2}$**

32. $5(m - 1) = -25$ **-4**

33. $-8a - 11 = 37$ **-6**

34. $\frac{7}{4}q - 2 = -5$ **$-\frac{12}{7}$**

35. $2(5 - n) = 8$ **1**

36. $-3(d - 7) = 6$ **5**

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7 Solving Inequalities in One Variable

Statements with **greater than** ($>$), **less than** ($<$), **greater than or equal to** ($\geq$), or **less than or equal to** ($\leq$) are **inequalities**.

- If any number is added or subtracted to each side of an inequality, the resulting inequality is true.

Example 1 **Solve each inequality.**

a. $x - 17 > 12$

$x - 17 > 12$	Original inequality
$x - 17 + 17 > 12 + 17$	Add 17 to each side.
$x > 29$	Simplify.

The solution set is $\{x \mid x > 29\}$.

b. $y + 11 \leq 5$

$y + 11 \leq 5$	Original inequality
$y + 11 - 11 \leq 5 - 11$	Subtract 11 from each side.
$y \leq -6$	Simplify.

The solution set is $\{y \mid y \leq -6\}$.

- If each side of an inequality is multiplied or divided by a positive number, the resulting inequality is true.

Example 2 **Solve each inequality.**

a. $\frac{t}{6} \geq 11$

$\frac{t}{6} \geq 11$	Original inequality
$(6)\frac{t}{6} \geq (6)11$	Multiply each side by 6.
$t \geq 66$	Simplify.

The solution set is $\{t \mid t > 66\}$.

b. $8p < 72$

$8p < 72$	Original inequality
$\frac{8p}{8} < \frac{72}{8}$	Divide each side by 8.
$p < 9$	Simplify.

The solution set is $\{p \mid p < 9\}$.

- If each side of an inequality is multiplied or divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is true.

Example 3 **Solve each inequality.**

a. $-5c > 30$

$-5c > 30$	Original inequality
$\frac{-5c}{-5} < \frac{30}{-5}$	Divide each side by -5. Change $>$ to $<$.
$c < -6$	Simplify.

The solution set is $\{c \mid c < -6\}$.

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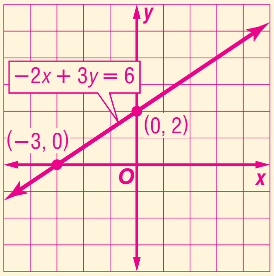

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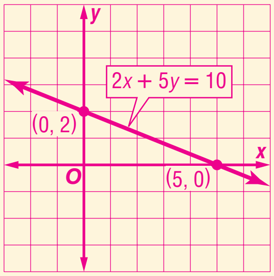

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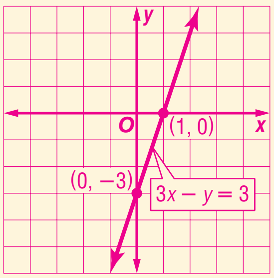

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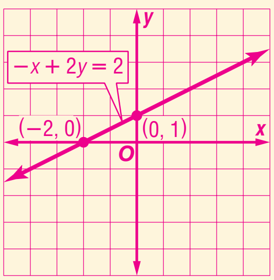

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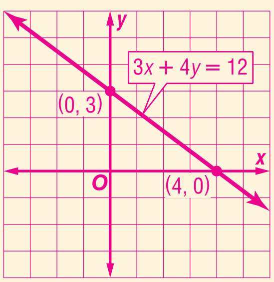

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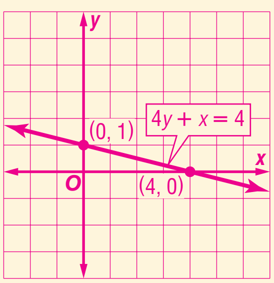

b. $-\frac{d}{13} \leq -4$

$-\frac{d}{13} \leq -4$	Original inequality
$(-13)\left(-\frac{d}{13}\right) \geq (-13)(-4)$	Multiply each side by −13. Change ≤ to ≥.
$d \geq 52$	Simplify.

The solution set is $\{d \mid d \geq 52\}$.

- Inequalities involving more than one operation can be solved by undoing the operations in the same way you would solve an equation with more than one operation.

Example 4 **Solve each inequality.**

a. $-6a + 13 < -7$

$-6a + 13 < -7$	Original inequality
$-6a + 13 - 13 < -7 - 13$	Subtract 13 from each side.
$-6a < -20$	Simplify.
$\frac{-6a}{-6} > \frac{-20}{-6}$	Divide each side by −6. Change < to >.
$a > \frac{10}{3}$	Simplify.

The solution set is $\left\{a \mid a > \frac{10}{3}\right\}$.

b. $4z + 7 \geq 8z - 1$

$4z + 7 \geq 8z - 1$	Original inequality.
$4z + 7 - 7 \geq 8z - 1 - 7$	Subtract 7 from each side.
$4z \geq 8z - 8$	Simplify.
$4z - 8z \geq 8z - 8 - 8z$	Subtract 8z from each side.
$-4z \geq -8$	Simplify.
$\frac{-4z}{-4} \leq \frac{-8}{-4}$	Divide each side by –4. Change ≥ to ≤ .
$z \leq 2$	Simplify.

The solution set is $\{z \mid z \leq 2\}$.

Exercises **Solve each inequality.**

1. $x - 7 < 6$ $\{x \mid x < 13\}$
2. $4c + 23 \leq -13$ $\{c \mid c \leq -9\}$
3. $-\frac{p}{5} \geq 14$ $\{p \mid p \leq -70\}$
4. $-\frac{a}{8} < 5$ $\{a \mid a > -40\}$
5. $\frac{t}{6} > -7$ $\{t \mid t > -42\}$
6. $\frac{a}{11} \leq 8$ $\{a \mid a \leq 88\}$
7. $d + 8 \leq 12$ $\{d \mid d \leq 4\}$
8. $m + 14 > 10$ $\{m \mid m > -4\}$
9. $2z - 9 < 7z + 1$ $\{z \mid z > -2\}$
10. $6t - 10 \geq 4t$ $\{t \mid t \geq 5\}$
11. $3z + 8 < 2$ $\{z \mid z < -2\}$
12. $a + 7 \geq -5$ $\{a \mid a \geq -12\}$
13. $m - 21 < 8$ $\{m \mid m < 29\}$
14. $x - 6 \geq 3$ $\{x \mid x \geq 9\}$
15. $-3b \leq 48$ $\{b \mid b \geq -16\}$
16. $4y < 20$ $\{y \mid y < 5\}$
17. $12k \geq -36$ $\{k \mid k \geq -3\}$
18. $-4h > 36$ $\{h \mid h < -9\}$
19. $\frac{2}{5}b - 6 \leq -2$ $\{b \mid b \leq 10\}$
20. $\frac{8}{3}t + 1 > -5$ $\left\{t \mid t > -\frac{9}{4}\right\}$
21. $7q + 3 \geq -4q + 25$ $\{q \mid q \geq 2\}$
22. $-3n - 8 > 2n + 7$ $\{n \mid n < -3\}$
23. $-3w + 1 \leq 8$ $\left\{w \mid w \geq -\frac{7}{3}\right\}$
24. $-\frac{4}{5}k - 17 > 11$ $\{k \mid k < -35\}$

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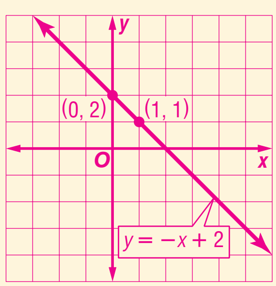

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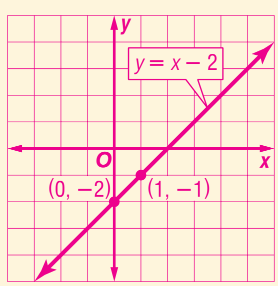

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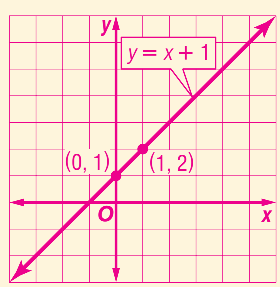

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8 Graphing Using Intercepts and Slope

- The x-coordinate of the point at which a line crosses the x-axis is called the **x-intercept**. The y-coordinate of the point at which a line crosses the y-axis is called the **y-intercept**. Since two points determine a line, one method of graphing a linear equation is to find these intercepts.

Example 1 **Determine the x-intercept and y-intercept of $4x - 3y = 12$. Then graph the equation.**

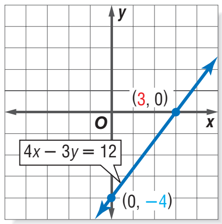
To find the x-intercept, let $y = 0$.

$4x - 3y = 12$	Original equation
$4x - 3(0) = 12$	Replace y with 0.
$4x = 12$	Simplify.
$x = 3$	Divide each side by 4.

To find the y-intercept, let $x = 0$.

$4x - 3y = 12$	Original equation
$4(0) - 3y = 12$	Replace x with 0.
$-3y = 12$	Divide each side by -3.
$y = -4$	Simplify.

Put a point on the x-axis at 3 and a point on the y-axis at -4. Draw the line through the two points.



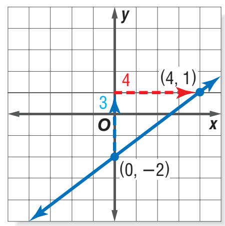
- A linear equation of the form $y = mx + b$ is in *slope-intercept* form, where m is the slope and b is the y-intercept. When an equation is written in this form, you can graph the equation quickly.

Example 2 **Graph $y = \frac{3}{4}x - 2$.**

Step 1 The y-intercept is -2. So, plot a point at $(0, -2)$.

Step 2 The slope is $\frac{3}{4}$. $\frac{\text{rise}}{\text{run}}$ From $(0, -2)$, move up 3 units and right 4 units. Plot a point.

Step 3 Draw a line connecting the points.



Exercises **Graph each equation using both intercepts.** **1–6. See margin.**

1. $-2x + 3y = 6$	2. $2x + 5y = 10$	3. $3x - y = 3$
4. $-x + 2y = 2$	5. $3x + 4y = 12$	6. $4y + x = 4$

Graph each equation using the slope and y-intercept. **7–12. See margin.**

7. $y = -x + 2$	8. $y = x - 2$	9. $y = x + 1$
10. $y = 3x - 1$	11. $y = -2x + 3$	12. $y = -3x - 1$

Graph each equation using either method. **13–21. See margin.**

13. $y = \frac{2}{3}x - 3$	14. $y = \frac{1}{2}x - 1$	15. $y = 2x - 2$
16. $-6x + y = 2$	17. $2y - x = -2$	18. $3x + 4y = -12$
19. $4x - 3y = 6$	20. $4x + y = 4$	21. $y = 2x - \frac{3}{2}$

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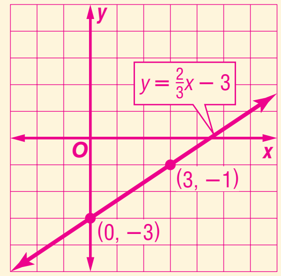

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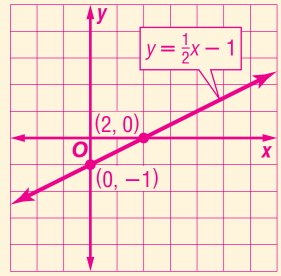

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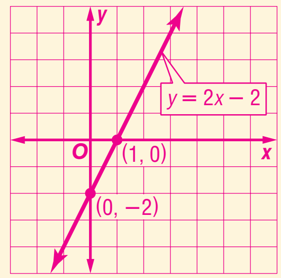

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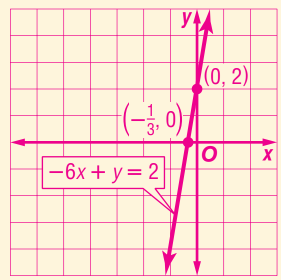

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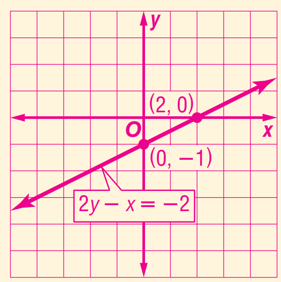

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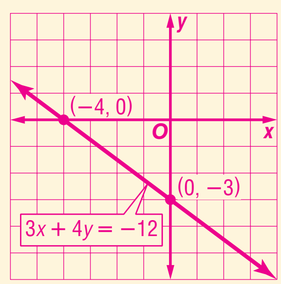

Answers continued on the following page.

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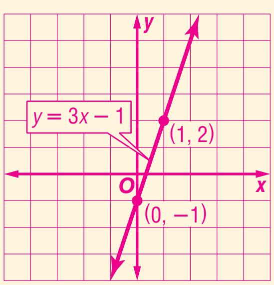

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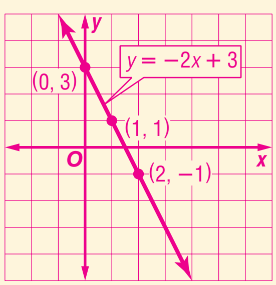

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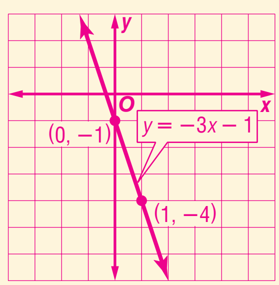

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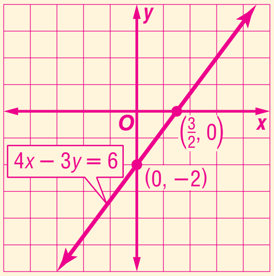

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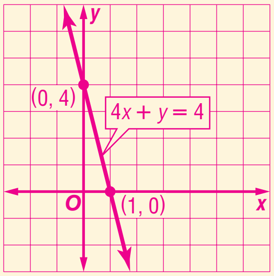

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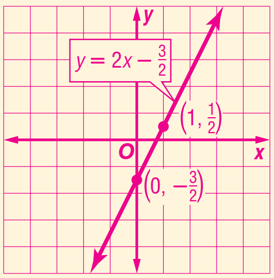
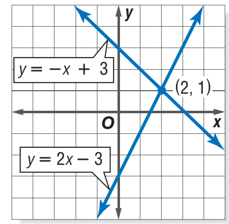


9 Solving Systems of Linear Equations

- Two or more equations that have common variables are called a **system of equations**. The solution of a system of equations in two variables is an ordered pair of numbers that satisfies both equations. A system of two linear equations can have zero, one, or an infinite number of solutions. There are three methods by which systems of equations can be solved: graphing, elimination, and substitution.

Example 1 **Solve each system of equations by graphing. Then determine whether each system has *no* solution, *one* solution, or *infinitely many* solutions.**

a. $y = -x + 3$
$y = 2x - 3$



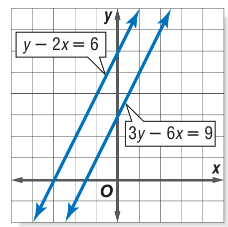
The graphs appear to intersect at (2, 1). Check this estimate by replacing x with 2 and y with 1 in each equation.

Check:

$y = -x + 3$	$y = 2x - 3$
$1 \stackrel{?}{=} -2 + 3$	$1 \stackrel{?}{=} 2(2) - 3$
$1 = 1$ ✓	$1 = 1$ ✓

The system has one solution at (2, 1).

b. $y - 2x = 6$
$3y - 6x = 9$



The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions of this system of equations. Notice that the lines have the same slope but different y-intercepts. Equations with the same slope *and* the same y-intercepts have an infinite number of solutions.

- It is difficult to determine the solution of a system when the two graphs intersect at noninteger values. There are algebraic methods by which an exact solution can be found. One such method is **substitution**.

Example 2 **Use substitution to solve the system of equations.**

$y = -4x$

$2y + 3x = 8$

Since $y = -4x$, substitute $-4x$ for y in the second equation.

$2y + 3x = 8$	Second equation
$2(-4x) + 3x = 3$	$y = -4x$
$-8x + 3x = 8$	Simplify.
$-5x = 8$	Combine like terms.
$\frac{-5x}{-5} = \frac{8}{-5}$	Divide each side by −5.
$x = -\frac{8}{5}$	Simplify.

Use $y = -4x$ to find the value of y.

$y = -4x$	First equation
$y = -4\left(-\frac{8}{5}\right)$	$x = -\frac{8}{5}$
$y = \frac{32}{5}$	Simplify.

The solution is $\left(-\frac{8}{5}, \frac{32}{5}\right)$.

- Sometimes adding or subtracting two equations together will eliminate one variable. Using this step to solve a system of equations is called **elimination**.

Example 3 **Use elimination to solve the system of equations.**

$\mathbf{3x + 5y = 7}$

$\mathbf{4x + 2y = 0}$

Either x or y can be eliminated. In this example, we will eliminate x.

$$3x + 5y = 7 \xrightarrow{\text{Multiply by 4.}} 12x + 20y = 28$$

$$4x + 2y = 0 \xrightarrow{\text{Multiply by } -3.} + \; -12x - 6y = 0$$

$$\begin{aligned} 14y &= 28 && \text{Add the equations.} \\ \frac{14y}{14} &= \frac{28}{14} && \text{Divide each side by 14.} \\ y &= 2 && \text{Simplify.} \end{aligned}$$

Now substitute 2 for y in either equation to find the value of x.

$$\begin{aligned} 4x + 2y &= 0 && \text{Second equation} \\ 4x + 2(2) &= 0 && y = 2 \\ 4x + 4 &= 0 && \text{Simplify.} \\ 4x + 4 - 4 &= 0 - 4 && \text{Subtract 4 from each side.} \\ 4x &= -4 && \text{Simplify.} \\ \frac{4x}{4} &= \frac{-4}{4} && \text{Divide each side by 4.} \\ x &= -1 && \text{Simplify.} \end{aligned}$$

The solution is $(-1, 2)$.

Exercises

Solve by graphing.

1. $y = -x + 2$; $y = -\frac{1}{2}x + 1$ **(2, 0)**

2. $y = 3x - 3$; $y = x + 1$ **(2, 3)**

3. $y - 2x = 1$; $2y - 4x = 1$ **no solution**

4. $2x - 4y = -2$; $-6x + 12y = 6$ **infinitely many solutions**

5. $4x + 3y = 12$; $3x - y = 9$ **(3, 0)**

6. $3y + x = -3$; $y - 3x = -1$ **(0, −1)**

Solve by substitution.

7. $-5x + 3y = 12$; $x + 2y = 8$ **(0, 4)**

8. $x - 4y = 22$; $2x + 5y = -21$ **(2, −5)**

9. $y + 5x = -3$; $3y - 2x = 8$ **(−1, 2)**

10. $y - 2x = 2$; $7y + 4x = 23$ $\left(\frac{1}{2}, 3\right)$

11. $2x - 3y = -8$; $-x + 2y = 5$ **(−1, 2)**

12. $4x + 2y = 5$; $3x - y = 10$ $\left(\frac{5}{2}, -\frac{5}{2}\right)$

Solve by elimination.

13. $-3x + y = 7$; $3x + 2y = 2$ $\left(-\frac{4}{3}, 3\right)$

14. $3x + 4y = -1$; $-9x - 4y = 13$ $\left(-2, \frac{5}{4}\right)$

15. $-4x + 5y = -11$; $2x + 3y = 11$ **(4, 1)**

16. $6x - 5y = 1$; $-2x + 9y = 7$ **(1, 1)**

17. $3x - 2y = 8$; $5x - 3y = 16$ **(8, 8)**

18. $4x + 7y = -17$; $3x + 2y = -3$ **(1, −3)**

Name an appropriate method to solve each system of equations. Then solve the system.

19. $4x - y = 11$; $2x - 3y = 3$ **elimination or substitution, (3, 1)**

20. $4x + 6y = 3$; $-10x - 15y = -4$ **elimination, no solution**

21. $3x - 2y = 6$; $5x - 5y = 5$ **graphing, (4, 3)**

22. $3y + x = 3$; $-2y + 5x = 15$ **elimination or substitution, (3, 0)**

23. $4x - 7y = 8$; $-2x + 5y = -1$ **elimination,** $\left(\frac{11}{2}, 2\right)$

24. $x + 3y = 6$; $4x - 2y = -32$ **elimination or substitution, (−6, 4)**

Prerequisite Skills

Prerequisite Skills 743

10 Square Roots and Simplifying Radicals

- A radical expression is an expression that contains a square root. The expression is in simplest form when the following three conditions have been met.
- No radicands have perfect square factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.
- The **Product Property** states that for two numbers a and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1 **Simplify.**

a. $\sqrt{45}$

$\sqrt{45} = \sqrt{3 \cdot 3 \cdot 5}$ Prime factorization of 45

$= \sqrt{3^2} \cdot \sqrt{5}$ Product Property of Square Roots

$= 3\sqrt{5}$ Simplify.

b. $\sqrt{3} \cdot \sqrt{3}$

$\sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3}$ Product Property

$= \sqrt{9}$ or 3 Simplify.

c. $\sqrt{6} \cdot \sqrt{15}$

$\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15}$ Product Property

$= \sqrt{3 \cdot 2 \cdot 3 \cdot 5}$ Prime factorization

$= \sqrt{3^2} \cdot \sqrt{10}$ Product Property

$= 3\sqrt{10}$ Simplify.

- For radical expressions in which the exponent of the variable inside the radical is *even* and the resulting simplified exponent is *odd*, you must use absolute value to ensure nonnegative results.

Example 2 $\sqrt{20x^3y^5z^6}$

$\sqrt{20x^3y^5z^6} = \sqrt{2^2 \cdot 5 \cdot x^3 \cdot y^5 \cdot z^6}$ Prime factorization

$= \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{x^3} \cdot \sqrt{y^5} \cdot \sqrt{z^6}$ Product Property

$= 2 \cdot \sqrt{5} \cdot x \cdot \sqrt{x} \cdot y^2 \cdot \sqrt{y} \cdot |z^3|$ Simplify.

$= 2xy^2|z^3|\sqrt{5xy}$ Simplify.

- The **Quotient Property** states that for any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example 3 **Simplify** $\sqrt{\frac{25}{16}}$.

$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}}$ Quotient Property

$= \frac{5}{4}$ Simplify.

- Rationalizing the denominator of a radical expression is a method used to eliminate radicals from the denominator of a fraction. To rationalize the denominator, multiply the expression by a fraction equivalent to 1 such that the resulting denominator is a perfect square.

Example 4 **Simplify.**

a. $\dfrac{2}{\sqrt{3}}$

$\dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\dfrac{\sqrt{3}}{\sqrt{3}}$.

$= \dfrac{2\sqrt{3}}{3}$ Simplify.

b. $\dfrac{\sqrt{13y}}{\sqrt{18}}$

$\dfrac{\sqrt{13y}}{\sqrt{18}} = \dfrac{\sqrt{13y}}{\sqrt{2 \cdot 3 \cdot 3}}$ Prime factorization

$= \dfrac{\sqrt{13y}}{3\sqrt{2}}$ Product Property

$= \dfrac{\sqrt{13y}}{3\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$ Multiply by $\dfrac{\sqrt{2}}{\sqrt{2}}$.

$= \dfrac{\sqrt{26y}}{6}$ Product Property

- Sometimes, conjugates are used to simplify radical expressions. Conjugates are binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$.

Example 5 **Simplify** $\dfrac{3}{5 - \sqrt{2}}$.

$\dfrac{3}{5 - \sqrt{2}} = \dfrac{3}{5 - \sqrt{2}} \cdot \dfrac{5 + \sqrt{2}}{5 + \sqrt{2}}$ $\dfrac{5 + \sqrt{2}}{5 + \sqrt{2}} = 1$

$= \dfrac{3(5 + \sqrt{2})}{5^2 - (\sqrt{2})^2}$ $(a - b)(a + b) = a^2 - b^2$

$= \dfrac{15 + 3\sqrt{2}}{25 - 2}$ Multiply. $(\sqrt{2})^2 = 2$

$= \dfrac{15 + 3\sqrt{2}}{23}$ Simplify.

Exercises **Simplify.** **8.** $2|a|b^2c^2\sqrt{14c}$ **19.** $\dfrac{6\sqrt{5} + 3\sqrt{10}}{2}$

1. $\sqrt{32}$ $4\sqrt{2}$ **2.** $\sqrt{75}$ $5\sqrt{3}$ **3.** $\sqrt{50} \cdot \sqrt{10}$ $10\sqrt{5}$ **4.** $\sqrt{12} \cdot \sqrt{20}$ $4\sqrt{15}$

5. $\sqrt{6} \cdot \sqrt{6}$ 6 **6.** $\sqrt{16} \cdot \sqrt{25}$ 20 **7.** $\sqrt{98x^3y^6}$ $7x|y^3|\sqrt{2x}$ **8.** $\sqrt{56a^2b^4c^5}$

9. $\sqrt{\dfrac{81}{49}}$ $\dfrac{9}{7}$ **10.** $\sqrt{\dfrac{121}{16}}$ $\dfrac{11}{4}$ **11.** $\sqrt{\dfrac{63}{8}}$ $\dfrac{3\sqrt{14}}{4}$ **12.** $\sqrt{\dfrac{288}{147}}$ $\dfrac{4\sqrt{6}}{7}$

13. $\dfrac{\sqrt{10p^3}}{\sqrt{27}}$ $\dfrac{p\sqrt{30p}}{9}$ **14.** $\dfrac{\sqrt{108}}{\sqrt{2q^6}}$ $\dfrac{3\sqrt{6}}{|q^3|}$ **15.** $\dfrac{4}{5 - 2\sqrt{3}}$ $\dfrac{20 + 8\sqrt{3}}{13}$ **16.** $\dfrac{7\sqrt{3}}{5 - 2\sqrt{6}}$ $35\sqrt{3} + 42\sqrt{2}$

17. $\dfrac{3}{\sqrt{48}}$ $\dfrac{\sqrt{3}}{4}$ **18.** $\dfrac{\sqrt{24}}{\sqrt{125}}$ $\dfrac{2\sqrt{30}}{25}$ **19.** $\dfrac{3\sqrt{5}}{2 - \sqrt{2}}$ **20.** $\dfrac{3}{-2 + \sqrt{13}}$ $\dfrac{2 + \sqrt{13}}{3}$

Prerequisite Skills
Prerequisite Skills

Prerequisite Skills 745

Prerequisite Skills

11 Multiplying Polynomials

- The **Product of Powers** rule states that for any number a and all integers m and n, $a^m \cdot a^n = a^{m+n}$.

Example 1 **Simplify each expression.**

a. $(4p^5)(p^4)$

$$\begin{aligned} (4p^5)(p^4) &= (4)(1)(p^5 \cdot p^4) && \text{Commutative and Associative Properties} \\ &= (4)(1)(p^{5+4}) && \text{Product of powers} \\ &= 4p^9 && \text{Simplify.} \end{aligned}$$

b. $(3yz^5)(-9y^2z^2)$

$$\begin{aligned} (3yz^5)(-9y^2z^2) &= (3)(-9)(y \cdot y^2)(z^5 \cdot z^2) && \text{Commutative and Associative Properties} \\ &= -27(y^{1+2})(z^{5+2}) && \text{Product of powers} \\ &= -27y^3z^7 && \text{Simplify.} \end{aligned}$$

- The Distributive Property can be used to multiply a monomial by a polynomial.

Example 2 **Simplify $3x^3(-4x^2 + x - 5)$.**

$$\begin{aligned} 3x^3(-4x^2 + x - 5) &= 3x^3(-4x^2) + 3x^3(x) - 3x^3(5) && \text{Distributive Property} \\ &= -12x^5 + 3x^4 - 15x^3 && \text{Multiply.} \end{aligned}$$

- To find the power of a power, multiply the exponents. This is called the **Power of a Power** rule.

Example 3 **Simplify each expression.**

a. $(-3x^2y^4)^3$

$$\begin{aligned} (-3x^2y^4)^3 &= (-3)^3(x^2)^3(y^4)^3 && \text{Power of a product} \\ &= -27x^6y^{12} && \text{Power of a power} \end{aligned}$$

b. $(xy)^3(-2x^4)^2$

$$\begin{aligned} (xy)^3(-2x^4)^2 &= x^3y^3(-2)^2(x^4)^2 && \text{Power of a product} \\ &= x^3y^3(4)x^8 && \text{Power of a power} \\ &= 4x^3 \cdot x^8 \cdot y^3 && \text{Commutative Property} \\ &= 4x^{11}y^3 && \text{Product of powers} \end{aligned}$$

- To multiply two binomials, find the sum of the products of

  F the *First* terms,
  O the *Outer* terms,
  I the *Inner* terms, and
  L the *Last* terms.

Example 4 **Find each product.**

a. $(2x - 3)(x + 1)$

$$\begin{aligned} (2x - 3)(x + 1) &= \overset{F}{(2x)(x)} + \overset{O}{(2x)(1)} + \overset{I}{(-3)(x)} + \overset{L}{(-3)(1)} && \text{FOIL method} \\ &= 2x^2 + 2x - 3x - 3 && \text{Multiply.} \\ &= 2x^2 - x - 3 && \text{Combine like terms.} \end{aligned}$$

b. $(x + 6)(x + 5)$

$$\begin{aligned} (x + 6)(x + 5) &= \overset{F}{(x)(x)} + \overset{O}{(x)(5)} + \overset{I}{(6)(x)} + \overset{L}{(6)(5)} && \text{FOIL method} \\ &= x^2 + 5x + 6x + 30 && \text{Multiply.} \\ &= x^2 + 11x + 30 && \text{Combine like terms.} \end{aligned}$$

746 Prerequisite Skills

- The Distributive Property can be used to multiply any two polynomials.

Example 5 **Find $(3x - 2)(2x^2 + 7x - 4)$.**

$$\begin{aligned}(3x - 2)(2x^2 + 7x - 4) &= 3x(2x^2 + 7x - 4) - 2(2x^2 + 7x - 4) && \text{Distributive Property}\\ &= 6x^3 + 21x^2 - 12x - 4x^2 - 14x + 8 && \text{Distributive Property}\\ &= 6x^3 + 17x^2 - 26x + 8 && \text{Combine like terms.}\end{aligned}$$

- Three special products are: $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$, and $(a + b)(a - b) = a^2 - b^2$.

Example 6 **Find each product.**

a. $(2x - z)^2$

$$\begin{aligned}(a - b)^2 &= a^2 - 2ab + b^2 && \text{Square of a difference}\\ (2x - z)^2 &= (2x)^2 - 2(2x)(z) + (z)^2 && a = 2x \text{ and } b = z\\ &= 4x^2 - 4xz + z^2 && \text{Simplify.}\end{aligned}$$

b. $(3x + 7)(3x - 7)$

$$\begin{aligned}(a + b)(a - b) &= a^2 - b^2 && \text{Product of sum and difference}\\ (3x + 7)(3x - 7) &= (3x)^2 - (7)^2 && a = 3x \text{ and } b = 7\\ &= 9x^2 - 49 && \text{Simplify.}\end{aligned}$$

Exercises **Find each product.**

1. $(3q^2)(q^5)$ $3q^7$
2. $(5m)(4m^3)$ $20m^4$
3. $\left(\frac{9}{2}c\right)(8c^5)$ $36c^6$
4. $(n^6)(10n^2)$ $10n^8$
5. $(fg^8)(15f^2g)$ $15f^3g^9$
6. $(6j^4k^4)(j^2k)$ $6j^6k^5$
7. $(2ab^3)(4a^2b^2)$ $8a^3b^5$
8. $\left(\frac{8}{5}x^3y\right)(4x^3y^2)$ $\frac{32}{5}x^6y^3$
9. $-2q^2(q^2 + 3)$ $-2q^4 - 6q^2$
10. $5p(p - 18)$ $5p^2 - 90p$
11. $15c(-3c^2 + 2c + 5)$ $-45c^3 + 30c^2 + 75c$
12. $8x(-4x^2 - x + 11)$ $-32x^3 - 8x^2 + 88x$
13. $4m^2(-2m^2 + 7m - 5)$ $-8m^4 + 28m^3 - 20m^2$
14. $8y^2(5y^3 - 2y + 1)$ $40y^5 - 16y^3 + 8y^2$
15. $\left(\frac{3}{2}m^3n^2\right)^2$ $\frac{9}{4}m^6n^4$
16. $(-2c^3d^2)^2$ $4c^6d^4$
17. $(-5wx^5)^3$ $-125w^3x^{15}$
18. $(6a^5b)^3$ $216a^{15}b^3$
19. $(k^2\ell)^3(13k^2)^2$ $169k^{10}\ell^3$
20. $(-5w^3x^2)^2(2w^5)^2$ $100w^{16}x^4$
21. $(-7y^3z^2)(4y^2)^4$ $-1792y^{11}z^2$
22. $\left(\frac{1}{2}p^2q^2\right)^2(4pq^3)^3$ $16p^7q^{13}$
23. $(m - 1)(m - 4)$ $m^2 - 5m + 4$
24. $(s - 7)(s - 2)$ $s^2 - 9s + 14$
25. $(x - 3)(x + 4)$ $x^2 + x - 12$
26. $(a + 3)(a - 6)$ $a^2 - 3a - 18$
27. $(5d + 3)(d - 4)$ $5d^2 - 17d - 12$
28. $(q + 2)(3q + 5)$ $3q^2 + 11q + 10$
29. $(2q + 3)(5q + 2)$ $10q^2 + 19q + 6$
30. $(2a - 3)(2a - 5)$ $4a^2 - 16a + 15$
31. $(d + 1)(d - 1)$ $d^2 - 1$
32. $(4a - 3)(4a + 3)$ $16a^2 - 9$
33. $(s - 5)^2$ $s^2 - 10s + 25$
34. $(3f - g)^2$ $9f^2 - 6fg + g^2$
35. $(2r - 5)^2$ $4r^2 - 20r + 25$
36. $\left(t + \frac{8}{3}\right)^2$ $t^2 + \frac{16}{3}t + \frac{64}{9}$
37. $(x + 4)(x^2 - 5x - 2)$ $x^3 - x^2 - 22x - 8$
38. $(x - 2)(x^2 + 3x - 7)$ $x^3 + x^2 - 13x + 14$
39. $(3b - 2)(3b^2 + b + 1)$ $9b^3 - 3b^2 + b - 2$
40. $(2j + 7)(j^2 - 2j + 4)$ $2j^3 + 3j^2 - 6j + 28$

Prerequisite Skills
Prerequisite Skills 747

Prerequisite Skills

12 Dividing Polynomials

- The **Quotient of Powers** rule states that for any nonzero number a and all integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.

- To find the power of a quotient, find the power of the numerator and the power of the denominator.

Example 1 **Simplify.**

a. $\frac{x^5y^8}{-xy^3}$

$$\frac{x^5y^8}{-xy^3} = \left(\frac{x^5}{-x}\right)\left(\frac{y^8}{y^3}\right) \quad \text{Group powers that have the same base.}$$

$$= -(x^{5-1})(y^{8-3}) \quad \text{Quotient of powers}$$

$$= -x^4y^5 \quad \text{Simplify.}$$

b. $\left(\frac{4z^3}{3}\right)^3$

$$\left(\frac{4z^3}{3}\right)^3 = \frac{(4z^3)^3}{3^3} \quad \text{Power of a quotient}$$

$$= \frac{4^3(z^3)^3}{3^3} \quad \text{Power of a product}$$

$$= \frac{64z^9}{27} \quad \text{Power of a product}$$

c. $\frac{w^{-2}x^4}{2w^{-5}}$

$$\frac{w^{-2}x^4}{2w^{-5}} = \frac{1}{2}\left(\frac{w^{-2}}{w^{-5}}\right)x^4 \quad \text{Group powers that have the same base.}$$

$$= \frac{1}{2}(w^{-2-(-5)})x^4 \quad \text{Quotient of powers}$$

$$= \frac{1}{2}w^3x^4 \quad \text{Simplify.}$$

- You can divide a polynomial by a monomial by separating the terms of the numerator.

Example 2 **Simplify** $\frac{15x^3 - 3x^2 + 12x}{3x}$.

$$\frac{15x^3 - 3x^2 + 12x}{3x} = \frac{15x^3}{3x} - \frac{3x^2}{3x} + \frac{12x}{3x} \quad \text{Divide each term by } 3x.$$

$$= 5x^2 - x + 4 \quad \text{Simplify.}$$

- Division can sometimes be performed using factoring.

Example 3 **Find** $(n^2 - 8n - 9) \div (n - 9)$.

$$(n^2 - 8n - 9) \div (n - 9) = \frac{n^2 - 8n - 9}{(n-9)} \quad \text{Write as a rational expression.}$$

$$= \frac{(n-9)(n+1)}{(n-9)} \quad \text{Factor the numerator.}$$

$$= \frac{\cancel{(n-9)}(n+1)}{\cancel{(n-9)}} \quad \text{Divide by the GCF.}$$

$$= n + 1 \quad \text{Simplify.}$$

748 Prerequisite Skills

- When you cannot factor, you can use a long division process similar to the one you use in arithmetic.

Example 4 **Find $(n^3 - 4n^2 - 9) \div (n - 3)$.**

In this case, there is no n term, so you must rename the dividend using 0 as the coefficient of the missing term.

$(n^3 - 4n^2 + 9) \div (n - 3) = (n^3 - 4n^2 + 0n + 9) \div (n - 3)$

Divide the first term of the dividend, n^3, by the first term of the divisor, n.

$$
\begin{array}{rll}
 & n^2 - n - 3 & \\
n - 3 \overline{)} & n^3 - 4n^2 + 0n + 12 & \\
 & \underline{(-)\ n^3 - 3n^2} & \text{Multiply } n^2 \text{ and } n - 3. \\
 & -n^2 + 0n & \text{Subtract and bring down } 0n. \\
 & \underline{(-) -n^2 + 3n} & \text{Multiply } -n \text{ and } n - 3. \\
 & -3n + 12 & \text{Subtract and bring down 12.} \\
 & \underline{(-) -3n + 9} & \text{Multiply } -3 \text{ and } n - 3. \\
 & 3 & \text{Subtract.}
\end{array}
$$

Therefore, $(n^3 - 4n^2 + 9) \div (n - 3) = n^2 - n - 3 + \frac{3}{n - 3}$. Since the quotient has a nonzero remainder, $n - 3$ is not a factor of $n^3 - 4n^2 + 9$.

Exercises **Find each quotient.**

1. $\frac{a^2c^2}{2a}$ $\frac{ac^2}{2}$
2. $\frac{5q^5r^3}{q^2r^2}$ $5q^3r$
3. $\frac{b^2d^5}{8b^{-2}d^3}$ $\frac{b^4d^2}{8}$
4. $\frac{5p^{-3}x}{2p^{-7}}$ $\frac{5}{2}p^4x$
5. $\frac{3r^{-3}s^2t^4}{2r^2st^{-3}}$ $\frac{3st^7}{2r^5}$
6. $\frac{3x^3y^{-1}z^5}{xyz^2}$ $\frac{3x^2z^3}{y^2}$
7. $\left(\frac{w^4}{6}\right)^3$ $\frac{w^{12}}{216}$
8. $\left(\frac{-3q^2}{5}\right)^3$ $\frac{-27q^6}{125}$
9. $\left(\frac{-2y^2}{7}\right)^2$ $\frac{4y^4}{49}$
10. $\left(\frac{5m^2}{3}\right)^4$ $\frac{625m^8}{81}$
11. $\frac{4z^2 - 16z - 36}{4z}$ $z - 4 - \frac{9}{z}$
12. $(5d^2 + 8d - 20) \div 10d$ $\frac{d}{2} + \frac{4}{5} + \frac{2}{d}$
13. $(p^3 - 12p^2 + 3p + 8) \div 4p$ $\frac{p^2}{4} - 3p + \frac{3}{4} + \frac{2}{p}$
14. $(b^3 + 4b^2 + 10) \div 2b$ $\frac{b^2}{2} + 2b + \frac{5}{b}$
15. $\frac{a^3 - 6a^2 + 4a - 3}{a^2}$ $a - 6 + \frac{4}{a} - \frac{3}{a^2}$
16. $\frac{8x^2y - 10xy^2 + 6x^3}{2x^2}$ $4y - \frac{5y^2}{x} + 3x$
17. $\frac{s^2 - 2s - 8}{s - 4}$ $s + 2$
18. $(r^2 + 9r + 20) \div (r + 5)$ $r + 4$
19. $(t^2 - 7t + 12) \div (t - 3)$ $t - 4$
20. $(c^2 + 3c - 54) \div (c + 9)$ $c - 6$
21. $(2q^2 - 9q - 5) \div (q - 5)$ $2q + 1$
22. $\frac{3z^2 - 2z - 5}{z + 1}$ $3z - 5$
23. $\frac{(m^3 + 3m^2 - 5m + 1)}{m - 1}$ $m^2 + 4m - 1$
24. $(d^3 - 2d^2 + 4d + 24) \div (d + 2)$ $d^2 - 4d + 12$
25. $(2j^3 + 5j + 26) \div (j + 2)$ $2j^2 - 4j + 13$
26. $\frac{2x^3 + 3x^2 - 176}{x - 4}$ $2x^2 + 11x + 44$
27. $(x^2 + 6x - 3) \div (x + 4)$ $x + 2 - \frac{11}{x - 4}$
28. $\frac{h^3 + 2h^2 - 6h + 1}{h - 2}$ $h^2 + 4h + 2 + \frac{5}{h - 2}$

Prerequisite Skills

Prerequisite Skills 749

Prerequisite Skills

Prerequisite Skills

13 Factoring to Solve Equations

- Some polynomials can be factored using the Distributive Property.

Example 1 **Factor $5t^2 + 15t$.**

Find the greatest common factor (GCF) of $5t^2$ and $15t$.

$5t^2 = 5 \cdot t \cdot t,\ 15t = 3 \cdot 5 \cdot t$ GCF: $5 \cdot t$ or $5t$

$5t^2 + 15t = 5t(t) + 5t(3)$ Rewrite each term using the GCF.

$= 5t(t + 3)$ Distributive Property

- To factor polynomials of the form $x^2 + bx + c$, find two integers m and n so that $mn = c$ and $m + n = b$. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

Example 2 **Factor each polynomial.**

a. $x^2 + 7x + 10$

In this equation, b is 7 and c is 10. Find two numbers with a product of 10 and with a sum of 7.

Both b and c are positive.

Factors of 10	Sum of Factors
1, 10	11
2, 5	7

$x^2 + 7x + 10 = (x + m)(x + n)$ The correct factors are 2 and 5.

$= (x + 2)(x + 5)$ Write the pattern; $m = 2$ and $n = 5$.

b. $x^2 - 8x + 15$

In this equation, b is -8 and c is 15. This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

b is negative and c is positive.

Factors of 15	Sum of Factors
−1, −15	−16
−3, −5	−8

$x^2 - 8x + 15 = (x + m)(x + n)$ The correct factors are -3 and -5.

$= (x - 3)(x - 5)$ Write the pattern; $m = -3$ and $n = -5$.

- To factor polynomials of the form $ax^2 + bx + c$, find two integers m and n with a product equal to ac and with a sum equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

c. $5x^2 - 19x - 4$

b is negative and c is negative.

In this equation, a is 5, b is -19, and c is -4.
Find two numbers with a product of -20 and with a sum of -19.

Factors of −20	Sum of Factors
−2, 10	8
2, −10	−8
−1, 20	19
1, −20	−19

The correct factors are 1 and -20.

$5x^2 - 19x - 4 = 5x^2 + mx + nx - 4$ Write the pattern.

$= 5x^2 + x + (-20)x - 4$ $m = 1$ and $n = -20$

$= (5x^2 + x) - (20x + 4)$ Group terms with common factors.

$= x(5x + 1) - 4(5x + 1)$ Factor the GCF from each group.

$= (x - 4)(5x + 1)$ Distributive Property

750 Prerequisite Skills

- Here are some special products.

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 3 **Factor each polynomial.**

a. $9x^2 + 6x + 1$

The first and last terms are perfect squares, and the middle term is equal to $2(3x)(1)$.

$9x^2 + 6x + 1 = (3x)^2 + 2(3x)(1) + 1^2$ Write as $a^2 + 2ab + b^2$.

$= (3x + 1)^2$ Factor using the pattern.

b. $x^2 - 9 = 0$

This is a difference of squares.

$x^2 - 9 = x^2 - (3)^2$ Write in the form $a^2 - b^2$.

$= (x - 3)(x + 3)$ Factor the difference of squares.

- The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$. Since 0 times any number is equal to zero, this implies that we can use factoring to solve equations.

Example 4 **Solve $x^2 - 5x + 4 = 0$ by factoring.**

Factor the polynomial. This expression is of the form $x^2 + bx + c$.

$x^2 - 5x + 4 = 0$ Original equation

$(x - 1)(x - 4) = 0$ Factor the polynomial.

If $ab = 0$, then $a = 0$, $b = 0$, or both equal 0. Let each factor equal 0.

$x - 1 = 0$ or $x - 4 = 0$

$x = 1$ $\quad$ $x = 4$

Exercises **Factor each polynomial.**

1. $u^2 - 12u$ **$u(u - 12)$**
2. $w^2 + 4w$ **$w(w + 4)$**
3. $7j^2 - 28j$ **$7j(j - 4)$**
4. $2g^2 + 24g$ **$2g(g + 12)$**
5. $6x^2 + 2x$ **$2x(3x + 1)$**
6. $5t^2 - 30t$ **$5t(t - 6)$**
7. $z^2 + 10z + 21$ **$(z + 7)(z + 3)$**
8. $n^2 + 8n + 15$ **$(n + 3)(n + 5)$**
9. $h^2 + 8h + 12$ **$(h + 2)(h + 6)$**
10. $x^2 + 14x + 48$ **$(x + 6)(x + 8)$**
11. $m^2 + 6m - 7$ **$(m - 1)(m + 7)$**
12. $b^2 + 2b - 24$ **$(b - 4)(b + 6)$**
13. $q^2 - 9q + 18$ **$(q - 3)(q - 6)$**
14. $p^2 - 5p + 6$ **$(p - 2)(p - 3)$**
15. $a^2 - 3a - 4$ **$(a - 4)(a + 1)$**
16. $k^2 - 4k - 32$ **$(k - 8)(k + 4)$**
17. $n^2 - 7n - 44$ **$(n - 11)(n + 4)$**
18. $y^2 - 3y - 88$ **$(y - 11)(y + 8)$**
19. $3z^2 + 4z - 4$ **$(3z - 2)(z + 2)$**
20. $2y^2 + 9y - 5$ **$(2y - 1)(y + 5)$**
21. $5x^2 + 7x + 2$ **$(5x + 2)(x + 1)$**
22. $3s^2 + 11s - 4$ **$(3s - 1)(s + 4)$**
23. $6r^2 - 5r + 1$ **$(2r - 1)(3r - 1)$**
24. $8a^2 + 15a - 2$ **$(8a - 1)(a + 2)$**
25. $w^2 - \frac{9}{4}$ **$\left(w + \frac{3}{2}\right)\left(w - \frac{3}{2}\right)$**
26. $c^2 - 64$ **$(c - 8)(c + 8)$**
27. $r^2 + 14r + 49$ **$(r + 7)^2$**
28. $b^2 + 18b + 81$ **$(b + 9)^2$**
29. $j^2 - 12j + 36$ **$(j - 6)^2$**
30. $4t^2 - 25$ **$(2t - 5)(2t + 5)$**

Solve each equation by factoring.

31. $10r^2 - 35r = 0$ **$0, \frac{7}{2}$**
32. $3x^2 + 15x = 0$ **$0, -5$**
33. $k^2 + 13k + 36 = 0$ **$-4, -9$**
34. $w^2 - 8w + 12 = 0$ **$2, 6$**
35. $c^2 - 5c - 14 = 0$ **$-2, 7$**
36. $z^2 - z - 42 = 0$ **$-6, 7$**
37. $2y^2 - 5y - 12 = 0$ **$-\frac{3}{2}, 4$**
38. $3b^2 - 4b - 15 = 0$ **$-\frac{5}{3}, 3$**
39. $t^2 + 12t + 36 = 0$ **-6**
40. $u^2 + 5u + \frac{25}{4} = 0$ **$-\frac{5}{2}$**
41. $q^2 - 8q + 16 = 0$ **4**
42. $a^2 - 6a + 9 = 0$ **3**

Prerequisite Skills 751

Prerequisite Skills

Prerequisite Skills

Prerequisite Skills

14 Operations with Matrices

- A **matrix** is a rectangular arrangement of numbers in rows and columns. Each entry in a matrix is called an **element**. A matrix is usually described by its **dimensions**, or the number of **rows** and **columns**, with the number of rows stated first.

- For example, matrix A has dimensions 3×2 and matrix B has dimensions 2×4.

$$\text{matrix } A = \begin{bmatrix} 6 & -2 \\ 0 & 5 \\ -4 & 10 \end{bmatrix} \qquad \text{matrix } B = \begin{bmatrix} 7 & -1 & -2 & 0 \\ 3 & 6 & -5 & 2 \end{bmatrix}$$

- If two matrices have the same dimensions, you can add or subtract them. To do this, add or subtract corresponding elements of the two matrices.

Example 1 If $A = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix}$, and $C = \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix}$, **find the sum and difference.**

a. $A + B$

$$A + B = \begin{bmatrix} 12 & 7 & -3 \\ 0 & -1 & -6 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 12 + (-3) & 7 + 0 & -3 + 5 \\ 0 + 2 & -1 + 7 & -6 + (-7) \end{bmatrix} \quad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 9 & 7 & 2 \\ 2 & 6 & -13 \end{bmatrix} \quad \text{Simplify.}$$

b. $B - C$

$$B - C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 7 & -7 \end{bmatrix} - \begin{bmatrix} 9 & 1 & -5 \\ 0 & -1 & 15 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} -3 - 9 & 0 - 1 & 5 - (-5) \\ 2 - 0 & 7 - (-1) & -7 - 15 \end{bmatrix} \quad \text{Definition of matrix subtraction}$$

$$= \begin{bmatrix} -12 & -1 & 10 \\ 2 & 8 & -22 \end{bmatrix} \quad \text{Simplify.}$$

- You can multiply any matrix by a constant called a *scalar*. This is called **scalar multiplication**. To perform scalar multiplication, multiply each element by the scalar.

Example 2 If $D = \begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix}$, **find $2D$.**

$$2D = 2\begin{bmatrix} -4 & 6 & -1 \\ 0 & 7 & 2 \\ -3 & -8 & -4 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 2(-4) & 2(6) & 2(-1) \\ 2(0) & 2(7) & 2(2) \\ 2(-3) & 2(-8) & 2(-4) \end{bmatrix} \quad \text{Definition of scalar multiplication}$$

$$= \begin{bmatrix} -8 & 12 & -2 \\ 0 & 14 & 4 \\ -6 & -16 & -8 \end{bmatrix} \quad \text{Simplify.}$$

752 Prerequisite Skills

- You can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix. The product of two matrices is found by multiplying columns and rows. The entry in the first row and first column of AB, the resulting product, is found by multiplying corresponding elements in the first row of A and the first column of B and then adding.

Example 3 **Find EF if $E = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix}$ and $F = \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$.**

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix}$$

Multiply the numbers in the first row of E by the numbers in the first column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & \\ & \end{bmatrix}$$

Multiply the numbers in the first row of E by the numbers in the second column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ & \end{bmatrix}$$

Multiply the numbers in the second row of E by the numbers in the first column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & \end{bmatrix}$$

Multiply the numbers in the second row of E by the numbers in the second column of F and add the products.

$$EF = \begin{bmatrix} 3 & -2 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 & 5 \\ 6 & -3 \end{bmatrix} = \begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix}$$

Simplify the matrix.

$$\begin{bmatrix} 3(-1) + (-2)(6) & 3(5) + (-2)(-3) \\ 0(-1) + 6(6) & 0(5) + 6(-3) \end{bmatrix} = \begin{bmatrix} -15 & 21 \\ 36 & -18 \end{bmatrix}$$

Exercises **If $A = \begin{bmatrix} 10 & -9 \\ 4 & -3 \\ -1 & 11 \end{bmatrix}$, $B = \begin{bmatrix} -1 & -3 \\ 2 & 8 \\ 7 & 6 \end{bmatrix}$, and $C = \begin{bmatrix} 8 & 0 \\ -2 & 2 \\ -10 & 6 \end{bmatrix}$, find each sum, difference, or product.** **1–24. See margin.**

1. $A + B$	2. $B + C$	3. $A - C$	4. $C - B$
5. $3A$	6. $5B$	7. $-4C$	8. $\frac{1}{2}C$
9. $2A + C$	10. $A - 5C$	11. $\frac{1}{2}C + B$	12. $3A - 3B$

If $X = \begin{bmatrix} 2 & -8 \\ 10 & 4 \end{bmatrix}$, $Y = \begin{bmatrix} -1 & 0 \\ 6 & -5 \end{bmatrix}$, and $Z = \begin{bmatrix} 4 & -8 \\ -7 & 0 \end{bmatrix}$, find each sum, difference, or product.

13. $X + Z$	14. $Y + Z$	15. $X - Y$	16. $3Y$
17. $-6X$	18. $\frac{1}{2}X + Z$	19. $5Z - 2Y$	20. XY
21. YZ	22. XZ	23. $\frac{1}{2}(XZ)$	24. $XY + 2Z$

Prerequisite Skills

Prerequisite Skills

1. $\begin{bmatrix} 9 & -12 \\ 6 & 5 \\ 6 & 17 \end{bmatrix}$ 2. $\begin{bmatrix} 7 & -3 \\ 0 & 10 \\ -3 & 12 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -9 \\ 6 & -5 \\ 9 & 5 \end{bmatrix}$ 4. $\begin{bmatrix} 9 & 3 \\ -4 & -6 \\ -17 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 30 & -27 \\ 12 & -9 \\ -3 & 33 \end{bmatrix}$ 6. $\begin{bmatrix} -5 & -15 \\ 10 & 40 \\ 35 & 30 \end{bmatrix}$

7. $\begin{bmatrix} -32 & 0 \\ 8 & -8 \\ 40 & -24 \end{bmatrix}$ 8. $\begin{bmatrix} 4 & 0 \\ -1 & 1 \\ -5 & 3 \end{bmatrix}$

9. $\begin{bmatrix} 28 & -18 \\ 6 & -4 \\ -12 & 28 \end{bmatrix}$ 10. $\begin{bmatrix} -30 & -9 \\ 14 & -13 \\ 49 & -19 \end{bmatrix}$

11. $\begin{bmatrix} 3 & -3 \\ 1 & 9 \\ 2 & 9 \end{bmatrix}$ 12. $\begin{bmatrix} 33 & -18 \\ 6 & -33 \\ -24 & 15 \end{bmatrix}$

13. $\begin{bmatrix} 6 & -16 \\ 3 & 4 \end{bmatrix}$ 14. $\begin{bmatrix} 3 & -8 \\ -1 & -5 \end{bmatrix}$

15. $\begin{bmatrix} 3 & -8 \\ 4 & 9 \end{bmatrix}$ 16. $\begin{bmatrix} -3 & 0 \\ 18 & -15 \end{bmatrix}$

17. $\begin{bmatrix} -12 & 48 \\ -60 & -24 \end{bmatrix}$ 18. $\begin{bmatrix} 5 & -12 \\ -2 & 2 \end{bmatrix}$

19. $\begin{bmatrix} 22 & -40 \\ -47 & 10 \end{bmatrix}$ 20. $\begin{bmatrix} -50 & 40 \\ 14 & -20 \end{bmatrix}$

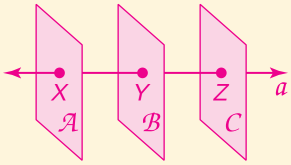
21. $\begin{bmatrix} -4 & 8 \\ 59 & -48 \end{bmatrix}$ 22. $\begin{bmatrix} 64 & -16 \\ 12 & -80 \end{bmatrix}$

23. $\begin{bmatrix} 32 & -8 \\ 6 & -40 \end{bmatrix}$ 24. $\begin{bmatrix} -42 & 24 \\ 0 & -20 \end{bmatrix}$

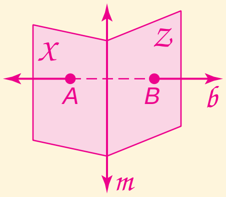
Prerequisite Skills 753

Lesson 1-1

8.



9.



Extra Practice

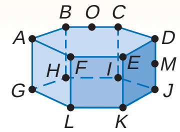
Extra Practice

Extra Practice

Lesson 1-1

(pages 6–12)

For Exercises 1–7, refer to the figure.



1. How many planes are shown in the figure? **8**
2. Name three collinear points. ***B*, *O*, *C* or *D*, *M*, *J***
3. Name all planes that contain point *G*. **planes *AFG*, *ABG*, and *GLK***
4. Name the intersection of plane *ABD* and plane *DJK*. **$\overleftrightarrow{DE}$**
5. Name two planes that do not intersect. **Sample answer: planes *ABD* and *GHJ***
6. Name a plane that contains $\overleftrightarrow{FK}$ and $\overleftrightarrow{EL}$. **plane *FEK***
7. Is the intersection of plane *ACD* and plane *EDJ* a point or a line? Explain. **A line; two planes intersect in a line, not a point.**

Draw and label a figure for each relationship. **8–9. See margin.**

8. Line *a* intersects planes $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ at three distinct points.
9. Planes $\mathcal{X}$ and $\mathcal{Z}$ intersect in line *m*. Line *b* intersects the two planes in two distinct points.

Lesson 1-2

(pages 13–19)

Find the precision for each measurement. Explain its meaning. **2. 0.5 mm; 85.5 to 86.5 mm**

1. 42 in. **$\frac{1}{2}$ in.; $41\frac{1}{2}$ to $42\frac{1}{2}$ in.**
2. 86 mm
3. 251 cm **0.5 cm; 250.5 to 251.5 cm**
4. 33.5 in. **0.05 in.; 33.45 to 33.55 in.**
5. $5\frac{1}{4}$ ft **$\frac{1}{8}$ ft; $5\frac{1}{8}$ to $5\frac{3}{8}$ ft**
6. 89 m **0.5 m; 88.5 to 89.5 m**

Find the value of the variable and *BC* if *B* is between *A* and *C*.

7. $AB = 4x$, $BC = 5x$; $AB = 16$ **$x = 4$; $BC = 20$**
8. $AB = 17$, $BC = 3m$, $AC = 32$ **$m = 5$; $BC = 15$**
9. $AB = 9a$, $BC = 12a$, $AC = 42$ **$a = 2$; $BC = 24$**
10. $AB = 25$, $BC = 3b$, $AC = 7b + 13$ **$b = 3$; $BC = 9$**
11. $AB = 5n + 5$, $BC = 2n$; $AC = 54$ **$n = 7$; $BC = 14$**
12. $AB = 6c - 8$, $BC = 3c + 1$, $AC = 65$ **$c = 8$; $BC = 25$**

Lesson 1-3

(pages 21–27)

Use the Pythagorean Theorem to find the distance between each pair of points.

1. $A(0, 0)$, $B(-3, 4)$ **5**
2. $C(-1, 2)$, $N(5, 10)$ **10**
3. $X(-6, -2)$, $Z(6, 3)$ **13**
4. $M(-5, -8)$, $O(3, 7)$ **17**
5. $T(-10, 2)$, $R(6, -10)$ **20**
6. $F(5, -6)$, $N(-5, 6)$ **$\sqrt{244} \approx 15.6$**

Use the Distance Formula to find the distance between each pair of points.

7. $D(0, 0)$, $M(8, -7)$ **$\sqrt{113} \approx 10.6$**
8. $X(-1, 1)$, $Y(1, -1)$ **$\sqrt{8} \approx 2.8$**
9. $Z(-4, 0)$, $A(-3, 7)$ **$\sqrt{50} \approx 7.1$**
10. $K(6, 6)$, $D(-3, -3)$ **$\sqrt{162} \approx 12.7$**
11. $T(-1, 3)$, $N(0, 2)$ **$\sqrt{2} \approx 1.4$**
12. $S(7, 2)$, $E(-6, 7)$ **$\sqrt{194} \approx 13.9$**

Find the coordinates of the midpoint of a segment having the given endpoints.

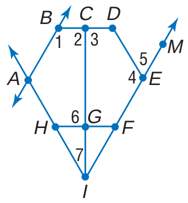
13. $A(0, 0)$, $D(-2, -8)$ **$(-1, -4)$**
14. $D(-4, -3)$, $E(2, 2)$ **$(-1, -0.5)$**
15. $K(-4, -5)$, $M(5, 4)$ **$(0.5, -0.5)$**
16. $R(-10, 5)$, $S(8, 4)$ **$(-1, 4.5)$**
17. $B(2.8, -3.4)$, $Z(1.2, 5.6)$ **$(2, 1.1)$**
18. $D(-6.2, 7)$, $K(3.4, -4.8)$ **$(-1.4, 1.1)$**

Find the coordinates of the missing endpoint given that *B* is the midpoint of $\overline{AC}$.

19. $C(0, 0)$, $B(5, -6)$ **$(10, -12)$**
20. $C(-7, -4)$, $B(3, 5)$ **$(13, 14)$**
21. $C(8, -4)$, $B(-10, 2)$ **$(-28, 8)$**
22. $C(6, 8)$, $B(-3, 5)$ **$(-12, 2)$**
23. $C(6, -8)$, $B(3, -4)$ **$(0, 0)$**
24. $C(-2, -4)$, $B(0, 5)$ **$(2, 14)$**

Lesson 1-4
(pages 29–36)

For Exercises 1–14, use the figure at the right.
Name the vertex of each angle.



1. $\angle 1$ **B**
2. $\angle 4$ **E**
3. $\angle 6$ **G**
4. $\angle 7$ **I**

Name the sides of each angle.

5. $\angle AIE$ $\overrightarrow{IA}, \overrightarrow{IE}$
6. $\angle 4$ $\overrightarrow{ED}, \overrightarrow{EF}$
7. $\angle 6$ $\overrightarrow{GC}, \overrightarrow{GH}$
8. $\angle AHF$ $\overrightarrow{HA}, \overrightarrow{HF}$

Write another name for each angle.

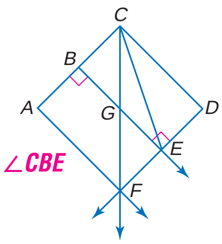
9. $\angle 3$ $\angle DCG$
10. $\angle DEF$ $\angle 4$
11. $\angle 2$ $\angle BCG$

Measure each angle and classify it as *right*, *acute*, or *obtuse*.

12. $\angle ABC$ **120°, obtuse**
13. $\angle CGF$ **90°, right**
14. $\angle HIF$ **60°, acute**

Lesson 1-5
(pages 37–43)

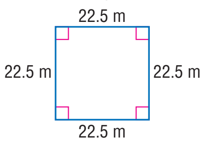
For Exercises 1–7, refer to the figure.



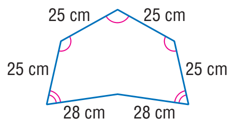
1. Name two acute vertical angles. **Sample answer: $\angle BGC$, $\angle FGE$**
2. Name two obtuse vertical angles. **Sample answer: $\angle BGF$, $\angle CGE$**
3. Name a pair of complementary adjacent angles. **Sample: $\angle BEC$, $\angle CED$**
4. Name a pair of supplementary adjacent angles. **Sample: $\angle CEF$, $\angle CED$**
5. Name a pair of congruent supplementary adjacent angles. **Sample: $\angle ABE$, $\angle CBE$**
6. If $m\angle BGC = 4x + 5$ and $m\angle FGE = 6x - 15$, find $m\angle BGF$. **135**
7. If $m\angle BCG = 5a + 5$, $m\angle GCE = 3a - 4$, and $m\angle ECD = 4a - 7$, find the value of a so that $\overline{AC} \perp \overline{CD}$. **8**
8. The measure of $\angle A$ is nine less than the measure of $\angle B$. If $\angle A$ and $\angle B$ form a linear pair, what are their measures? **85.5, 94.5**
9. The measure of an angle's complement is 17 more than the measure of the angle. Find the measure of the angle and its complement. **36.5, 53.5**

Lesson 1-6
(pages 45–50)

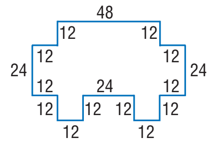
Name each polygon by its number of sides. Classify it as *convex* or *concave* and *regular* or *irregular*. Then find the perimeter.

1.


quadrilateral; convex; regular; 90 m

2.


hexagon; concave; irregular; 156 cm

3.


All measurements in inches.

16-gon; concave; irregular; 264 in.

Find the perimeter of each polygon.

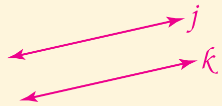
4. triangle with vertices at $X(3, 3)$, $Y(-2, 1)$, and $Z(1, -3)$ **≈16.7 units**
5. pentagon with vertices at $P(-2, 3)$, $E(-5, 0)$, $N(-2, -4)$, $T(2, -1)$, and $A(2, 2)$ **≈21.4 units**
6. hexagon with vertices at $H(0, 4)$, $E(-3, 2)$, $X(-3, -2)$, $G(0, -5)$, $O(5, -2)$, and $N(5, 2)$ **≈27.1 units**

Extra Practice

Extra Practice 755

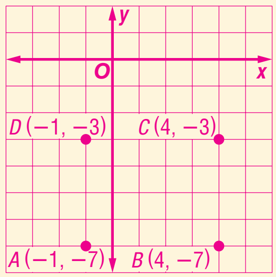
Lesson 2-1

1.



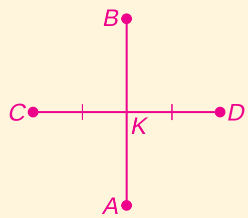
lines j and k do not intersect.

2.



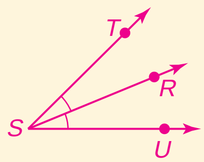
ABCD is a rectangle.

3.



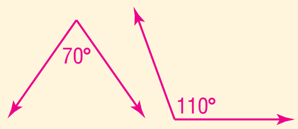
$CK = KD$

4.



$\angle TSR \cong \angle RSU$

8.



Lesson 2-2

1. $(-3)^2 = 9$ and a robin is a fish; false
2. $(-3)^2 = 9$ or a robin is a fish; true
3. $(-3)^2 = 9$ and an acute angle measures less than 90°; true
4. $(-3)^2 = 9$ or an acute angle measures less than 90°; true
5. $(-3)^2 \neq 9$ or a robin is a fish; false
6. $(-3)^2 = 9$ or an acute angle measures 90° or more; true
7. A robin is a fish and an acute angle measures less than 90°; false
8. $(-3)^2 = 9$ and a robin is a fish, or an acute angle measures less than 90°; true
9. $(-3)^2 \neq 9$ or an acute angle measures 90° or more; false

Extra Practice

Lesson 2-1 *(pages 62–66)*

Make a conjecture based on the given information. Draw a figure to illustrate your conjecture. 1–4. See margin.

1. Lines j and k are parallel.
2. $A(-1, -7)$, $B(4, -7)$, $C(4, -3)$, $D(-1, -3)$
3. $\overline{AB}$ bisects $\overline{CD}$ at K.
4. $\overrightarrow{SR}$ is an angle bisector of $\angle TSU$.

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture. 6. False; sample counterexample: $r = 0.5$

5. **Given:** EFG is an equilateral triangle.
   Conjecture: $EF = FG$ true
6. **Given:** r is a rational number.
   Conjecture: r is a whole number.
7. **Given:** n is a whole number.
   Conjecture: n is a rational number. true
8. **Given:** $\angle 1$ and $\angle 2$ are supplementary angles.
   Conjecture: $\angle 1$ and $\angle 2$ form a linear pair.
   False; see margin for counterexample.

Lesson 2-2 *(pages 67–74)*

Use the following statements to write a compound statement for each conjunction and disjunction. Then find its truth value. 1–9. See margin.

p: $(-3)^2 = 9$ **q: A robin is a fish.** **r: An acute angle measures less than 90°.**

1. p and q
2. p or q
3. p and r
4. p or r
5. $\sim p$ or q
6. p or $\sim r$
7. $q \wedge r$
8. $(p \wedge q) \vee r$
9. $\sim p \vee \sim r$

Copy and complete each truth table.

10.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

11.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Lesson 2-3 *(pages 75–80)*

Identify the hypothesis and conclusion of each statement. 1–4. See margin.

1. If no sides of a triangle are equal, then it is a scalene triangle.
2. If it rains today, you will be wearing your raincoat.
3. If $6 - x = 11$, then $x = -5$.
4. If you are in college, you are at least 18 years old.

Write each statement in if-then form. 5–8. See margin.

5. The sum of the measures of two supplementary angles is 180.
6. A triangle with two congruent sides is an isosceles triangle.
7. Two lines that do not intersect are parallel lines.
8. A Saint Bernard is a dog.

Write the converse, inverse, and contrapositive of each conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample. 9–12. See margin.

9. All triangles are polygons.
10. If two angles are congruent angles, then they have the same measure.
11. If three points lie on the same line, then they are collinear.
12. If $\overleftrightarrow{PQ}$ is a perpendicular bisector of $\overline{LM}$, then a right angle is formed.

756 Extra Practice

Lesson 2-3

1. H: no sides of a triangle are equal; C: it is a scalene triangle
2. H: it rains today; C: you will be wearing your raincoat
3. H: $6 - x = 11$; C: $x = -5$
4. H: you are in college; C: you are at least 18 years old
5. If two angles are supplementary, then the sum of their measures is 180.
6. If a triangle has two congruent sides, then it is an isosceles triangle.
7. If two lines do not intersect, then they are parallel lines.
8. If an animal is a Saint Bernard, then it is a dog.

Lesson 2-4

(pages 82–87)

Use the Law of Syllogism to determine whether a valid conclusion can be reached from each set of statements. If a valid conclusion is possible, write it. If not, write *no conclusion*.

1. (1) If it rains, then the field will be muddy. **See margin.**
(2) If the field is muddy, then the game will be cancelled.

2. (1) If you read a book, then you enjoy reading.
(2) If you are in the 10th grade, then you passed the 9th grade. **no conclusion**

Determine if statement (3) follows from statements (1) and (2) by the Law of Detachment or the Law of Syllogism. If it does, state which law was used. If it does not, write *invalid*.

3. (1) If it snows outside, you will wear your winter coat.
(2) It is snowing outside.
(3) You will wear your winter coat. **yes; Law of Detachment**

4. (1) Two complementary angles are both acute angles.
(2) $\angle 1$ and $\angle 2$ are acute angles.
(3) $\angle 1$ and $\angle 2$ are complementary angles. **invalid**

Lesson 2-5

(pages 89–93)

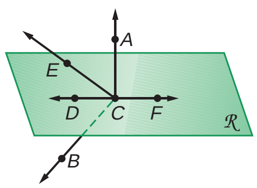
Determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

1. $\overleftrightarrow{RS}$ is perpendicular to $\overleftrightarrow{PS}$. **Sometimes; $\overleftrightarrow{RS}$ and $\overleftrightarrow{PS}$ could intersect to form a 45° angle.**

2. Three points will lie on one line. **Sometimes; if they are collinear, then they lie on one line.**

3. Points B and C are in plane $\mathcal{K}$. A line perpendicular to line BC is in plane $\mathcal{K}$. **Sometimes; the line could lie in a plane perpendicular to plane $\mathcal{K}$.**

For Exercises 4–7, use the figure at the right. In the figure, $\overleftrightarrow{EC}$ and $\overleftrightarrow{CD}$ are in plane $\mathcal{R}$, and F is on $\overleftrightarrow{CD}$. State the postulate that can be used to show each statement is true. **4–7. See margin.**



4. $\overleftrightarrow{DF}$ lies in plane $\mathcal{R}$.

5. E and C are collinear.

6. D, F, and E are coplanar.

7. E and F are collinear.

Lesson 2-6

(pages 94–100)

State the property that justifies each statement.

1. If $x - 5 = 6$, then $x = 11$. **Addition Property**

2. If $AB = CD$ and $CD = EF$, then $AB = EF$. **Transitive Property**

3. If $a - b = r$, then $r = a - b$. **Symmetric Property**

4. Copy and complete the following proof.

Given: $\frac{5x - 1}{8} = 3$

Prove: $x = 5$

Proof:

Statements	Reasons
a. ? **a.** $\frac{5x-1}{8} = 3$	a. Given
b. ? **b.** $8\left(\frac{5x-1}{8}\right) = 8(3)$	b. Multiplication Prop.
c. $5x - 1 = 24$	c. ? **Dist. Prop. and Substitution**
d. $5x = 25$	d. ? **Addition Prop.**
e. ? **$x = 5$**	e. Division Property

11. Converse: If three points are collinear, then they lie on the same line; true

Inverse: If three points do not lie on the same line, then they are not collinear; true

Contrapositive: If three points are not collinear, then they do not lie on the same line; true

12. Converse: If a right angle is formed by $\overleftrightarrow{PQ}$ and $\overline{LM}$, then $\overleftrightarrow{PQ}$ is a perpendicular bisector of $\overline{LM}$; false; $\overleftrightarrow{PQ}$ may not pass through the midpoint of $\overline{LM}$.

Inverse: If $\overleftrightarrow{PQ}$ is not a perpendicular bisector of $\overline{LM}$, then a right angle is not formed; false; $\overleftrightarrow{PQ}$ could be perpendicular to $\overline{LM}$, without bisecting $\overline{LM}$.

Contrapositive: If a right angle is not formed by $\overleftrightarrow{PQ}$ and $\overline{LM}$, then $\overleftrightarrow{PQ}$ is not a perpendicular bisector of $\overline{LM}$; true

Lesson 2-4

1. If it rains then the game will be cancelled.

Lesson 2-5

4. If two points lie in a plane, then the entire line containing those points lies in that plane.

5. Through any two points, there is exactly one line.

6. Through any three points not on the same line, there is exactly one plane.

7. Through any two points, there is exactly one line.

9. Converse: If a figure is a polygon, then it is a triangle; false; pentagons are polygons but are not triangles.

Inverse: If a figure is not a triangle, then it is not a polygon; false; a hexagon is not a triangle, but it is a polygon.

Contrapositive: If a figure is not a polygon, then it is not a triangle; true

10. Converse: If two angles have the same measure, then they are congruent angles; true

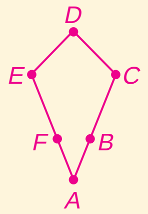
Inverse: If two angles are not congruent angles, then they do not have the same measure; true

Contrapositive: If two angles do not have the same measure, then they are not congruent angles; true

Lesson 2-7

9. Given: $\overline{AB} \cong \overline{AF}$, $\overline{AF} \cong \overline{ED}$, $\overline{ED} \cong \overline{CD}$

Prove: $\overline{AB} \cong \overline{CD}$



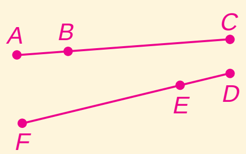
Proof:

Statements (Reasons)

1. $\overline{AB} \cong \overline{AF}$, $\overline{AF} \cong \overline{ED}$ (Given)
2. $\overline{AB} \cong \overline{ED}$ (Transitive)
3. $\overline{ED} \cong \overline{CD}$ (Given)
4. $\overline{AB} \cong \overline{CD}$ (Transitive)

10. Given: $AC = DF$, $AB = DE$

Prove: $BC = EF$



Proof:

Statements (Reasons)

1. $AC = AB + BC$ and $DF = DE + EF$ (Segment Addition Postulate)
2. $AC = DF$ (Given)
3. $AB + BC = DE + EF$ (Substitution)
4. $AB = DE$ (Given)
5. $BC = EF$ (Subtraction)

Extra Practice

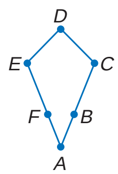
Lesson 2-7 (pages 101–106)

Justify each statement with a property of equality or a property of congruence.

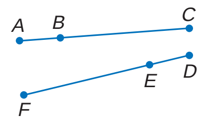
1. If $CD = OP$, then $CD + GH = OP + GH$. **Addition**
2. If $\overline{MN} \cong \overline{PQ}$, then $\overline{PQ} \cong \overline{MN}$. **Symmetric**
3. If $\overline{TU} \cong \overline{JK}$ and $\overline{JK} \cong \overline{DF}$, then $\overline{TU} \cong \overline{DF}$. **Transitive**
4. If $AB = 10$ and $CD = 10$, then $AB = CD$. **Substitution**
5. $\overline{XB} \cong \overline{XB}$ **Reflexive**
6. If $GH = RS$, then $GH - VW = RS - VW$. **Subtraction**
7. If $EF = XY$, then $EF + KL = XY + KL$. **Addition**
8. If $\overline{JK} \cong \overline{XY}$ and $\overline{XY} \cong \overline{LM}$, then $\overline{JK} \cong \overline{LM}$. **Transitive**

Write a two-column proof. **9–10. See margin.**

9. **Given:** $\overline{AB} \cong \overline{AF}$, $\overline{AF} \cong \overline{ED}$, $\overline{ED} \cong \overline{CD}$
Prove: $\overline{AB} \cong \overline{CD}$



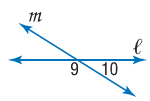
10. **Given:** $AC = DF$, $AB = DE$
Prove: $BC = EF$



Lesson 2-8 (pages 107–114)

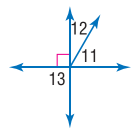
Find the measure of each numbered angle.

1. $m\angle 9 = 141 + x$
$m\angle 10 = 25 + x$



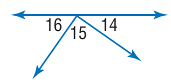
$m\angle 9 = 148$, $m\angle 10 = 32$

2. $m\angle 11 = x + 40$
$m\angle 12 = x + 10$
$m\angle 13 = 3x + 30$



$m\angle 11 = 60$, $m\angle 12 = 30$, $m\angle 13 = 90$

3. $m\angle 14 = x + 25$
$m\angle 15 = 4x + 50$
$m\angle 16 = x + 45$



$m\angle 14 = 35$, $m\angle 15 = 90$, $m\angle 16 = 55$

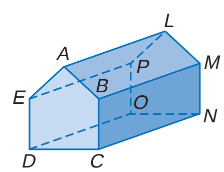
Determine whether the following statements are *always*, *sometimes*, or *never* true.

4. Two angles that are complementary are congruent. **sometimes**
5. Two angles that form a linear pair are complementary. **never**
6. Two congruent angles are supplementary. **sometimes**
7. Perpendicular lines form four right angles. **always**
8. Two right angles are supplementary. **always**
9. Two lines intersect to form four right angles. **sometimes**

Lesson 3-1 (pages 126–131)

For Exercises 1–3, refer to the figure at the right.

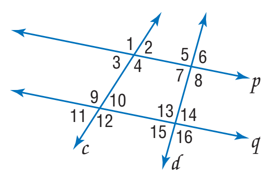
1. Name all segments parallel to $\overline{AE}$. **$\overline{LP}$**
2. Name all planes intersecting plane *BCN*.
3. Name all segments skew to $\overline{DC}$.



2. ABM, OCN, ABC, LMN, AEP
3. $\overline{BM}$, $\overline{AL}$, $\overline{EP}$, $\overline{OP}$, $\overline{PL}$, $\overline{LM}$, $\overline{MN}$

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

4. $\angle 2$ and $\angle 5$ **cons. int.**
5. $\angle 9$ and $\angle 13$ **corresponding**
6. $\angle 12$ and $\angle 13$ **alt. int.**
7. $\angle 3$ and $\angle 6$ **alt. ext.**

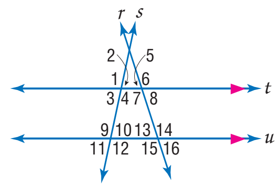


Lesson 3-2

(pages 133–138)

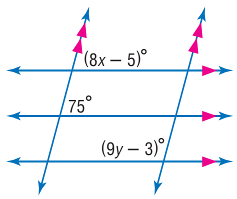
In the figure, $m\angle 5 = 72$ and $m\angle 9 = 102$. Find the measure of each angle.

1. $m\angle 1$ **102**
2. $m\angle 13$ **72**
3. $m\angle 4$ **102**
4. $m\angle 10$ **78**
5. $m\angle 7$ **108**
6. $m\angle 16$ **72**



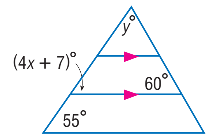
Find x and y in each figure.

7.



$x = 10$; $y = 12$

8.



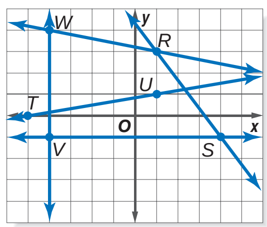
$x = 12$; $y = 65$

Lesson 3-3

(pages 139–144)

Find the slope of each line.

1. $\overleftrightarrow{RS}$ **$-\frac{4}{3}$**
2. $\overleftrightarrow{TU}$ **$\frac{1}{6}$**
3. $\overleftrightarrow{WV}$ **undefined**
4. $\overleftrightarrow{WR}$ **$-\frac{1}{5}$**
5. a line parallel to $\overleftrightarrow{TU}$ **$\frac{1}{6}$**
6. a line perpendicular to $\overleftrightarrow{WR}$ **5**
7. a line perpendicular to $\overleftrightarrow{WV}$ **0**



Determine whether $\overleftrightarrow{RS}$ and $\overleftrightarrow{TU}$ are *parallel*, *perpendicular*, or *neither*.

8. $R(3, 5)$, $S(5, 6)$, $T(-2, 0)$, $U(4, 3)$ **parallel**
9. $R(5, 11)$, $S(2, 2)$, $T(-1, 0)$, $U(2, 1)$ **neither**
10. $R(-1, 4)$, $S(-3, 7)$, $T(5, -1)$, $U(8, 1)$ **perpendicular**
11. $R(-2, 5)$, $S(-4, 1)$, $T(3, 3)$, $U(1, 5)$ **neither**

Lesson 3-4

(pages 145–150)

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

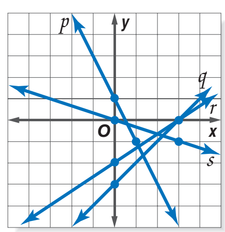
1. $m = 1$, y-intercept: -5 **$y = x - 5$**
2. $m = -\frac{1}{2}$, y-intercept: $\frac{1}{2}$ **$y = -\frac{1}{2}x + \frac{1}{2}$**
3. $m = 3$, $b = -\frac{1}{4}$ **$y = 3x - \frac{1}{4}$**

Write an equation in point-slope form of the line having the given slope that contains the given point.

4. $m = 3$, $(-2, 4)$ **$y - 4 = 3(x + 2)$**
5. $m = -4$, $(0, 3)$ **$y - 3 = -4x$**
6. $m = \frac{2}{3}$, $(5, -7)$ **$y + 7 = \frac{2}{3}(x - 5)$**

For Exercises 7–14, use the graph at the right. Write an equation in slope-intercept form for each line.

7. p **$y = -2x + 1$**
8. q **$y = x - 3$**
9. r **$y = \frac{2}{3}x - 2$**
10. s **$y = -\frac{1}{3}x$**
11. parallel to line q, contains $(2, -5)$ **$y = x - 7$**
12. perpendicular to line r, contains $(0, 1)$ **$y = -\frac{3}{2}x + 1$**
13. parallel to line s, contains $(-2, -2)$ **$y = -\frac{1}{3}x - \frac{8}{3}$**
14. perpendicular to line p, contains $(0, 0)$ **$y = \frac{1}{2}x$**



Extra Practice

Extra Practice

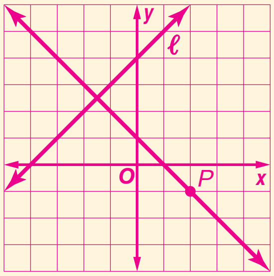
Extra Practice 759

Lesson 3-5

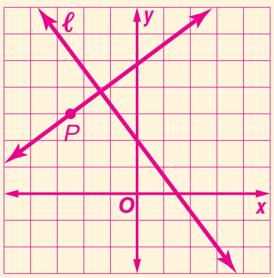
1. $c \parallel d$; $\cong$ alternate exterior $\angle$s
2. none
3. $c \parallel d$; $\cong$ alternate interior $\angle$s
4. $c \parallel d$; supplementary consecutive interior $\angle$s

Lesson 3-6

7. $d = \frac{7\sqrt{2}}{2}$;



8. $d = 1.4$:

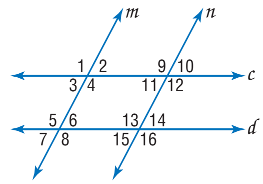


Extra Practice

Lesson 3-5 *(pages 151–157)*

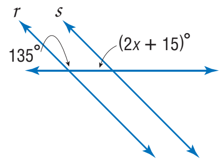
Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

1. $\angle 9 \cong \angle 16$
2. $\angle 10 \cong \angle 16$
3. $\angle 12 \cong \angle 13$
4. $m\angle 12 + m\angle 14 = 180$ **1–4. See margin.**



Find x so that $r \parallel s$.

5.



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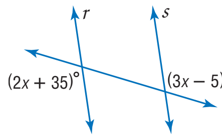
6.



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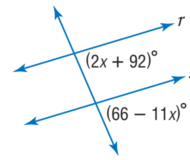
7.



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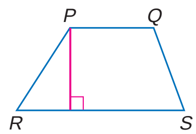
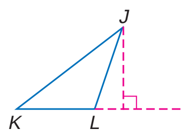
Lesson 3-6 *(pages 159–164)*

Copy each figure. Draw the segment that represents the distance indicated.

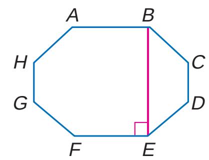
1. P to $\overleftrightarrow{RS}$



2. J to $\overleftrightarrow{KL}$



3. B to $\overleftrightarrow{FE}$



Find the distance between each pair of parallel lines.

4. $y = \frac{2}{3}x - 2$ **2.08**
   $y = \frac{2}{3}x + \frac{1}{2}$
5. $y = 2x + 4$ **4.02**
   $y - 2x = -5$
6. $x + 4y = -6$ **2.43**
   $x + 4y = 4$

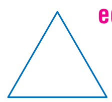
COORDINATE GEOMETRY Construct a line perpendicular to ℓ through P. Then find the distance from P to ℓ. **7–8. See margin.**

7. Line ℓ contains points $(0, 4)$ and $(-4, 0)$. Point P has coordinates $(2, -1)$.
8. Line ℓ contains points $(3, -2)$ and $(0, 2)$. Point P has coordinates $(-2.5, 3)$.

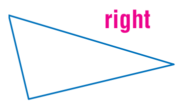
Lesson 4-1 *(pages 178–183)*

Use a protractor to classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

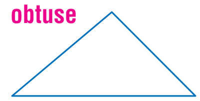
1. **equiangular**



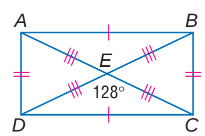
2. **right**



3. **obtuse**



Identify the indicated type of triangles in the figure if $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$, $\overline{AE} \cong \overline{BE} \cong \overline{EC} \cong \overline{ED}$, and $m\angle BAD = m\angle ABC = m\angle BCD = m\angle ADC = 90$.



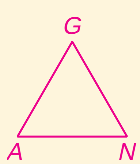
4. right
5. obtuse **$\triangle ABE$, $\triangle CDE$**
6. acute **$\triangle BEC$, $\triangle AED$**
7. isosceles **$\triangle ABE$, $\triangle CDE$, $\triangle BEC$, $\triangle AED$**

4. $\triangle DAB$, $\triangle ABC$, $\triangle BCD$, $\triangle ADC$

8. Find a and the measure of each side of equilateral triangle MNO if $MN = 5a$, $NO = 4a + 6$, and $MO = 7a - 12$. **$a = 6$; $MN = NO = MO = 30$**
9. Triangle TAC is an isosceles triangle with $\overline{TA} \cong \overline{AC}$. Find b, TA, AC, and TC if $TA = 3b + 1$, $AC = 4b - 11$, and $TC = 6b - 2$. **$b = 12$; $TA = AC = 37$, $TC = 70$**

Lesson 4-3

5. Given: $\triangle ANG \cong \triangle NGA$, $\triangle NGA \cong \triangle GAN$
   Prove: $\triangle AGN$ is equilateral and equiangular.



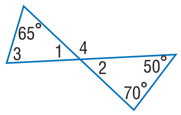
Proof: Statements (Reasons)

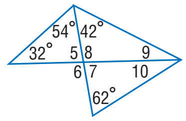
1. $\triangle ANG \cong \triangle NGA$ (Given)
2. $\overline{AN} \cong \overline{NG}$, $\angle A \cong \angle N$ (CPCTC)
3. $\triangle NGA \cong \triangle GAN$ (Given)
4. $\overline{NG} \cong \overline{GA}$, $\angle N \cong \angle G$ (CPCTC)
5. $\overline{AN} \cong \overline{NG} \cong \overline{GA}$ (Transitive Property of $\cong$)
6. $\triangle AGN$ is equilateral. (Def. of equilateral $\triangle$)
7. $\angle A \cong \angle N \cong \angle G$ (Transitive Property of $\cong$)
8. $\triangle AGN$ is equiangular. (Def. of equiangular $\triangle$)

Lesson 4-2 *(pages 185–191)*

Find the measure of each angle.

1. $\angle 1$ **60**
2. $\angle 2$ **60**
3. $\angle 3$ **55**
4. $\angle 4$ **120**
5. $\angle 5$ **94**
6. $\angle 6$ **86**
7. $\angle 7$ **94**
8. $\angle 8$ **86**
9. $\angle 9$ **52**
10. $\angle 10$ **24**

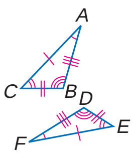




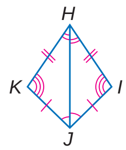
Lesson 4-3 *(pages 192–198)*

Identify the congruent triangles in each figure.

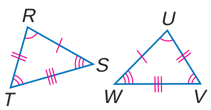
1. **$\triangle ABC \cong \triangle FDE$**



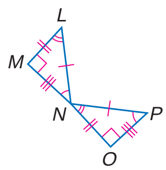
2. **$\triangle JKH \cong \triangle JIH$**



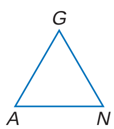
3. **$\triangle RTS \cong \triangle UVW$**



4. **$\triangle LMN \cong \triangle NOP$**



5. Write a two-column proof. **See margin.**
   Given: $\triangle ANG \cong \triangle NGA$
   $\triangle NGA \cong \triangle GAN$
   Prove: $\triangle AGN$ is equilateral and equiangular.



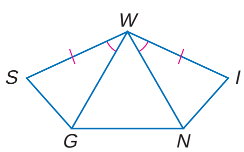
Lesson 4-4 *(pages 200–206)*

Determine whether $\triangle RST \cong \triangle JKL$ given the coordinates of the vertices. Explain.

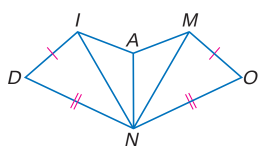
1. $R(-6, 2)$, $S(-4, 4)$, $T(-2, 2)$, $J(6, -2)$, $K(4, -4)$, $L(2, -2)$ **Yes; see margin for explanation.**
2. $R(-6, 3)$, $S(-4, 7)$, $T(-2, 3)$, $J(2, 3)$, $K(5, 7)$, $L(6, 3)$ **No; see margin for explanation.**

Write a two-column proof. **3–4. See margin.**

3. **Given:** $\triangle GWN$ is equilateral.
   $\overline{WS} \cong \overline{WI}$
   $\angle SWG \cong \angle IWN$
   Prove: $\triangle SWG \cong \triangle IWN$



4. **Given:** $\triangle ANM \cong \triangle ANI$
   $\overline{DI} \cong \overline{OM}$
   $\overline{ND} \cong \overline{NO}$
   Prove: $\triangle DIN \cong \triangle OMN$



Lesson 4-4

1. $RS = \sqrt{(-6-(-4))^2 + (4-2)^2}$
$= \sqrt{4+4}$ or $\sqrt{8}$

$ST = \sqrt{(-4-(-2))^2 + (4-2)^2}$
$= \sqrt{4+4}$ or $\sqrt{8}$

$RT = \sqrt{(-6-(-2))^2 + (2-2)^2}$
$= \sqrt{16}$ or 4

$JK = \sqrt{(6-4)^2 + (-2-(-4))^2}$
$= \sqrt{4+4}$ or $\sqrt{8}$

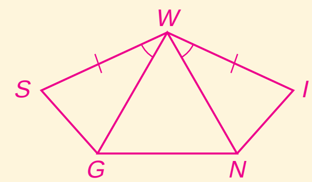
$KL = \sqrt{(4-2)^2 + (-4-(-2))^2}$
$= \sqrt{4+4}$ or $\sqrt{8}$

$JL = \sqrt{(6-2)^2 + (-2-(-2))^2}$
$= \sqrt{16}$ or 4

$RS = JK$, $ST = KL$, and $RT = JL$. By definition of congruent segments, all corresponding segments are congruent. Therefore, $\triangle RST \cong \triangle JKL$.

2. RS
$= \sqrt{(-6-(-4))^2 + (3-7)^2}$
$= \sqrt{4+16}$ or $\sqrt{20}$
$JK = \sqrt{(2-5)^2 + (3-7)^2}$
$= \sqrt{9+16}$ or 5
Since, $RS \neq JK$ the triangles are not congruent.

3. Given: $\triangle GWN$ is equilateral.
$\overline{WS} \cong \overline{WI}$
$\angle SWG \cong \angle IWN$
Prove: $\triangle SWG \cong \triangle IWN$

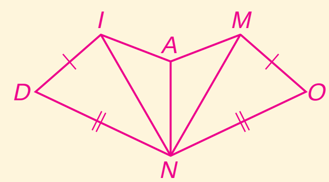


Proof:

Statements (Reasons)

1. $\triangle GWN$ is equilateral. (Given)
2. $\overline{WG} \cong \overline{WN}$ (Def. of equilateral triangle)
3. $\overline{WS} \cong \overline{WI}$ (Given)
4. $\angle SWG \cong \angle IWN$ (Given)
5. $\triangle SWG \cong \triangle IWN$ (SAS)

4. Given: $\triangle ANM \cong \triangle ANI$
$\overline{DI} \cong \overline{OM}$
$\overline{ND} \cong \overline{NO}$
Prove: $\triangle DIN \cong \triangle OMN$



Proof:

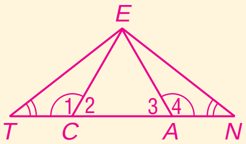
Statements (Reasons)

1. $\triangle ANM \cong \triangle ANI$ (Given)
2. $\overline{IN} \cong \overline{MN}$ (CPCTC)
3. $\overline{DI} \cong \overline{OM}$ (Given)
4. $\overline{ND} \cong \overline{NO}$ (Given)
5. $\triangle DIN \cong \triangle OMN$ (SSS)

Lesson 4-5

1. Given: $\triangle TEN$ is isosceles with base $\overline{TN}$. $\angle 1 \cong \angle 4$, $\angle T \cong \angle N$

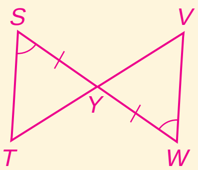
   Prove: $\triangle TEC \cong \triangle NEA$



Proof: If $\triangle TEN$ is isosceles with base $\overline{TN}$, then $\overline{TE} \cong \overline{NE}$. Since $\angle 1 \cong \angle 4$ and $\angle T \cong \angle N$ are given, then $\triangle TEC \cong \triangle NEA$ by AAS.

2. Given: $\angle S \cong \angle W$, $\overline{SY} \cong \overline{YW}$

   Prove: $\overline{ST} \cong \overline{WV}$



Proof: $\angle S \cong \angle W$ and $\overline{SY} \cong \overline{YW}$ are given and $\angle SYT \cong \angle WYV$ since vertical angles are congruent. Then $\triangle SYT \cong \triangle WYV$ by ASA and $\overline{ST} \cong \overline{WV}$ by CPCTC.

3. Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

   Prove: $\overline{PT} \cong \overline{LX}$

   Proof:

   $\angle 1 \cong \angle 2$ (Given); $\angle 3 \cong \angle 4$ (Given); $\overline{TX} \cong \overline{TX}$ (Reflexive Prop.) → $\triangle PXT \cong \triangle LTX$ (SAS) → $\overline{PT} \cong \overline{LX}$ (CPCTC)

4. Given: $\overline{FP} \parallel \overline{ML}$, $\overline{FL} \parallel \overline{MP}$

   Prove: $\overline{MP} \cong \overline{FL}$

   Proof:

   $\overline{FP} \parallel \overline{ML}$ (Given) → $\angle 3 \cong \angle 4$ (Alt. Int. ∠s Th.); $\overline{FL} \parallel \overline{MP}$ (Given) → $\angle 1 \cong \angle 2$ (Alt. Int. ∠s Th.); $\overline{PL} \cong \overline{PL}$ (Reflex. Prop.) → $\triangle FLP \cong \triangle MPL$ (ASA) → $\overline{MP} \cong \overline{FL}$ (CPCTC)

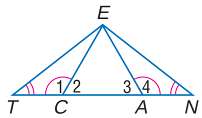
Extra Practice

Lesson 4-5
(pages 207–213)

Write a paragraph proof. 1–2. See margin.

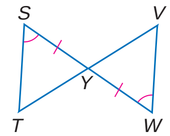
1. **Given:** $\triangle TEN$ is isosceles with base $\overline{TN}$. $\angle 1 \cong \angle 4$, $\angle T \cong \angle N$

   Prove: $\triangle TEC \cong \triangle NEA$



2. **Given:** $\angle S \cong \angle W$, $\overline{SY} \cong \overline{YW}$

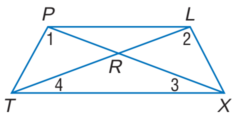
   Prove: $\overline{ST} \cong \overline{WV}$



Write a flow proof. 3–4. See margin.

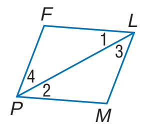
3. **Given:** $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

   Prove: $\overline{PT} \cong \overline{LX}$



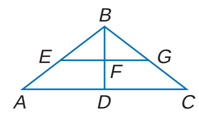
4. **Given:** $\overline{FP} \parallel \overline{ML}$, $\overline{FL} \parallel \overline{MP}$

   Prove: $\overline{MP} \cong \overline{FL}$



Lesson 4-6
(pages 216–221)

Refer to the figure for Exercises 1–6.



1. If $\overline{AD} \cong \overline{BD}$, name two congruent angles. $\angle DAB \cong \angle DBA$
2. If $\overline{BF} \cong \overline{FG}$, name two congruent angles. $\angle FBG \cong \angle FGB$
3. If $\overline{BE} \cong \overline{BG}$, name two congruent angles. $\angle BEF \cong \angle BGF$
4. If $\angle FBE \cong \angle FEB$, name two congruent segments. $\overline{FB} \cong \overline{FE}$
5. If $\angle BCA \cong \angle BAC$, name two congruent segments. $\overline{BA} \cong \overline{BC}$
6. If $\angle DBC \cong \angle BCD$, name two congruent segments. $\overline{BD} \cong \overline{CD}$

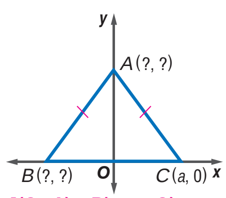
Lesson 4-7
(pages 222–226)

Position and label each triangle on the coordinate plane. 1–4. See margin for sample answers.

1. isosceles $\triangle ABC$ with base $\overline{BC}$ that is r units long
2. equilateral $\triangle XYZ$ with sides $4b$ units long
3. isosceles right $\triangle RST$ with hypotenuse $\overline{ST}$ and legs $(3 + a)$ units long
4. equilateral $\triangle CDE$ with base $\overline{DE}$ $\frac{1}{4}b$ units long.

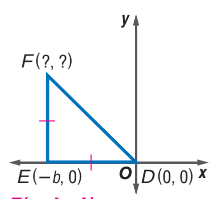
Name the missing coordinates of each triangle.

5.



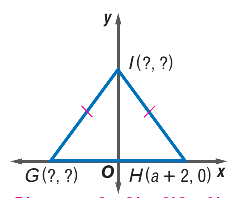
$A(0, b)$, $B(-a, 0)$

6.



$F(-b, b)$

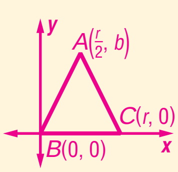
7.



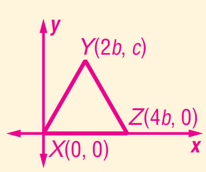
$G(-a - 2, 0)$, $I(0, b)$

Lesson 4-7

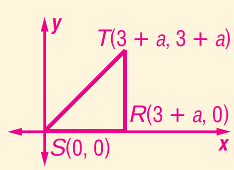
1.



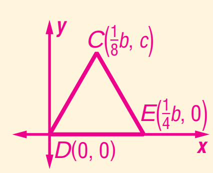
2.



3.



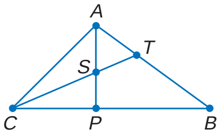
4.



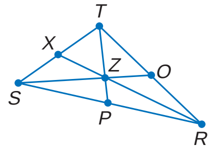
Lesson 5-1

(pages 238–246)

For Exercises 1–4, refer to the figures at the right.



1. Suppose $CP = 7x - 1$ and $PB = 6x + 3$. If S is the circumcenter of $\triangle ABC$, find x and CP. **4; 27**
2. Suppose $m\angle ACT = 15a - 8$ and $m\angle ACB = 74$. If S is the incenter of $\triangle ABC$, find a and $m\angle ACT$. **3; 37**
3. Suppose $TO = 7b + 5$, $OR = 13b - 10$, and $TR = 18b$. If Z is the centroid of $\triangle TRS$, find b and TR. **2.5; 45**
4. Suppose $XR = 19n - 14$ and $ZR = 10n + 4$. If Z is the centroid of $\triangle TRS$, find n and ZR. **5; 54**



State whether each sentence is *always*, *sometimes*, or *never* true.

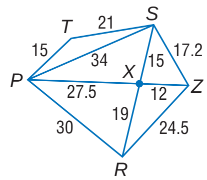
5. The circumcenter and incenter of a triangle are the same point. **sometimes**
6. The three altitudes of a triangle intersect at a point inside the triangle. **sometimes**
7. In an equilateral triangle, the circumcenter, incenter, and centroid are the same point. **always**
8. The incenter is inside of a triangle. **always**

Lesson 5-2

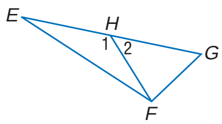
(pages 247–254)

Determine the relationship between the measures of the given angles.

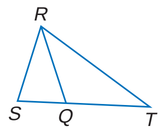
1. $\angle TPS, \angle TSP$ **$m\angle TPS > m\angle TSP$**
2. $\angle PRZ, \angle ZPR$ **$m\angle PRZ > m\angle ZPR$**
3. $\angle SPZ, \angle SZP$ **$m\angle SPZ < m\angle SZP$**
4. $\angle SPR, \angle SRP$ **$m\angle SPR = m\angle SRP$**



5. **Given:** $FH > FG$ **See margin.**
   Prove: $m\angle 1 > m\angle 2$



6. **Given:** $\overline{RQ}$ bisects $\angle SRT$. **See margin.**
   Prove: $m\angle SQR > m\angle SRQ$



Lesson 5-3

(pages 255–260)

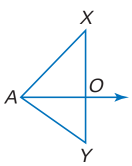
State the assumption you would make to start an indirect proof of each statement.

1. $\angle ABC \cong \angle XYZ$ **$\angle ABC \not\cong \angle XYZ$**
2. An angle bisector of an equilateral triangle is also a median.
3. $\overrightarrow{RS}$ bisects $\angle ARC$ **$\overrightarrow{RS}$ does not bisect $\angle ARC$.**

2. An angle bisector of an equilateral triangle is not a median.

Write an indirect proof. **4–5. See margin.**

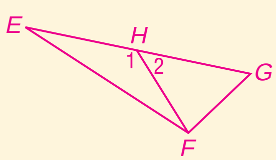
4. **Given:** $\angle AOY \cong \angle AOX$
   $\overline{XO} \not\cong \overline{YO}$
   Prove: $\overrightarrow{AO}$ is not the angle bisector of $\angle XAY$.



5. **Given:** $\triangle RUN$
   Prove: There can be no more than one right angle in $\triangle RUN$.

Lesson 5-2

5. Given: $FH > FG$
   Prove: $m\angle 1 > m\angle 2$

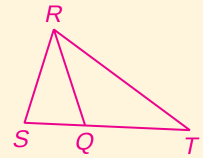


Proof:

Statements (Reasons)

1. $FH > FG$ (Given)
2. $m\angle FGH > m\angle 2$ (If one side of a △ is longer than another, the ∠ opp. the longer side > than the ∠ opp. the shorter side.)
3. $m\angle 1 > m\angle FGH$ (Exterior Angle Inequality Theorem)
4. $m\angle 1 > m\angle 2$ (Transitive Prop. of Inequality)

6. Given: $\overrightarrow{RQ}$ bisects $\angle SRT$.
   Prove: $m\angle SQR > m\angle SRQ$



Proof:

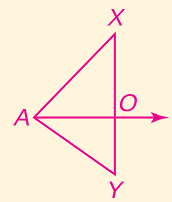
Statements (Reasons)

1. $\overrightarrow{RQ}$ bisects $\angle SRT$. (Given)
2. $\angle SRQ \cong \angle QRT$ (Def. ∠ bisector)
3. $m\angle SRQ = m\angle QRT$ (Def. ≅ ∠s)
4. $m\angle SQR > m\angle QRT$ (Exterior Angle Inequality Theorem)
5. $m\angle SQR > m\angle SRQ$ (Subst.)

Extra Practice

Lesson 5-3

4. Given: $\angle AOY \cong \angle AOX$, $\overline{XO} \not\cong \overline{YO}$
   Prove: $\overrightarrow{AO}$ is not the angle bisector of $\angle XAY$.



Proof:

Step 1: Assume $\overrightarrow{AO}$ is the angle bisector of $\angle XAY$.

Step 2: If $\overrightarrow{AO}$ is the angle bisector of $\angle XAY$, then $\angle XAO \cong \angle YAO$. $\angle AOY \cong \angle AOX$ by given and $\overline{AO} \cong \overline{AO}$ by reflexive. Then $\triangle XAO \cong \triangle YAO$ by ASA. $\overline{XO} \cong \overline{YO}$ by CPCTC.

Step 3: This conclusion contradicts the given fact $\overline{XO} \not\cong \overline{YO}$. Thus, $\overrightarrow{AO}$ is not the angle bisector of $\angle XAY$.

5. Given: $\triangle RUN$
   Prove: There can be no more than one right angle in $\triangle RUN$.

Proof:

Step 1: Assume $\triangle RUN$ has two right angles.

Step 2: By the Angle Sum Theorem, $m\angle R + m\angle U + m\angle N = 180$. If you substitute 90 for two of the ∠ measures, since the △ has two right ∠s, then $90 + 90 + m\angle N = 180$. Then, $180 + m\angle N = 180$.

Step 3: This conclusion means that $m\angle N = 0$. This is not possible if $\triangle RUN$ is a △. Thus, there can be no more than one right ∠ in $\triangle RUN$.

Extra Practice

Lesson 5-4

(pages 261–266)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*.

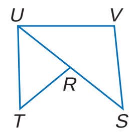
1. 2, 2, 6 **no**
2. 2, 3, 4 **yes**
3. 6, 8, 10 **yes**
4. 1, 1, 2 **no**
5. 15, 20, 30 **yes**
6. 1, 3, 5 **no**
7. 2.5, 3.5, 6.5 **no**
8. 0.3, 0.4, 0.5 **yes**

Find the range for the measure of the third side of a triangle given the measures of two sides.

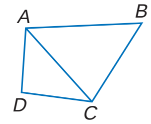
9. 6 and 10 $4 < n < 16$
10. 2 and 5 $3 < n < 7$
11. 20 and 12 $8 < n < 32$
12. 8 and 8 $0 < n < 16$
13. 18 and 36 $18 < n < 54$
14. 32 and 34 $2 < n < 66$
15. 2 and 29 $27 < n < 31$
16. 80 and 25 $55 < n < 105$

Write a two-column proof. **17–18. See margin.**

17. **Given:** $RS = RT$
Prove: $UV + VS > UT$



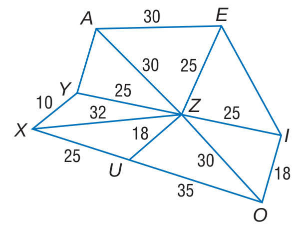
18. **Given:** quadrilateral $ABCD$
Prove: $AD + CD + AB > BC$



Lesson 5-5

(pages 267–273)

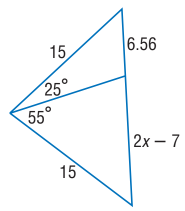
Write an inequality relating the given pair of angle or segment measures.



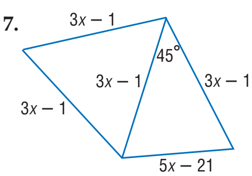
1. XZ, OZ $XZ > OZ$
2. $m\angle ZIO$, $m\angle ZUX$ $m\angle ZIO < m\angle ZUX$
3. $m\angle AEZ$, $m\angle AZE$ $m\angle AEZ = m\angle AZE$
4. IO, AE $IO < AE$
5. $m\angle AZE$, $m\angle IZO$ $m\angle AZE > m\angle IZO$

Write an inequality to describe the possible values of x.

6. $6.78 < x < 15.22$



7. $4.2 < x < 10$



Lesson 6-1

(pages 282–287)

1. **ARCHITECTURE** The ratio of the height of a model of a house to the actual house is 1:63. If the width of the model is 16 inches, find the width of the actual house in feet. **84 ft**

2. **CONSTRUCTION** A 64-inch long board is divided into lengths in the ratio 2:3. What are the two lengths into which the board is divided? **25.6 in., 38.4 in.**

ALGEBRA **Solve each proportion.**

3. $\frac{x+4}{26} = -\frac{1}{3}$ $-\frac{38}{3}$
4. $\frac{3x+1}{14} = \frac{5}{7}$ 3
5. $\frac{x-3}{4} = \frac{x+1}{5}$ 19
6. $\frac{2x+2}{2x-1} = \frac{1}{3}$ $-\frac{7}{4}$

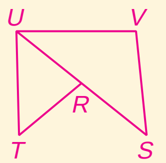
7. Find the measures of the sides of a triangle if the ratio of the measures of three sides of a triangle is 9:6:5, and its perimeter is 100 inches. **45 in., 30 in., 25 in.**

8. Find the measures of the angles in a triangle if the ratio of the measures of the three angles is 13:16:21. **46.8, 57.6, 75.6**

Extra Practice

Lesson 5-4

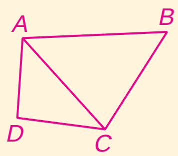
17. **Given:** $RS = RT$
Prove: $UV + VS > UT$



Proof:

Statements (Reasons)
1. $RS = RT$ (Given)
2. $UV + VS > US$ (Triangle Inequality Theorem)
3. $US = UR + RS$ (Segment Addition Postulate)
4. $UV + VS > UR + RS$ (Substitution)
5. $UV + VS > UR + RT$ (Substitution)
6. $UR + RT > UT$ (Triangle Inequality Theorem)
7. $UV + VS > UT$ (Transitive Property of Inequality)

18. **Given:** quadrilateral $ABCD$
Prove: $AD + CD + AB > BC$



Proof:

Statements (Reasons)
1. quadrilateral $ABCD$ (Given)
2. Draw $\overline{AC}$. (Through any 2 pts. there is 1 line.)
3. $AD + CD > AC$; $AB + AC > BC$ (Triangle Inequality Theorem)
4. $AC > BC - AB$ (Subtraction Prop. of Inequality)
5. $AD + CD > BC - AB$ (Transitive Prop. of Inequality)
6. $AD + CD + AB > BC$ (Addition Prop. of Inequality)

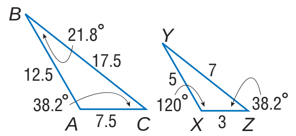
764 Extra Practice

Lesson 6-2

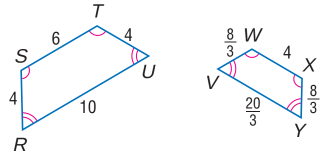
(pages 289–297)

Determine whether each pair of figures is similar. Justify your answer. **1–2. See margin.**

1.



2.



For Exercises 3 and 4, use $\triangle RST$ with vertices $R(3, 6)$, $S(1, 2)$, and $T(3, -1)$. Explain. **3–4. See margin.**

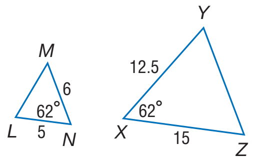
3. If the coordinates of each vertex are decreased by 3, describe the new figure. Is it similar to $\triangle RST$?
4. If the coordinates of each vertex are multiplied by 0.5, describe the new figure. Is it similar to $\triangle RST$?

Lesson 6-3

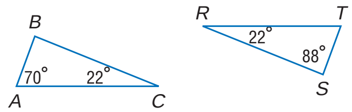
(pages 298–306)

Determine whether each pair of triangles is similar. Justify your answer. **1–2. See margin.**

1.

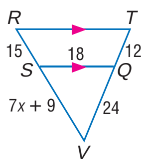


2.

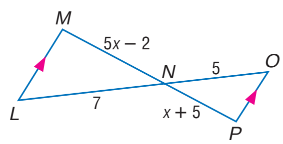


ALGEBRA **Identify the similar triangles. Find x and the measures of the indicated sides.**

3. RT and SV



4. PN and MN

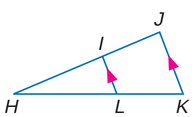


3–4. See margin.

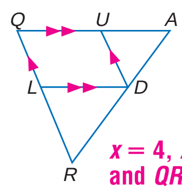
Lesson 6-4

(pages 307–315)

1. If $HI = 28$, $LH = 21$, and $LK = 8$, find IJ. $10\frac{2}{3}$

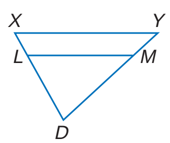


2. Find x, AD, DR, and QR if $AU = 15$, $QU = 25$, $AD = 3x + 6$, $DR = 8x - 2$, and $UD = 15$.



$x = 4$, $AD = 18$, $DR = 30$, and $QR = 40$

Find x so that $\overline{XY} \parallel \overline{LM}$.



3. $XL = 3$, $YM = 5$, $LD = 9$, $MD = x + 3$ **12**
4. $YM = 3$, $LD = 3x + 1$, $XL = 4$, $MD = x + 7$ **5**
5. $MD = 5x - 6$, $YM = 3$, $LD = 5x + 1$, $XL = 5$ **3.3**

2. $\angle S \cong \angle W$, $\angle T \cong \angle X$, $\angle U \cong \angle Y$, $\angle R \cong \angle V$.
   All of the corresponding angles are congruent. Now determine whether corresponding sides are proportional.
   $\frac{RS}{VW} = \frac{4}{\frac{8}{3}} = 1.5$
   $\frac{ST}{WX} = \frac{6}{4} = 1.5$
   $\frac{TU}{XY} = \frac{4}{\frac{8}{3}} = 1.5$
   $\frac{RU}{VY} = \frac{10}{\frac{20}{3}} = 1.5$
   The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so polygon $RSTU \sim$ polygon $VWXY$.
3. Yes; the new triangle is congruent and similar to the original, but shifted to the left 3 units and down 3 units.
4. Yes; the new triangle is similar to the original, but the length of each side is one half the length of the corresponding sides of the original triangle.

Lesson 6-3

1. Yes; $\triangle LNM \sim \triangle YXZ$; SAS Similarity
2. Yes; $\triangle ABC \sim \triangle TSR$; AA Similarity.
3. $\triangle RTV \sim \triangle SQV$; $x = 3$; $RT = 27$; $SV = 30$
4. $\triangle MNL \sim \triangle PNO$; $x = 2.5$; $PN = 7.5$; $MN = 10.5$

Lesson 6-2

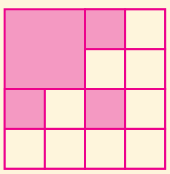
1. $m\angle A = 180 - 21.8 - 38.2 = 120$, so $m\angle A = m\angle X$. Therefore $\angle A \cong \angle X$.
   $m\angle Y = 180 - 120 - 38.2 = 21.8$, so $m\angle Y = m\angle B$. Therefore $\angle Y \cong \angle B$.
   $m\angle C = m\angle Z$, therefore $\angle C \cong \angle Z$.

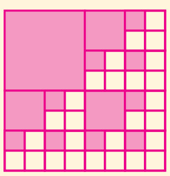
All of the corresponding angles are congruent. Now determine whether corresponding sides are proportional.

$\frac{AB}{XY} = \frac{12.5}{5} = 2.5$ $\quad \frac{BC}{YZ} = \frac{17.5}{7} = 2.5$ $\quad \frac{AC}{XZ} = \frac{7.5}{3} = 2.5$

The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so $\triangle ABC \sim \triangle XYZ$.

Lesson 6-6

1. 

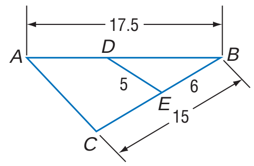
2. 

3. converges to 1
4. approaches positive infinity
5. converges to 0
6. approaches positive infinity

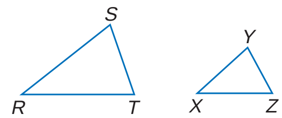
Lesson 6-5 *(pages 316–323)*

Find the perimeter of each triangle.

1. $\triangle ABC$ if $\triangle ABC \sim \triangle DBE$, $AB = 17.5$, $BC = 15$, $BE = 6$, and $DE = 5$ **45**



2. $\triangle RST$ if $\triangle RST \sim \triangle XYZ$, $RT = 12$, $XZ = 8$, and the perimeter of $\triangle XYZ = 22$ **33**

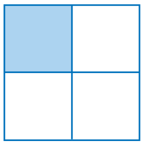


3. $\triangle LMN$ if $\triangle LMN \sim \triangle NXY$, $NX = 14$, $YX = 11$, $YN = 9$, and $LN = 27$ **102**

4. $\triangle GHI$ if $\triangle ABC \sim \triangle GHI$, $AB = 6$, $GH = 10$, and the perimeter of $\triangle ABC = 25$ **$41\frac{2}{3}$**

Lesson 6-6 *(pages 325–331)*

Stage 1 of a fractal is shown drawn on grid paper. Stage 1 is made by dividing a square into 4 congruent squares and shading the top left-hand square.



1. Draw Stage 2 by repeating the Stage 1 process in each of the 3 remaining unshaded squares. How many shaded squares are at this stage? **4**
2. Draw Stage 3 by repeating the Stage 1 process in each of the unshaded squares in Stage 2. How many shaded squares are at this stage? **13**

1–2. See margin for fractals.

Find the value of each expression. Then, use that value as the next x in the expression. Repeat the process and describe your observations. **3–6. See margin.**

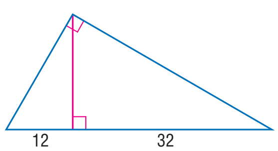
3. $x^{\frac{1}{4}}$, where x initially equals 6
4. 4^x, where x initially equals 0.4
5. x^3, where x initially equals 0.5
6. 3^x, where x initially equals 10

Lesson 7-1 *(pages 342–348)*

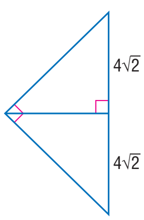
Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.

1. 8 and 12 **$4\sqrt{6} \approx 9.8$**
2. 15 and 20 **$10\sqrt{3} \approx 17.3$**
3. 1 and 2 **$\sqrt{2} \approx 1.4$**
4. 4 and 16 **8**
5. $3\sqrt{2}$ and $6\sqrt{2}$ **6**
6. $\frac{1}{2}$ and 10 **$\sqrt{5} \approx 2.2$**
7. $\frac{3}{8}$ and $\frac{1}{2}$ **$\frac{\sqrt{3}}{4} \approx 0.4$**
8. $\frac{\sqrt{2}}{2}$ and $\frac{3\sqrt{2}}{2}$ **$\frac{\sqrt{6}}{2} \approx 1.2$**
9. $\frac{1}{10}$ and $\frac{7}{10}$ **$\frac{\sqrt{7}}{10} \approx 0.3$**

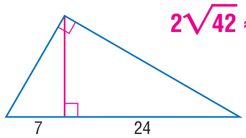
Find the altitude of each triangle.

10. 

$8\sqrt{6} \approx 19.6$

11. 

$4\sqrt{2} \approx 5.7$

12. 

$2\sqrt{42} \approx 13.0$

766 Extra Practice

Lesson 7-2
(pages 350–356)

Determine whether $\triangle DEF$ is a right triangle for the given vertices. Explain. **1–4. See margin.**

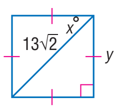
1. $D(0, 1), E(3, 2), F(2, 3)$
2. $D(-2, 2), E(3, -1), F(-4, -3)$
3. $D(2, -1), E(-2, -4), F(-4, -1)$
4. $D(1, 2), E(5, -2), F(-2, -1)$

Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean triple. **5–13. See margin.**

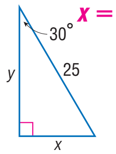
5. 1, 1, 2
6. 21, 28, 35
7. 3, 5, 7
8. 2, 5, 7
9. 24, 45, 51
10. $\frac{1}{3}, \frac{5}{3}, \frac{\sqrt{26}}{3}$
11. $\frac{6}{11}, \frac{8}{11}, \frac{10}{11}$
12. $\frac{1}{2}, \frac{1}{2}, 1$
13. $\frac{\sqrt{6}}{3}, \frac{\sqrt{10}}{5}, \frac{\sqrt{240}}{15}$

Lesson 7-3
(pages 357–363)

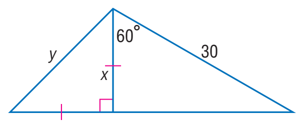
Find the measures of x and y.

1. 

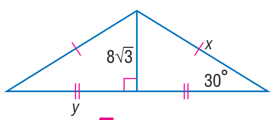
$x = 45,\ y = 13$

2. 

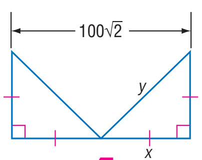
$x = 12.5,\ y = 12.5\sqrt{3}$

3. 

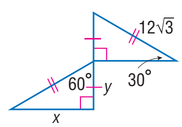
$x = 15,\ y = 15\sqrt{2}$

4. 

$x = 16\sqrt{3},\ y = 24$

5. 

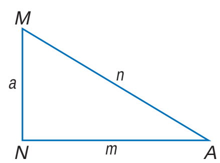
$x = 50\sqrt{2},\ y = 100$

6. 

$x = 18,\ y = 6\sqrt{3}$

Lesson 7-4
(pages 364–370)

Use $\triangle MAN$ with right angle N to find $\sin M$, $\cos M$, $\tan M$, $\sin A$, $\cos A$, and $\tan A$. Express each ratio as a fraction, and as a decimal to the nearest hundredth. **1–4. See margin.**

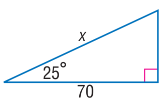


1. $m = 21, a = 28, n = 35$
2. $m = \sqrt{2}, a = \sqrt{3}, n = \sqrt{5}$
3. $m = \frac{\sqrt{2}}{2}, a = \frac{\sqrt{2}}{2}, n = 1$
4. $m = 3\sqrt{5}, a = 5\sqrt{3}, n = 2\sqrt{30}$

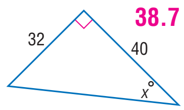
Find the measure of each angle to the nearest tenth of a degree.

5. $\cos A = 0.6293$ **51.0**
6. $\sin B = 0.5664$ **34.5**
7. $\tan C = 0.2665$ **14.9**
8. $\sin D = 0.9352$ **69.3**
9. $\tan M = 0.0808$ **4.6**
10. $\cos R = 0.1097$ **83.7**

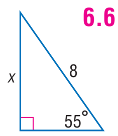
Find x. Round to the nearest tenth.

11. 

77.2

12. 

38.7

13. 

6.6

Extra Practice

Lesson 7-2

1. yes; $DE = \sqrt{10}$, $EF = \sqrt{2}$, $DF = \sqrt{8}$; $(\sqrt{2})^2 + (\sqrt{8})^2 = (\sqrt{10})^2$
2. no; $DE = \sqrt{34}$, $EF = \sqrt{53}$, $DF = \sqrt{29}$; $(\sqrt{29})^2 + (\sqrt{34})^2 \neq (\sqrt{53})^2$
3. no; $DE = 5$, $EF = \sqrt{13}$, $DF = 6$; $(\sqrt{13})^2 + 5^2 \neq 6^2$
4. yes; $DE = \sqrt{32}$, $EF = \sqrt{50}$, $DF = \sqrt{18}$; $(\sqrt{18})^2 + (\sqrt{32})^2 = (\sqrt{50})^2$
5. no; no
6. yes; yes
7. no; no
8. no; no
9. yes, yes
10. yes; no
11. yes; no
12. no; no
13. yes; no

Lesson 7-4

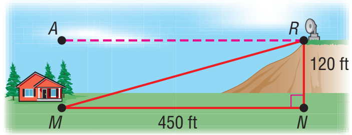
1. $\frac{3}{5} = 0.60$; $\frac{4}{5} = 0.80$; $\frac{3}{4} = 0.75$; $\frac{4}{5} = 0.80$; $\frac{3}{5} = 0.60$; $\frac{4}{3} = 1.33$
2. $\frac{\sqrt{10}}{5} \approx 0.63$; $\frac{\sqrt{15}}{5} \approx 0.77$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{15}}{5} \approx 0.77$; $\frac{\sqrt{10}}{5} \approx 0.63$; $\frac{\sqrt{6}}{2} \approx 1.22$
3. $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00
4. $\frac{\sqrt{6}}{4} \approx 0.61$; $\frac{\sqrt{10}}{4} \approx 0.79$; $\frac{\sqrt{15}}{5} \approx 0.77$; $\frac{\sqrt{10}}{4} \approx 0.79$; $\frac{\sqrt{6}}{4} \approx 0.61$; $\frac{\sqrt{15}}{3} \approx 1.29$

Extra Practice

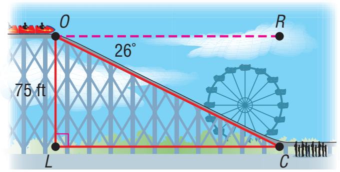
Extra Practice

Lesson 7-5
(pages 371–376)

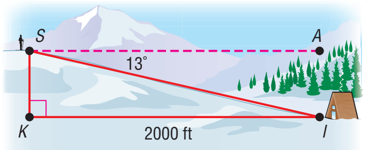
1. **COMMUNICATIONS** A house is located below a hill that has a satellite dish. If $MN = 450$ feet and $RN = 120$ feet, what is the measure of the angle of elevation to the top of the hill? **about 14.9**



2. **AMUSEMENT PARKS** Mandy is at the top of the Mighty Screamer roller coaster. Her friend Bryn is at the bottom of the coaster waiting for the next ride. If the angle of depression from Mandy to Bryn is 26° and OL is 75 feet, what is the distance from L to C? **about 153.8 ft**



3. **SKIING** Mitchell is at the top of the Bridger Peak ski run. His brother Scott is looking up from the ski lodge at I. If the angle of elevation from Scott to Mitchell is 13° and the distance from K to I is 2000 ft, what is the length of the ski run SI? **about 2052.6 ft**



Lesson 7-6
(pages 377–383)

Find each measure using the given measures from $\triangle ANG$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $m\angle N = 32$, $m\angle A = 47$, and $n = 15$, find a. **20.7**
2. If $a = 10.5$, $m\angle N = 26$, $m\angle A = 75$, find n. **4.8**
3. If $n = 18.6$, $a = 20.5$, $m\angle A = 65$, find $m\angle N$. **55**
4. If $a = 57.8$, $n = 43.2$, $m\angle A = 33$, find $m\angle N$. **24**

Solve each $\triangle AKX$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

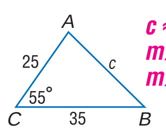
5. $m\angle X = 62$, $a = 28.5$, $m\angle K = 33$ **$m\angle A = 85$, $x \approx 25.3$, $k \approx 15.6$**
6. $k = 3.6$, $x = 3.7$, $m\angle X = 55$ **$m\angle K \approx 53$, $m\angle A \approx 72$, $a \approx 4.3$**
7. $m\angle K = 35$, $m\angle A = 65$, $x = 50$ **$m\angle X = 80$, $a \approx 46.0$, $k \approx 29.1$**
8. $m\angle A = 122$, $m\angle X = 15$, $a = 33.2$ **$m\angle K = 43$, $k \approx 26.7$, $x \approx 10.1$**

Lesson 7-7
(pages 385–390)

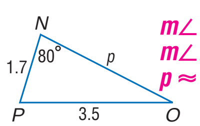
In $\triangle CDE$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

1. $c = 100$, $d = 125$, $e = 150$; $m\angle E$ **82.8**
2. $c = 5$, $d = 6$, $e = 9$; $m\angle C$ **31.6**
3. $c = 1.2$, $d = 3.5$, $e = 4$; $m\angle D$ **57.3**
4. $c = 42.5$, $d = 50$, $e = 81.3$; $m\angle E$ **122.8**

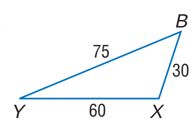
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

5.


$c \approx 29.1$
$m\angle A \approx 80$
$m\angle B \approx 45$

6.


$m\angle O \approx 29$
$m\angle P \approx 71$
$p \approx 3.4$

7.


$m\angle B \approx 50$
$m\angle X \approx 108$
$m\angle Y \approx 22$

768 Extra Practice

Lesson 8-1 *(pages 404–409)*

Find the sum of the measures of the interior angles of each convex polygon.

1. 25-gon **4140**
2. 30-gon **5040**
3. 22-gon **3600**
4. 17-gon **2700**
5. $5a$-gon **$180(5a - 2)$**
6. b-gon **$180(b - 2)$**

The measure of an interior angle of a regular polygon is given. Find the number of sides in each polygon.

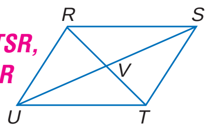
7. 156 **15**
8. 168 **30**
9. 162 **20**

Find the measures of an interior angle and an exterior angle given the number of sides of a regular polygon. Round to the nearest tenth.

10. 15 **156, 24**
11. 13 **152.3, 27.7**
12. 42 **171.4, 8.6**

Lesson 8-2 *(pages 411–416)*

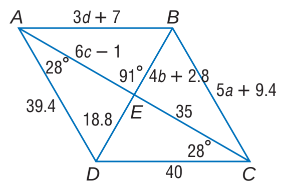
Complete each statement about ▱$RSTU$. Justify your answer.



1. $\angle SRU \cong$ __?__ **$\angle UTS$**
2. $\angle UTS$ is supplementary to __?__ **$\angle TSR$, $\angle TUR$**
3. $\overline{RU} \parallel$ __?__ **$\overline{ST}$**
4. $\overline{RU} \cong$ __?__ **$\overline{ST}$**
5. $\triangle RST \cong$ __?__ **$\triangle TUR$**
6. $\overline{SV} \cong$ __?__ **$\overline{VU}$**

1–6. See margin for justification.

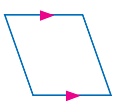
ALGEBRA **Use ▱$ABCD$ to find each measure or value.**

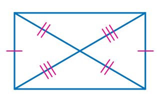


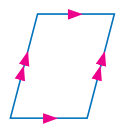
7. $m\angle BAE =$ __?__ **28**
8. $m\angle BCE =$ __?__ **28**
9. $m\angle BEC =$ __?__ **89**
10. $m\angle CED =$ __?__ **91**
11. $m\angle ABE =$ __?__ **61**
12. $m\angle EBC =$ __?__ **63**
13. $a =$ __?__ **6**
14. $b =$ __?__ **4**
15. $c =$ __?__ **6**
16. $d =$ __?__ **11**

Lesson 8-3 *(pages 417–423)*

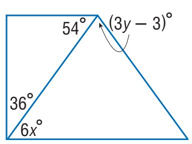
Determine whether each quadrilateral is a parallelogram. Justify your answer. **1–3. See margin for justification.**

1. 
no

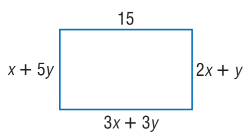
2. 
yes

3. 
yes

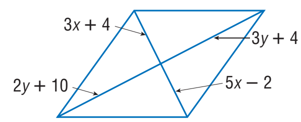
ALGEBRA **Find x and y so that each quadrilateral is a parallelogram.**

4. 

$x = 9$, $y = 13$

5. 

$x = 4$, $y = 1$

6. 

$x = 3$, $y = 6$

Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

7. $L(-3, 2)$, $M(5, 2)$, $N(3, -6)$, $O(-5, -6)$; Slope Formula **yes**
8. $W(-5, 6)$, $X(2, 5)$, $Y(-3, -4)$, $Z(-8, -2)$; Distance Formula **no**
9. $Q(-5, 4)$, $R(0, 6)$, $S(3, -1)$, $T(-2, -3)$; Midpoint Formula **yes**
10. $G(-5, 0)$, $H(-13, 5)$, $I(-10, 9)$, $J(-2, 4)$; Distance and Slope Formulas **yes**

Lesson 8-2

1. $\angle UTS$; opp. $\angle$s of ▱ are $\cong$.
2. $\angle TSR$; cons. $\angle$s in ▱ are suppl.
3. Opp. sides of ▱ are parallel.
4. Opp. sides of ▱ are $\cong$.
5. Diag. of ▱ separates ▱ into 2 $\cong$ $\triangle$s.
6. Diag. of ▱ bisect each other.

Lesson 8-3

1. Only one pair of sides is shown to be parallel.
2. If diag. bisect each other, then quad. is ▱.
3. If both pairs of opp. sides are $\parallel$, then quad. is ▱.

Extra Practice 769

Lesson 8-6

1a. $\overline{AD} \parallel \overline{BC}$; *ABCD* is a trapezoid.

1b. $\overline{AB} \cong \overline{CD}$; *ABCD* is an isosceles trapezoid.

2a. $\overline{QR} \parallel \overline{ST}$; *QRST* is a trapezoid.

2b. $\overline{QT} \not\cong \overline{RS}$; *QRST* is not an isosceles trapezoid.

3a. $\overline{ON} \parallel \overline{LM}$; *LMNO* is a trapezoid.

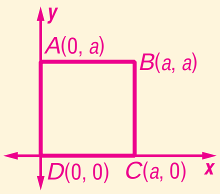
3b. $\overline{LO} \cong \overline{MN}$; *LMNO* is an isosceles trapezoid.

4a. $\overline{WX} \parallel \overline{ZY}$; *WXYZ* is a trapezoid.

4b. $\overline{WZ} \not\cong \overline{XY}$; *WXYZ* is not an isosceles trapezoid.

Lesson 8-7

3. Given: *ABCD* is a square.
Prove: $\overline{AC} \cong \overline{BD}$



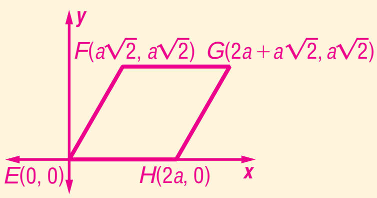
Proof:

$AC = \sqrt{(a-0)^2 + (0-a)^2}$
$= \sqrt{a^2 + a^2}$
$= \sqrt{2a^2}$

$BD = \sqrt{(a-0)^2 + (a-0)^2}$
$= \sqrt{a^2 + a^2}$
$= \sqrt{2a^2}$

$AC = BD$
$\overline{AC} \cong \overline{BD}$

4. Given: *EFGH* is a quadrilateral.
Prove: *EFGH* is a rhombus.



Proof:

$EF = \sqrt{(a\sqrt{2} - 0)^2 + (a\sqrt{2} - 0)^2}$
$= \sqrt{2a^2 + 2a^2}$
$= \sqrt{4a^2}$ or $2a$

$FG = \sqrt{((2a + a\sqrt{2}) - a\sqrt{2})^2 + (a\sqrt{2} - a\sqrt{2})^2}$
$= \sqrt{(2a)^2 + 0^2}$
$= \sqrt{4a^2}$ or $2a$

$GH = \sqrt{((2a + a\sqrt{2}) - 2a)^2 + (a\sqrt{2} - 0)^2}$
$= \sqrt{2a^2 + 2a^2}$
$= \sqrt{4a^2}$ or $2a$

$EH = \sqrt{(2a - 0)^2 + (0 - 0)^2}$
$= \sqrt{(2a)^2 + 0^2}$
$= \sqrt{4a^2}$ or $2a$

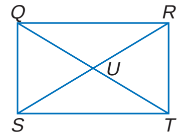
$EF = FG = GH = EH$
$\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{EH}$
Since all four sides are congruent, *EFGH* is a rhombus.

Extra Practice

Lesson 8-4

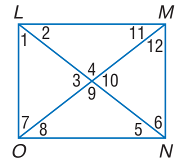
(pages 424–430)

ALGEBRA **Refer to rectangle *QRST*.**



1. If $QU = 2x + 3$ and $UT = 4x - 9$, find SU. **15**
2. If $RU = 3x - 6$ and $UT = x + 9$, find RS. **33**
3. If $QS = 3x + 40$ and $RT = 16 - 3x$, find QS. **28**
4. If $m\angle STQ = 5x + 3$ and $m\angle RTQ = 3 - x$, find x. **21**
5. If $m\angle SRQ = x^2 + 6$ and $m\angle RST = 36 - x$, find $m\angle SRT$. **48 or 59**
6. If $m\angle TQR = x^2 + 16$ and $m\angle QTR = x + 32$, find $m\angle TQS$. **25 or 38**

Find each measure in rectangle *LMNO* if $m\angle 5 = 38$.

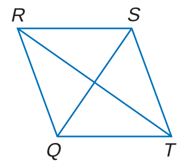


7. $m\angle 1$ **52**	8. $m\angle 2$ **38**	9. $m\angle 3$ **76**
10. $m\angle 4$ **104**	11. $m\angle 6$ **52**	12. $m\angle 7$ **52**
13. $m\angle 8$ **38**	14. $m\angle 9$ **104**	15. $m\angle 10$ **76**
16. $m\angle 11$ **38**	17. $m\angle 12$ **52**	18. $m\angle OLM$ **90**

Lesson 8-5

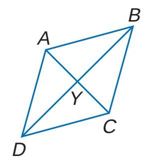
(pages 431–437)

In rhombus *QRST*, $m\angle QRS = m\angle TSR - 40$ and $TS = 15$.



1. Find $m\angle TSQ$. **55**
2. Find $m\angle QRS$. **70**
3. Find $m\angle SRT$. **35**
4. Find QR. **15**

ALGEBRA **Use rhombus *ABCD* with $AY = 6$, $DY = 3r + 3$, and $BY = \dfrac{10r - 4}{2}$.**



5. Find $m\angle ACB$. **60**
6. Find $m\angle ABD$. **30**
7. Find BY. **10.5**
8. Find AC. **12**

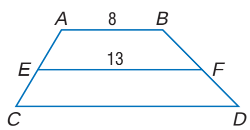
Lesson 8-6

(pages 439–445)

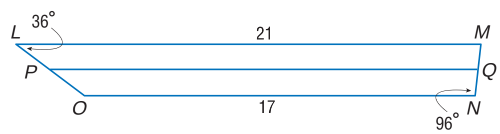
COORDINATE GEOMETRY **For each quadrilateral with the given vertices,**
a. verify that the quadrilateral is a trapezoid, and
b. determine whether the figure is an isosceles trapezoid. **1–4. See margin.**

1. $A(0, 9)$, $B(3, 4)$, $C(-5, 4)$, $D(-2, 9)$
2. $Q(1, 4)$, $R(4, 6)$, $S(10, 7)$, $T(1, 1)$
3. $L(1, 2)$, $M(4, -1)$, $N(3, -5)$, $O(-3, 1)$
4. $W(1, -2)$, $X(3, -1)$, $Y(7, -2)$, $Z(1, -5)$

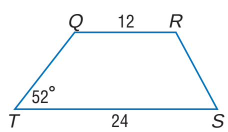
5. For trapezoid *ABDC*, *E* and *F* are midpoints of the legs. Find *CD*. **18**



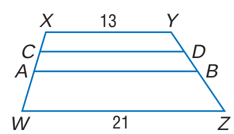
6. For trapezoid *LMNO*, *P* and *Q* are midpoints of the legs. Find *PQ*, $m\angle M$, and $m\angle O$. **19, 84, 144**



7. For isosceles trapezoid *QRST*, find the length of the median, $m\angle S$, and $m\angle R$. **18, 52, 128**



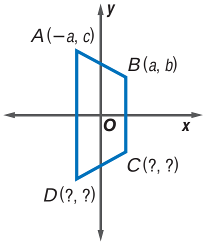
8. For trapezoid *XYZW*, *A* and *B* are midpoints of the legs. For trapezoid *XYBA*, *C* and *D* are midpoints of the legs. Find *CD*. **15**



Lesson 8-7 (pages 447–451)

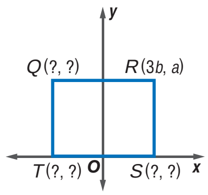
Name the missing coordinates for each quadrilateral.

1. isosceles trapezoid $ABCD$



$C(a, -b)$, $D(-a, -c)$

2. rectangle $QRST$



$S(3b, 0)$, $T(-3b, 0)$, $Q(-3b, a)$

Position and label each figure on the coordinate plane. Then write a coordinate proof for each of the following. **3–4. See margin.**

3. The diagonals of a square are congruent.
4. Quadrilateral $EFGH$ with vertices $E(0, 0)$, $F(a\sqrt{2}, a\sqrt{2})$, $G(2a + a\sqrt{2}, a\sqrt{2})$, and $H(2a, 0)$ is a rhombus.

Lesson 9-1 (pages 463–469)

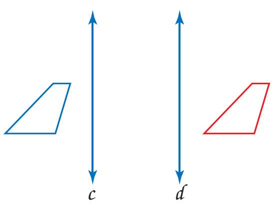
COORDINATE GEOMETRY **Graph each figure and its image under the given reflection.**

1. $\triangle ABN$ with vertices $A(2, 2)$, $B(3, -2)$, and $N(-3, -1)$ in the x-axis **1–7. See margin.**
2. rectangle $BARN$ with vertices $B(3, 3)$, $A(3, -4)$, $R(-1, -4)$, and $N(-1, 3)$ in the line $y = x$
3. trapezoid $ZOID$ with vertices $Z(2, 3)$, $O(2, -4)$, $I(-3, -3)$, and $D(-3, 1)$ in the origin
4. $\triangle PQR$ with vertices $P(-2, 1)$, $Q(2, -2)$, and $R(-3, -4)$ in the y-axis
5. square $BDFH$ with vertices $B(-4, 4)$, $D(-1, 4)$, $F(-1, 1)$, and $H(-4, 1)$ in the origin
6. quadrilateral $QUAD$ with vertices $Q(1, 3)$, $U(3, 1)$, $A(-1, 0)$, and $D(-3, 4)$ in the line $y = -1$
7. $\triangle CAB$ with vertices $C(0, 4)$, $A(1, -3)$, and $B(-4, 0)$ in the line $x = -2$

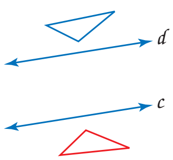
Lesson 9-2 (pages 470–475)

In each figure, $c \parallel d$. Determine whether the red figure is a translation image of the blue figure. Write *yes* or *no*. Explain your answer. **1–3. See margin.**

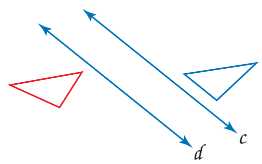
1.



2.



3.



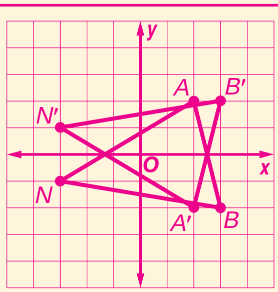
COORDINATE GEOMETRY **Graph each figure and its image under the given translation.** **4–8. See p. 781A.**

4. $\overline{LM}$ with endpoints $L(2, 3)$ and $M(-4, 1)$ under the translation $(x, y) \to (x + 2, y + 1)$
5. $\triangle DEF$ with vertices $D(1, 2)$, $E(-2, 1)$, and $F(-3, -1)$ under the translation $(x, y) \to (x - 1, y - 3)$
6. quadrilateral $WXYZ$ with vertices $W(1, 1)$, $X(-2, 3)$, $Y(-3, -2)$, and $Z(2, -2)$ under the translation $(x, y) \to (x + 1, y - 1)$
7. pentagon $ABCDE$ with vertices $A(1, 3)$, $B(-1, 1)$, $C(-1, -2)$, $D(3, -2)$, and $E(3, 1)$ under the translation $(x, y) \to (x - 2, y + 3)$
8. $\triangle RST$ with vertices $R(-4, 3)$, $S(-2, -3)$, and $T(2, -1)$ under the translation $(x, y) \to (x + 3, y - 2)$

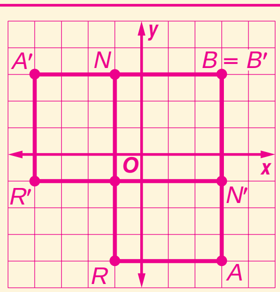
Extra Practice

Lesson 9-1

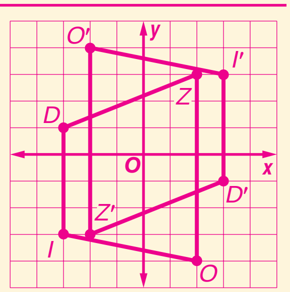
1.



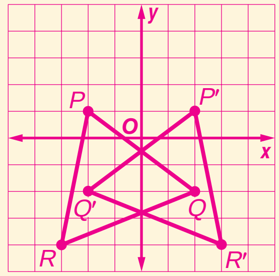
2.



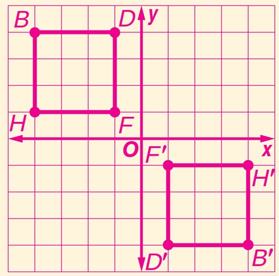
3.



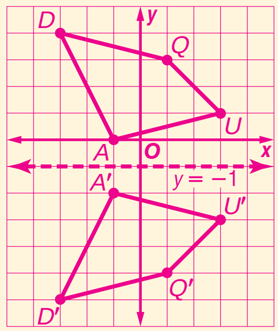
4.



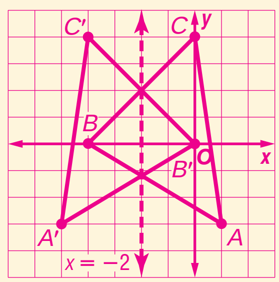
5.



6.



7.



Lesson 9-2

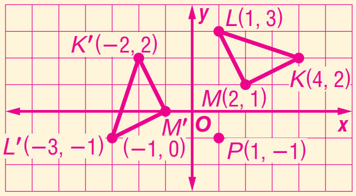
1. **Yes; it is one reflection after another with respect to the two parallel lines.**
2. **No; the figure has a different orientation.**
3. **No; it is not one reflection after another with respect to the two parallel lines.**

Extra Practice

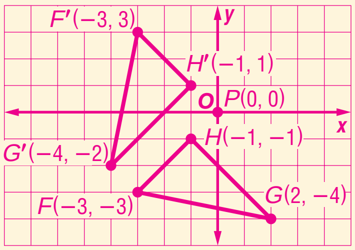
Extra Practice 771

Lesson 9-3

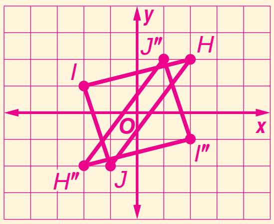
1.



2.

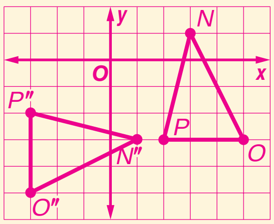


3.



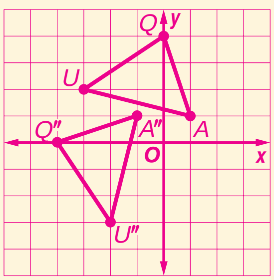
H"(−2, −2), I"(2, −1), and J"(1, 2); 180°

4.



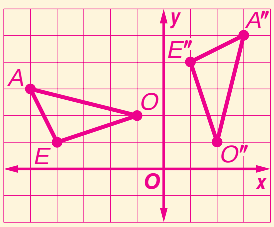
N"(1, −3), O"(−3, −5), and P"(−3, −2); 90° clockwise

5.



Q"(−4, 0), U"(−2, −3), and A"(−1, 1); 90° counterclockwise

6.



A"(3, 5), E"(1, 4), and O"(2, 1); 90° clockwise

Lesson 9-3 *(pages 476–482)*

COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the vertices with coordinates. **1–2. See margin.**

1. $\triangle KLM$ with vertices $K(4, 2)$, $L(1, 3)$, and $M(2, 1)$ counterclockwise about the point $P(1, -1)$
2. $\triangle FGH$ with vertices $F(-3, -3)$, $G(2, -4)$, and $H(-1, -1)$ clockwise about the point $P(0, 0)$

COORDINATE GEOMETRY Draw the rotation image of each triangle by reflecting the triangle in the given lines. State the coordinates of the rotation image and the angle of rotation. **3–6. See margin.**

3. $\triangle HIJ$ with vertices $H(2, 2)$, $I(-2, 1)$, and $J(-1, -2)$, reflected in the x-axis and then in the y-axis
4. $\triangle NOP$ with vertices $N(3, 1)$, $O(5, -3)$, and $P(2, -3)$, reflected in the y-axis and then in the line $y = x$
5. $\triangle QUA$ with vertices $Q(0, 4)$, $U(-3, 2)$, and $A(1, 1)$, reflected in the x-axis and then in the line $y = x$
6. $\triangle AEO$ with vertices $A(-5, 3)$, $E(-4, 1)$, and $O(-1, 2)$, reflected in the line $y = -x$ and then in the y-axis

Lesson 9-4 *(pages 483–488)*

Determine whether a semi-regular tessellation can be created from each set of figures. Assume each figure has a side length of 1 unit.

1. regular hexagons and squares **no**
2. squares and regular pentagons **no**
3. regular hexagons and regular octagons **no**

Determine whether each statement is *always*, *sometimes*, or *never* true.

4. Any right isosceles triangle forms a uniform tessellation. **sometimes**
5. A semi-regular tessellation is uniform. **always**
6. A polygon that is not regular can tessellate the plane. **sometimes**
7. If the measure of one interior angle of a regular polygon is greater than 120, it cannot tessellate the plane. **always**

Lesson 9-5 *(pages 490–497)*

Find the measure of the dilation image or the preimage of $\overline{OM}$ with the given scale factor.

1. $OM = 1$, $r = -2$ $\mathbf{O'M' = 2}$
2. $OM = 3$, $r = \frac{1}{3}$ $\mathbf{O'M' = 1}$
3. $O'M' = \frac{3}{4}$, $r = 3$ $\mathbf{OM = \frac{1}{4}}$
4. $OM = \frac{7}{8}$, $r = -\frac{5}{7}$ $\mathbf{O'M' = \frac{5}{8}}$
5. $O'M' = 4$, $r = -\frac{2}{3}$ $\mathbf{OM = 6}$
6. $O'M' = 4.5$, $r = -1.5$ $\mathbf{OM = 3}$

COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with scale factor $r = 3$. Then graph a dilation with $r = \frac{1}{3}$. **7–10. See p. 781A.**

7. $T(1, 1)$, $R(-1, 2)$, $I(-2, 0)$
8. $E(2, 1)$, $I(3, -3)$, $O(-1, -2)$
9. $A(0, -1)$, $B(-1, 1)$, $C(0, 2)$, $D(1, 1)$
10. $B(1, 0)$, $D(2, 0)$, $F(3, -2)$, $H(0, -2)$

772 Extra Practice

Lesson 9-6

(pages 498–505)

Find the magnitude and direction of $\overrightarrow{XY}$ for the given coordinates. **1–6. See margin.**

1. $X(1, 1)$, $Y(-2, 3)$
2. $X(-1, -1)$, $Y(2, 2)$
3. $X(-5, 4)$, $Y(-2, -3)$
4. $X(2, 1)$, $Y(-4, -4)$
5. $X(-2, -1)$, $Y(2, -2)$
6. $X(3, -1)$, $Y(-3, 1)$

Graph the image of each figure under a translation by the given vector. **7–9. See margin.**

7. $\triangle HIJ$ with vertices $H(2, 3)$, $I(-4, 2)$, $J(-1, 1)$; $\vec{a} = \langle 1, 3\rangle$
8. quadrilateral $RSTW$ with vertices $R(4, 0)$, $S(0, 1)$, $T(-2, -2)$, $W(3, -1)$; $\vec{x} = \langle -3, 4\rangle$
9. pentagon $AEIOU$ with vertices $A(-1, 3)$, $E(2, 3)$, $I(2, 0)$, $O(-1, -2)$, $U(-3, 0)$; $\vec{b} = \langle -2, -1\rangle$

10. $\sqrt{74} \approx 8.6, 54.5°$ 11. $3\sqrt{5} \approx 6.7, 116.6°$ 12. $\sqrt{41} \approx 6.4, 321.3°$

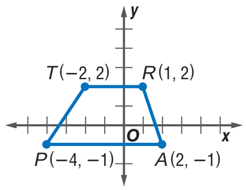
Find the magnitude and direction of each resultant for the given vectors.

10. $\vec{c} = \langle 2, 3\rangle$, $\vec{d} = \langle 3, 4\rangle$
11. $\vec{a} = \langle 1, 3\rangle$, $\vec{b} = \langle -4, 3\rangle$
12. $\vec{x} = \langle 1, 2\rangle$, $\vec{y} = \langle 4, -6\rangle$
13. $\vec{s} = \langle 2, 5\rangle$, $\vec{t} = \langle -6, -8\rangle$ **5, 216.9°**
14. $\vec{m} = \langle 2, -3\rangle$, $\vec{n} = \langle -2, 3\rangle$ **0, 0°**
15. $\vec{u} = \langle -7, 2\rangle$, $\vec{v} = \langle 4, 1\rangle$ **$3\sqrt{2} \approx 4.2, 135°$**

Lesson 9-7

(pages 506–511)

Find the coordinates of the image under the stated transformation. **1–4. See margin.**



1. reflection in the x-axis
2. rotation 90° clockwise about the origin
3. translation $(x, y) \rightarrow (x - 4, y + 3)$
4. dilation by scale factor -4

Use a matrix to find the coordinates of the vertices of the image of each figure after the stated transformation. **5–10. See margin.**

5. $\triangle DEF$ with $D(2, 4)$, $E(-2, -4)$, and $F(4, -6)$; dilation by a scale factor of 2.5
6. $\triangle RST$ with $R(3, 4)$, $S(-6, -2)$, and $T(5, -3)$; reflection in the x-axis
7. quadrilateral $CDEF$ with $C(1, 1)$, $D(-2, 5)$, $E(-2, 0)$, and $F(-1, -2)$; rotation of 90° counterclockwise
8. quadrilateral $WXYZ$ with $W(0, 4)$, $X(-5, 0)$, $Y(0, -3)$, and $Z(5, -2)$; translation $(x, y) \rightarrow (x + 1, y - 4)$
9. quadrilateral $JKLM$ with $J(-6, -2)$, $K(-2, -8)$, $L(4, -4)$, and $M(6, 6)$; dilation by a scale factor of $-\frac{1}{2}$
10. pentagon $ABCDE$ with $A(2, 2)$, $B(0, 4)$, $C(-3, 2)$, $D(-3, -4)$, and $E(2, -4)$; reflection in the line $y = x$

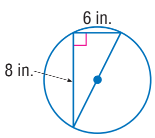
Lesson 10-1

(pages 522–528)

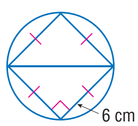
The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.

1. $r = 18$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **36 in., 113.10 in.**
2. $d = 34.2$ ft, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **17.1 ft, 107.44 ft**
3. $C = 12\pi$ m, $r = \underline{\ ?\ }$, $d = \underline{\ ?\ }$ **6 m, 12 m**
4. $C = 84.8$ mi, $r = \underline{\ ?\ }$, $d = \underline{\ ?\ }$ **13.50 mi, 26.99 mi**
5. $d = 8.7$ cm, $r = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **4.35 cm, 27.33 cm**
6. $r = 3b$ in., $d = \underline{\ ?\ }$, $C = \underline{\ ?\ }$ **$6b$ in., $18.85b$ in.**

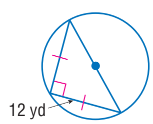
Find the exact circumference of each circle.

7. 

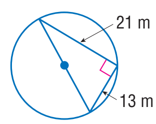
10π in.

8. 

$6\sqrt{2}\pi$ cm

9. 

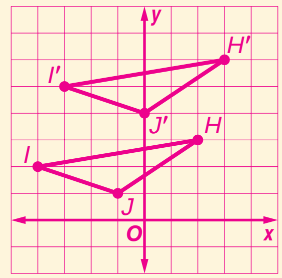
$12\sqrt{2}\pi$ yd

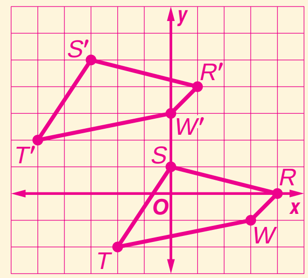
10. 

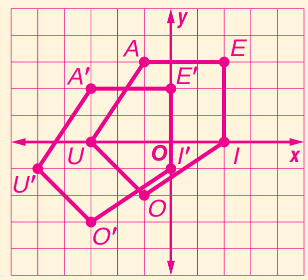
$\sqrt{610}\pi$ m

Extra Practice

Lesson 9-6

1. $\sqrt{13} \approx 3.6, 146.3°$
2. $3\sqrt{2} \approx 4.2, 45°$
3. $\sqrt{58} \approx 7.6, 293.2°$
4. $\sqrt{61} \approx 7.8, 219.8°$
5. $\sqrt{17} \approx 4.1, 346.0°$
6. $2\sqrt{10} \approx 6.3, 161.6°$
7. 

8. 

9. 

Lesson 9-7

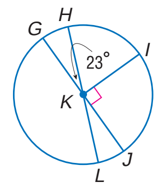
1. $R'(1, -2)$, $T'(-2, -2)$, $P'(-4, 1)$, $A'(2, 1)$
2. $R'(2, -1)$, $T'(2, 2)$, $P'(-1, 4)$, $A'(-1, -2)$
3. $R'(-3, 5)$, $T'(-6, 5)$, $P'(-8, 2)$, $A'(-2, 2)$
4. $R'(-4, -8)$, $T'(8, -8)$, $P'(16, 4)$, $A'(-8, 4)$
5. $D'(5, 10)$, $E'(-5, -10)$, $F'(10, -15)$
6. $R'(3, -4)$, $S'(-6, 2)$, $T'(5, 3)$
7. $C'(-1, 1)$, $D'(-5, -2)$, $E'(0, -2)$, $F'(2, -1)$
8. $W'(1, 0)$, $X'(-4, -4)$, $Y'(1, -7)$, $Z'(6, -6)$
9. $J'(3, 1)$, $K'(1, 4)$, $L'(-2, 2)$, $M'(-3, -3)$
10. $A'(2, 2)$, $B'(4, 0)$, $C'(2, -3)$, $D'(-4, -3)$, $E'(-4, 2)$

Lesson 10-2

(pages 529–535)

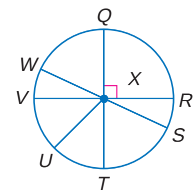
Find each measure.

1. $m\angle GKI$ **90**
2. $m\angle LKJ$ **23**
3. $m\angle LKI$ **113**
4. $m\angle LKG$ **157**
5. $m\angle HKI$ **67**
6. $m\angle HKJ$ **157**



In $\odot X$, $\overline{WS}$, $\overline{VR}$, and $\overline{QT}$ are diameters, $m\angle WXV = 25$ and $m\angle VXU = 45$. Find each measure.

7. $m\widehat{QR}$ **90**
8. $m\widehat{QW}$ **65**
9. $m\widehat{TU}$ **45**
10. $m\widehat{WRV}$ **335**
11. $m\widehat{SV}$ **155**
12. $m\widehat{TRW}$ **245**

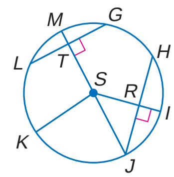


Lesson 10-3

(pages 536–543)

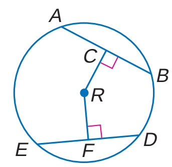
In $\odot S$, $HJ = 22$, $LG = 18$, $m\widehat{IJ} = 35$, and $m\widehat{LM} = 30$. Find each measure.

1. HR **11**
2. RJ **11**
3. LT **9**
4. TG **9**
5. $m\widehat{HJ}$ **70**
6. $m\widehat{LG}$ **60**
7. $m\widehat{MG}$ **30**
8. $m\widehat{HI}$ **35**



In $\odot R$, $CR = RF$, and $ED = 30$. Find each measure.

9. AB **30**
10. EF **15**
11. DF **15**
12. BC **15**

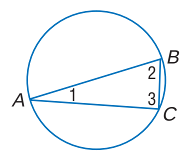


Lesson 10-4

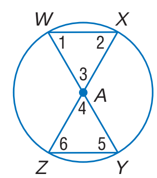
(pages 544–551)

Find the measure of each numbered angle for each figure. **1–6. See margin.**

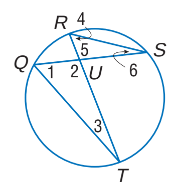
1. $m\widehat{AB} = 176$, and $m\widehat{BC} = 42$



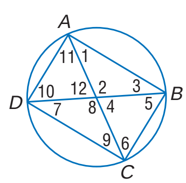
2. $\overline{WX} \cong \overline{ZY}$, and $m\widehat{ZW} = 120$



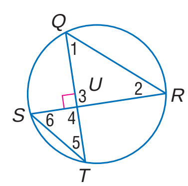
3. $m\widehat{QR} = 40$, and $m\widehat{TS} = 110$



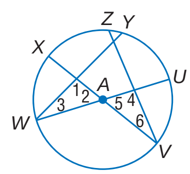
4. $\square ABCD$ is a rectangle, and $m\widehat{BC} = 70$.



5. $m\widehat{TR} = 100$, and $\overline{SR} \perp \overline{QT}$



6. $m\widehat{UY} = m\widehat{XZ} = 56$ and $m\widehat{UV} = m\widehat{XW} = 56$



7. Rhombus $ABCD$ is inscribed in a circle. What can you conclude about $\overline{BD}$? **It is a diameter of the circle.**
8. Triangle RST is inscribed in a circle. If the measure of $\widehat{RS}$ is 170, what is the measure of $\angle T$? **85**

Extra Practice

Extra Practice

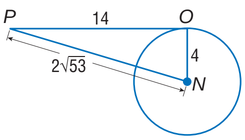
Lesson 10-4

1. $m\angle 1 = 21$, $m\angle 2 = 71$, $m\angle 3 = 88$
2. $m\angle 1 = 60$, $m\angle 2 = 60$, $m\angle 3 = 60$, $m\angle 4 = 60$, $m\angle 5 = 60$, $m\angle 6 = 60$
3. $m\angle 1 = 55$, $m\angle 2 = 105$, $m\angle 3 = 20$, $m\angle 4 = 55$, $m\angle 5 = 105$, $m\angle 6 = 20$
4. $m\angle 1 = 35$, $m\angle 2 = 110$, $m\angle 3 = 35$, $m\angle 4 = 70$, $m\angle 5 = 55$, $m\angle 6 = 55$, $m\angle 7 = 35$, $m\angle 8 = 110$, $m\angle 9 = 35$, $m\angle 10 = 55$, $m\angle 11 = 55$, $m\angle 12 = 70$
5. $m\angle 1 = 50$, $m\angle 2 = 40$, $m\angle 3 = 90$, $m\angle 4 = 90$, $m\angle 5 = 40$, $m\angle 6 = 50$
6. $m\angle 1 = 96$, $m\angle 2 = 56$, $m\angle 3 = 28$, $m\angle 4 = 96$, $m\angle 5 = 56$, $m\angle 6 = 28$

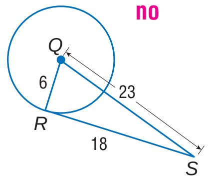
774 Extra Practice

Lesson 10-5
(pages 552–558)

Determine whether each segment is tangent to the given circle.

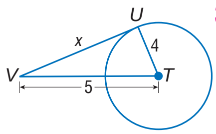
1. 

yes

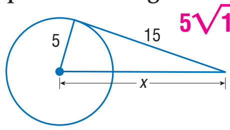
2. 

no

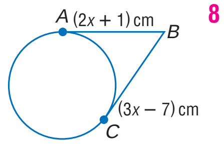
Find x. Assume that segments that appear to be tangent are tangent.

3. 

3

4. 

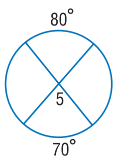
$5\sqrt{10}$

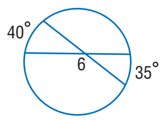
5. 

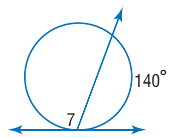
8

Lesson 10-6
(pages 561–568)

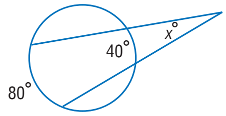
Find each measure.

1. $m\angle 5$ 75


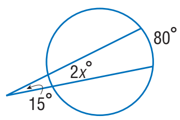
2. $m\angle 6$ 142.5


3. $m\angle 7$ 110


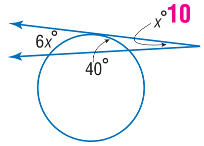
Find x. Assume that any segment that appears to be tangent is tangent.

4. 

20

5. 

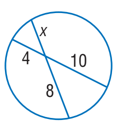
25

6. 

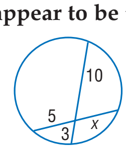
10

Lesson 10-7
(pages 569–574)

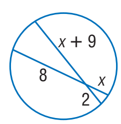
Find x. Assume that segments that appear to be tangent are tangent.

1. 

5

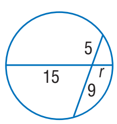
2. 

6

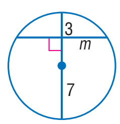
3. 

3

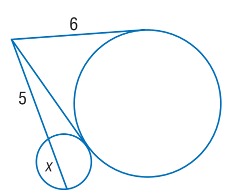
Find each variable to the nearest tenth.

4. 

3.0

5. 

5.7

6. 

2.2

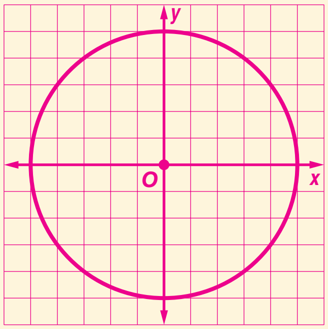
Extra Practice

Extra Practice 775

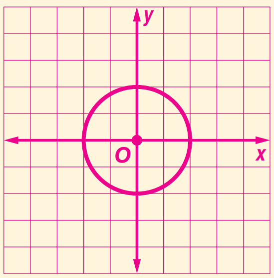
Lesson 10-8

1. $(x-1)^2+(y+2)^2=4$
2. $x^2+y^2=16$
3. $(x+3)^2+(y+4)^2=11$
4. $(x-3)^2+(y+1)^2=9$
5. $(x-6)^2+(y-12)^2=49$
6. $(x-4)^2+y^2=16$
7. $(x-6)^2+(y+6)^2=121$
8. $(x+5)^2+(y-1)^2=1$

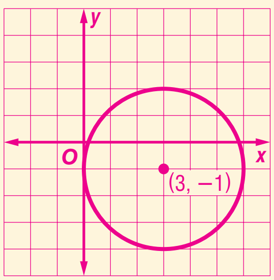
9.



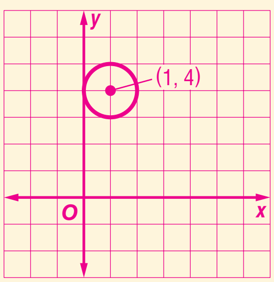
10.



11.



12.



Extra Practice

Lesson 10-8

(pages 575–580)

Write an equation for each circle. 1–8. See margin.

1. center at $(1, -2)$, $r = 2$
2. center at origin, $r = 4$
3. center at $(-3, -4)$, $r = \sqrt{11}$
4. center at $(3, -1)$, $d = 6$
5. center at $(6, 12)$, $r = 7$
6. center at $(4, 0)$, $d = 8$
7. center at $(6, -6)$, $d = 22$
8. center at $(-5, 1)$, $d = 2$

Graph each equation. 9–12. See margin.

9. $x^2 + y^2 = 25$
10. $x^2 + y^2 - 3 = 1$
11. $(x - 3)^2 + (y + 1)^2 = 9$
12. $(x - 1)^2 + (y - 4)^2 = 1$

13. Find the radius of a circle whose equation is $(x + 3)^2 + (y - 1)^2 = r^2$ and contains $(-2, 1)$. 1

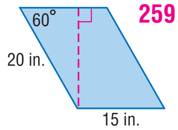
14. Find the radius of a circle whose equation is $(x - 4)^2 + (y - 3)^2 = r^2$ and contains $(8, 3)$. 4

Lesson 11-1

(pages 595–600)

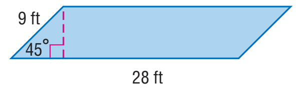
Find the area and perimeter of each parallelogram. Round to the nearest tenth if necessary.

1.



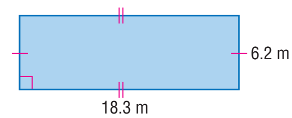
259.8 in^2, 70 in.

2.



178.2 ft^2, 74 ft

3.



113.5 m^2, 49 m

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

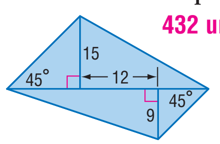
4. $Q(-3, 3)$, $R(-1, 3)$, $S(-1, 1)$, $T(-3, 1)$ square, 4 units^2
5. $A(-7, -6)$, $B(-2, -6)$, $C(-2, -3)$, $D(-7, -3)$ 5. rectangle, 15 units^2
6. $L(5, 3)$, $M(8, 3)$, $N(9, 7)$, $O(6, 7)$ parallelogram, 12 units^2
7. $W(-1, -2)$, $X(-1, 1)$, $Y(2, 1)$, $Z(2, -2)$ square, 9 units^2

Lesson 11-2

(pages 601–609)

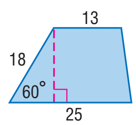
Find the area of each quadrilateral.

1.



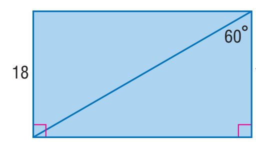
432 units^2

2.



296.2 units^2

3.



561.2 units^2

COORDINATE GEOMETRY **Find the area of trapezoid *ABCD* given the coordinates of the vertices.**

4. $A(1, 1)$, $B(2, 3)$, $C(4, 3)$, $D(7, 1)$ 8 units^2
5. $A(-2, 2)$, $B(2, 2)$, $C(7, -3)$, $D(-4, -3)$ 37.5 units^2
6. $A(1, -1)$, $B(4, -1)$, $C(8, 5)$, $D(1, 5)$ 30 units^2
7. $A(-2, 2)$, $B(4, 2)$, $C(3, -2)$, $D(1, -2)$ 16 units^2

COORDINATE GEOMETRY **Find the area of rhombus *LMNO* given the coordinates of the vertices.**

8. $L(-3, 0)$, $M(1, -2)$, $N(-3, -4)$, $O(-7, -2)$ 16 units^2
9. $L(-3, -2)$, $M(-4, 2)$, $N(-3, 6)$, $O(-2, 2)$ 8 units^2
10. $L(-1, -4)$, $M(3, 4)$, $N(-1, 12)$, $O(-5, 4)$ 64 units^2
11. $L(-2, -2)$, $M(4, 4)$, $N(10, -2)$, $O(4, -8)$ 72 units^2

Lesson 11-3

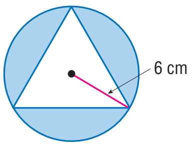
(pages 610–616)

Find the area of each regular polygon. Round to the nearest tenth.

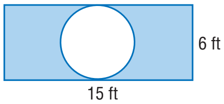
1. a square with perimeter 54 feet **$182.3\ ft^2$**
2. a triangle with side length 9 inches **$35.1\ inches^2$**
3. an octagon with side length 6 feet **$173.8\ ft^2$**
4. a decagon with apothem length of 22 centimeters **$1572.6\ cm^2$**

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

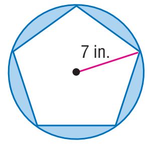
5. **$66.3\ cm^2$**



6. **$61.7\ ft^2$**



7. **$37.4\ in^2$**

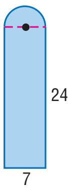


Lesson 11-4

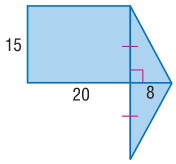
(pages 617–621)

Find the area of each figure. Round to the nearest tenth if necessary.

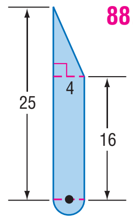
1. **$187.2\ units^2$**



2. **$420\ units^2$**



3. **$88.3\ units^2$**



COORDINATE GEOMETRY **The vertices of an irregular figure are given. Find the area of each figure.**

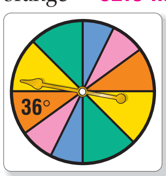
4. $R(0, 5)$, $S(3, 3)$, $T(3, 0)$ **$4.5\ units^2$**
5. $A(-5, -3)$, $B(-3, 0)$, $C(2, -1)$, $D(2, -3)$ **$15.5\ units^2$**
6. $L(-1, 4)$, $M(3, 2)$, $N(3, -1)$, $O(-1, -2)$, $P(-3, 1)$ **$24\ units^2$**

Lesson 11-5

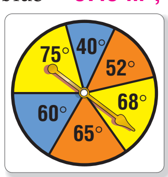
(pages 622–627)

Find the total area of the sectors of the indicated color. Then find the probability of spinning the color indicated if the diameter of each spinner is 20 inches.

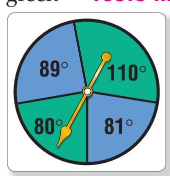
1. orange **$\approx 62.8\ in^2$; 0.20**



2. blue **$\approx 87.3\ in^2$; ≈ 0.28**

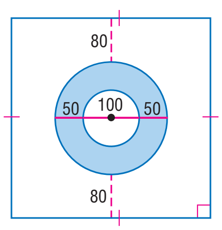


3. green **$\approx 165.8\ in^2$; ≈ 0.53**



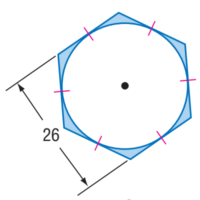
Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region.

4.



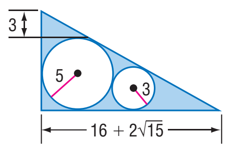
$\approx 23{,}561.9\ units^2$, ≈ 0.18

5.



$\approx 54.5\ units^2$, ≈ 0.09

6.

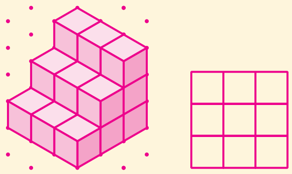


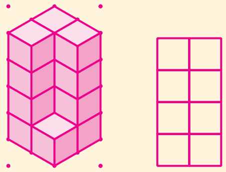
$\approx 47.6\ units^2$; ≈ 0.31

Extra Practice

Extra Practice 777

Lesson 12-1

1. 

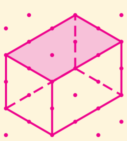
2. 

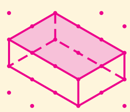
3. pentagonal pyramid; base: *OXNEP*; faces: *OXNEP*, △*PET*, △*ETN*, △*NTX*, △*XTO*, △*OTP*; edges: $\overline{PE}$, $\overline{EN}$, $\overline{NX}$, $\overline{XO}$, $\overline{OP}$, $\overline{TP}$, $\overline{TE}$, $\overline{TN}$, $\overline{TX}$, $\overline{TO}$; vertices: *T*, *P*, *E*, *N*, *X*, *O*

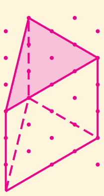
4. cone; base: circle *L*; vertex: *Z*

5. octagonal prism; bases: *ABCDEFGH*, *STUVWXYZ*; faces: *ABCDEFGH*, *STUVWXYZ*, *ABXY*, *BCWX*, *CDVW*, *DEUV*, *FEUT*, *GFTS*, *HGSZ*, *HAYZ*; edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, $\overline{AH}$, $\overline{ST}$, $\overline{TU}$, $\overline{UV}$, $\overline{VW}$, $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, $\overline{ZS}$, $\overline{AY}$, $\overline{BX}$, $\overline{CW}$, $\overline{DV}$, $\overline{EU}$, $\overline{FT}$, $\overline{GS}$, $\overline{HZ}$; vertices: *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *S*, *T*, *U*, *V*, *W*, *X*, *Y*, *Z*

Lesson 12-2

1. 

2. 

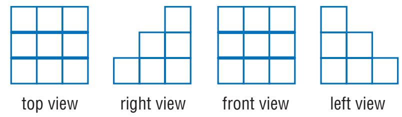
3. 

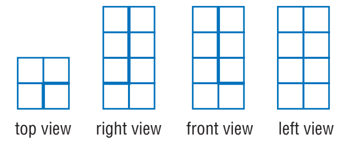
Extra Practice

Lesson 12-1

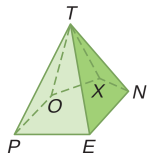
(pages 636–642)

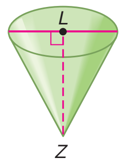
Draw the back view and corner view of a figure given its orthogonal drawing. 1–2. See margin.

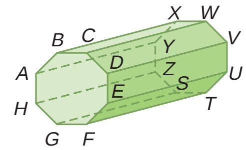
1. 

2. 

Identify each solid. Name the bases, faces, edges, and vertices. 3–5. See margin.

3. 

4. 

5. 

Lesson 12-2

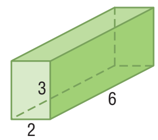
(pages 643–648)

Sketch each solid using isometric dot paper. 1–4. See margin.

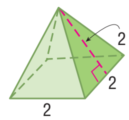
1. rectangular prism 2 units high, 3 units long, and 2 units wide
2. rectangular prism 1 unit high, 2 units long, and 3 units wide
3. triangular prism 3 units high with bases that are right triangles with legs 3 units and 4 units long
4. triangular prism 5 units high with bases that are right triangles with legs 4 units and 6 units long

5–7. See margin for nets.

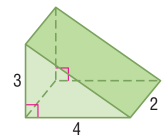
For each solid, draw a net and find the surface area. Round to the nearest tenth if necessary.

5. 

72 units^2

6. 

12 units^2

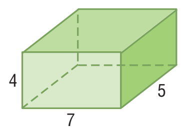
7. 

36 units^2

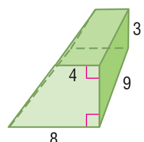
Lesson 12-3

(pages 649–654)

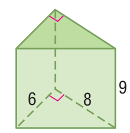
Find the lateral area and the surface area of each prism. Round to the nearest tenth if necessary.

1. 

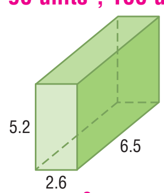
96 units^2; 166 units^2

2. 

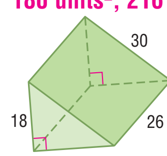
180 units^2; 216 units^2

3. 

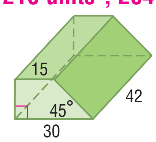
216 units^2; 264 units^2

4. 

94.6 units^2; 128.4 units^2

5. 

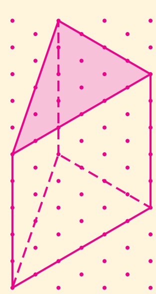
1872 units^2; 2304 units^2

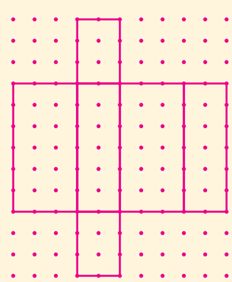
6. 

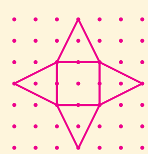
3411.0 units^2; 4086.0 units^2

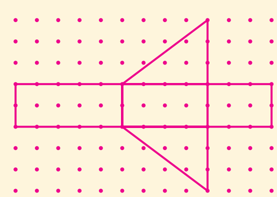
7. The surface area of a right triangular prism is 228 square inches. The base is a right triangle with legs measuring 6 inches and 8 inches. Find the height of the prism. 7.5 in.

8. The surface area of a right triangular prism with height 18 inches is 1380 square inches. The base is a right triangle with a leg measuring 15 inches and a hypotenuse of length 25 inches. Find the length of the other leg of the base. 20 in.

4. 

5. 

6. 

7. 

Lesson 12-4

(pages 655–659)

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. $r = 2$ ft, $h = 3.5$ ft **69.1 ft^2**

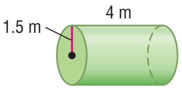
2. $d = 15$ in., $h = 20$ in. **1295.9 in^2**

3. $r = 3.7$ m, $h = 6.2$ m **230.2 m^2**

4. $d = 19$ mm, $h = 32$ mm **2477.1 mm^2**

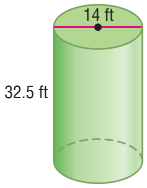
Find the surface area of each cylinder. Round to the nearest tenth.

5.



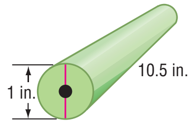
51.8 m^2

6.



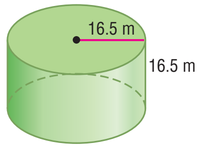
1737.3 ft^2

7.



34.6 in^2

8.



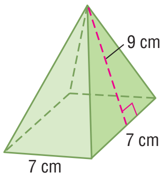
3421.2 m^2

Lesson 12-5

(pages 660–665)

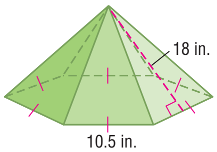
Find the surface area of each regular pyramid. Round to the nearest tenth.

1.



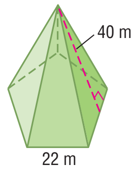
175 cm^2

2.



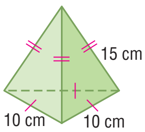
853.4 in^2

3.



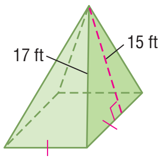
3032.7 m^2

4.



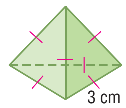
255.4 cm^2

5.



736 ft^2

6.



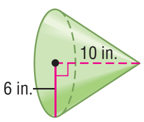
15.6 cm^2

Lesson 12-6

(pages 666–670)

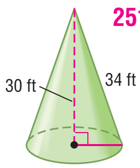
Find the surface area of each cone. Round to the nearest tenth.

1.



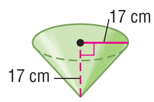
332.9 in^2

2.



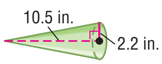
2513.3 ft^2

3.



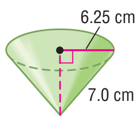
2191.9 cm^2

4.



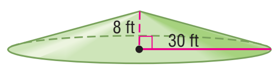
89.4 in^2

5.



260.2 cm^2

6.



5753.7 ft^2

7. Find the surface area of a cone if the height is 28 inches and the slant height is 40 inches. **6153.2 in^2**

8. Find the surface area of a cone if the height is 7.5 centimeters and the radius is 2.5 centimeters. **81.7 cm^2**

Extra Practice

Extra Practice 779

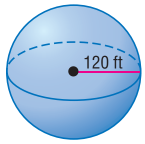
Extra Practice

Extra Practice

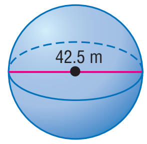
Lesson 12-7

(pages 671–676)

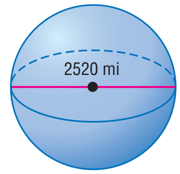
Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

1.


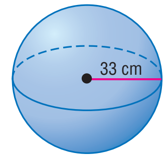
180,955.7 ft^2

2.


5674.5 m^2

3.


19,950,370.0 mi^2

4.


13,684.8 cm^2

5. a hemisphere with the circumference of a great circle 14.1 cm **47.5 cm^2**

6. a sphere with the circumference of a great circle 50.3 in. **805.4 in^2**

7. a sphere with the area of a great circle 98.5 m^2 **394 m^2**

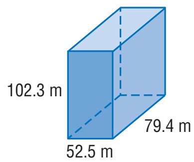
8. a hemisphere with the circumference of a great circle 3.1 in. **2.3 in^2**

9. a hemisphere with the area of a great circle 31,415.9 ft^2 **94,247.7 ft^2**

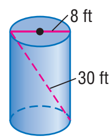
Lesson 13-1

(pages 688–694)

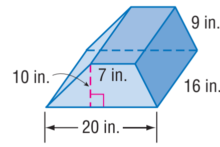
Find the volume of each prism or cylinder. Round to the nearest tenth if necessary.

1.


426,437.6 m^3

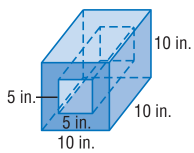
2.


5102.4 ft^3

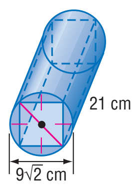
3.


2160 in^3

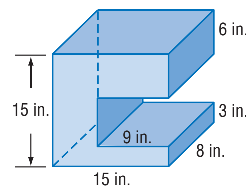
Find the volume of each solid to the nearest tenth.

4.


750 in^3

5.


970.9 cm^3

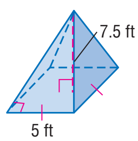
6.


1368 in^3

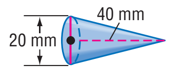
Lesson 13-2

(pages 696–701)

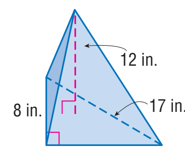
Find the volume of each cone or pyramid. Round to the nearest tenth if necessary.

1.


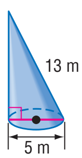
62.5 ft^3

2.


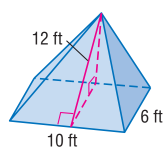
4188.8 mm^3

3.


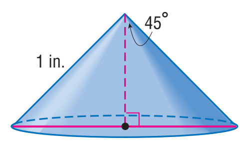
240 in^3

4.


78.5 m^3

5.


207.8 m^3

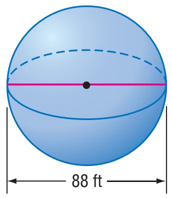
6.


0.4 in^3

Lesson 13-3
(pages 702–706)

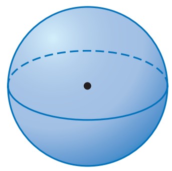
Find the volume of each sphere or hemisphere. Round to the nearest tenth.

1.



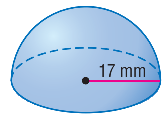
356,817.9 ft^3

2. $C = 4$ m



1.1 m^3

3.



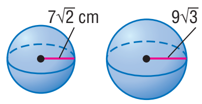
10,289.8 mm^3

4. The diameter of the sphere is 3 cm. **14.1 cm^3**
5. The radius of the hemisphere is $7\sqrt{2}$ m. **2031.9 m^3**
6. The diameter of the hemisphere is 90 ft. **190,851.8 ft^3**
7. The radius of the sphere is 0.5 in. **0.5 in^3**

Lesson 13-4
(pages 707–713)

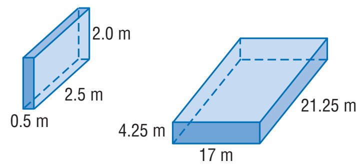
Determine whether each pair of solids are *similar, congruent,* or *neither.*

1.



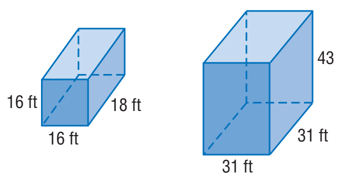
similar

2.



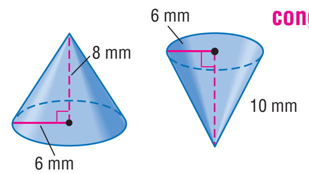
similar

3.



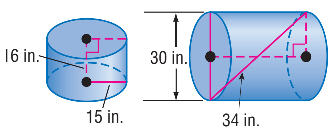
neither

4.



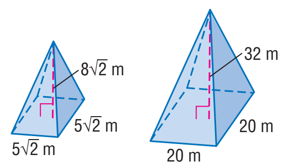
congruent

5.



congruent

6.



similar

Lesson 13-5
(pages 714–719)

Graph the rectangular solid that contains the given point and the origin. Label the coordinates of each vertex. **1–6. See margin.**

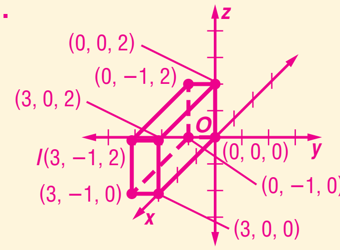
1. $A(3, -3, -3)$
2. $E(-1, 2, -3)$
3. $I(3, -1, 2)$
4. $Z(2, -1, 3)$
5. $Q(-4, -2, -4)$
6. $Y(-3, 1, -4)$

Determine the distance between each pair of points. Then determine the coordinates of the midpoint, *M*, of the segment joining the pair of points. **7–12. See margin.**

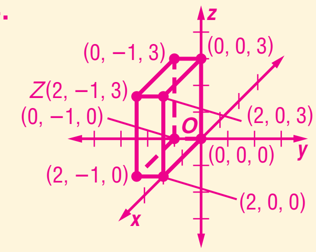
7. $A(-3, 3, 1)$ and $B(3, -3, -1)$
8. $O(2, -1, -3)$ and $P(-2, 4, -4)$
9. $D(0, -5, -3)$ and $E(0, 5, 3)$
10. $J(-1, 3, 5)$ and $K(3, -5, -3)$
11. $A(2, 1, 6)$ and $Z(-4, -5, -3)$
12. $S(-8, 3, -5)$ and $T(6, -1, 2)$

Extra Practice

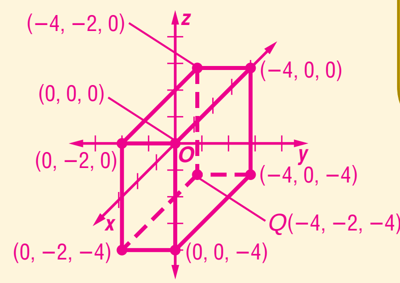
3.



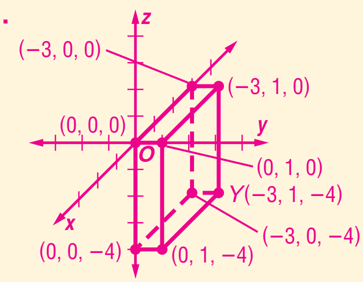
4.



5.



6.



7. $AB = 2\sqrt{19}$; $(0, 0, 0)$

8. $OP = \sqrt{42}$; $(0, 1.5, -3.5)$

9. $DE = 2\sqrt{34}$; $(0, 0, 0)$

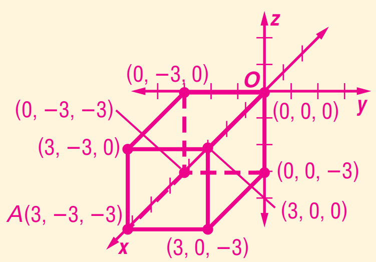
10. $JK = 12$; $(1, -1, 1)$

11. $AZ = 3\sqrt{17}$; $(-1, -2, 1.5)$

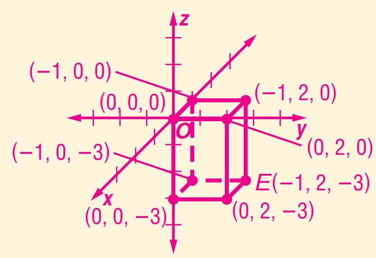
12. $ST = 3\sqrt{29}$; $(-1, 1, -1.5)$

Lesson 13-5

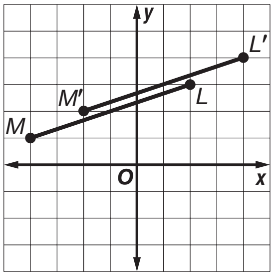
1.



2.



Extra Practice
Page 771, Lesson 9-2

4.


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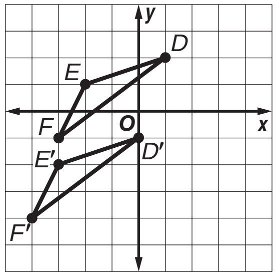

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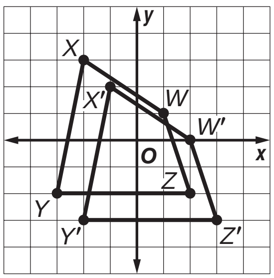

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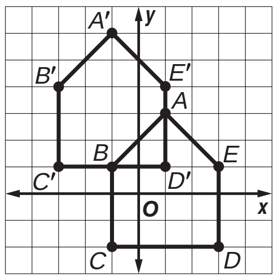

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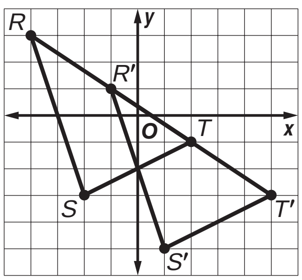

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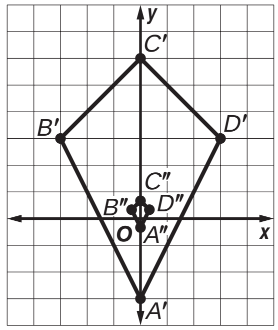

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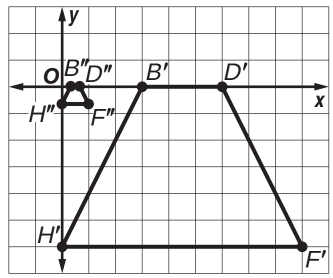

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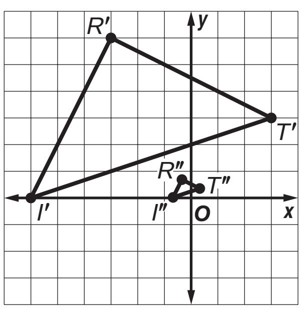
7.


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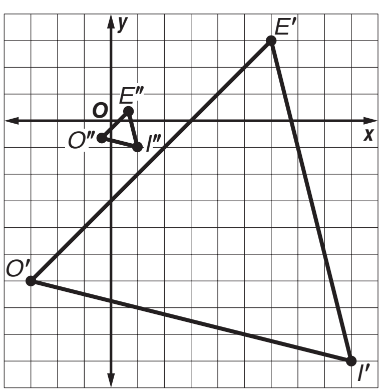
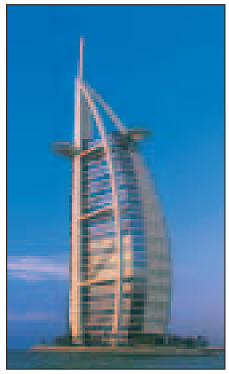


Notes

Mixed Problem Solving and Proof

Chapter 1 Points, Lines, Planes, and Angles
(pages 4–59)

ARCHITECTURE **For Exercises 1–4, use the following information.**
The Burj Al Arab in Dubai, United Arab Emirates, is one of the world's tallest hotels. *(Lesson 1-1)*



1. Trace the outline of the building on your paper.
2. Label three different planes suggested by the outline.
3. Highlight three lines in your drawing that, when extended, do not intersect.
4. Label three points on your sketch. Determine if they are coplanar and collinear.

1–4. See margin.

SKYSCRAPERS **For Exercises 5–7, use the following information.** *(Lesson 1-2)*

Tallest Buildings in San Antonio, TX	
Name	**Height (ft)**
Tower of the Americas	622
Marriot Rivercenter	546
Weston Centre	444
Tower Life	404

Source: www.skyscrapers.com

5. What is the precision for the measures of the heights of the buildings? **0.5 ft**
6. What does the precision mean for the measure of the Tower of the Americas?
7. What is the difference in height between Weston Centre and Tower Life? **39–41 ft**

6. The height is between 621.5 and 622.5 ft.

PERIMETER **For Exercises 8–11, use the following information.** *(Lesson 1-3)* **10. 18.5 units**
The coordinates of the vertices of $\triangle ABC$ are $A(0, 6)$, $B(-6, -2)$, and $C(8, -4)$. Round to the nearest tenth.

8. Find the perimeter of $\triangle ABC$. **36.9 units**
9. Find the coordinates of the midpoints of each side of $\triangle ABC$. **(−3, 2), (1, −3), (4, 1)**
10. Suppose the midpoints are connected to form a triangle. Find the perimeter of this triangle.
11. Compare the perimeters of the two triangles. **See margin.**

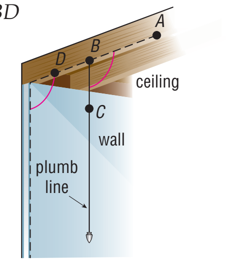
12. **TRANSPORTATION** Mile markers are used to name the exits on Interstate 70 in Kansas. The exit for Hays is 3 miles farther than halfway between Exits 128 and 184. What is the exit number for the Hays exit? *(Lesson 1-3)* **159**

13. **ENTERTAINMENT** The Ferris wheel at the Navy Pier in Chicago has forty gondolas. What is the measure of an angle with a vertex that is the center of the wheel and with sides that are two consecutive spokes on the wheel? Assume that the gondolas are equally spaced. *(Lesson 1-4)* **9**

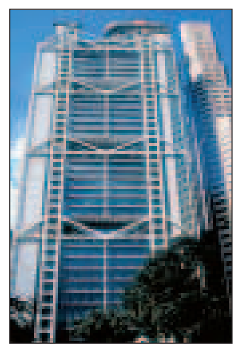
CONSTRUCTION **For Exercises 14–15, use the following information.**
A framer is installing a cathedral ceiling in a newly built home. A protractor and a plumb bob are used to check the angle at the joint between the ceiling and wall. The wall is vertical, so the angle between the vertical plumb line and the ceiling is the same as the angle between the wall and the ceiling. *(Lesson 1-5)*

14. How are $\angle ABC$ and $\angle CBD$ related?
15. If $m\angle ABC = 110$, what is $m\angle CBD$? **70**

14. They form a linear pair and are supplementary.



STRUCTURES **For Exercises 16–17, use the following information.** *(Lesson 1-6)*
The picture shows the Hongkong and Shanghai Bank located in Hong Kong, China.



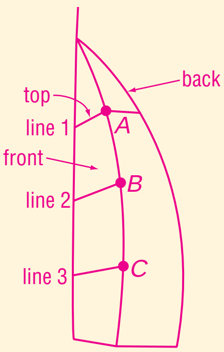
16. Name five different polygons suggested by the picture.
17. Classify each polygon you identified as *convex* or *concave* and *regular* or *irregular*.

16–17. See margin.

Mixed Problem Solving and Proof

Chapter 1

1–3. Sample answer:



4. See figure for Exercises 1–3; points *A*, *B*, and *C* might be coplanar, but they are not collinear.

11. $\triangle ABC$ has a perimeter twice that of the smaller triangle.

16. Sample answer: isosceles triangle, rectangle, pentagon, hexagon, square

17. triangle: convex irregular; rectangle: convex irregular; pentagon: convex irregular; hexagon: concave irregular; square: convex regular

Chapter 2

2. Sample answer: In 2010, California will have about 245 people per square mile. In 2010, Michigan will have about 185 people per square mile.

6. The Hatter is correct; Alice exchanged the hypothesis and conclusion.

8. then she should not accept it and should notify airline personnel immediately

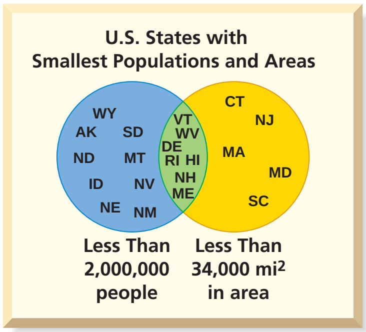
POPULATION For Exercises 1–2, use the table showing the population density for various states in 1960, 1980, and 2000. The figures represent the number of people per square mile. *(Lesson 2-1)*

State	1960	1980	2000
CA	100.4	151.4	217.2
CT	520.6	637.8	702.9
DE	225.2	307.6	401.0
HI	98.5	150.1	188.6
MI	137.7	162.6	175.0

Source: U.S. Census Bureau

1. Find a counterexample for the following statement. The population density for each state in the table increased by at least 30 during each 20-year period. **MI for both periods**
2. Write two conjectures for the year 2010. **See margin.**

STATES For Exercises 3–5, refer to the Venn diagram. *(Lesson 2-2)*



Source: *World Almanac*

3. How many states have less than 2,000,000 people? **16 states**
4. How many states have less than 34,000 square miles in area? **12 states**
5. How many states have less than 2,000,000 people and are less than 34,000 square miles in area? **7 states**

LITERATURE For Exercises 6–7, use the following quote from Lewis Carroll's *Alice's Adventures in Wonderland*. *(Lesson 2-3)*

"Then you should say what you mean," the March Hare went on.
"I do," Alice hastily replied; "at least—at least I mean what I say—that's the same thing, you know."
"Not the same thing a bit!" said the Hatter.

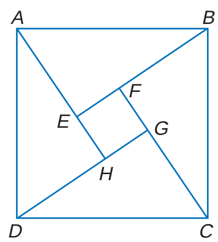
6. Who is correct? Explain. **See margin.**
7. How are the phrases *say what you mean* and *mean what you say* related? **They are converses of each other.**

8. **AIRLINE SAFETY** Airports in the United States post a sign stating *If any unknown person attempts to give you any items including luggage to transport on your flight, do not accept it and notify airline personnel immediately.* Write a valid conclusion to the hypothesis, *If a person Candace does not know attempts to give her an item to take on her flight, . . .* *(Lesson 2-4)* **See margin.**

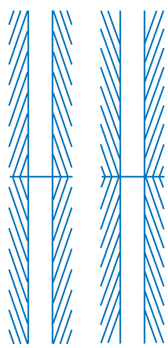
9. **PROOF** Write a paragraph proof to show that $\overline{AB} \cong \overline{CD}$ if B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. *(Lesson 2–5)* **See margin.**

10. **CONSTRUCTION** Engineers consider the expansion and contraction of materials used in construction. The coefficient of linear expansion, k, is dependent on the change in length and the change in temperature and is found by the formula, $k = \frac{\Delta\ell}{\ell(T - t)}$. Solve this formula for T and justify each step. *(Lesson 2-6)* **See margin.**

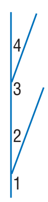
11. **PROOF** Write a two-column proof. *(Lesson 2-7)*
Given: $ABCD$ has 4 congruent sides. $DH = BF = AE$; $EH = FE$ **See margin.**
Prove: $AB + BE + AE = AD + AH + DH$



ILLUSIONS This drawing was created by German psychologist Wilhelm Wundt. *(Lesson 2-8)*

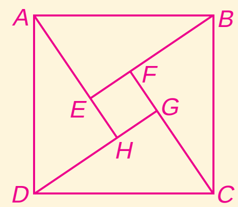


12. Describe the relationship between each pair of vertical lines. **12–13. See margin.**
13. A close-up of the angular lines is shown below. If $\angle 4 \cong \angle 2$, write a two-column proof to show that $\angle 3 \cong \angle 1$.



10. Given: $k = \frac{\Delta\ell}{\ell(T - t)}$
Prove: $T = \frac{\Delta\ell}{k\ell} + t$
Proof:
Statements (Reasons)
1. $k = \frac{\Delta\ell}{\ell(T - t)}$ (Given)
2. $k(T - t) = \frac{\Delta\ell}{\ell}$ (Mult. Prop.)
3. $T - t = \frac{\Delta\ell}{k\ell}$ (Division Prop.)
4. $T = \frac{\Delta\ell}{k\ell} + t$ (Addition Prop.)

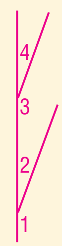
11. Given: $ABCD$ has 4 $\cong$ sides. $DH = BF = AE$; $EH = FE$
Prove: $AB + BE + AE = AD + AH + DH$



Proof:
Statements (Reasons)
1. $DH = BF = AE$; $EH = FE$ (Given)
2. $BE = BF + FE$; $AE + EH = AH$ (Segment Add. Prop.)
3. $BF + FE = AH$ (Substitution)
4. $BF + FE = AE + EH$ (Addition Prop.)
5. $BE = AH$ (Transitive Prop.)
6. $ABCD$ has 4 $\cong$ sides. (Given)
7. $AB = AD$ (Def. of $\cong$ segments)
8. $AB + BE = AD + AH$ (Addition Prop.)
9. $AB + BE + AE = AD + AH + DH$ (Addition Prop.)

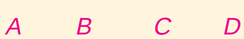
12. The vertical lines are parallel. The first pair of vertical lines appear to curve inward, the second pair appear to curve outward.

13. Given: $\angle 4 \cong \angle 2$
Prove: $\angle 3 \cong \angle 1$



Proof:
Statements (Reasons)
1. $\angle 4 \cong \angle 2$ (Given)
2. $\angle 4$ and $\angle 3$ form a linear pair; $\angle 2$ and $\angle 1$ form a linear pair. (Def. of linear pair)
3. $\angle 4$ and $\angle 3$ are supplementary; $\angle 2$ and $\angle 1$ are supplementary. (Supplement Theorem)
4. $\angle 3 \cong \angle 1$ (∠s suppl. to $\cong$ ∠s are $\cong$.)

9. Given: B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$.
Prove: $\overline{AB} \cong \overline{CD}$



Proof: By the definition of midpoint, $AB = BC$ and $BC = CD$. By the Transitive Property, $AB = CD$. By definition of congruence, $\overline{AB} \cong \overline{CD}$.

Chapter 3

1. Alternate interior angles are congruent, so $\angle 1 \cong \angle 2$.

11. Given: $\overline{MQ} \parallel \overline{NP}$
$\angle 4 \cong \angle 3$
Prove: $\angle 1 \cong \angle 5$
Proof:
Statements (Reasons)
1. $\overline{MQ} \parallel \overline{NP}$; $\angle 4 \cong \angle 3$ (Given)
2. $\angle 3 \cong \angle 5$ (Alt. Int. $\angle$s Theorem)
3. $\angle 4 \cong \angle 5$ (Transitive Prop.)
4. $\angle 1 \cong \angle 4$ (Corres. $\angle$s Post.)
5. $\angle 1 \cong \angle 5$ (Transitive Prop.)

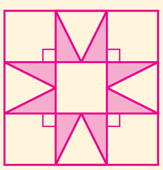
15. If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel.

16. Given: $\angle 1 \cong \angle 3$, $\overline{AB} \parallel \overline{DC}$
Prove: $\overline{BC} \parallel \overline{AD}$
Proof:
Statements (Reasons)
1. $\overline{AB} \parallel \overline{DC}$ (Given)
2. $\angle 1 \cong \angle 4$ (Alt. Int. $\angle$s Theorem)
3. $\angle 1 \cong \angle 3$ (Given)
4. $\angle 4 \cong \angle 3$ (Transitive Prop.)
5. $\overline{BC} \parallel \overline{AD}$ (If corr. $\angle$s are $\cong$, then lines are $\parallel$.)

17. The shortest distance is a perpendicular segment. You cannot walk this route because there are no streets that exactly follow this route and you cannot walk through or over buildings.

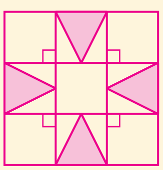
Chapter 4

1. The triangles appear to be scalene. One leg looks longer than the other leg.



(Shading should be red.)

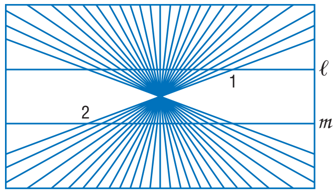
2. The triangles appear to be isosceles. Two of the sides appear to be the same length.



(Shading should be blue.)

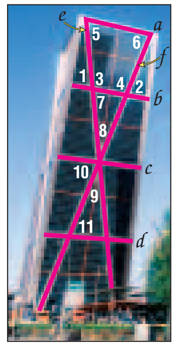
Mixed Problem Solving and Proof

1. **OPTICAL ILLUSIONS** Lines ℓ and m are parallel, but appear to be bowed due to the transversals drawn through ℓ and m. Make a conjecture about the relationship between $\angle 1$ and $\angle 2$. *(Lesson 3-1)*



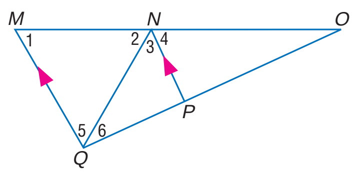
See margin.

ARCHITECTURE **For Exercises 2–10, use the following information.**
The picture shows one of two towers of the Puerta de Europa in Madrid, Spain. Lines a, b, c, and d are parallel. The lines are cut by transversals e and f. If $m\angle 1 = m\angle 2 = 75$, find the measure of each angle. *(Lesson 3-2)*



2. $\angle 3$ 105
3. $\angle 4$ 105
4. $\angle 5$ 75
5. $\angle 6$ 75
6. $\angle 7$ 75
7. $\angle 8$ 30
8. $\angle 9$ 30
9. $\angle 10$ 75
10. $\angle 11$ 75

11. **PROOF** Write a two-column proof. *(Lesson 3-2)*
Given: $\overline{MQ} \parallel \overline{NP}$ See margin.
$\angle 4 \cong \angle 3$
Prove: $\angle 1 \cong \angle 5$



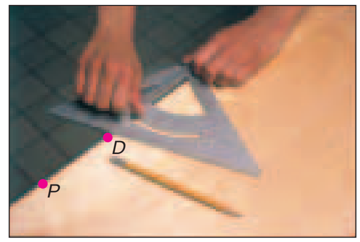
12. **EDUCATION** Between 1995 and 2000, the average cost for tuition and fees for American universities increased by an average rate of \$84.20 per year. In 2000, the average cost was \$2600. If costs increase at the same rate, what will the total average cost be in 2010? *(Lesson 3-3)* \$3442

RECREATION **For Exercises 13 and 14, use the following information.** *(Lesson 3-4)*
The Three Forks community swimming pool holds 74,800 gallons of water. At the end of the summer, the pool is drained and winterized.

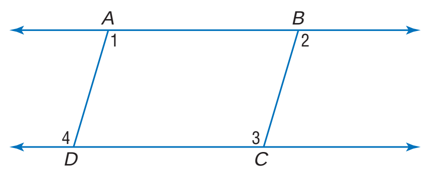
13. If the pool drains at the rate of 1200 gallons per hour, write an equation to describe the number of gallons left after x hours. $y = 74{,}800 - 1200x$

14. How many hours will it take to drain the pool? $62\frac{1}{3}$ h

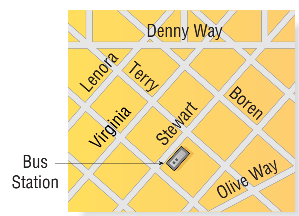
15. **CONSTRUCTION** An *engineer and carpenter square* is used to draw parallel line segments. Martin makes two cuts at an angle of 120° with the edge of the wood through points D and P. Explain why these cuts will be parallel. *(Lesson 3-5)* See margin.



16. **PROOF** Write a two-column proof. *(Lesson 3-5)*
Given: $\angle 1 \cong \angle 3$
$\overline{AB} \parallel \overline{DC}$
Prove: $\overline{BC} \parallel \overline{AD}$ See margin.

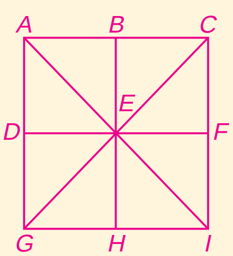


17. **CITIES** The map shows a portion of Seattle, Washington. Describe a segment that represents the shortest distance from the Bus Station to Denny Way. Can you walk the route indicated by your segment? Explain. *(Lesson 3-6)* See margin.



4. $\triangle BED \cong \triangle CFG$;
$\triangle BJH \cong \triangle CKM$;
$\triangle BPN \cong \triangle CQS$;
$\triangle DIH \cong \triangle GLM$;
$\triangle DON \cong \triangle GRS$

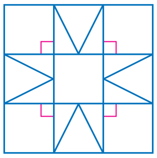
7. Given: $\overline{AC} \cong \overline{CI} \cong \overline{IG} \cong \overline{AG}$; $\overline{AI} \cong \overline{GC}$
Prove: $\triangle ACI \cong \triangle CAG$



Proof:

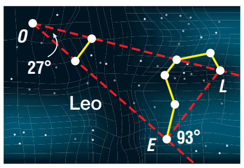
$\overline{CI} \cong \overline{AG}$ (Given)
$\overline{AI} \cong \overline{GC}$ (Given)
$\overline{CA} \cong \overline{CA}$ (Reflexive Prop.)
→ $\triangle ACI \cong \triangle CAG$ (SSS)

QUILTING **For Exercises 1 and 2, trace the quilt pattern square below.** *(Lesson 4-1)*

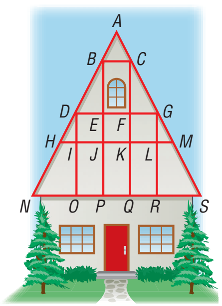


1. Shade all right triangles red. Do these triangles appear to be scalene or isosceles? Explain.
2. Shade all acute triangles blue. Do these triangles appear to be scalene, isoscles, or equilateral? Explain. **1–2. See margin.**

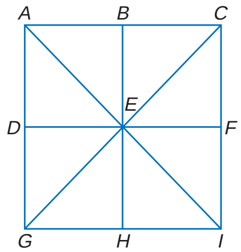
3. **ASTRONOMY** Leo is a constellation that represents a lion. Three of the brighter stars in the constellation form $\triangle LEO$. If the angles have measures as shown in the figure, find $m\angle OLE$. *(Lesson 4-2)* **66**



4. **ARCHITECTURE** The diagram shows an A-frame house with various points labeled. Assume that segments and angles that appear to be congruent in the diagram are congruent. Indicate which triangles are congruent. *(Lesson 4-3)* **See margin.**

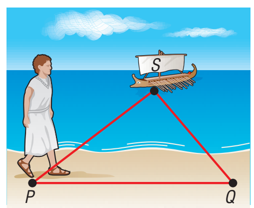


RECREATION **For Exercises 5–7, use the following information.**
Tapatan is a game played in the Philippines on a square board, like the one shown at the top right. Players take turns placing each of their three pieces on a different point of intersection. After all the pieces have been played, the players take turns moving a piece along a line to another intersection. A piece cannot jump over another piece. A player who gets all their pieces in a straight line wins. Point E bisects all four line segments that pass through it. All sides are congruent, and the diagonals are congruent. Suppose a letter is assigned to each intersection. *(Lesson 4-4)*

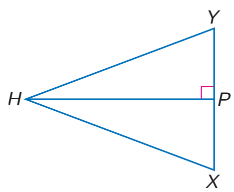


5. Is $\triangle GHE \cong \triangle CBE$? Explain. **yes; SAS**
6. Is $\triangle AEG \cong \triangle IEG$? Explain. **yes; SSS or SAS**
7. Write a flow proof to show that $\triangle ACI \cong \triangle CAG$. **See margin.**

8. **HISTORY** It is said that Thales determined the distance from the shore to the Greek ships by sighting the angle to the ship from a point P on the shore, walking to point Q, and then sighting the angle to the ship from Q. He then reproduced the angles on the other side of $\overline{PQ}$ and continued these lines until they intersected. Is this method valid? Explain. *(Lesson 4-5)* **See margin.**



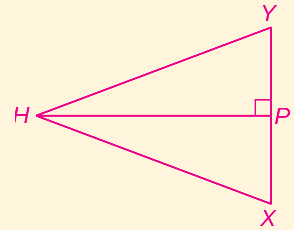
9. **PROOF** Write a two-column proof. *(Lesson 4-6)* **See margin.**
Given: $\overline{PH}$ bisects $\angle YHX$.
$\overline{PH} \perp \overline{YX}$
Prove: $\triangle YHX$ is an isosceles triangle.



10. **PROOF** $\triangle ABC$ is a right isosceles triangle with hypotenuse $\overline{AB}$. M is the midpoint of $\overline{AB}$. Write a coordinate proof to show that $\overline{CM}$ is perpendicular to $\overline{AB}$. *(Lesson 4-7)* **See margin.**

Mixed Problem Solving and Proof

9. Given: $\overline{PH}$ **bisects** $\angle YHX$, $\overline{PH} \perp \overline{YX}$
Prove: $\triangle YHX$ **is an isosceles triangle.**

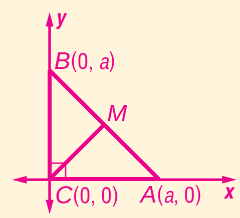


Proof:

Statements (Reasons)

1. $\overline{PH}$ bisects $\angle YHX$. (Given)
2. $\angle YHP \cong \angle XHP$ (Def. of $\angle$ bisector)
3. $\overline{PH} \perp \overline{YX}$ (Given)
4. $\angle YPH$ and $\angle XPH$ are rt. $\angle$s (Def. of $\perp$ lines)
5. $\angle YPH \cong \angle XPH$ (All rt. $\angle$s are $\cong$.)
6. $\angle Y \cong \angle X$ (Third $\angle$ Th.)
7. $\overline{HX} \cong \overline{HY}$ (Conv. of Isos. $\triangle$ Th.)
8. $\triangle YHX$ is an isosceles triangle. (Def. of isos. $\triangle$)

10. Given: $\triangle ABC$ **is a right isosceles triangle.** M **is the midpoint of** $\overline{AB}$.
Prove: $\overline{CM} \perp \overline{AB}$



Proof: Place the triangle so that the vertices are $A(a, 0)$, $B(0, a)$, and $C(0, 0)$.

By the Midpoint Formula, the coordinates of M are
$\left(\frac{0+a}{2}, \frac{a+0}{2}\right)$ **or** $\left(\frac{a}{2}, \frac{a}{2}\right)$.

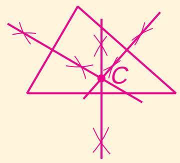
Find the slopes of $\overline{AB}$ and $\overline{CM}$.

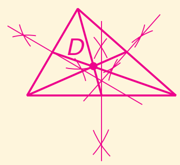
Slope of $\overline{AB} = \frac{0-a}{a-0} = \frac{-a}{a} = -1$

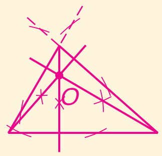
Slope of $\overline{CM} = \frac{\frac{a}{2}-0}{\frac{a}{2}-0} = \frac{\frac{a}{2}}{\frac{a}{2}} = 1$

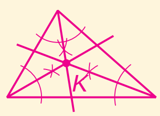
The product of the slopes is -1, so $\overline{CM} \perp \overline{AB}$.

8. Yes, the method is valid. Thales sighted $\angle SPQ$ and $\angle SQP$. He then constructed $\angle QPA$ congruent to $\angle SPQ$ and $\angle PQA$ congruent to $\angle SQP$. $\triangle SPQ$ and $\triangle APQ$ share the side $\overline{PQ}$. Since $\angle QPA \cong \angle SPQ$, $\angle PQA \cong \angle SQP$, and $\overline{PQ} \cong \overline{PQ}$, $\triangle SPQ \cong \triangle APQ$ by the ASA Postulate.

1. 

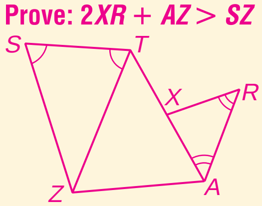
2. 

3. 

4. 

9. Given: $x + y > 634$
Prove: $x > 317$ or $y > 317$
Proof:
Step 1: Assume $x < 317$ and $y < 317$.
Step 2: $x + y < 634$
Step 3: This contradicts the fact that $2x + y > 634$. Therefore, at least one of the legs was longer than 317 miles.

11. Given: $\angle ZST \cong \angle ZTS$
$\angle XRA \cong \angle XAR$
$TA = 2AX$
Prove: $2XR + AZ > SZ$



Proof:
Statements (Reasons)
1. $\angle ZST \cong \angle ZTS$ (Given)
2. $\overline{SZ} \cong \overline{TZ}$ (Isos. $\triangle$ Th.)
3. $SZ = TZ$ (Def. of $\cong$)
4. $TA + AZ > TZ$ ($\triangle$ Inequal. Th.)
5. $TA = 2AX$ (Given)
6. $2AX + AZ > TZ$ (Substitution)
7. $\angle XRA \cong \angle XAR$ (Given)
8. $\overline{XR} \cong \overline{XA}$ (Isos. $\triangle$ Th.)
9. $XR = XA$ (Def. of $\cong$)
10. $2XR + AZ > TZ$ (Substitution)
11. $2XR + AZ > SZ$ (Substitution)

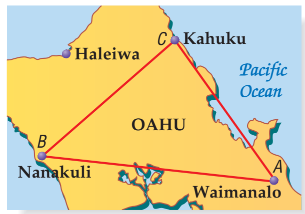
Mixed Problem Solving and Proof

CONSTRUCTION **For Exercises 1–4, draw a large, acute scalene triangle. Use a compass and straightedge to make the required constructions.** *(Lesson 5-1)*

1. Find the circumcenter. Label it C.
2. Find the centroid of the triangle. Label it D.
3. Find the orthocenter. Label it O.
4. Find the incenter of the triangle. Label it I.

1–4. See margin.

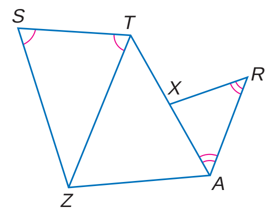
RECREATION **For Exercises 5–7, use the following information.** *(Lesson 5-2)*
Kailey plans to fly over the route marked on the map of Oahu in Hawaii.



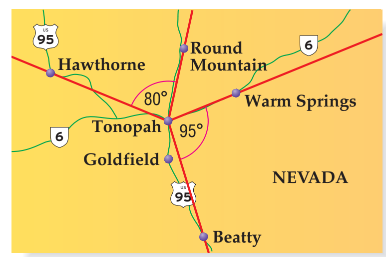
5. The measure of angle A is two degrees more than the measure of angle B. The measure of angle C is fourteen degrees less than twice the measure of angle B. What are the measures of the three angles? $m\angle A = 50$, $m\angle B = 48$, $m\angle C = 82$
6. Write the lengths of the legs of Kailey's trip in order from least to greatest. AC, BC, BA
7. The length of the entire trip is about 68 miles. The middle leg is 11 miles greater than one-half the length of the shortest leg. The longest leg is 12 miles greater than three-fourths of the shortest leg. What are the lengths of the legs of the trip? 20 mi, 21 mi, 27 mi
8. **LAW** A man is accused of comitting a crime. If the man is telling the truth when he says, "I work every Tuesday from 3:00 P.M. to 11:00 P.M.," what fact about the crime could be used to prove by indirect reasoning that the man was innocent? *(Lesson 5-3)* that the crime was committed on Tuesday between 3:00 P.M. and 11:00 P.M.

TRAVEL **For Exercises 9 and 10, use the following information.**
The total air distance to fly from Bozeman, Montana, to Salt Lake City, Utah, to Boise, Idaho is just over 634 miles.

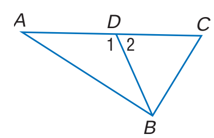
9. Write an indirect proof to show that at least one of the legs of the trip is longer than 317 miles. *(Lesson 5-3)* See margin.
10. The air distance from Bozeman to Salt Lake City is 341 miles and the distance from Salt Lake to Boise is 294 miles. Find the range for the distance from Bozeman to Boise. *(Lesson 5-4)* $47 < n < 635$
11. **PROOF** Write a two-column proof.
Given: $\angle ZST \cong \angle ZTS$
$\angle XRA \cong \angle XAR$
$TA = 2AX$
Prove: $2XR + AZ > SZ$
(Lesson 5-4) See margin.



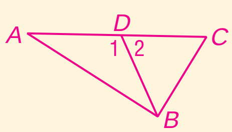
12. **GEOGRAPHY** The map shows a portion of Nevada. The distance from Tonopah to Round Mountain is the same as the distance from Tonopah to Warm Springs. The distance from Tonopah to Hawthorne is the same as the distance from Tonopah to Beatty. Use the angle measures to determine which distance is greater, Round Mountain to Hawthorne or Warm Springs to Beatty. *(Lesson 5-5)* Warm Springs to Beatty



13. **PROOF** Write a two-column proof. *(Lesson 5-5)*
Given: $\overline{DB}$ is a median of $\triangle ABC$.
$m\angle 1 > m\angle 2$
Prove: $m\angle C > m\angle A$
See margin.



13. Given: $\overline{DB}$ is a median of $\triangle ABC$.
$m\angle 1 > m\angle 2$
Prove: $m\angle C > m\angle A$



Proof:
Statements (Reasons)
1. $\overline{DB}$ is a median of $\triangle ABC$; $m\angle 1 > m\angle 2$ (Given)
2. D is the midpoint of $\overline{AC}$. (Def. of median)
3. $\overline{AD} \cong \overline{DC}$ (Midpoint Theorem)
4. $\overline{DB} \cong \overline{DB}$ (Reflexive Property)
5. $AB > BC$ (SAS Inequality)
6. $m\angle C > m\angle A$ (If one side of a $\triangle$ is longer than another, the $\angle$ opp. the longer side $>$ the $\angle$ opp. the shorter side.)

1. **TOYS** In 2000, \$34,554,554,000 was spent on toys in the U.S. The U.S. population in 2000 was 281,421,906, with 21.4% of the population 14 years and under. If all of the toys purchased in 2000 were for children 14 years and under, what was the average amount spent per child? *(Lesson 6-1)* **about \$573.77**

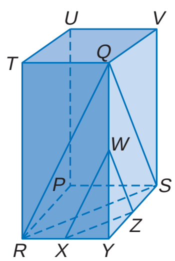
QUILTING **For Exercises 2–4, use the following information.** *(Lesson 6-2)*
Felicia found a pattern for a quilt square. The pattern measures three-quarters of an inch on a side. Felicia wants to make a quilt that is 77 inches by 110 inches when finished.

2. If Felicia wants to use only whole quilt squares, what is the greatest side length she can use for each square? **11 in.**
3. How many quilt squares will she need for the quilt? **70 squares**
4. By what scale factor will she need to increase the pattern for the quilt square? $\frac{44}{3}$

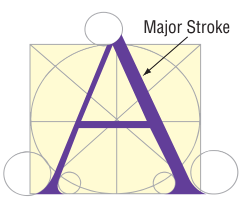
PROOF **For Exercises 5 and 6, write a paragraph proof.** *(Lesson 6-3)* **5–6. See margin.**

5. **Given:** $\triangle WYX \sim \triangle QYR$, $\triangle ZYX \sim \triangle SYR$
Prove: $\triangle WYZ \sim \triangle QYS$

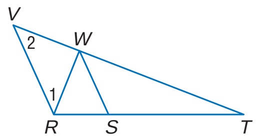
6. **Given:** $\overline{WX} \parallel \overline{QR}$, $\overline{ZX} \parallel \overline{SR}$
Prove: $\overline{WZ} \parallel \overline{QS}$



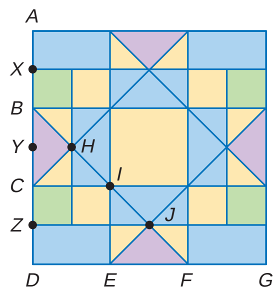
HISTORY **For Exercises 7 and 8, use the following information.** *(Lesson 6-4)*
In the fifteenth century, mathematicians and artists tried to construct the perfect letter. Damiano da Moile used a square as a frame to design the letter "A" as shown in the diagram. The thickness of the major stroke of the letter was to be $\frac{1}{12}$ of the height of the letter.



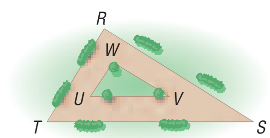
7. Explain why the bar through the middle of the A is half the length between the outside bottom corners of the sides of the letter. **See margin.**
8. If the letter were 3 centimeters tall, how wide would the major stroke of the A be? **0.25 cm**
9. **PROOF** Write a two-column proof. *(Lesson 6-5)*
Given: $\overline{WS}$ bisects $\angle RWT$. $\angle 1 \cong \angle 2$ **See margin.**
Prove: $\frac{VW}{WT} = \frac{RS}{ST}$



ART **For Exercises 10 and 11, use the diagram of a square mosaic tile.** $AB = BC = CD = \frac{1}{3}AD$ **and** $DE = EF = FG = \frac{1}{3}DG$. *(Lesson 6-5)*



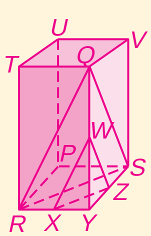
10. What is the ratio of the perimeter of $\triangle BDF$ to the perimeter of $\triangle BCI$? Explain.
11. Find two triangles such that the ratio of their perimeters is 2 : 3. Explain. **10–11. See margin.**
12. **TRACK** A triangular track is laid out as shown. $\triangle RST \sim \triangle WVU$. If $UV = 500$ feet, $VW = 400$ feet, $UW = 300$ feet, and $ST = 1000$ feet, find the perimeter of $\triangle RST$. *(Lesson 6-5)* **2400 ft**



13. **BANKING** Ashante has \$5000 in a savings account with a yearly interest rate of 2.5%. The interest is compounded twice per year. What will be the amount in the savings account after 5 years? *(Lesson 6-6)* **\$5661.35**

Chapter 6

5. Given: $\triangle WYX \sim \triangle QYR$, $\triangle ZYX \sim \triangle SYR$
Prove: $\triangle WYZ \sim \triangle QYS$

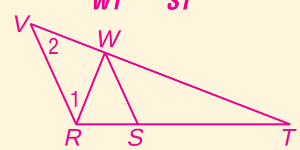


Proof: It is given that $\triangle WYX \sim \triangle QYR$ and $\triangle ZYX \sim \triangle SYR$. By definition of similar polygons we know that $\frac{WY}{QY} = \frac{YX}{YR}$ and $\frac{YX}{YR} = \frac{ZY}{SY}$. Then $\frac{WY}{QY} = \frac{ZY}{SY}$ by the Transitive Property. $\angle WYZ \cong \angle QYS$ because congruence of angles is reflexive. Therefore, $\triangle WYZ \sim \triangle QYS$ by SAS Similarity.

6. Given: $\overline{WX} \parallel \overline{QR}$, $\overline{ZX} \parallel \overline{SR}$
Prove: $\overline{WZ} \parallel \overline{QS}$
Proof: We are given that $\overline{WX} \parallel \overline{QR}$, $\overline{ZX} \parallel \overline{SR}$. By the Corresponding Angles Postulate, $\angle XWY \cong \angle RQY$ and $\angle YXZ \cong \angle YRS$. By the Reflexive Property, $\angle QYS \cong \angle QYS$, $\angle QYR \cong \angle QYR$ and $\angle RYS \cong \angle RYS$. $\triangle QYR \sim \triangle WYX$ and $\triangle YRS \sim \triangle YXZ$ by AA Similarity. By the definition of similar triangles, $\frac{WY}{QY} = \frac{YX}{YR}$ and $\frac{YX}{YR} = \frac{ZY}{SY}$. $\frac{WY}{QY} = \frac{ZY}{SY}$ by the Transitive Property. $\triangle WYZ \sim \triangle QYS$ by SAS Similarity. By the definition of similar triangles $\angle YWZ \cong \angle YQS$. $\overline{WZ} \parallel \overline{QS}$ by the Corresponding Angles Postulate.

7. The bar connects the midpoints of each leg of the letter and is parallel to the base. Therefore, the length of the bar is one-half the length of the base because a midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

9. Given: $\overline{WS}$ bisects $\angle RWT$, $\angle 1 \cong \angle 2$
Prove: $\frac{VW}{WT} = \frac{RS}{ST}$



Proof:

Statements (Reasons)
1. $\overline{WS}$ bisects $\angle RWT$ (Given)
2. $\frac{RW}{WT} = \frac{RS}{ST}$ ($\angle$ Bisector Th.)
3. $\angle 1 \cong \angle 2$ (Given)
4. $\overline{RW} \cong \overline{VW}$ (Conv. of Isos. $\triangle$ Th.)
5. $RW = VW$ (Def. of $\cong$)
6. $\frac{VW}{WT} = \frac{RS}{ST}$ (Substitution)

10. Since $\triangle BDF \sim \triangle BCI$ and the ratio of side lengths is 2:1, the ratio of perimeters will be 2:1 by the Proportional Perimeters Theorem.

11. Sample answer: $\triangle BCI \sim \triangle BZJ$ and both are isosceles right triangles with a ratio of side length of 2:3. By the Proportional Perimeters Theorem, the ratio of their perimeters will be 2:3.

Chapter 7

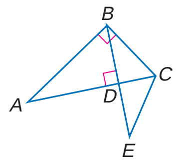
1. Given: D is the midpoint of $\overline{BE}$, $\overline{BD}$ is an altitude of right triangle ABC
   Prove: $\frac{AD}{DE} = \frac{DE}{DC}$
   Proof:
   Statements (Reasons)
   1. $\overline{BD}$ is an altitude of right triangle ABC. (Given)
   2. $\frac{AD}{DB} = \frac{DB}{DC}$ (The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.)
   3. D is the midpoint of $\overline{BE}$. (Given)
   4. $DB = DE$ (Def. of midpoint)
   5. $\frac{AD}{DE} = \frac{DE}{DC}$ (Substitution)
3. No; the measures do not satisfy the Pythagorean Theorem since $(2.7)^2 + (3.0)^2 \neq (5.3)^2$.
8. $AE \approx 339.4$ ft, $EB = 300$ ft, $CF \approx 134.2$ ft, $DF \approx 84.9$ ft

Chapter 8

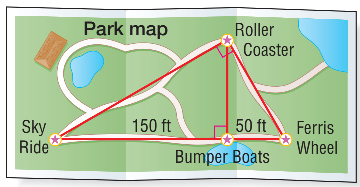
3. Sample answer: Make sure that opposite sides are congruent or make sure that opposite angles are congruent.
4. Given: $\square ABCD$, $\overline{AE} \cong \overline{CF}$
   Prove: Quadrilateral $EBFD$ is a $\square$.
   Proof:
   Statements (Reasons)
   1. $\square ABCD$, $\overline{AE} \cong \overline{CF}$ (Given)
   2. $\overline{AB} \cong \overline{DC}$ (Opp. sides of a $\square$ are $\cong$.)
   3. $\angle A \cong \angle C$ (Opp. $\angle$s of a $\square$ are $\cong$.)
   4. $\triangle BAE \cong \triangle DCF$ (SAS)
   5. $\overline{EB} \cong \overline{DF}$, $\angle BEA \cong \angle DFC$ (CPCTC)
   6. $\overline{BC} \parallel \overline{AD}$ (Def. of $\square$)
   7. $\angle DFC \cong \angle FDE$ (Alt. Int. $\angle$s Th.)
   8. $\angle BEA \cong \angle FDE$ (Trans. Prop.)
   9. $\overline{EB} \parallel \overline{DF}$ (Corres. $\angle$s Post.)
   10. Quadrilateral $EBFD$ is a $\square$. (If one pair of opp. sides is $\parallel$ and $\cong$, then the quad. is a $\square$.)
5. The legs are made so that they will bisect each other, so the quadrilateral formed by the ends of the legs is always a parallelogram. Therefore, the top of the stand is parallel to the floor.

Chapter 7 Right Triangles and Trigonometry *(pages 340–399)*

1. **PROOF** Write a two-column proof. *(Lesson 7-1)*
   Given: D is the midpoint of $\overline{BE}$, $\overline{BD}$ is an altitude of right triangle $\triangle ABC$ **See margin.**
   Prove: $\frac{AD}{DE} = \frac{DE}{DC}$

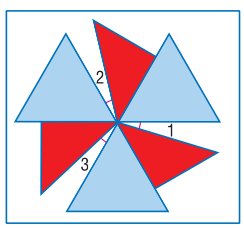


2. **AMUSEMENT PARKS** The map shows the locations of four rides at an amusement park. Find the length of the path from the roller coaster to the bumper boats. Round to the nearest tenth. *(Lesson 7-1)* **86.6 ft**



3. **CONSTRUCTION** Carlotta drew a diagram of a right triangular brace with side measures of 2.7 centimeters, 3.0 centimeters, and 5.3 centimeters. Is the diagram correct? Explain. *(Lesson 7-2)* **See margin.**

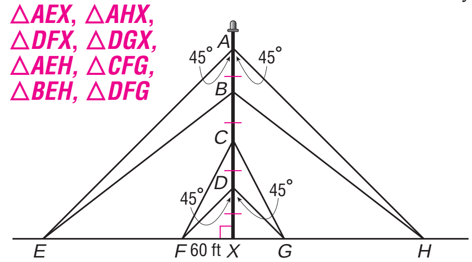
DESIGN For Exercises 4–5, use the following information. *(Lesson 7-3)*
Kwan designed the pinwheel. The blue triangles are congruent equilateral triangles each with an altitude of 4 inches. The red triangles are congruent isosceles right triangles. The hypotenuse of a red triangle is congruent to a side of a blue triangle.



4. If angles 1, 2, and 3 are congruent, find the measure of each angle. **15**
5. Find the perimeter of the pinwheel. Round to the nearest inch. **55 in.**

COMMUNICATION For Exercises 6–9, use the following information. *(Lesson 7-4)*
The diagram shows a radio tower secured by four pairs of guy wires that are equally spaced apart with $DX = 60$ feet. Round to the nearest tenth if necessary.

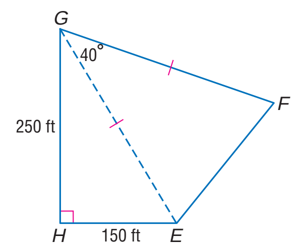
6. $\triangle AEX$, $\triangle AHX$, $\triangle DFX$, $\triangle DGX$, $\triangle AEH$, $\triangle CFG$, $\triangle BEH$, $\triangle DFG$



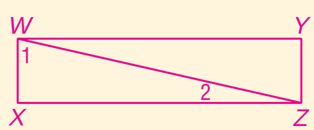
6. Name the isosceles triangles in the diagram.
7. Find $m\angle BEX$ and $m\angle CFX$. **36.9, 63.4**
8. Find AE, EB, CF, and DF. **See margin.**
9. Find the total amount of wire used to support the tower. **1717 ft**
10. **METEOROLOGY** A searchlight is 6500 feet from a weather station. If the angle of elevation to the spot of light on the clouds above the station is 47°, how high is the cloud ceiling? *(Lesson 7-5)* **≈ 6970 ft**

GARDENING For Exercises 11 and 12, use the information below. *(Lesson 7-6)*
A flower bed at Magic City Rose Garden is in the shape of an obtuse scalene triangle with the shortest side measuring 7.5 feet. Another side measures 14 feet and the measure of the opposite angle is 103°.

11. Find the measures of the other angles of the triangle. Round to the nearest degree. **31, 46**
12. Find the perimeter of the garden. Round to the nearest tenth. **31.8 ft**
13. **HOUSING** Mr. and Mrs. Abbott bought a lot at the end of a cul-de-sac. They want to build a fence on three sides of the lot, excluding $\overline{HE}$. To the nearest foot, how much fencing will they need to buy? *(Lesson 7-7)* **741 ft**



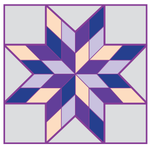
6. Given: $\square WXZY$, $\angle 1$ and $\angle 2$ are complementary.
   Prove: $WXZY$ is a rectangle.



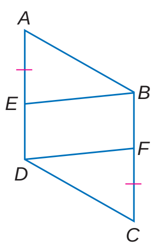
Proof:
Statements (Reasons)
1. $\square WXZY$, $\angle 1$ and $\angle 2$ are complementary (Given)
2. $m\angle 1 + m\angle 2 = 90$ (Def. of complementary $\angle$s)
3. $m\angle 1 + m\angle 2 + m\angle X = 180$ (Angle Sum Th.)
4. $90 + m\angle X = 180$ (Substitution)
5. $m\angle X = 90$ (Subtraction)
6. $\angle X \cong \angle Y$ (Opp. $\angle$s of a $\square$ are $\cong$.)
7. $m\angle Y = 90$ (Substitution)

ENGINEERING **For Exercises 1–2, use the following information.**
The London Eye in London, England, is the world's largest observation wheel. The ride has 32 enclosed capsules for riders. *(Lesson 8-1)*

1. Suppose each capsule is connected with a straight piece of metal forming a 32-gon. Find the sum of the measures of the interior angles. **5400**
2. What is the measure of one interior angle of the 32-gon? **168.75**
3. **QUILTING** The quilt square shown is called the Lone Star pattern. Describe two ways that the quilter could ensure that the pieces will fit properly. *(Lesson 8-2)* **See margin.**

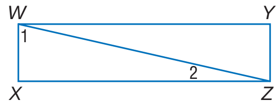


4. **PROOF** Write a two-column proof. *(Lesson 8-3)*
Given: $\square ABCD$, $\overline{AE} \cong \overline{CF}$
Prove: Quadrilateral $EBFD$ is a parallelogram.
See margin.

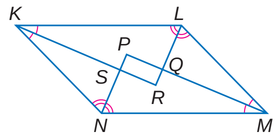


5. **MUSIC** Why will the keyboard stand shown always remain parallel to the floor? *(Lesson 8-3)*
See margin.

6. **PROOF** Write a two-column proof. *(Lesson 8-4)*
Given: $\square WXZY$, $\angle 1$ and $\angle 2$ are complementary.
Prove: $WXZY$ is a rectangle. **See margin.**

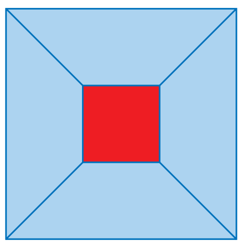


7. **PROOF** Write a paragraph proof. *(Lesson 8-4)*
Given: $\square KLMN$
Prove: $PQRS$ is a rectangle. **See margin.**



8. **CONSTRUCTION** Mr. Redwing is building a sandbox. He placed stakes at what he believes will be the four vertices of a square with a distance of 5 feet between each stake. How can he be sure that the sandbox will be a square? *(Lesson 8-5)* **See margin.**

DESIGN **For Exercises 9 and 10, use the square floor tile design shown below.** *(Lesson 8-6)*



9. Explain how you know that the trapezoids in the design are isosceles. **See margin.**
10. The perimeter of the floor tile is 48 inches, and the perimeter of the interior red square is 16 inches. Find the perimeter of one trapezoid. **$16 + 8\sqrt{2}$ in. ≈ 27.3 in.**
11. **PROOF** Position a quadrilateral on the coordinate plane with vertices $Q(-a, 0)$, $R(a, 0)$, $S(b, c)$, and $T(-b, c)$. Prove that the quadrilateral is an isosceles trapezoid. *(Lesson 8-7)*
See margin.

John D. Norman/CORBIS

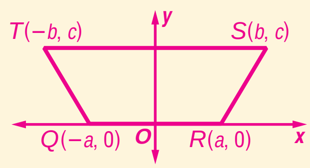
8. $\angle X$ and $\angle XWY$ are suppl., $\angle X$ and $\angle XZY$ are suppl. (Cons. ∠s in ▱ are suppl.)
9. $m\angle X + m\angle XWY = 180$, $m\angle X + m\angle XZY = 180$ (Def. of suppl. ∠s)
10. $90 + m\angle XWY = 180$, $90 + m\angle XZY = 180$ (Substitution)
11. $m\angle XWY = 90$, $m\angle XZY = 90$ (Subtraction)
12. $\angle X$, $\angle Y$, $\angle XWY$, and $\angle XZY$ are rt. ∠s (Def. rt. ∠)
13. $WXZY$ is a rect. (Def. of rect.)

7. Given: $\square KLMN$
Prove: $PQRS$ is a rectangle.
Proof: The diagram indicates that $\angle KNS \cong \angle SNM \cong \angle MLQ \cong \angle QLK$ and $\angle NKS \cong \angle SKL \cong \angle LMQ \cong \angle QMN$ in $\square KLMN$. Since $\triangle KLR$, $\triangle KNS$, $\triangle MLQ$, and $\triangle MNP$ all have two angles congruent, the third angles are congruent by the Third Angle Theorem. So $\angle QRS \cong \angle KSN \cong \angle MQL \cong \angle SPQ$. Since they are vertical angles, $\angle KSN \cong \angle PSR$ and $\angle MQL \cong \angle PQR$. Therefore, $\angle QRS \cong \angle PSR \cong \angle PQR \cong \angle SPQ$. $PQRS$ is a parallelogram since if both pairs of opposite angles are congruent, the quadrilateral is a parallelogram. $\angle KSN$ and $\angle KSP$ form a linear pair and are therefore supplementary angles. $\angle KSP$ and $\angle PSR$ form a linear pair and are supplementary angles. Therefore, $\angle KSN$ and $\angle PSR$ are supplementary. Since they are also congruent, each is a right angle. If a parallelogram has one right angle, it has four right angles. Therefore, $PQRS$ is a rectangle.

8. Sample answer: He should measure the angles at the vertices to see if they are 90 or he can check to see if the diagonals are congruent.

9. The legs of the trapezoids are part of the diagonals of the square. The diagonals of a square bisect opposite angles, so each base angle of a trapezoid measures 45°. One pair of sides is parallel and the base angles are congruent.

11. Given: Quadrilateral $QRST$
Prove: $QRST$ is an isosceles trapezoid



Proof:

$$TQ = \sqrt{(-b-(-a))^2 + (c-0)^2} = \sqrt{b^2 - 2ab + a^2 + c^2}$$

$$SR = \sqrt{(b-a)^2 + (c-0)^2} = \sqrt{b^2 - 2ab + a^2 + c^2}$$

Slope of $\overline{TS} = \frac{c-c}{b-(-b)} = \frac{0}{2b}$ or 0.

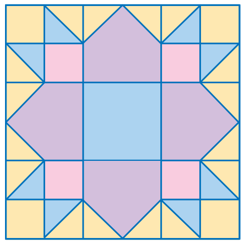
Slope of $\overline{QR} = \frac{0-0}{a-(-a)} = \frac{0}{2a}$ or 0.

Slope of $\overline{TQ} = \frac{c-0}{-b-(-a)}$ or $\frac{c}{-b+a}$.

Slope of $\overline{SR} = \frac{c-0}{b-a}$ or $\frac{c}{b-a}$.

Exactly one pair of opposite sides are parallel. The legs are congruent. $QRST$ is an isosceles trapezoid.

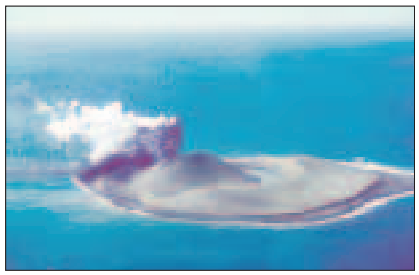
QUILTING **For Exercises 1 and 2, use the diagram of a quilt square.** *(Lesson 9-1)*



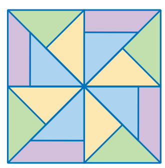
1. How many lines of symmetry are there for the entire quilt square? **4**

2. Consider different sections of the quilt square. Describe at least three different lines of reflection and the figures reflected in those lines. **See margin.**

3. **ENVIRONMENT** A cloud of dense gas and dust pours out of Surtsey, a volcanic island off the south coast of Iceland. If the cloud blows 40 miles north and then 30 miles east, make a sketch to show the translation of the smoke particles. Then find the distance of the shortest path that would take the particles to the same position. *(Lesson 9-2)* **See margin.**



ART **For Exercises 4–7, use the mosaic tile.**



4. Identify the order and magnitude of rotation that takes a yellow triangle to a blue triangle. *(Lesson 9-3)*

5. Identify the order and magnitude of rotation that takes a blue triangle to a yellow triangle. *(Lesson 9-3)* **4–5. See margin.**

6. Identify the magnitude of rotation that takes a trapezoid to a consecutive trapezoid. *(Lesson 9-3)* **90°**

7. Can the mosaic tile tessellate the plane? Explain. *(Lesson 9-4)* **See margin.**

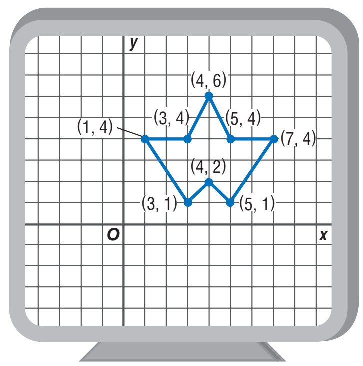
8. **CRAFTS** Eduardo found a pattern for cross-stitch on the Internet. The pattern measures 2 inches by 3 inches. He would like to enlarge the piece to 4 inches by 6 inches. The copy machine available to him enlarges 150% or less by increments of whole number percents. Find two whole number percents by which he can consecutively enlarge the piece and get as close to the desired dimensions as possible without exceeding them. *(Lesson 9-5)*
Sample answer: 150% followed by 133%

AVIATION **For Exercises 9 and 10, use the following information.** *(Lesson 9-6)*
A small aircraft flies due south at an average speed of 190 miles per hour. The wind is blowing due west at 30 miles per hour.

9. Draw a diagram using vectors to represent this situation. **See margin.**

10. Find the resultant velocity and direction of the plane. **about 192.4 mph; about 9.0° west of due south**

GRAPHICS **For Exercises 11–14, use the graphic shown on the computer screen.** *(Lesson 9-7)*



11–14. See margin.

11. Suppose you want the figure to move to Quadrant III but be upside down. Write two matrices that make this transformation, if they are applied consecutively.

12. Write one matrix that can be used to do the same transformation as in Exercise 11. What type of transformation is this?

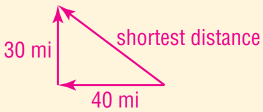
13. Compare the two matrices in Exercise 11 to the matrix in Exercise 12. What do you notice?

14. Write the vertex matrix for the figure in Quadrant III and graph it on the coordinate plane.

Chapter 9

2. Sample answer: Look at the upper right-hand square containing two squares and four triangles. The blue triangles are reflections over a line representing the diagonal of the square. The purple pentagon is formed by reflecting a trapezoid over a line through the center of the square surrounding the pentagon. Any small pink square is a reflection of a small yellow square reflected over a diagonal of the larger square.

3. 50 mi;

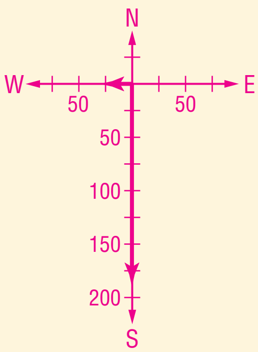


4. either 45° clockwise or 45° counterclockwise

5. either 45° clockwise or 45° counterclockwise

7. Yes; the measure of one interior angle is 90, which is a factor of 360. So, a square can tessellate the plane.

9.

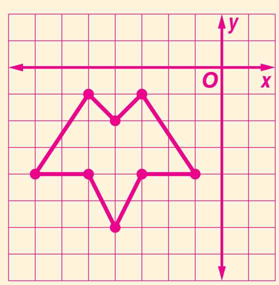


11. Sample answer: The matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ will produce the vertices for a reflection of the figure in the y-axis. Then the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ will produce the vertices for a reflection of the second figure in the x-axis. This figure will be upside down.

12. The matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ will produce the vertices for a 180° rotation about the origin. The figure will be upside down and in Quadrant III.

13. The matrix for Exercise 12 has the first row entries for the first matrix used in Exercise 11 and the second row entries for the second matrix used in 11.

14. $\begin{bmatrix} -4 & -5 & -7 & -5 & -4 & -3 & -1 & -3 \\ -6 & -4 & -4 & -1 & -2 & -1 & -4 & -4 \end{bmatrix}$;



1. **CYCLING** A bicycle tire travels about 50.27 inches during one rotation of the wheel. What is the diameter of the tire? *(Lesson 10-1)* **about 16 in.**

SPACE **For Exercises 2–4, use the following information.** *(Lesson 10-2)*
School children were recently surveyed about what they believe to be the most important reason to explore Mars. They were given five choices and the table below shows the results.

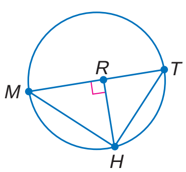
Reason to Visit Mars	Number of Students
Learn about life beyond Earth	910
Learn more about Earth	234
Seek potential for human inhabitance	624
Use as a base for further exploration	364
Increase human knowledge	468

Source: *USA TODAY*

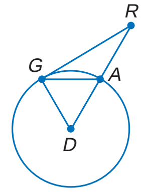
2. If you were to construct a circle graph of this data, how many degrees would be allotted to each category? **2–4. See margin.**
3. Describe the type of arc associated with each category.
4. Construct a circle graph for these data.

5. **CRAFTS** Yvonne uses wooden spheres to make paperweights to sell at craft shows. She cuts off a flat surface for each base. If the original sphere has a radius of 4 centimeters and the diameter of the flat surface is 6 centimeters, what is the height of the paperweight? *(Lesson 10-3)* **about 6.6 cm**

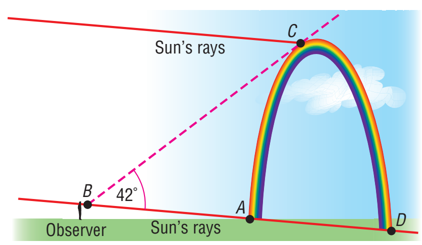
6. **PROOF** Write a two-column proof. *(Lesson 10-4)*
Given: $\overparen{MHT}$ is a semicircle.
$\overline{RH} \perp \overline{TM}$
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$ **See margin.**



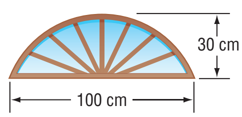
7. **PROOF** Write a paragraph proof. *(Lesson 10-5)*
Given: $\overline{GR}$ is tangent to $\odot D$ at G. **See margin.**
$\overline{AG} \cong \overline{DG}$
Prove: $\overline{AG}$ bisects $\overline{RD}$.



8. **METEOROLOGY** A rainbow is really a full circle with a center at a point in the sky directly opposite the Sun. The position of a rainbow varies according to the viewer's position, but its angular size, $\angle ABC$, is always 42°. If $m\overparen{CD} = 160$, find the measure of the visible part of the rainbow, $m\overparen{AC}$. *(Lesson 10-6)* **76**

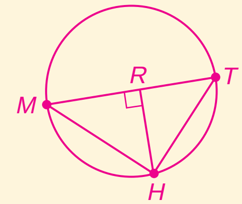


9. **CONSTRUCTION** An arch over an entrance is 100 centimeters wide and 30 centimeters high. Find the radius of the circle that contains the arch. *(Lesson 10-7)* **about 56.7 cm**



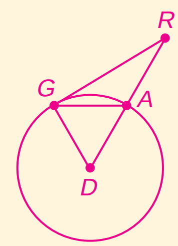
10. **SPACE** Objects that have been left behind in Earth's orbit from space missions are called "space junk." These objects are a hazard to current space missions and satellites. Eighty percent of space junk orbits Earth at a distance of 1,200 miles from the surface of Earth, which has a diameter of 7,926 miles. Write an equation to model the orbit of 80% of space junk with Earth's center at the origin. *(Lesson 10-8)*
$x^2 + y^2 = 26{,}656{,}569$

6. Given: $\overparen{MHT}$ **is a semicircle,** $\overline{RH} \perp \overline{TM}$
Prove: $\frac{TR}{RH} = \frac{TH}{HM}$



Proof:
Statements (Reasons)
1. $\overparen{MHT}$ is a semicircle, $\overline{RH} \perp \overline{TM}$ (Given)
2. $\angle THM$ is a rt. $\angle$. (If an inscribed $\angle$ intercepts a semicircle, the $\angle$ is a rt. $\angle$.)
3. $\angle TRH$ is a rt. $\angle$. (Def. $\perp$ lines)
4. $\angle THM \cong \angle TRH$ (All rt. $\angle$s are $\cong$.)
5. $\angle T \cong \angle T$ (Reflexive Prop.)
6. $\triangle TRH \sim \triangle THM$ (AA Sim.)
7. $\frac{TR}{RH} = \frac{TH}{HM}$ (Def. $\sim \triangle$s)

7. Given: $\overline{GR}$ **is tangent to** $\odot D$ **at** G. $\overline{AG} \cong \overline{DG}$
Prove: $\overline{AG}$ **bisects** $\overline{RD}$.

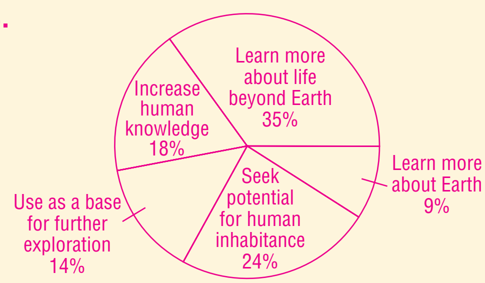


Proof: Since $\overline{DA}$ is a radius, $\overline{DG} \cong \overline{DA}$. Since $\overline{AG} \cong \overline{DG} \cong \overline{DA}$, $\triangle GDA$ is equilateral. Therefore, each angle has a measure of 60. Since $\overline{GR}$ is tangent to $\odot D$, $m\angle RGD = 90$. Since $m\angle AGD = 60$, then by the Angle Addition Postulate, $m\angle RGA = 30$. If $m\angle DAG = 60$, then $m\angle RAG = 120$. Then $m\angle R = 30$. Then, $\triangle RAG$ is isosceles, and $\overline{RA} \cong \overline{AG}$. By the Transitive Property, $\overline{RA} \cong \overline{DA}$. Therefore, $\overline{AG}$ bisects $\overline{RD}$.

Chapter 10

2. Learn about life beyond Earth: 126°; Learn more about Earth: 32.4°; Seek potential for human inhabitance: 86.4°; Use as a base for further exploration: 50.4°; Increase human knowledge: 64.8°
3. All of the categories are represented by minor arcs.

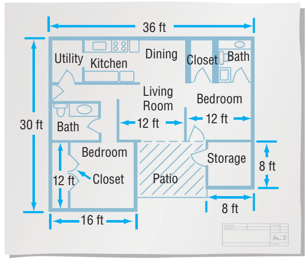
4.



9. The total for the black tiles is greater. For the red tiles, there are 4 hexagons and 5 squares for a perimeter of $2[4(4 + 2\sqrt{2}) + 5 \cdot 4] = (72 + 16\sqrt{2})$ feet. For the black tiles, there are 8 squares and 8 triangles for a perimeter of $2[(8 \cdot 4 + 8(2 + \sqrt{2})] = (96 + 16\sqrt{2})$ feet.

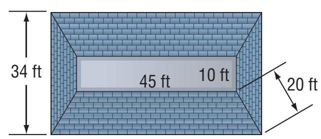
Mixed Problem Solving and Proof

REMODELING **For Exercises 1–3, use the following information.**
The diagram shows the floor plan of the home that the Summers are buying. They want to replace the patio with a larger sunroom to increase their living space by one-third. *(Lesson 11-1)*

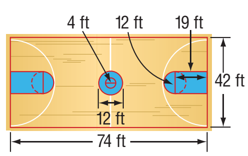


1. Excluding the patio and storage area, how many square feet of living area are in the current house? **840 ft^2**
2. What area should be added to the house to increase the living area by one-third? **280 ft^2**
3. The Summers want to connect the bedroom and storage area with the sunroom. What will be the dimensions of the sunroom? **12 ft by 23.3 ft**

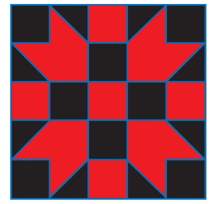
HOME REPAIR **For Exercises 4 and 5, use the following information.**
Scott needs to replace the shingles on the roof of his house. The roof is composed of two large isosceles trapezoids, two smaller isosceles trapezoids, and a rectangle. Each trapezoid has the same height. *(Lesson 11-2)*



4. Find the height of the trapezoids. **16 ft**
5. Find the area of the roof covered by shingles. **2528 ft^2**
6. **SPORTS** The Moore High School basketball team wants to paint their basketball court as shown. They want the center circle and the free throw areas painted blue. What is the area of the court that they will paint blue? *(Lesson 11-3)* **682.19 ft^2**

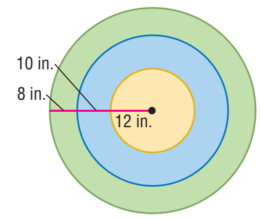


MUSEUMS **For Exercises 7–9, use the following information.**
The Hyalite Hills Museum plans to install the square mosaic pattern shown below in the entry hall. It is 10 feet on each side with each small black or red square tile measuring 2 feet on each side. *(Lesson 11-4)*



7. Find the area of black tiles. **48 ft^2**
8. Find the area of red tiles. **52 ft^2**
9. Which is greater, the total perimeter of the red tiles or the total perimeter of the black tiles? Explain. **See margin.**

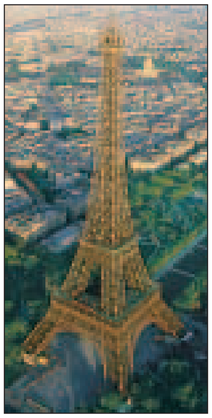
10. **GAMES** If the dart lands on the target, find the probability that it lands in the blue region. *(Lesson 11-5)* **≈0.378**



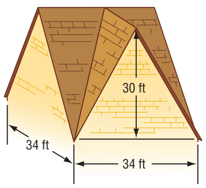
11. **ACCOMMODATIONS** The convention center in Washington, D.C., lies in the northwest sector of the city between New York and Massachusetts Avenues, which intersect at a 130° angle. If the amount of hotel space is evenly distributed over an area with that intersection as the center and a radius of 1.5 miles, what is the probability that a vistor, randomly assigned to a hotel, will be housed in the sector containing the convention center? *(Lesson 11-5)* **$\frac{13}{36}$ or 36.1%**

792 Mixed Problem Solving and Proof

1. **ARCHITECTURE** Sketch an orthogonal drawing of the Eiffel Tower. *(Lesson 12-1)* **See margin.**



2. **CONSTRUCTION** The roof shown below is a hip-and-valley style. Use the dimensions given to find the area of the roof that would need to be shingled. *(Lesson 12-2)* **about 2344.8 ft^2**



3. **AERONAUTICAL ENGINEERING** The surface area of the wing on an aircraft is used to determine a design factor known as wing loading. If the total weight of the aircraft and its load is w and the total surface area of its wings is s, then the formula for the wing loading factor, ℓ, is $\ell = \frac{w}{s}$. If the wing loading factor is exceeded, the pilot must either reduce the fuel load or remove passengers or cargo. Find the wing loading factor for a plane if it had a take-off weight of 750 pounds and the surface area of the wings was 532 square feet. *(Lesson 12-2)* **about 1.41**

4. **MANUFACTURING** Many baking pans are given a special nonstick coating. A rectangular cake pan is 9 inches by 13 inches by 2 inches deep. What is the area of the inside of the pan that needs to be coated? *(Lesson 12-3)* **205 in^2**

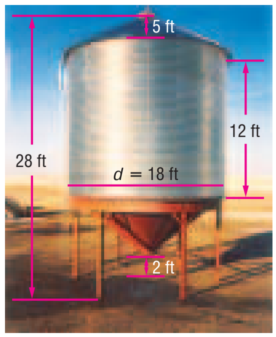
5. **COMMUNICATIONS** Coaxial cable is used to transmit long-distance telephone calls, cable television programming, and other communications. A typical coaxial cable contains 22 copper tubes and has a diameter of 3 inches. What is the lateral area of a coaxial cable that is 500 feet long? *(Lesson 12-4)* **about 392.7 ft^2**

COLLECTIONS **For Exercises 6 and 7, use the following information.**
Soledad collects unique salt-and-pepper shakers. She inherited a pair of tetrahedral shakers from her mother. *(Lesson 12-5)*

6. Each edge of a shaker measures 3 centimeters. Make a sketch of one shaker. **See margin.**

7. Find the total surface area of one shaker. **about 15.6 cm^2**

8. **FARMING** The picture below shows a combination hopper cone and bin used by farmers to store grain after harvest. The cone at the bottom of the bin allows the grain to be emptied more easily. Use the dimensions shown in the diagram to find the entire surface area of the bin with a conical top and bottom. Write the exact answer and the answer rounded to the nearest square foot. *(Lesson 12-6)*
$\pi(216 + 9\sqrt{106} + 81\sqrt{2}) \approx 1330$ **ft^2**



GEOGRAPHY **For Exercises 9–11, use the following information.**
Joaquin is buying Dennis a globe for his birthday. The globe has a diameter of 16 inches. *(Lesson 12-7)*

9. What is the surface area of the globe? **804.2 in^2**

10. If the diameter of Earth is 7926 miles, find the surface area of Earth. **197,359,487.5 mi^2**

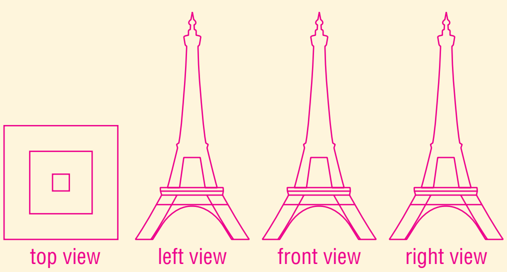
11. The continent of Africa occupies about 11,700,000 square miles. How many square inches will be used to represent Africa on the globe? **about 47.7 in^2**

Mixed Problem Solving and Proof

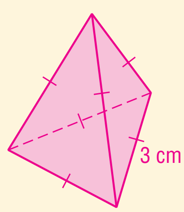
(t)Yann Arthus-Bertrand/CORBIS, (c)courtesy M-K Distributors, Conrad MT, (b)Aaron Haupt

Chapter 12

1.



6.



Mixed Problem Solving and Proof

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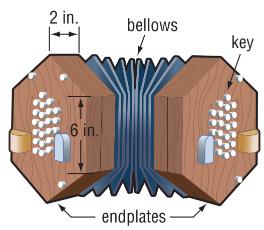
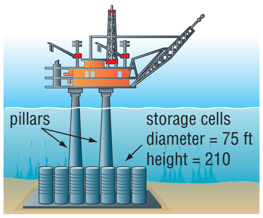
1. **METEOROLOGY** The TIROS weather satellites were a series of weather satellites, the first being launched on April 1, 1960. These satellites carried television and infrared cameras and were covered by solar cells. If the cylinder-shaped body of a TIROS had a diameter of 42 inches and a height of 19 inches, what was the volume available for carrying instruments and cameras? Round to the nearest tenth. *(Lesson 13-1)* **26,323.4 in^3**

2. **SPACECRAFT** The smallest manned spacecraft, used by astronauts for jobs outside the Space Shuttle, is the Manned Maneuvering Unit. It is 4 feet tall, 2 feet 8 inches wide, and 3 feet 8 inches deep. Find the volume of this spacecraft in cubic feet. Round to the nearest tenth. *(Lesson 13-1)* **39.1 ft^3**

3. **MUSIC** To play a concertina, you push and pull the end plates and press the keys. The air pressure causes vibrations of the metal reeds that make the notes. When fully expanded, the concertina is 36 inches from end to end. If the concertina is compressed, it is 7 inches from end to end. Find the volume of air in the instrument when it is fully expanded and when it is compressed. (*Hint:* Each endplate is a regular hexagonal prism and contains no air.) *(Lesson 13-1)* **2993.0 in^3; 280.6 in^3**

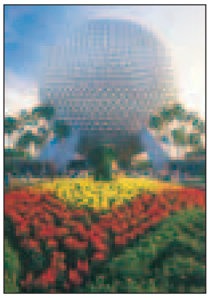


4. **ENGINEERING** The base of an oil drilling platform is made up of 24 concrete cylindrical cells. Twenty of the cells are used for oil storage. The pillars that support the platform deck rest on the four other cells. Find the total volume of the storage cells. *(Lesson 13-1)* **18,555,031.6 ft^3**



5. **HOME BUSINESS** Jodi has a home-based business selling homemade candies. She is designing a pyramid-shaped box for the candy. The base is a square measuring 14.5 centimeters on a side. The slant height of the pyramid is 16 centimeters. Find the volume of the box. Round to the nearest cubic centimeter. *(Lesson 13-2)* **1000 cm^3**

ENTERTAINMENT **For Exercises 6–10, use the following information.**
Some people think that the Spaceship Earth geosphere at Epcot® in Disney World resembles a golf ball. The building is a sphere measuring 165 feet in diameter. A typical golf ball has a diameter of approximately 1.5 inches.



6. Find the volume of Spaceship Earth. Round to the nearest cubic foot. *(Lesson 13-3)* **2,352,071 ft^3**
7. Find the volume of a golf ball. Round to the nearest tenth. *(Lesson 13-3)* **1.8 in^3**
8. What is the scale factor that compares Spaceship Earth to a golf ball? *(Lesson 13-4)* **1320 to 1**
9. What is the ratio of the volume of Spaceship Earth to the volume of a golf ball? *(Lesson 13-4)*
10. Suppose a six-foot-tall golfer plays golf with a 1.5 inch diameter golf ball. If the ratio between golfer and ball remains the same, how tall would a golfer need to be to use Spaceship Earth as a golf ball? *(Lesson 13-4)* **7920 ft tall**

9. 1320^3 to 1 or 2,299,968,000 to 1

ASTRONOMY **For Exercises 11 and 12, use the following information.**
A museum has set aside a children's room containing objects suspended from the ceiling to resemble planets and stars. Suppose an imaginary coordinate system is placed in the room with the center of the room at (0, 0, 0). Three particular stars are located at $S(-10, 5, 3)$, $T(3, -8, -1)$, and $R(-7, -4, -2)$, where the coordinates represent the distance in feet from the center of the room. *(Lesson 13-5)*

11. Find the distance between each pair of stars.
12. Which star is farthest from the center of the room? **the star located at *S***

11. $ST = \sqrt{354}$ ft, $TR = 3\sqrt{13}$ ft, $SR = \sqrt{115}$ ft

R. Stuart Westmorland/Stock Boston

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Becoming a Better Test-Taker

At some time in your life, you will have to take a standardized test. Sometimes this test may determine if you go on to the next grade or course, or even if you will graduate from high school. This section of your textbook is dedicated to making you a better test-taker.

TYPES OF TEST QUESTIONS In the following pages, you will see examples of four types of questions commonly seen on standardized tests. A description of each type of question is shown in the table below.

Type of Question	Description	See Pages
multiple choice	Four or five possible answer choices are given from which you choose the best answer.	796–797
gridded response	You solve the problem. Then you enter the answer in aspecial grid and color in the corresponding circles.	798–801
short response	You solve the problem, showing your work and/or explaining your reasoning.	802–805
extended response	You solve a multi-part problem, showing your work and/or explaining your reasoning.	806–810

PRACTICE After being introduced to each type of question, you can practice that type of question. Each set of practice questions is divided into five sections that represent the categories most commonly assessed on standardized tests.

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

USING A CALCULATOR On some tests, you are permitted to use a calculator. You should check with your teacher to determine if calculator use is permitted on the test you will be taking, and, if so, what type of calculator can be used.

TEST-TAKING TIPS In addition to the Test-Taking Tips like the one shown at the right, here are some additional thoughts that might help you.

- Get a good night's rest before the test. Cramming the night before does not improve your results.
- Budget your time when taking a test. Don't dwell on problems that you cannot solve. Just make sure to leave that question blank on your answer sheet.
- Watch for key words like NOT and EXCEPT. Also look for order words like LEAST, GREATEST, FIRST, and LAST.

Test-Taking Tip
If you are allowed to use a calculator, make sure you are familiar with how it works so that you won't waste time trying to figure out the calculator when taking the test.

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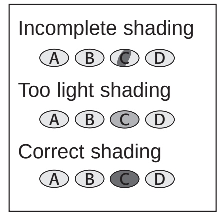
Preparing for Standardized Tests

Multiple-Choice Questions



Multiple-choice questions are the most common type of question on standardized tests. These questions are sometimes called *selected-response questions*. You are asked to choose the best answer from four or five possible answers.

To record a multiple-choice answer, you may be asked to shade in a bubble that is a circle or an oval or just to write the letter of your choice. Always make sure that your shading is dark enough and completely covers the bubble.

Sometimes a question does not provide you with a figure that represents the problem. Drawing a diagram may help you to solve the problem. Once you draw the diagram, you may be able to eliminate some of the possibilities by using your knowledge of mathematics. Another answer choice might be that the correct answer is not given.

Example

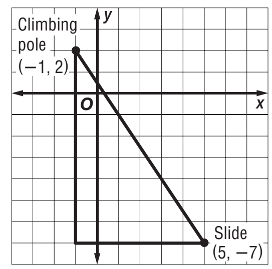
A coordinate plane is superimposed on a map of a playground. Each side of each square represents 1 meter. The slide is located at (5, –7), and the climbing pole is located at (–1, 2). What is the distance between the slide and the pole?

Strategy

Diagrams
Draw a diagram of the playground.

(A) $\sqrt{15}$ **m** **(B)** **6 m** **(C)** **9 m** **(D)** $9\sqrt{13}$ **m** **(E)** **none of these**

Draw a diagram of the playground on a coordinate plane. Notice that the difference in the *x*-coordinates is 6 meters and the difference in the *y*-coordinates is 9 meters.



Since the two points are two vertices of a right triangle, the distance between the two points must be greater than either of these values. So we can eliminate Choices B and C.

Use the Distance Formula or the Pythagorean Theorem to find the distance between the slide and the climbing pole. Let's use the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$6^2 + 9^2 = c^2$	Substitution
$36 + 81 = c^2$	
$117 = c^2$	
$3\sqrt{13} = c$	Take the square root of each side and simplify.

So, the distance between the slide and pole is $3\sqrt{13}$ meters. Since this is not listed as choice A, B, C, or D, the answer is Choice E.

If you are short on time, you can test each answer choice to find the correct answer. Sometimes you can make an educated guess about which answer choice to try first.

Multiple-Choice Practice

Choose the best answer.

Number and Operations

1. Carmen designed a rectangular banner that was 5 feet by 8 feet for a local business. The owner of the business asked her to make a larger banner measuring 10 feet by 20 feet. What was the percent increase in size from the first banner to the second banner? **D**

Ⓐ 4% Ⓑ 20%

Ⓒ 80% Ⓓ 400%

2. A roller coaster casts a shadow 57 yards long. Next to the roller coaster is a 35-foot tree with a shadow that is 20 feet long at the same time of day. What is the height of the roller coaster to the nearest whole foot? **C**

Ⓐ 98 ft Ⓑ 100 ft

Ⓒ 299 ft Ⓓ 388 ft

Algebra

3. At Speedy Car Rental, it costs \$32 per day to rent a car and then \$0.08 per mile. If y is the total cost of renting the car and x is the number of miles, which equation describes the relation between x and y? **C**

Ⓐ $y = 32x + 0.08$ Ⓑ $y = 32x - 0.08$

Ⓒ $y = 0.08x + 32$ Ⓓ $y = 0.08x - 32$

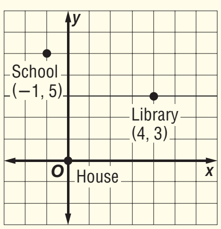
4. Eric plotted his house, school, and the library on a coordinate plane. Each side of each square represents one mile. What is the distance from his house to the library? **B**

Ⓐ $\sqrt{24}$ mi

Ⓑ 5 mi

Ⓒ $\sqrt{26}$ mi

Ⓓ $\sqrt{29}$ mi



Geometry

5. The grounds outside of the Custer County Museum contain a garden shaped like a right triangle. One leg of the triangle measures 8 feet, and the area of the garden is 18 square feet. What is the length of the other leg? **E**

Ⓐ 2.25 in. Ⓑ 4.5 in. Ⓒ 13.5 in.

Ⓓ 27 in. Ⓔ 54 in.

Test-Taking Tip

Questions 2, 5 and 7
The units of measure given in the question may not be the same as those given in the answer choices. Check that your solution is in the proper unit.

6. The circumference of a circle is equal to the perimeter of a regular hexagon with sides that measure 22 inches. What is the length of the radius of the circle to the nearest inch? Use 3.14 for π. **C**

Ⓐ 7 in. Ⓑ 14 in. Ⓒ 21 in.

Ⓓ 24 in. Ⓔ 28 in.

Measurement

7. Eduardo is planning to install carpeting in a rectangular room that measures 12 feet 6 inches by 18 feet. How many square yards of carpet does he need for the project? **A**

Ⓐ 25 yd^2 Ⓑ 50 yd^2

Ⓒ 225 yd^2 Ⓓ 300 yd^2

8. Marva is comparing two containers. One is a cylinder with diameter 14 centimeters and height 30 centimeters. The other is a cone with radius 15 centimeters and height 14 centimeters. What is the ratio of the volume of the cylinder to the volume of the cone? **C**

Ⓐ 3 to 1 Ⓑ 2 to 1

Ⓒ 7 to 5 Ⓓ 7 to 10

Data Analysis and Probability

9. Refer to the table. Which statement is true about this set of data? **D**

Country	Spending per Person
Japan	\$8622
United States	\$8098
Switzerland	\$6827
Norway	\$6563
Germany	\$5841
Denmark	\$5778

Source: *Top 10 of Everything 2003*

Ⓐ The median is less than the mean.

Ⓑ The mean is less than the median.

Ⓒ The range is 2844.

Ⓓ A and C are true.

Ⓔ B and C are true.

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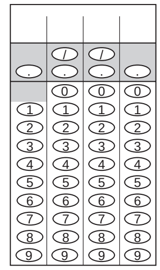
Gridded-Response Questions

Gridded-response questions are another type of question on standardized tests. These questions are sometimes called *student-produced response* or *grid-in*, because you must create the answer yourself, not just choose from four or five possible answers.



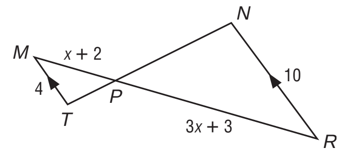
For gridded response, you must mark your answer on a grid printed on an answer sheet. The grid contains a row of four or five boxes at the top, two rows of ovals or circles with decimal and fraction symbols, and four or five columns of ovals, numbered 0–9. Since there is no negative symbol on the grid, answers are never negative. An example of a grid from an answer sheet is shown at the right.

How do you correctly fill in the grid?

Example 1 **In the diagram, $\triangle MPT \sim \triangle RPN$. Find PR.**

What do you need to find?

You need to find the value of x so that you can substitute it into the expression $3x + 3$ to find PR. Since the triangles are similar, write a proportion to solve for x.



$\frac{MT}{RN} = \frac{PM}{PR}$	Definition of similar polygons
$\frac{4}{10} = \frac{x + 2}{3x + 3}$	Substitution
$4(3x + 3) = 10(x + 2)$	Cross products
$12x + 12 = 10x + 20$	Distributive Property
$2x = 8$	Subtract 12 and $10x$ from each side.
$x = 4$	Divide each side by 2.

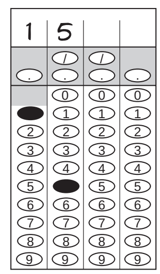
Now find PR.

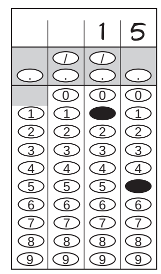
$PR = 3x + 3$

$= 3(4) + 3$ or 15

How do you fill in the grid for the answer?

- Write your answer in the answer boxes.
- Write only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.
- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.





Many gridded-response questions result in an answer that is a fraction or a decimal. These values can also be filled in on the grid.

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How do you grid decimals and fractions?

Example 2 **A triangle has a base of length 1 inch and a height of 1 inch. What is the area of the triangle in square inches?**

Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.

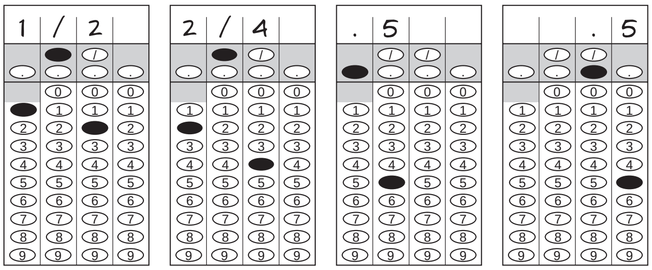
$A = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(1)(1)$ — Substitution

$= \frac{1}{2}$ or 0.5 — Simplify.

How do you grid the answer?

You can either grid the fraction or the decimal. Be sure to write the decimal point or fraction bar in the answer box. The following are acceptable answer responses.

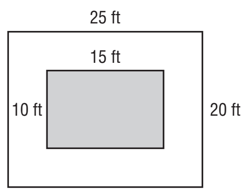


Do not leave a blank answer box in the middle of an answer.

Sometimes an answer is an improper fraction. Never change the improper fraction to a mixed number. Instead, grid either the improper fraction or the equivalent decimal.

How do you grid mixed numbers?

Example 3 **The shaded region of the rectangular garden will contain roses. What is the ratio of the area of the garden to the area of the shaded region?**



Strategy

Formulas

If you are unsure of a formula, check the reference sheet.

First, find the area of the garden.

$A = \ell w$

$= 25(20)$ or 500

Then find the area of the shaded region.

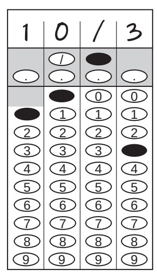
$A = \ell w$

$= 15(10)$ or 150

Write the ratio of the areas as a fraction.

$\frac{\text{area of garden}}{\text{area of shaded region}} = \frac{500}{150}$ or $\frac{10}{3}$

Leave the answer as the improper fraction $\frac{10}{3}$, as there is no way to correctly grid $3\frac{1}{3}$.



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Gridded-Response Practice

Solve each problem and complete the grid.

Number and Operations

1. A large rectangular meeting room is being planned for a community center. Before building the center, the planning board decides to increase the area of the original room by 40%. When the room is finally built, budget cuts force the second plan to be reduced in area by 25%. What is the ratio of the area of the room that is built to the area of the original room? **1.05**

2. Greenville has a spherical tank for the city's water supply. Due to increasing population, they plan to build another spherical water tank with a radius twice that of the current tank. How many times as great will the volume of the new tank be as the volume of the current tank? **8**

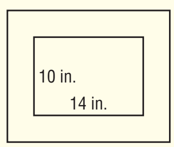
3. In Earth's history, the Precambrian period was about 4600 million years ago. If this number of years is written in scientific notation, what is the exponent for the power of 10? **9**

4. A virus is a type of microorganism so small it must be viewed with an electron microscope. The largest shape of virus has a length of about 0.0003 millimeter. To the nearest whole number, how many viruses would fit end to end on the head of a pin measuring 1 millimeter? **3333**

Preparing for Standardized Tests

Algebra

5. Kaia has a painting that measures 10 inches by 14 inches. She wants to make her own frame that has an equal width on all sides. She wants the total area of the painting and frame to be 285 square inches. What will be the width of the frame in inches? **5/2 or 2.5**

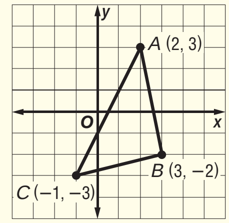


Test-Taking Tip

Question 1

Remember that you have to grid the decimal point or fraction bar in your answer. If your answer does not fit on the grid, convert to a fraction or decimal. If your answer still cannot be gridded, then check your computations.

6. The diagram shows a triangle graphed on a coordinate plane. If $\overline{AB}$ is extended, what is the value of the y-intercept? **13**



7. Tyree networks computers in homes and offices. In many cases, he needs to connect each computer to every other computer with a wire. The table shows the number of wires he needs to connect various numbers of computers. Use the table to determine how many wires are needed to connect 20 computers. **190**

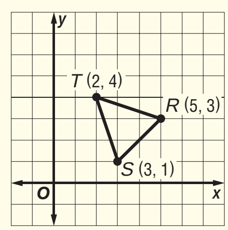
Computers	Wires	Computers	Wires
1	0	5	10
2	1	6	15
3	3	7	21
4	6	8	28

8. A line perpendicular to $9x - 10y = -10$ passes through $(-1, 4)$. Find the x-intercept of the line. **13/5 or 2.6**

9. Find the positive solution of $6x^2 - 7x = 5$. **5/3**

Geometry

10. The diagram shows $\triangle RST$ on the coordinate plane. The triangle is first rotated 90° counterclockwise about the origin and then reflected in the y-axis. What is the x-coordinate of the image of T after the two transformations? **4**



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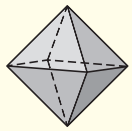
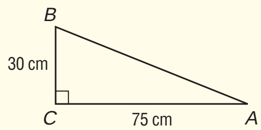
11. An octahedron is a solid with eight faces that are all equilateral triangles. How many edges does the octahedron have? **12**

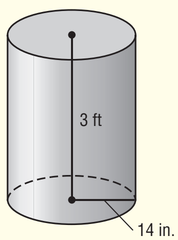


12. Find the measure of $\angle A$ to the nearest tenth of a degree. **21.8**

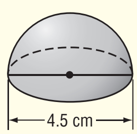


Measurement

13. The Pep Club plans to decorate some large garbage barrels for Spirit Week. They will cover only the sides of the barrels with decorated paper. How many square feet of paper will they need to cover 8 barrels like the one in the diagram? Use 3.14 for π. Round to the nearest square foot. **176**



14. Kara makes decorative paperweights. One of her favorites is a hemisphere with a diameter of 4.5 centimeters. What is the surface area of the hemisphere including the bottom on which it rests? Use 3.14 for π. Round to the nearest tenth of a square centimeter. **47.7**



15. The record for the fastest land speed of a car traveling for one mile is approximately 763 miles per hour. The car was powered by two jet engines. What was the speed of the car in feet per second? Round to the nearest whole number. **1119**

16. On average, a B-777 aircraft uses 5335 gallons of fuel on a 2.5-hour flight. At this rate, how much fuel will be needed for a 45-minute flight? Round to the nearest gallon. **1601**

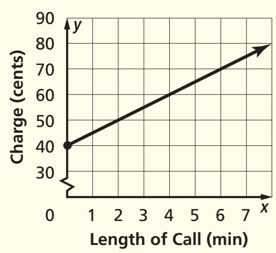
Data Analysis and Probability

17. The table shows the heights of the tallest buildings in Kansas City, Missouri. To the nearest tenth, what is the positive difference between the median and the mean of the data? **6.0**

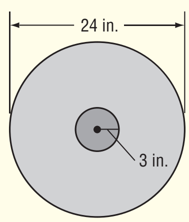
Name	Height (m)
One Kansas City Place	193
Town Pavilion	180
Hyatt Regency	154
Power and Light Building	147
City Hall	135
1201 Walnut	130

Source: skyscrapers.com

18. A long-distance telephone service charges 40 cents per call and 5 cents per minute. If a function model is written for the graph, what is the rate of change of the function? **5**



19. In a dart game, the dart must land within the innermost circle on the dartboard to win a prize. If a dart hits the board, what is the probability, as a percent, that it will hit the innermost circle? **6.25**



Preparing for Standardized Tests

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Preparing for Standardized Tests

Short-Response Questions

Short-response questions require you to provide a solution to the problem, as well as any method, explanation, and/or justification you used to arrive at the solution. These are sometimes called *constructed-response, open-response, open-ended, free-response,* or *student-produced questions.* The following is a sample rubric, or scoring guide, for scoring short-response questions.

Credit	Score	Criteria
Full	2	Full credit: The answer is correct and a full explanation is provided that shows each step in arriving at the final answer.
Partial	1	Partial credit: There are two different ways to receive partial credit. • The answer is correct, but the explanation provided is incomplete or incorrect. • The answer is incorrect, but the explanation and method of solving the problem is correct.
None	0	No credit: Either an answer is not provided or the answer does not make sense.

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Example

Mr. Solberg wants to buy all the lawn fertilizer he will need for this season. His front yard is a rectangle measuring 55 feet by 32 feet. His back yard is a rectangle measuring 75 feet by 54 feet. Two sizes of fertilizer are available—one that covers 5000 square feet and another covering 15,000 square feet. He needs to apply the fertilizer four times during the season. How many bags of each size should he buy to have the least amount of waste?

Full Credit Solution

Find the area of each part of the lawn and multiply by 4 since the fertilizer is to be applied 4 times. Each portion of the lawn is a rectangle, so $A = lw$.

$$4[(55 \times 32) + (75 \times 54)] = 23{,}240 \text{ ft}^2$$

If Mr. Solberg buys 2 bags that cover 15,000 ft^2, he will have too much fertilizer. If he buys 1 large bag, he will still need to cover 23,240 − 15,000 or 8240 ft^2.

The steps, calculations, and reasoning are clearly stated.

Find how many small bags it takes to cover 82400 ft^2.

$$8240 \div 5000 = 1.648$$

Since he cannot buy a fraction of a bag, he will need to buy 2 of the bags that cover 5000 ft^2 each.

Mr. Solberg needs to buy 1 bag that covers 15,000 square feet and 2 bags that cover 5000 square feet each.

The solution of the problem is clearly stated.

Strategy

Estimation

Use estimation to check your solution.

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Partial Credit Solution

In this sample solution, the answer is correct. However, there is no justification for any of the calculations.

There is not an explanation of how 23,240 was obtained.

> 23,240
>
> $23{,}240 - 15{,}000 = 8240$
>
> $8240 \div 5000 = 1.648$
>
> Mr. Solberg needs to buy 1 large bag and 2 small bags.

Partial Credit Solution

In this sample solution, the answer is incorrect. However, after the first statement all of the calculations and reasoning are correct.

The first step of multiplying the area by 4 was left out.

> First find the total number of square feet of lawn.
> Find the area of each part of the yard.
>
> $(55 \times 32) + (75 \times 54) = 5810 \text{ ft}^2$
>
> The area of the lawn is greater than 5000 ft^2, which is the amount covered by the smaller bag, but buying the bag that covers 15,000 ft^2 would result in too much waste.
>
> $5810 \div 5000 = 1.162$
>
> Therefore, Mr. Solberg will need to buy 2 of the smaller bags of fertilizer.

No Credit Solution

In this sample solution, the response is incorrect and incomplete.

The wrong operations are used, so the answer is incorrect. Also, there are no units of measure given with any of the calculations.

> $55 + 75 = 130$
>
> $32 + 54 = 86$
>
> $130 \times 86 \times 4 = 44{,}720$
>
> $44{,}720 \div 15{,}000 = 2.98$
>
> Mr. Solberg will need 3 bags of fertilizer.

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5. 

7. πr^2 is the area of the base, $2\pi rh$ is the area of the sides, and $2\pi r^2$ is the area of the hemisphere; $\pi r(3r + 2h)$.

Short-Response Practice

Solve each problem. Show all your work.

Number and Operations

1. In 2000, approximately $191 billion in merchandise was sold by a popular retail chain store in the United States. The population at that time was 281,421,906. Estimate the average amount of merchandise bought from this store by each person in the U.S. **about $680**

2. At a theme park, three educational movies run continuously all day long. At 9 A.M., the three shows begin. One runs for 15 minutes, the second for 18 minutes, and the third for 25 minutes. At what time will the movies all begin at the same time again? **4:30 P.M.**

3. Ming found a sweater on sale for 20% off the original price. However, the store was offering a special promotion, where all sale items were discounted an additional 60%. What was the total percent discount for the sweater? **68%**

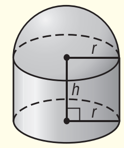
4. The serial number of a DVD player consists of three letters of the alphabet followed by five digits. The first two letters can be any letter, but the third letter cannot be *O*. The first digit cannot be zero. How many serial numbers are possible with this system? **1,521,000,000**

Algebra

5. Solve and graph $2x - 9 \leq 5x + 4$. $x \geq -\frac{13}{3}$; **See margin for graph.**

6. Vance rents rafts for trips on the Jefferson River. You have to reserve the raft and provide a $15 deposit in advance. Then the charge is $7.50 per hour. Write an equation that can be used to find the charge for any amount of time, where y is the total charge in dollars and x is the amount of time in hours. $y = 15 + 7.50x$

7. Hector is working on the design for the container shown below that consists of a cylinder with a hemisphere on top. He has written the expression $\pi r^2 + 2\pi rh + 2\pi r^2$ to represent the surface area of any size container of this shape. Explain the meaning of each term of the expression. **See margin.**

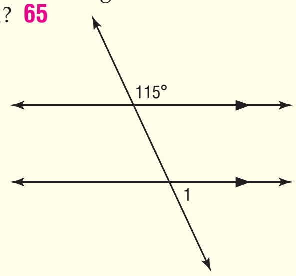


8. Find all solutions of the equation $6x^2 + 13x = 5$. $-\frac{5}{2}, \frac{1}{3}$

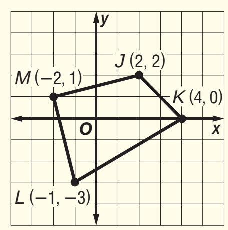
9. In 1999, there were 2,192,070 farms in the U.S., while in 2001, there were 2,157,780 farms. Let x represent years since 1999 and y represent the total number of farms in the U.S. Suppose the number of farms continues to decrease at the same rate as from 1999 to 2001. Write an equation that models the number of farms for any year after 1999. $y = 2{,}192{,}070 - 17{,}145x$

Geometry

10. Refer to the diagram. What is the measure of $\angle 1$? **65**



11. Quadrilateral *JKLM* is to be reflected in the line $y = x$. What are the coordinates of the vertices of the image? $J'(2, 2)$, $K'(0, 4)$, $L'(-3, -1)$, $M'(1, -2)$



> **Test-Taking Tip**
>
> A B C D
>
> **Question 4**
>
> Be sure to completely and carefully read the problem before beginning any calculations. If you read too quickly, you may miss a key piece of information.

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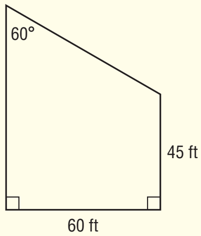
12. Write an equation in standard form for a circle that has a diameter with endpoints at $(-3, 2)$ and $(4, -5)$. $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = 98$

13. In the Columbia Village subdivision, an unusually shaped lot, shown below, will be used for a small park. Find the exact perimeter of the lot. $(150 + 60\sqrt{3})$ ft

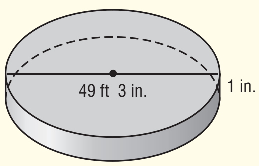


Measurement

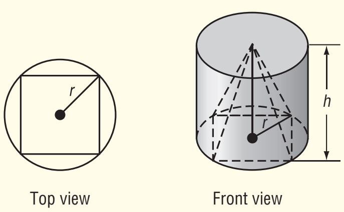
14. The Astronomical Unit (AU) is the distance from Earth to the Sun. It is usually rounded to 93,000,000 miles. The star Alpha Centauri is 25,556,250 million miles from Earth. What is this distance in AU? **about 274,798 AU**

15. Linesse handpaints unique designs on shirts and sells them. It takes her about 4.5 hours to create a design. At this rate, how many shirts can she design if she works 22 days per month for an average of 6.5 hours per day? **between 31 and 32 shirts**

16. The world's largest pancake was made in England in 1994. To the nearest cubic foot, what was the volume of the pancake? **159 ft^3**



17. Find the ratio of the volume of the cylinder to the volume of the pyramid. **3π to 2**



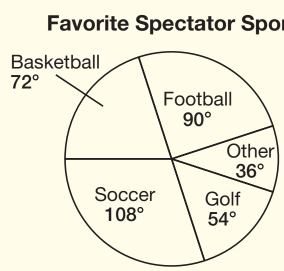
Data Analysis and Probability

18. The table shows the winning times for the Olympic men's 1000-meter speed skating event. Make a scatter plot of the data and describe the pattern in the data. Times are rounded to the nearest second. **See margin.**

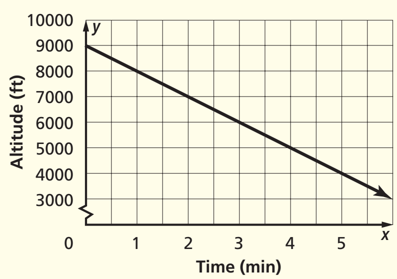
Men's 1000-m Speed Skating Event		
Year	Country	Time(s)
1976	U.S.	79
1980	U.S.	75
1984	Canada	76
1988	USSR	73
1992	Germany	75
1994	U.S.	72
1998	Netherlands	71
2002	Netherlands	67

Source: *The World Almanac*

19. Bradley surveyed 70 people about their favorite spectator sport. If a person is chosen at random from the people surveyed, what is the probability that the person's favorite spectator sport is basketball? **20% or 0.2**



20. The graph shows the altitude of a small airplane. Write a function to model the graph. Explain what the model means in terms of the altitude of the airplane. **See margin.**



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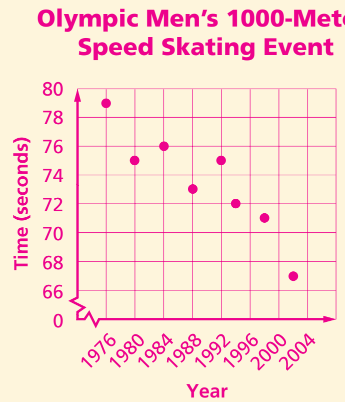
18. Sample answer: Times have been decreasing since 1992.



20. $y = 9000 - 1000x$; 9000 is the greatest altitude reached by the plane during this flight. The rate of change is -1000, which means the altitude is decreasing steadily by 1000 feet per minute.

Preparing for Standardized Tests

Extended-Response Questions

Extended-response questions are often called *open-ended* or *constructed-response questions.* Most extended-response questions have multiple parts. You must answer all parts to receive full credit.

Extended-response questions are similar to short-response questions in that you must show all of your work in solving the problem. A rubric is also used to determine whether you receive full, partial, or no credit. The following is a sample rubric for scoring extended-response questions.

Credit	Score	Criteria
Full	4	Full credit: A correct solution is given that is supported by well-developed, accurate explanations.
Partial	3, 2, 1	Partial credit: A generally correct solution is given that may contain minor flaws in reasoning or computation or an incomplete solution. The more correct the solution, the greater the score.
None	0	No credit: An incorrect solution is given indicating no mathematical understanding of the concept, or no solution is given.

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Make sure that when the problem says to *Show your work,* you show every part of your solution including figures, sketches of graphing calculator screens, or the reasoning behind your computations.

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Example

Polygon $WXYZ$ with vertices $W(-3, 2)$, $X(4, 4)$, $Y(3, -1)$, and $Z(-2, -3)$ is a figure represented on a coordinate plane to be used in the graphics for a video game. Various transformations will be performed on the polygon to use for the game.

a. Graph $WXYZ$ and its image $W'X'Y'Z'$ under a reflection in the y-axis. Be sure to label all of the vertices.

b. Describe how the coordinates of the vertices of $WXYZ$ relate to the coordinates of the vertices of $W'X'Y'Z'$.

c. Another transformation is performed on $WXYZ$. This time, the vertices of the image $W'X'Y'Z'$ are $W'(2, -3)$, $X'(4, 4)$, $Y'(-1, 3)$, and $Z'(-3, -2)$. Graph $WXYZ$ and its image under this transformation. What transformation produced $W'X'Y'Z'$?

Strategy

Make a List
Write notes about what to include in your answer for each part of the question.

Full Credit Solution

Part a A complete graph includes labels for the axes and origin and labels for the vertices, including letter names and coordinates.

- The vertices of the polygon should be correctly graphed and labeled.
- The vertices of the image should be located such that the transformation shows a reflection in the y-axis.
- The vertices of the polygons should be connected correctly. Optionally, the polygon and its image could be graphed in two contrasting colors.

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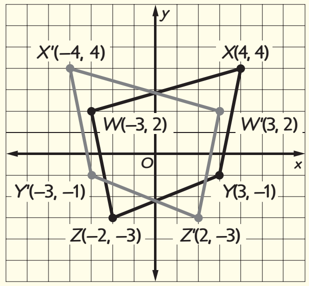
The first step of doubling the square footage for two coats of paint was left out.

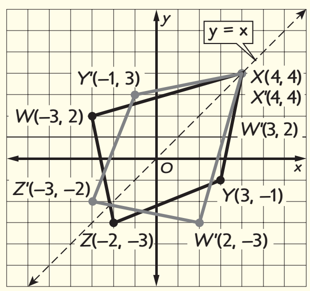


Part b

The coordinates of W and W' are $(-3, 2)$ and $(3, 2)$. The x-coordinates are the opposite of each other and the y-coordinates are the same. For any point (a, b), the coordinates of the reflection in the y-axis are (-a, b).

Part c

For full credit, the graph in Part C must also be accurate, which is true for this graph.

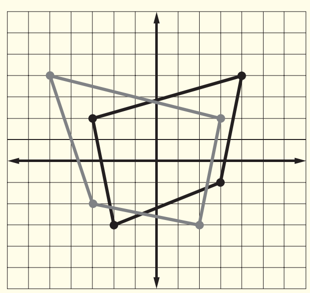


The coordinates of Z and Z' have been switched. In other words, for any point (a, b), the coordinates of the reflection in the y-axis are (b, a). Since X and X' are the same point, the polygon has been reflected in the line $y = x$.

Partial Credit Solution

Part a This sample graph includes no labels for the axes and for the vertices of the polygon and its image. Two of the image points have been incorrectly graphed.

More credit would have been given if all of the points were reflected correctly. The images for *X* and *Y* are not correct.



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Part b Partial credit is given because the reasoning is correct, but the reasoning was based on the incorrect graph in Part a.

> *For two of the points, W and Z, the y-coordinates are the same and the x-coordinates are opposites. But, for points X and Y, there is no clear relationship.*

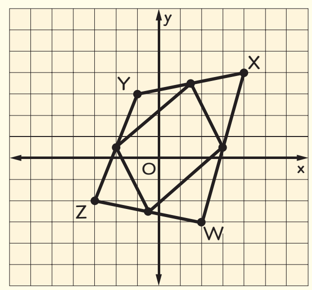
Part c Full credit is given for Part c. The graph supplied by the student was identical to the graph shown for the full credit solution for Part c. The explanation below is correct, but slightly different from the previous answer for Part c.

> *I noticed that point X and point X' were the same. I also guessed that this was a reflection, but not in either axis. I played around with my ruler until I found a line that was the line of reflection. The transformation from WXYZ to W'X'Y'Z' was a reflection in the line $y = x$.*

This sample answer might have received a score of 2 or 1, depending on the judgment of the scorer. Had the student graphed all points correctly and gotten Part b correct, the score would probably have been a 3.

No Credit Solution

Part a The sample answer below includes no labels on the axes or the coordinates of the vertices of the polygon. The polygon *WXYZ* has three vertices graphed incorrectly. The polygon that was graphed is not reflected correctly either.



Part b

> *I don't see any way that the coordinates relate.*

Part c

> *It is a reduction because it gets smaller.*

In this sample answer, the student does not understand how to graph points on a coordinate plane and also does not understand the reflection of figures in an axis or other line.

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Extended-Response Practice

Solve each problem. Show all your work.

Number and Operations

1. Refer to the table.

Population		
City	**1990**	**2000**
Phoenix, AZ	983,403	1,321,045
Austin, TX	465,622	656,562
Charlotte, NC	395,934	540,828
Mesa, AZ	288,091	396,375
Las Vegas, NV	258,295	478,434

Source: census.gov

a. For which city was the increase in population the greatest? What was the increase?

b. For which city was the percent of increase in population the greatest? What was the percent increase?

c. Suppose that the population increase of a city was 30%. If the population in 2000 was 346,668, find the population in 1990. **1a–c. See margin.**

2. Molecules are the smallest units of a particular substance that still have the same properties as that substance. The diameter of a molecule is measured in angstroms (Å). Express each value in scientific notation.

a. An angstrom is exactly 10^{-8} centimeter. A centimeter is approximately equal to 0.3937 inch. What is the approximate measure of an angstrom in inches?

b. How many angstroms are in one inch?

c. If a molecule has a diameter of 2 angstroms, how many of these molecules placed side by side would fit on an eraser measuring $\frac{1}{4}$ inch? **2a–c. See margin.**

Algebra

3. The Marshalls are building a rectangular in-ground pool in their backyard. The pool will be 24 feet by 29 feet. They want to build a deck of equal width all around the pool. The final area of the pool and deck will be 1800 square feet. **3a–c. See margin.**

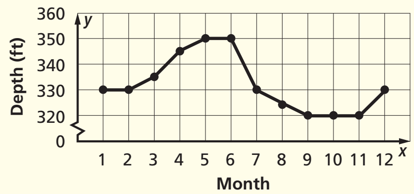
a. Draw and label a diagram.

b. Write an equation that can be used to find the width of the deck.

c. Find the width of the deck.

4. The depth of a reservoir was measured on the first day of each month. (Jan. = 1, Feb. = 2, and so on.)

Depth of the Reservoir



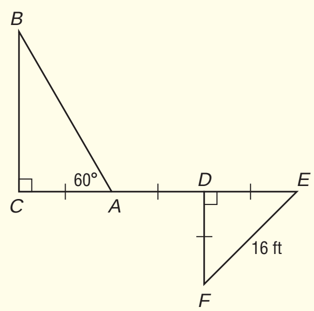
a. What is the slope of the line joining the points with x-coordinates 6 and 7? What does the slope represent?

b. Write an equation for the segment of the graph from 5 to 6. What is the slope of the line and what does this represent in terms of the reservoir?

c. What was the lowest depth of the reservoir? When was this depth first measured and recorded? **4a–c. See margin.**

Geometry

5. The Silver City Marching Band is planning to create this formation with the members.



a. Find the missing side measures of $\triangle EDF$. Explain.

b. Find the missing side measures of $\triangle ABC$. Explain.

c. Find the total distance of the path: A to B to C to A to D to E to F to D.

d. The director wants to place one person at each point A, B, C, D, E, and F. He then wants to place other band members approximately one foot apart on all segments of the formation. How many people should he place on each segment of the formation? How many total people will he need? **5a–d. See margin.**

1. A. Las Vegas at 220,139

B. Las Vegas at about 85.2%

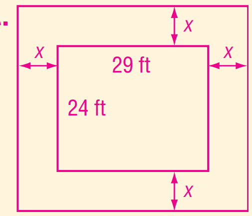
C. About 266,667

2. A. $10^{-8} \times 0.3937$ is the number of inches. This can be rewritten as 3.937×10^{-9} inches.

B. 3.937×10^{-9} inches = 1 Å, so 1 inch = 1 ÷ (3.937×10^{-9}) $\approx 2.54 \times 10^{8}$ Å.

C. If 1 inch contains 2.54×10^{8} Å, then one-quarter inch contains $(2.54 \times 10^{8}) \div 4$ or 6.35×10^{7} Å. If each molecule measures 2 Å, then there are $(6.35 \times 10^{7}) \div 2$ or 3.175×10^{7} of these molecules across the eraser.

3. A.



B. $1800 = (24 + 2x)(29 + 2x)$

C. 8 feet

4. A. The slope is −20. This means that the depth of the reservoir dropped by 20 feet in one month from the first day of June to the first day of July.

B. $y = 350$; the slope is 0. The water depth did not change from the first day of May to the first day of June.

C. 320 feet; it was measured on the first day of September.

5. A. $ED = DF = 8\sqrt{2} \approx 11.3$ feet, since $\triangle EDF$ is a 45°–45°–90° triangle.

B. $AC = 8\sqrt{2}$ since it is congruent to ED. Then, since $\triangle ABC$ is a 30°–60°–90° triangle, $AB = 16\sqrt{2} \approx 22.6$ feet, and $BC = 8\sqrt{6} \approx 19.6$ feet.

C. $22.6 + 19.6 + 11.3 + 11.3 + 11.3 + 16 + 11.3 \approx 103.4$ ft

D. Sample answer: 6 at the points, 15 on EF, 10 on each of ED, DF, DA and AC, 19 on BC, and 22 on AB. The total will be 102 people. (Depending upon how students decide to round the number of feet and place the students, the answer could vary slightly.)

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6. A. 420 cm^3

B. approximately 502.7 cm^3

C. Using the approximation of Part B, about 20% increase.

7. A. Sample answers with time given in days.

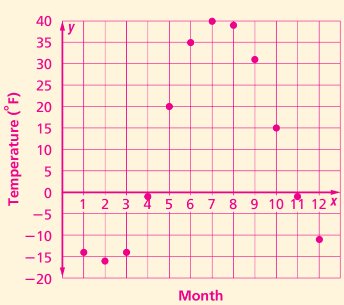
Planet	Time
Mercury	230 days
Venus	270 days
Mars	415 days
Jupiter	1003.3 days

B. Sample answer: Write the distance in scientific notation; for example, 138 million miles is 1.38×10^8. Then write 25,000 as 2.5×10^4. $1.38 \div 2.5 = 0.552$ and $10^{8-4} = 10^4$. $0.552 \times 10^4 = 5520$. This is the number of hours of the trip.

C. Neptune; sample explanation: 13.3 years is 116,508 hours. Multiply 116,508 by 25,000 to get 2.9127×10^9 miles, which is approximately the distance to Neptune.

8. A.

Temperatures for Barrow



B. Sample answer: The points suggest a curve that increases from February to June and July and then decreases back to December.

C. 10.25

D. Sample answer: If the line $y = 10.25$ is drawn on the same coordinate plane as the scatter plot, half of the graph lies below the line and half lies above the line.

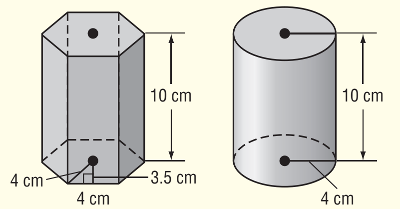
9. A. $\frac{1}{49}$

B. $\frac{8}{49}\left(\frac{24}{49}\right) = \frac{192}{2401}$

Preparing for Standardized Tests

Measurement

6. Two containers have been designed. One is a hexagonal prism, and the other is a cylinder.



a. What is the volume of the hexagonal prism?

b. What is the volume of the cylinder?

c. What is the percent of increase in volume from the prism to the cylinder? **6a–c. See margin.**

7. Kabrena is working on a project about the solar system. The table shows the maximum distances from Earth to the other planets in millions of miles.

Distance from Earth to Other Planets

Planet	Distance	Planet	Distance
Mercury	138	Saturn	1031
Venus	162	Uranus	1962
Mars	249	Neptune	2913
Jupiter	602	Pluto	4681

Source: *The World Almanac*

a. The maximum speed of the Apollo moon missions spacecraft was about 25,000 miles per hour. Make a table showing the time it would take a spacecraft traveling at this speed to reach each of the four closest planets.

b. Describe how to use scientific notation to calculate the time it takes to reach any planet.

c. Which planet would it take approximately 13.3 years to reach? Explain. **7a–c. See margin.**

Test-Taking Tip

Question 6
While preparing to take a standardized test, familiarize yourself with the formulas for surface area and volume of common three-dimensional figures.

Data Analysis and Probability

8. The table shows the average monthly temperatures in Barrow, Alaska. The months are given numerical values from 1-12. (Jan. = 1, Feb. = 2, and so on.)

Average Monthly Temperature

Month	°F	Month	°F
1	–14	7	40
2	–16	8	39
3	–14	9	31
4	–1	10	15
5	20	11	–1
6	35	12	–11

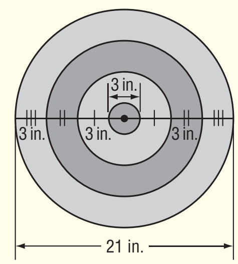
a. Make a scatter plot of the data. Let x be the numerical value assigned to the month and y be the temperature.

b. Describe any trends shown in the graph.

c. Find the mean of the temperature data.

d. Describe any relationship between the mean of the data and the scatter plot. **8a–d. See margin.**

9. A dart game is played using the board shown. The inner circle is pink, the next ring is blue, the next red, and the largest ring is green. A dart must land on the board during each round of play. **9a–c. See margin.**



a. What is the probability that a dart landing on the board hits the pink circle?

b. What is the probability that the first dart thrown lands in the blue ring and the second dart lands in the green ring?

c. Suppose players throw a dart twice. For which outcome of two darts would you award the most expensive prize? Explain your reasoning.

C. Sample answer: The least probability for two darts is for each of them to land in the pink circle. The most expensive prize should be for P(pink) followed by P(pink).

810 Preparing for Standardized Tests

Postulates, Theorems, and Corollaries

Chapter 2 Reasoning and Proof

Postulate 2.1 Through any two points, there is exactly one line. (p. 89)

Postulate 2.2 Through any three points not on the same line, there is exactly one plane. (p. 89)

Postulate 2.3 A line contains at least two points. (p. 90)

Postulate 2.4 A plane contains at least three points not on the same line. (p. 90)

Postulate 2.5 If two points lie in a plane, then the entire line containing those points lies in that plane. (p. 90)

Postulate 2.6 If two lines intersect, then their intersection is exactly one point. (p. 90)

Postulate 2.7 If two planes intersect, then their intersection is a line. (p. 90)

Theorem 2.1 **Midpoint Theorem** If M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$. (p. 91)

Postulate 2.8 **Ruler Postulate** The points on any line or line segment can be paired with real numbers so that, given any two points A and B on a line, A corresponds to zero, and B corresponds to a positive real number. (p. 101)

Postulate 2.9 **Segment Addition Postulate** If B is between A and C, then $AB + BC = AC$. If $AB + BC = AC$, then B is between A and C. (p. 102)

Theorem 2.2 Congruence of segments is reflexive, symmetric, and transitive. (p. 102)

Postulate 2.10 **Protractor Postulate** Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r. (p. 107)

Postulate 2.11 **Angle Addition Postulate** If R is in the interior of $\angle PQS$, then $m\angle PQR + m\angle RQS = m\angle PQS$. If $m\angle PQR + m\angle RQS = m\angle PQS$, then R is in the interior of $\angle PQS$. (p. 107)

Theorem 2.3 **Supplement Theorem** If two angles form a linear pair, then they are supplementary angles. (p. 108)

Theorem 2.4 **Complement Theorem** If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. (p. 108)

Theorem 2.5 Congruence of angles is reflexive, symmetric, and transitive. (p. 108)

Theorem 2.6 Angles supplementary to the same angle or to congruent angles are congruent. (p. 109) Abbreviation: $\measuredangle$s suppl. to same $\angle$ or $\cong$ $\measuredangle$s are $\cong$.

Theorem 2.7 Angles complementary to the same angle or to congruent angles are congruent. (p. 109) Abbreviation: $\measuredangle$s compl. to same $\angle$ or $\cong$ $\measuredangle$s are $\cong$.

Theorem 2.8 **Vertical Angle Theorem** If two angles are vertical angles, then they are congruent. (p. 110)

Theorem 2.9 Perpendicular lines intersect to form four right angles. (p. 110)

Theorem 2.10 All right angles are congruent. (p. 110)

Theorem 2.11 Perpendicular lines form congruent adjacent angles. (p. 110)

Theorem 2.12 If two angles are congruent and supplementary, then each angle is a right angle. (p. 110)

Theorem 2.13 If two congruent angles form a linear pair, then they are right angles. (p. 110)

Chapter 3 Perpendicular and Parallel Lines

Postulate 3.1 **Corresponding Angles Postulate** If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent. (p. 133)

Theorem 3.1 **Alternate Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent. (p. 134)

Theorem 3.2 **Consecutive Interior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary. (p. 134)

Theorem 3.3 **Alternate Exterior Angles Theorem** If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent. (p. 134)

Theorem 3.4 **Perpendicular Transversal Theorem** In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other. (p. 134)

Postulate 3.2 Two nonvertical lines have the same slope if and only if they are parallel. (p. 141)

Postulate 3.3 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1. (p. 141)

Postulate 3.4 If two lines in a plane are cut by a transversal so that corresponding angles are congruent, then the lines are parallel. (p. 151) Abbreviation: If corr. ∡s are $\cong$, lines are $\parallel$.

Postulate 3.5 **Parallel Postulate** If there is a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line. (p. 152)

Theorem 3.5 If two lines in a plane are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel. (p. 152)
Abbreviation: If alt. ext. ∡s are $\cong$, then lines are $\parallel$.

Theorem 3.6 If two lines in a plane are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the lines are parallel. (p. 152)
Abbreviation: If cons. int. ∡s are suppl., then lines are $\parallel$.

Theorem 3.7 If two lines in a plane are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel. (p. 152)
Abbreviation: If alt. int. ∡s are $\cong$, then lines are $\parallel$.

Theorem 3.8 In a plane, if two lines are perpendicular to the same line, then they are parallel. (p. 152) Abbreviation: If 2 lines are $\perp$ to the same line, then lines are $\parallel$.

Theorem 3.9 In a plane, if two lines are each equidistant from a third line, then the two lines are parallel to each other. (p. 161)

Chapter 4 Congruent Triangles

Theorem 4.1 **Angle Sum Theorem** The sum of the measures of the angles of a triangle is 180. (p. 185)

Theorem 4.2 **Third Angle Theorem** If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent. (p. 186)

Theorem 4.3 **Exterior Angle Theorem** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. (p. 186)

Corollary 4.1 The acute angles of a right triangle are complementary. (p. 188)

Corollary 4.2 There can be at most one right or obtuse angle in a triangle. (p. 188)

Theorem 4.4 Congruence of triangles is reflexive, symmetric, and transitive. (p. 193)

Postulate 4.1 **Side-Side-Side Congruence (SSS)** If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent. (p. 201)

Postulate 4.2 **Side-Angle-Side Congruence (SAS)** If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (p. 202)

Postulate 4.3 **Angle-Side-Angle Congruence (ASA)** If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, the triangles are congruent. (p. 207)

Theorem 4.5 **Angle-Angle-Side Congruence (AAS)** If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent. (p. 208)

Theorem 4.6 **Leg-Leg Congruence (LL)** If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent. (p. 214)

Theorem 4.7 **Hypotenuse-Angle Congruence (HA)** If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent. (p. 215)

Theorem 4.8 **Leg-Angle Congruence (LA)** If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent. (p. 215)

Postulate 4.4 **Hypotenuse-Leg Congruence (HL)** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent. (p. 215)

Theorem 4.9 **Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (p. 216)

Theorem 4.10 If two angles of a triangle are congruent, then the sides opposite those angles are congruent. (p. 218) Abbreviation: Conv. of Isos. △Th.

Corollary 4.3 A triangle is equilateral if and only if it is equiangular. (p. 218)

Corollary 4.4 Each angle of an equilateral triangle measures 60°. (p. 218)

Chapter 5 Relationships in Triangles

Theorem 5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. (p. 238)

Theorem 5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment. (p. 238)

Theorem 5.3 **Circumcenter Theorem** The circumcenter of a triangle is equidistant from the vertices of the triangle. (p. 239)

Theorem 5.4 Any point on the angle bisector is equidistant from the sides of the angle. (p. 239)

Theorem 5.5 Any point equidistant from the sides of an angle lies on the angle bisector. (p. 239)

Theorem 5.6 **Incenter Theorem** The incenter of a triangle is equidistant from each side of the triangle. (p. 240)

Theorem 5.7 **Centroid Theorem** The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median. (p. 240)

Theorem 5.8 **Exterior Angle Inequality Theorem** If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles. (p. 248)

Theorem 5.9 If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side. (p. 249)

Theorem 5.10 If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle. (p. 250)

Theorem 5.11 **Triangle Inequality Theorem** The sum of the lengths of any two sides of a triangle is greater than the length of the third side. (p. 261)

Theorem 5.12 The perpendicular segment from a point to a line is the shortest segment from the point to the line. (p. 262)

Corollary 5.1 The perpendicular segment from a point to a plane is the shortest segment from the point to the plane. (p. 263)

Theorem 5.13 **SAS Inequality/Hinge Theorem** If two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle. (p. 267)

Theorem 5.14 **SSS Inequality** If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle. (p. 268)

Chapter 6 Proportions and Similarity

Postulate 6.1 **Angle-Angle (AA) Similarity** If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. (p. 298)

Theorem 6.1 **Side-Side-Side (SSS) Similarity** If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar. (p. 299)

Theorem 6.2 **Side-Angle-Side (SAS) Similarity** If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (p. 299)

Theorem 6.3 Similarity of triangles is reflexive, symmetric, and transitive. (p. 300)

Theorem 6.4	**Triangle Proportionality Theorem** If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths. (p. 307)
Theorem 6.5	**Converse of the Triangle Proportionality Theorem** If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side. (p. 308)
Theorem 6.6	**Triangle Midsegment Theorem** A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side. (p. 308)
Corollary 6.1	If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally. (p. 309)
Corollary 6.2	If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal. (p. 309)
Theorem 6.7	**Proportional Perimeters Theorem** If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides. (p. 316)
Theorem 6.8	If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. (p. 317) Abbreviation: $\sim \triangle$s have corr. altitudes proportional to the corr. sides.
Theorem 6.9	If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides. (p. 317) Abbreviation: $\sim \triangle$s have corr. $\angle$ bisectors proportional to the corr. sides.
Theorem 6.10	If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides. (p. 317) Abbreviation: $\sim \triangle$s have corr. medians proportional to the corr. sides.
Theorem 6.11	**Angle Bisector Theorem** An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides. (p. 319)

Chapter 7 Right Triangles and Trigonometry

Theorem 7.1	If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other. (p. 343)
Theorem 7.2	The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. (p. 343)
Theorem 7.3	If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg. (p. 344)
Theorem 7.4	**Pythagorean Theorem** In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse. (p. 350)
Theorem 7.5	**Converse of the Pythagorean Theorem** If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle. (p. 351)
Theorem 7.6	In a $45°$-$45°$-$90°$ triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg. (p. 357)

Theorem 7.7 In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg. (p. 359)

Chapter 8 Quadrilaterals

Theorem 8.1 **Interior Angle Sum Theorem** If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$. (p. 404)

Theorem 8.2 **Exterior Angle Sum Theorem** If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360. (p. 406)

Theorem 8.3 Opposite sides of a parallelogram are congruent. (p. 412)
Abbreviation: Opp. sides of ▱ are ≅.

Theorem 8.4 Opposite angles of a parallelogram are congruent. (p. 412)
Abbreviation: Opp. ∠s of ▱ are ≅.

Theorem 8.5 Consecutive angles in a parallelogram are supplementary. (p. 412)
Abbreviation: Cons. ∠s in ▱ are suppl.

Theorem 8.6 If a parallelogram has one right angle, it has four right angles. (p. 412)
Abbreviation: If ▱ has 1 rt. ∠, it has 4 rt. ∠s.

Theorem 8.7 The diagonals of a parallelogram bisect each other. (p. 413)
Abbreviation: Diag. of ▱ bisect each other.

Theorem 8.8 The diagonal of a parallelogram separates the parallelogram into two congruent triangles. (p. 414) Abbreviation: Diag. of ▱ separates ▱ into 2 ≅ △s.

Theorem 8.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 418) Abbreviation: If both pairs of opp. sides are ≅ , then quad. is ▱.

Theorem 8.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (p. 418) Abbreviation: If both pairs of opp. ∠s are ≅, then quad. is ▱.

Theorem 8.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (p. 418) Abbreviation: If diag. bisect each other, then quad. is ▱.

Theorem 8.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram. (p. 418)
Abbreviation: If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.

Theorem 8.13 If a parallelogram is a rectangle, then the diagonals are congruent. (p. 424)
Abbreviation: If ▱ is rectangle, diag. are ≅.

Theorem 8.14 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (p. 426) Abbreviation: If diagonals of ▱ are ≅, ▱ is a rectangle.

Theorem 8.15 The diagonals of a rhombus are perpendicular. (p. 431)

Theorem 8.16 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (p. 431)

Theorem 8.17 Each diagonal of a rhombus bisects a pair of opposite angles. (p. 431)

Theorem 8.18 Both pairs of base angles of an isosceles trapezoid are congruent. (p. 439)

Theorem 8.19 The diagonals of an isosceles trapezoid are congruent. (p. 439)

Theorem 8.20 The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases. (p. 441)

Chapter 9 Transformations

Postulate 9.1 In a given rotation, if A is the preimage, A' is the image, and P is the center of rotation, then the measure of the angle of rotation, $\angle APA'$ is twice the measure of the acute or right angle formed by the intersecting lines of reflection. (p. 477)

Corollary 9.1 Reflecting an image successively in two perpendicular lines results in a 180° rotation. (p. 477)

Theorem 9.1 If a dilation with center C and a scale factor of r transforms A to E and B to D, then $ED = |r|(AB)$. (p. 491)

Theorem 9.2 If $P(x, y)$ is the preimage of a dilation centered at the origin with a scale factor r, then the image is $P'(rx, ry)$. (p. 492)

Chapter 10 Circles

Theorem 10.1 Two arcs are congruent if and only if their corresponding central angles are congruent. (p. 530)

Postulate 10.1 **Arc Addition Postulate** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. (p. 531)

Theorem 10.2 In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. (p. 536)
Abbreviations: In ⊙, 2 minor arcs are ≅, *iff* corr. chords are ≅.
In ⊙, 2 chords are ≅, *iff* corr. minor arcs are ≅.

Theorem 10.3 In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc. (p. 537)

Theorem 10.4 In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center. (p. 539)

Theorem 10.5 If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle). (p. 544)

Theorem 10.6 If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent. (p. 546) Abbreviations: Inscribed ∡ of same arc are ≅. Inscribed ∡ of ≅ arcs are ≅.

Theorem 10.7 If an inscribed angle intercepts a semicircle, the angle is a right angle. (p. 547)

Theorem 10.8 If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. (p. 548)

Theorem 10.9 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. (p. 553)

Postulates, Theorems, and Corollaries

Theorem 10.10 If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle. (p. 553)

Theorem 10.11 If two segments from the same exterior point are tangent to a circle, then they are congruent. (p. 554)

Theorem 10.12 If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle. (p. 561)

Theorem 10.13 If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc. (p. 562)

Theorem 10.14 If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs. (p. 563)

Theorem 10.15 If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal. (p. 569)

Theorem 10.16 If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment. (p. 570)

Theorem 10.17 If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment. (p. 571)

Chapter 11 Area of Polygons And Circles

Postulate 11.1 Congruent figures have equal areas. (p. 603)

Postulate 11.2 The area of a region is the sum of the areas of all of its nonoverlapping parts. (p. 619)

Chapter 13 Volume

Theorem 13.1 If two solids are similar with a scale factor of $a : b$, then the surface areas have a ratio of $a^2 : b^2$, and the volumes have a ratio of $a^3 : b^3$. (p. 709)

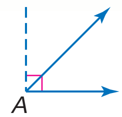
Postulates, Theorems, and Corollaries

Glossary/Glosario

English | Español

A

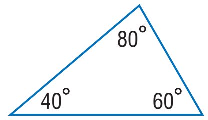
acute angle (p. 30) An angle with a degree measure less than 90.



$0 < m\angle A < 90$

ángulo agudo Ángulo cuya medida en grados es menos de 90.

acute triangle (p. 178) A triangle in which all of the angles are acute angles.



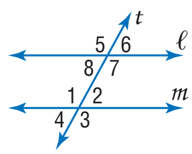
three acute angles
tres ángulos agudos

triángulo acutángulo Triángulo cuyos ángulos son todos agudos.

adjacent angles (p. 37) Two angles that lie in the same plane, have a common vertex and a common side, but no common interior points.

ángulos adyacentes Dos ángulos que yacen sobre el mismo plano, tienen el mismo vértice y un lado en común, pero ningún punto interior.

alternate exterior angles (p. 128) In the figure, transversal t intersects lines ℓ and m. $\angle 5$ and $\angle 3$, and $\angle 6$ and $\angle 4$ are alternate exterior angles.



ángulos alternos externos En la figura, la transversal t interseca las rectas ℓ y m. $\angle 5$ y $\angle 3$, y $\angle 6$ y $\angle 4$ son ángulos alternos externos.

alternate interior angles (p. 128) In the figure above, transversal t intersects lines ℓ and m. $\angle 1$ and $\angle 7$, and $\angle 2$ and $\angle 8$ are alternate interior angles.

ángulos alternos internos En la figura anterior, la transversal t interseca las rectas ℓ y m. $\angle 1$ y $\angle 7$, y $\angle 2$ y $\angle 8$ son ángulos alternos internos .

altitude **1.** (p. 241) In a triangle, a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to that side. **2.** (pp. 649, 655) In a prism or cylinder, a segment perpendicular to the bases with an endpoint in each plane. **3.** (pp. 660, 666) In a pyramid or cone, the segment that has the vertex as one endpoint and is perpendicular to the base.

altura **1.** En un triángulo, segmento trazado desde el vértice de un triángulo hasta el lado opuesto y que es perpendicular a dicho lado. **2.** El segmento perpendicular a las bases de prismas y cilindros que tiene un extremo en cada plano. **3.** El segmento que tiene un extremo en el vértice de pirámides y conos y que es perpendicular a la base.

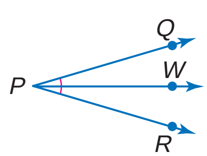
ambiguous case of the Law of Sines (p. 384) Given the measures of two sides and a nonincluded angle, there exist two possible triangles.

caso ambiguo de la ley de los senos Dadas las medidas de dos lados y de un ángulo no incluido, existen dos triángulos posibles.

angle (p. 29) The intersection of two noncollinear rays at a common endpoint. The rays are called *sides* and the common endpoint is called the *vertex*.

ángulo La intersección de dos semirrectas no colineales en un punto común. Las semirrectas se llaman *lados* y el punto común se llama *vértice.*

angle bisector (p. 32) A ray that divides an angle into two congruent angles.



$\overrightarrow{PW}$ is the bisector of $\angle P$.
$\overrightarrow{PW}$ es la bisectriz del $\angle P$.

bisectriz de un ángulo Semirrecta que divide un ángulo en dos ángulos congruentes.

Glossary/Glosario

angle of depression (p. 372) The angle between the line of sight and the horizontal when an observer looks downward.

ángulo de depresión Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia abajo.

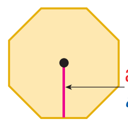
angle of elevation (p. 371) The angle between the line of sight and the horizontal when an observer looks upward.

ángulo de elevación Ángulo formado por la horizontal y la línea de visión de un observador que mira hacia arriba.

angle of rotation (p. 476) The angle through which a preimage is rotated to form the image.

ángulo de rotación El ángulo a través del cual se rota una preimagen para formar la imagen.

apothem (p. 610) A segment that is drawn from the center of a regular polygon perpendicular to a side of the polygon.



apotema Segmento perpendicular trazado desde el centro de un polígono regular hasta uno de sus lados.

arc (p. 530) A part of a circle that is defined by two endpoints.

arco Parte de un círculo definida por los dos extremos de una recta.

axis **1.** (p. 655) In a cylinder, the segment with endpoints that are the centers of the bases. **2.** (p. 666) In a cone, the segment with endpoints that are the vertex and the center of the base.

eje **1.** El segmento en un cilindro cuyos extremos forman el centro de las bases. **2.** El segmento en un cono cuyos extremos forman el vértice y el centro de la base.

B

between (p. 14) For any two points *A* and *B* on a line, there is another point *C* between *A* and *B* if and only if *A*, *B*, and *C* are collinear and $AC + CB = AB$.

ubicado entre Para cualquier par de puntos *A* y *B* de una recta, existe un punto *C* ubicado entre *A* y *B* si y sólo si *A*, *B* y *C* son colineales y $AC + CB = AB$.

biconditional (p. 81) The conjunction of a conditional statement and its converse.

bicondicional La conjunción entre un enunciado condicional y su recíproco.

C

center of rotation (p. 476) A fixed point around which shapes move in a circular motion to a new position.

centro de rotación Punto fijo alrededor del cual gira una figura hasta alcanzar una posición determinada.

central angle (p. 529) An angle that intersects a circle in two points and has its vertex at the center of the circle.

ángulo central Ángulo que interseca un círculo en dos puntos y cuyo vértice se localiza en el centro del círculo.

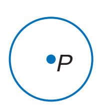
centroid (p. 240) The point of concurrency of the medians of a triangle.

centroide Punto de intersección de las medianas de un triángulo.

chord **1.** (p. 522) For a given circle, a segment with endpoints that are on the circle. **2.** (p. 671) For a given sphere, a segment with endpoints that are on the sphere.

cuerda **1.** Segmento cuyos extremos están en un círculo. **2.** Segmento cuyos extremos están en una esfera.

circle (p. 522) The locus of all points in a plane equidistant from a given point called the *center* of the circle.



P is the center of the circle.
P es el centro del círculo.

círculo Lugar geométrico formado por el conjunto de puntos en un plano, equidistantes de un punto dado llamado *centro*.

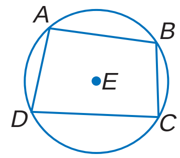
circumcenter (p. 238) The point of concurrency of the perpendicular bisectors of a triangle.

circuncentro Punto de intersección de las mediatrices de un triángulo.

circumference (p. 523) The distance around a circle.

circunferencia Distancia alrededor de un círculo.

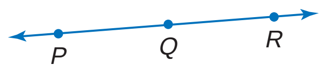
circumscribed (p. 537) A circle is circumscribed about a polygon if the circle contains all the vertices of the polygon.



$\odot E$ is circumscribed about quadrilateral *ABCD*.
$\odot E$ está circunscrito al cuadrilátero ABCD.

circunscrito Un polígono está circunscrito a un círculo si todos sus vértices están contenidos en el círculo.

collinear (p. 6) Points that lie on the same line.



P, *Q*, and *R* are collinear.
P, Q y R son colineales.

colineal Puntos que yacen en la misma recta.

column matrix (p. 506) A matrix containing one column often used to represent an ordered pair or a vector, such as $\langle x, y\rangle = \begin{bmatrix} x \\ y \end{bmatrix}$.

matriz columna Matriz formada por una sola columna y que se usa para representar pares ordenados o vectores como, por ejemplo, $\langle x, y\rangle = \begin{bmatrix} x \\ y \end{bmatrix}$.

complementary angles (p. 39) Two angles with measures that have a sum of 90.

ángulos complementarios Dos ángulos cuya suma es igual a 90 grados.

component form (p. 498) A vector expressed as an ordered pair, ⟨change in x, change in y⟩.

componente Vector representado en forma de par ordenado, ⟨cambio en x, cambio en y⟩.

composition of reflections (p. 471) Successive reflections in parallel lines.

composición de reflexiones Reflexiones sucesivas en rectas paralelas.

compound statement (p. 67) A statement formed by joining two or more statements.

enunciado compuesto Enunciado formado por la unión de dos o más enunciados.

concave polygon (p. 45) A polygon for which there is a line containing a side of the polygon that also contains a point in the interior of the polygon.

polígono cóncavo Polígono para el cual existe una recta que contiene un lado del polígono y un punto interior del polígono.

conclusion (p. 75) In a conditional statement, the statement that immediately follows the word *then*.

conclusión Parte del enunciado condicional que está escrita después de la palabra *entonces*.

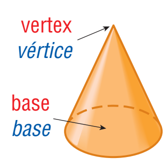
concurrent lines (p. 238) Three or more lines that intersect at a common point.

rectas concurrentes Tres o más rectas que se intersecan en un punto común.

conditional statement (p. 75) A statement that can be written in *if-then form*.

enunciado condicional Enunciado escrito en la forma *si-entonces*.

cone (p. 666) A solid with a circular base, a vertex not contained in the same plane as the base, and a lateral surface area composed of all points in the segments connecting the vertex to the edge of the base.



cono Sólido de base circular cuyo vértice no se localiza en el mismo plano que la base y cuya superficie lateral está formada por todos los segmentos que unen el vértice con los límites de la base.

congruence transformations (p. 194) A mapping for which a geometric figure and its image are congruent.

transformación de congruencia Transformación en un plano en la que la figura geométrica y su imagen son congruentes.

congruent (p. 15) Having the same measure.

congruente Que miden lo mismo.

congruent arcs (p. 530) Arcs of the same circle or congruent circles that have the same measure.

arcos congruentes Arcos de un mismo círculo, o de círculos congruentes, que tienen la misma medida.

congruent solids (p. 707) Two solids are congruent if all of the following conditions are met.
1. The corresponding angles are congruent.
2. Corresponding edges are congruent.
3. Corresponding faces are congruent.
4. The volumes are congruent.

sólidos congruentes Dos sólidos son congruentes si cumplen todas las siguientes condiciones:
1. Los ángulos correspondientes son congruentes.
2. Las aristas correspondientes son congruentes.
3. Las caras correspondientes son congruentes.
4. Los volúmenes son congruentes.

congruent triangles (p. 192) Triangles that have their corresponding parts congruent.

triángulos congruentes Triángulos cuyas partes correspondientes son congruentes.

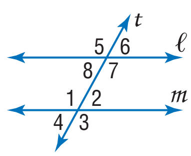
conjecture (p. 62) An educated guess based on known information.

conjetura Juicio basado en información conocida.

conjunction (p. 68) A compound statement formed by joining two or more statements with the word *and*.

conjunción Enunciado compuesto que se obtiene al unir dos o más enunciados con la palabra *y*.

consecutive interior angles (p. 128) In the figure, transversal t intersects lines ℓ and m. There are two pairs of consecutive interior angles: $\angle 8$ and $\angle 1$, and $\angle 7$ and $\angle 2$.



ángulos internos consecutivos En la figura, la transversal t interseca las rectas ℓ y m. La figura presenta dos pares de ángulos consecutivos internos: $\angle 8$ y $\angle 1$, y $\angle 7$ y $\angle 2$.

construction (p. 15) A method of creating geometric figures without the benefit of measuring tools. Generally, only a pencil, straightedge, and compass are used.

construcción Método para dibujar figuras geométricas sin el uso de instrumentos de medición. En general, sólo requiere de un lápiz, una regla sin escala y un compás.

contrapositive (p. 77) The statement formed by negating both the hypothesis and conclusion of the converse of a conditional statement.

antítesis Enunciado formado por la negación de la hipótesis y la conclusión del recíproco de un enunciado condicional dado.

converse (p. 77) The statement formed by exchanging the hypothesis and conclusion of a conditional statement.

recíproco Enunciado que se obtiene al intercambiar la hipótesis y la conclusión de un enunciado condicional dado.

convex polygon (p. 45) A polygon for which there is no line that contains both a side of the polygon and a point in the interior of the polygon.

polígono convexo Polígono para el cual no existe recta alguna que contenga un lado del polígono y un punto en el interior del polígono.

coordinate proof (p. 222) A proof that uses figures in the coordinate plane and algebra to prove geometric concepts.

prueba de coordenadas Demostración que usa álgebra y figuras en el plano de coordenadas para demostrar conceptos geométricos.

coplanar (p. 6) Points that lie in the same plane.

coplanar Puntos que yacen en un mismo plano.

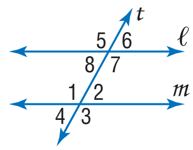
corner view (p. 636) The view from a corner of a three-dimensional figure, also called the *perspective view.*

vista de esquina Vista de una figura tridimensional desde una esquina. También se conoce como *vista de perspectiva.*

corollary (p. 188) A statement that can be easily proved using a theorem is called a corollary of that theorem.

corolario La afirmación que puede demostrarse fácilmente mediante un teorema se conoce como corolario de dicho teorema.

corresponding angles (p. 128) In the figure, transversal t intersects lines ℓ and m. There are four pairs of corresponding angles: $\angle 5$ and $\angle 1$, $\angle 8$ and $\angle 4$, $\angle 6$ and $\angle 2$, and $\angle 7$ and $\angle 3$.



ángulos correspondientes En la figura, la transversal t interseca las rectas ℓ y m. La figura muestra cuatro pares de ángulos correspondientes: $\angle 5$ y $\angle 1$, $\angle 8$ y $\angle 4$, $\angle 6$ y $\angle 2$, y $\angle 7$ y $\angle 3$.

cosine (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg adjacent to the acute angle to the measure of the hypotenuse.

coseno Para un ángulo agudo de un triángulo rectángulo, la razón entre la medida del cateto adyacente al ángulo agudo y la medida de la hipotenusa de un triángulo rectángulo.

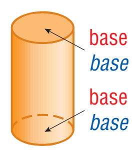
counterexample (p. 63) An example used to show that a given statement is not always true.

contraejemplo Ejemplo que se usa para demostrar que un enunciado dado no siempre es verdadero.

cross products (p. 283) In the proportion $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, the cross products are ad and bc. The proportion is true if and only if the cross products are equal.

productos cruzados En la proporción, $\frac{a}{b} = \frac{c}{d}$, donde $b \neq 0$ y $d \neq 0$, los productos cruzados son ad y bc. La proporción es verdadera si y sólo si los productos cruzados son iguales.

cylinder (p. 638) A figure with bases that are formed by congruent circles in parallel planes.



cilindro Figura cuyas bases son círculos congruentes localizados en planos paralelos.

D

deductive argument (p. 94) A proof formed by a group of algebraic steps used to solve a problem.

argumento deductivo Demostración que consta del conjunto de pasos algebraicos que se usan para resolver un problema.

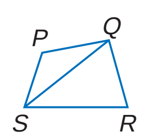
deductive reasoning (p. 82) A system of reasoning that uses facts, rules, definitions, or properties to reach logical conclusions.

razonamiento deductivo Sistema de razonamiento que emplea hechos, reglas, definiciones y propiedades para obtener conclusiones lógicas.

degree (p. 29) A unit of measure used in measuring angles and arcs. An arc of a circle with a measure of 1° is $\frac{1}{360}$ of the entire circle.

grado Unidad de medida que se usa para medir ángulos y arcos. El arco de un círculo que mide 1° equivale a $\frac{1}{360}$ del círculo completo.

diagonal (p. 404) In a polygon, a segment that connects nonconsecutive vertices of the polygon.



$\overline{SQ}$ is a diagonal.
$\overline{SQ}$ es una diagonal.

diagonal Recta que une vértices no consecutivos de un polígono.

diameter **1.** (p. 522) In a circle, a chord that passes through the center of the circle. **2.** (p. 671) In a sphere, a segment that contains the center of the sphere, and has endpoints that are on the sphere.

diámetro **1.** Cuerda que pasa por el centro de un círculo. **2.** Segmento que incluye el centro de una esfera y cuyos extremos se localizan en la esfera.

dilation (p. 490) A transformation determined by a center point C and a scale factor k. When $k > 0$, the image P' of P is the point on $\overrightarrow{CP}$ such that $CP' = |k| \cdot CP$. When $k < 0$, the image P' of P is the point on the ray opposite $\overrightarrow{CP}$ such that $CP' = k \cdot CP$.

dilatación Transformación determinada por un punto central C y un factor de escala k. Cuando $k > 0$, la imagen P' de P es el punto en $\overrightarrow{CP}$ tal que $CP' = |k| \cdot CP$. Cuando $k < 0$, la imagen P' de P es el punto en la semirrecta opuesta $\overrightarrow{CP}$ tal que $CP' = k \cdot CP$.

direct isometry (p. 481) An isometry in which the image of a figure is found by moving the figure intact within the plane.

isometría directa Isometría en la cual se obtiene la imagen de una figura, al mover la figura intacta junto con su plano.

direction (p. 498) The measure of the angle that a vector forms with the positive x-axis or any other horizontal line.

dirección Medida del ángulo que forma un vector con el eje positivo x o con cualquier otra recta horizontal.

disjunction (p. 68) A compound statement formed by joining two or more statements with the word *or*.

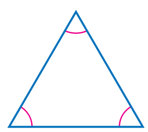
disyunción Enunciado compuesto que se forma al unir dos o más enunciados con la palabra *o*.

E

equal vectors (p. 499) Vectors that have the same magnitude and direction.

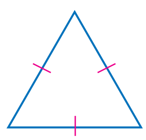
vectores iguales Vectores que poseen la misma magnitud y dirección.

equiangular triangle (p. 178) A triangle with all angles congruent.



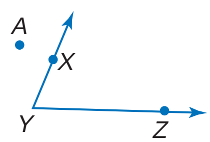
triángulo equiangular Triángulo cuyos ángulos son congruentes entre sí.

equilateral triangle (p. 179) A triangle with all sides congruent.



triángulo equilátero Triángulo cuyos lados son congruentes entre sí.

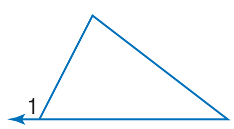
exterior (p. 29) A point is in the exterior of an angle if it is neither on the angle nor in the interior of the angle.



A is in the exterior of $\angle XYZ$.
A está en el exterior del $\angle XYZ$.

exterior Un punto yace en el exterior de un ángulo si no se localiza ni en el ángulo ni en el interior del ángulo.

exterior angle (p. 186) An angle formed by one side of a triangle and the extension of another side.



$\angle 1$ is an exterior angle.
$\angle 1$ es un ángulo externo.

ángulo externo Ángulo formado por un lado de un triángulo y la extensión de otro de sus lados.

extremes (p. 283) In $\frac{a}{b} = \frac{c}{d}$, the numbers a and d.

extremos Los números a y d en $\frac{a}{b} = \frac{c}{d}$.

F

flow proof (p. 187) A proof that organizes statements in logical order, starting with the given statements. Each statement is written in a box with the reason verifying the statement written below the box. Arrows are used to indicate the order of the statements.

demostración de flujo Demostración en que se ordenan los enunciados en orden lógico, empezando con los enunciados dados. Cada enunciado se escribe en una casilla y debajo de cada casilla se escribe el argumento que verifica el enunciado. El orden de los enunciados se indica mediante flechas.

fractal (p. 325) A figure generated by repeating a special sequence of steps infinitely often. Fractals often exhibit self-similarity.

fractal Figura que se obtiene mediante la repetición infinita de una sucesión particular de pasos. Los fractales a menudo exhiben autosemejanza.

G

geometric mean (p. 342) For any positive numbers a and b, the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

media geométrica Para todo número positivo a y b, existe un número positivo x tal que $\frac{a}{x} = \frac{x}{b}$.

geometric probability (p. 622) Using the principles of length and area to find the probability of an event.

probabilidad geométrica El uso de los principios de longitud y área para calcular la probabilidad de un evento.

glide reflection (p. 475) A composition of a translation and a reflection in a line parallel to the direction of the translation.

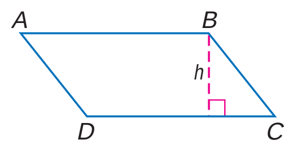
reflexión de deslizamiento Composición que consta de una traslación y una reflexión realizadas sobre una recta paralela a la dirección de la traslación.

great circle (p. 671) For a given sphere, the intersection of the sphere and a plane that contains the center of the sphere.

círculo máximo La intersección entre una esfera dada y un plano que contiene el centro de la esfera.

H

height of a parallelogram (p. 595) The length of an altitude of a parallelogram.



h is the height of parallelogram *ABCD.*
H es la altura del paralelogramo ABCD.

altura de un paralelogramo La longitud de la altura de un paralelogramo.

hemisphere (p. 672) One of the two congruent parts into which a great circle separates a sphere.

hemisferio Cada una de las dos partes congruentes en que un círculo máximo divide una esfera.

hypothesis (p. 75) In a conditional statement, the statement that immediately follows the word *if*.

hipótesis El enunciado escrito a continuación de la palabra *si* en un enunciado condicional.

I

if-then statement (p. 75) A compound statement of the form “if A, then B”, where A and B are statements.

enunciado si-entonces Enunciado compuesto de la forma “si A, entonces B”, donde A y B son enunciados.

incenter (p. 240) The point of concurrency of the angle bisectors of a triangle.

incentro Punto de intersección de las bisectrices interiores de un triángulo.

included angle (p. 201) In a triangle, the angle formed by two sides is the included angle for those two sides.

ángulo incluido En un triángulo, el ángulo formado por dos lados cualesquiera del triángulo es el ángulo incluido de esos dos lados.

included side (p. 207) The side of a triangle that is a side of each of two angles.

lado incluido El lado de un triángulo que es común a de sus dos ángulos.

indirect isometry (p. 481) An isometry that cannot be performed by maintaining the orientation of the points, as in a direct isometry.

isometría indirecta Tipo de isometría que no se puede obtener manteniendo la orientación de los puntos, como ocurre durante la isometría directa.

indirect proof (p. 255) In an indirect proof, one assumes that the statement to be proved is false. One then uses logical reasoning to deduce that a statement contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

demostración indirecta En una demostración indirecta, se asume que el enunciado por demostrar es falso. Después, se deduce lógicamente que existe un enunciado que contradice un postulado, un teorema o una de las conjeturas. Una vez hallada una contradicción, se concluye que el enunciado que se suponía falso debe ser, en realidad, verdadero.

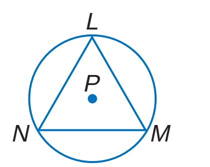
indirect reasoning (p. 255) Reasoning that assumes that the conclusion is false and then shows that this assumption leads to a contradiction of the hypothesis or some other accepted fact, like a postulate, theorem, or corollary. Then, since the assumption has been proved false, the conclusion must be true.

razonamiento indirecto Razonamiento en que primero se asume que la conclusión es falsa y, después, se demuestra que esto contradice la hipótesis o un hecho aceptado como un postulado, un teorema o un corolario. Finalmente, dado que se ha demostrado que la conjetura es falsa, entonces la conclusión debe ser verdadera.

inductive reasoning (p. 62) Reasoning that uses a number of specific examples to arrive at a plausible generalization or prediction. Conclusions arrived at by inductive reasoning lack the logical certainty of those arrived at by deductive reasoning.

razonamiento inductivo Razonamiento que usa varios ejemplos específicos para lograr una generalización o una predicción creíble. Las conclusiones obtenidas mediante el razonamiento inductivo carecen de la certidumbre lógica de aquellas obtenidas mediante el razonamiento deductivo.

inscribed (p. 537) A polygon is inscribed in a circle if each of its vertices lie on the circle.



$\triangle LMN$ is inscribed in $\odot P$.
$\triangle LMN$ *está inscrito en* $\odot P$.

inscrito Un polígono está inscrito en un círculo si todos sus vértices yacen en el círculo.

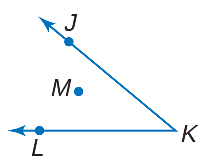
intercepted (p. 544) An angle intercepts an arc if and only if each of the following conditions are met.

1. The endpoints of the arc lie on the angle.
2. All points of the arc except the endpoints are in the interior of the circle.
3. Each side of the angle contains an endpoint of the arc.

intersecado Un ángulo interseca un arco si y sólo si se cumplen todas las siguientes condiciones.

1. Los extremos del arco yacen en el ángulo.
2. Todos los puntos del arco, exceptuando sus extremos, yacen en el interior del círculo.
3. Cada lado del ángulo contiene un extremo del arco.

interior (p. 29) A point is in the interior of an angle if it does not lie on the angle itself and it lies on a segment with endpoints that are on the sides of the angle.



M is in the interior of $\angle JKL$.
M *está en el interior del* $\angle JKL$.

interior Un punto se localiza en el interior de un ángulo, si no yace en el ángulo mismo y si está en un segmento cuyos extremos yacen en los lados del ángulo.

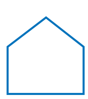
inverse (p. 77) The statement formed by negating both the hypothesis and conclusion of a conditional statement.

inversa Enunciado que se obtiene al negar la hipótesis y la conclusión de un enunciado condicional.

irregular figure (p. 617) A figure that cannot be classified as a single polygon.

figura irregular Figura que no se puede clasificar como un solo polígono.

irregular polygon (p. 618) A polygon that is not regular.

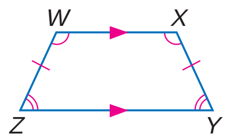


polígono irregular Polígono que no es regular.

isometry (p. 463) A mapping for which the original figure and its image are congruent.

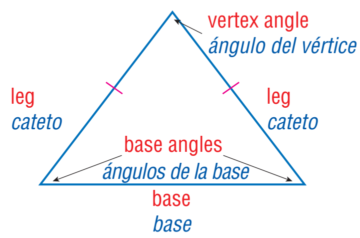
isometría Transformación en que la figura original y su imagen son congruentes.

isosceles trapezoid (p. 439) A trapezoid in which the legs are congruent, both pairs of base angles are congruent, and the diagonals are congruent.



trapecio isósceles Trapecio cuyos catetos son congruentes, ambos pares de ángulos son congruentes y las diagonales son congruentes.

isosceles triangle (p. 179) A triangle with at least two sides congruent. The congruent sides are called *legs*. The angles opposite the legs are *base angles*. The angle formed by the two legs is the *vertex angle*. The side opposite the vertex angle is the *base*.



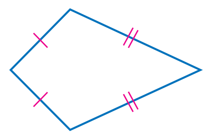
triángulo isósceles Triángulo que tiene por lo menos dos lados congruentes. Los lados congruentes se llaman *catetos*. Los ángulos opuestos a los catetos son los *ángulos de la base*. El ángulo formado por los dos catetos es el *ángulo del vértice*. Los lados opuestos al ángulo del vértice forman la *base*.

iteration (p. 325) A process of repeating the same procedure over and over again.

iteración Proceso de repetir el mismo procedimiento una y otra vez.

K

kite (p. 438) A quadrilateral with exactly two distinct pairs of adjacent congruent sides.



cometa Cuadrilátero que tiene exactamente dospares de lados congruentes adyacentes distintivos.

L

lateral area (p. 649) For prisms, pyramids, cylinders, and cones, the area of the figure, not including the bases.

área lateral En prismas, pirámides, cilindros y conos, es el área de la figura, sin incluir el área de las bases.

lateral edges **1.** (p. 649) In a prism, the intersection of two adjacent lateral faces. **2.** (p. 660) In a pyramid, lateral edges are the edges of the lateral faces that join the vertex to vertices of the base.

aristas laterales **1.** En un prisma, la intersección de dos caras laterales adyacentes. **2.** En una pirámide, las aristas de las caras laterales que unen el vértice de la pirámide con los vértices de la base.

lateral faces **1.** (p. 649) In a prism, the faces that are not bases. **2.** (p. 660) In a pyramid, faces that intersect at the vertex.

caras laterales **1.** En un prisma, las caras que no forman las bases. **2.** En una pirámide, las caras que se intersecan en el vértice.

Law of Cosines (p. 385) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively. Then the following equations are true.
$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

ley de los cosenos Sea $\triangle ABC$ cualquier triángulo donde a, b y c son las medidas de los lados opuestos a los ángulos que miden A, B y C respectivamente. Entonces las siguientes ecuaciones son ciertas.
$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

Law of Detachment (p. 82) If $p \rightarrow q$ is a true conditional and p is true, then q is also true.

ley de indiferencia Si $p \rightarrow q$ es un enunciado condicional verdadero y p es verdadero, entonces q es verdadero también.

Glossary/Glosario

Law of Sines (p. 377) Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite the angles with measures A, B, and C respectively. Then, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

ley de los senos Sea $\triangle ABC$ cualquier triángulo donde a, b y c representan las medidas de los lados opuestos a los ángulos A, B y C respectivamente. Entonces, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Law of Syllogism (p. 83) If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

ley del silogismo Si $p \rightarrow q$ y $q \rightarrow r$ son enunciados condicionales verdaderos, entonces $p \rightarrow r$ también es verdadero.

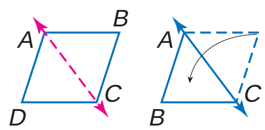
line (p. 6) A basic undefined term of geometry. A line is made up of points and has no thickness or width. In a figure, a line is shown with an arrowhead at each end. Lines are usually named by lowercase script letters or by writing capital letters for two points on the line, with a double arrow over the pair of letters.

recta Término primitivo en geometría. Una recta está formada por puntos y carece de grosor o ancho. En una figura, una recta se representa con una flecha en cada extremo. Por lo general, se designan con letras minúsculas o con las dos letras mayúsculas de dos puntos sobre la línea. Se escribe una flecha doble sobre el par de letras mayúsculas.

line of reflection (p. 463) A line through a figure that separates the figure into two mirror images.

línea de reflexión Línea que divide una figura en dos imágenes especulares.

line of symmetry (p. 466) A line that can be drawn through a plane figure so that the figure on one side is the reflection image of the figure on the opposite side.



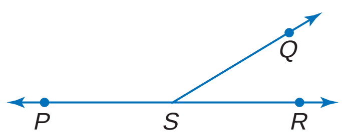
$\overleftrightarrow{AC}$ is a line of symmetry.
$\overleftrightarrow{AC}$ es un eje de simetría.

eje de simetría Recta que se traza a través de una figura plana, de modo que un lado de la figura es la imagen reflejada del lado opuesto.

line segment (p. 13) A measurable part of a line that consists of two points, called endpoints, and all of the points between them.

segmento de recta Sección medible de una recta. Consta de dos puntos, llamados extremos, y todos los puntos localizados entre ellos.

linear pair (p. 37) A pair of adjacent angles whose non-common sides are opposite rays.



$\angle PSQ$ and $\angle QSR$ are a linear pair.
$\angle PSQ$ y $\angle QSR$ forman un par lineal.

par lineal Par de ángulos adyacentes cuyos lados no comunes forman semirrectas opuestas.

locus (p. 11) The set of points that satisfy a given condition.

lugar geométrico Conjunto de puntos que satisfacen una condición dada.

logically equivalent (p. 77) Statements that have the same truth values.

equivalente lógico Enunciados que poseen el mismo valor de verdad.

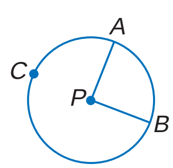
M

magnitude (p. 498) The length of a vector.

magnitud La longitud de un vector.

major arc (p. 530) An arc with a measure greater than 180.

$\widehat{ACB}$ is a major arc.



arco mayor Arco que mide más de 180°.

$\widehat{ACB}$ es un arco mayor.

matrix logic (p. 88) A method of deductive reasoning that uses a table to solve problems.

lógica matricial Método de razonamiento deductivo que utiliza una tabla para resolver problemas.

means (p. 283) In $\frac{a}{b} = \frac{c}{d}$, the numbers b and c.

medios Los números b y c en la proporción $\frac{a}{b} = \frac{c}{d}$.

median **1.** (p. 240) In a triangle, a line segment with endpoints that are a vertex of a triangle and the midpoint of the side opposite the vertex. **2.** (p. 440) In a trapezoid, the segment that joins the midpoints of the legs.

mediana **1.** Segmento de recta de un triángulo cuyos extremos son un vértice del triángulo y el punto medio del lado opuesto a dicho vértice. **2.** Segmento que une los puntos medios de los catetos de un trapecio.

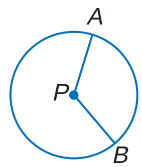
midpoint (p. 22) The point halfway between the endpoints of a segment.

punto medio Punto que es equidistante entre los extremos de un segmento.

midsegment (p. 308) A segment with endpoints that are the midpoints of two sides of a triangle.

segmento medio Segmento cuyos extremos son los puntos medios de dos lados de un triángulo.

minor arc (p. 530) An arc with a measure less than 180. $\overset{\frown}{AB}$ is a minor arc.



arco menor Arco que mide menos de 180°. $\overset{\frown}{AB}$ es un arco menor.

N

negation (p. 67) If a statement is represented by p, then *not* p is the negation of the statement.

negación Si p representa un enunciado, entonces *no* p representa la negación del enunciado.

net (p. 644) A two-dimensional figure that when folded forms the surfaces of a three-dimensional object.

red Figura bidimensional que al ser plegada forma las superficies de un objeto tridimensional.

***n*-gon** (p. 46) A polygon with n sides.

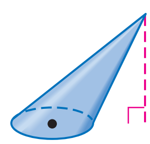
enágono Polígono con n lados.

non-Euclidean geometry (p. 165) The study of geometrical systems that are not in accordance with the Parallel Postulate of Euclidean geometry.

geometría no euclidiana El estudio de sistemas geométricos que no satisfacen el Postulado de las Paralelas de la geometría euclidiana.

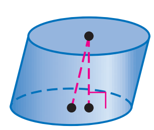
O

oblique cone (p. 666) A cone that is not a right cone.



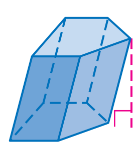
cono oblicuo Cono que no es un cono recto.

oblique cylinder (p. 655) A cylinder that is not a right cylinder.



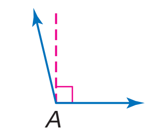
cilindro oblicuo Cilindro que no es un cilindro recto.

oblique prism (p. 649) A prism in which the lateral edges are not perpendicular to the bases.



prisma oblicuo Prisma cuyas aristas laterales no son perpendiculares a las bases.

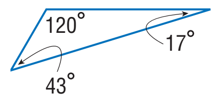
obtuse angle (p. 30) An angle with degree measure greater than 90 and less than 180.



$90 < m\angle A < 180$

ángulo obtuso Ángulo que mide más de 90° y menos de 180°.

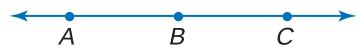
obtuse triangle (p. 178) A triangle with an obtuse angle.



one obtuse angle
un ángulo obtuso

triángulo obtusángulo Triángulo que tiene un ángulo obtuso.

opposite rays (p. 29) Two rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$ such that B is between A and C.



semirrectas opuestas Dos semirrectas $\overrightarrow{BA}$ y $\overrightarrow{BC}$ tales que B se localiza entre A y C.

ordered triple (p. 714) Three numbers given in a specific order used to locate points in space.

triple ordenado Tres números dados en un orden específico que sirven para ubicar puntos en el espacio.

orthocenter (p. 240) The point of concurrency of the altitudes of a triangle.

ortocentro Punto de intersección de las alturas de un triángulo.

orthogonal drawing (p. 636) The two-dimensional top view, left view, front view, and right view of a three-dimensional object.

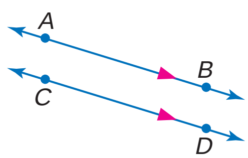
vista ortogonal Vista bidimensional desde arriba, desde la izquierda, desde el frente o desde la derecha de un cuerpo tridimensional.

P

paragraph proof (p. 90) An informal proof written in the form of a paragraph that explains why a conjecture for a given situation is true.

demostración de párrafo Demostración informal escrita en forma de párrafo que explica por qué una conjetura acerca de una situación dada es verdadera.

parallel lines (p. 126) Coplanar lines that do not intersect.



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

rectas paralelas Rectas coplanares que no se intersecan.

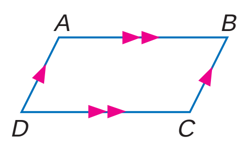
parallel planes (p. 126) Planes that do not intersect.

planos paralelos Planos que no se intersecan.

parallel vectors (p. 499) Vectors that have the same or opposite direction.

vectores paralelos Vectores que tienen la misma dirección o la dirección opuesta.

parallelogram (p. 411) A quadrilateral with parallel opposite sides. Any side of a parallelogram may be called a *base*.



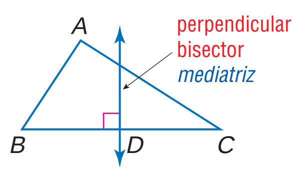
$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$

paralelogramo Cuadrilátero cuyos lados opuestos son paralelos entre sí. Cualquier lado del paralelogramo puede ser la *base*.

perimeter (p. 46) The sum of the lengths of the sides of a polygon.

perímetro La suma de la longitud de los lados de un polígono.

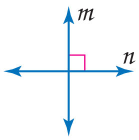
perpendicular bisector (p. 238) In a triangle, a line, segment, or ray that passes through the midpoint of a side and is perpendicular to that side.



D is the midpoint of $\overline{BC}$.
D es el punto medio de $\overline{BC}$.

mediatriz Recta, segmento o semirrecta que atraviesa el punto medio del lado de un triángulo y que es perpendicular a dicho lado.

perpendicular lines (p. 40) Lines that form right angles.



line $m \perp$ line n
recta $m \perp$ recta n

rectas perpendiculares Rectas que forman ángulos rectos.

perspective view (p. 636) The view of a three-dimensional figure from the corner.

vista de perspectiva Vista de una figura tridimensional desde una de sus esquinas.

pi (π) (p. 524) An irrational number represented by the ratio of the circumference of a circle to the diameter of the circle.

pi (π) Número irracional representado por la razón entre la circunferencia de un círculo y su diámetro.

plane (p. 6) A basic undefined term of geometry. A plane is a flat surface made up of points that has no depth and extends indefinitely in all directions. In a figure, a plane is often represented by a shaded, slanted 4-sided figure. Planes are usually named by a capital script letter or by three noncollinear points on the plane.

plano Término primitivo en geometría. Es una superficie formada por puntos y sin profundidad que se extiende indefinidamente en todas direcciones. Los planos a menudo se representan con un cuadrilátero inclinado y sombreado. Los planos en general se designan con una letra mayúscula o con tres puntos no colineales del plano.

plane Euclidean geometry (p. 165) Geometry based on Euclid's axioms dealing with a system of points, lines, and planes.

geometría del plano euclidiano Geometría basada en los axiomas de Euclides, los que integran un sistema de puntos, rectas y planos.

Platonic Solids (p. 637) The five regular polyhedra: tetrahedron, hexahedron, octahedron, dodecahedron, or icosahedron.

sólidos platónicos Cualquiera de los siguientes cinco poliedros regulares: tetraedro, hexaedro, octaedro, dodecaedro e icosaedro.

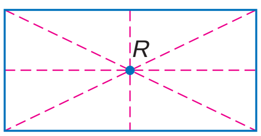
point (p. 6) A basic undefined term of geometry. A point is a location. In a figure, points are represented by a dot. Points are named by capital letters.

punto Término primitivo en geometría. Un punto representa un lugar o localización. En una figura, se representa con una marca puntual. Los puntos se designan con letras mayúsculas.

point of concurrency (p. 238) The point of intersection of concurrent lines.

punto de concurrencia Punto de intersección de rectas concurrentes.

point of symmetry (p. 466) The common point of reflection for all points of a figure.



R is a point of symmetry.
R es un punto de simetría.

punto de simetría El punto común de reflexión de todos los puntos de una figura.

point of tangency (p. 552) For a line that intersects a circle in only one point, the point at which they intersect.

punto de tangencia Punto de intersección de una recta que interseca un círculo en un solo punto, el punto en donde se intersecan.

point-slope form (p. 145) An equation of the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of any point on the line and m is the slope of the line.

forma punto-pendiente Ecuación de la forma $y - y_1 = m(x - x_1)$, donde (x_1, y_1) representan las coordenadas de un punto cualquiera sobre la recta y m representa la pendiente de la recta.

polygon (p. 45) A closed figure formed by a finite number of coplanar segments called *sides* such that the following conditions are met.
1. The sides that have a common endpoint are noncollinear.
2. Each side intersects exactly two other sides, but only at their endpoints, called the *vertices*.

polígono Figura cerrada formada por un número finito de segmentos coplanares llamados *lados*, y que satisface las siguientes condiciones:
1. Los lados que tienen un extremo común son no colineales.
2. Cada lado interseca exactamente dos lados, pero sólo en sus extremos, formando los *vértices*.

polyhedrons (p. 637) Closed three-dimensional figures made up of flat polygonal regions. The flat regions formed by the polygons and their interiors are called *faces*. Pairs of faces intersect in segments called *edges*. Points where three or more edges intersect are called *vertices*.

poliedro Figura tridimensional cerrada formada por regiones poligonales planas. Las regiones planas definidas por un polígono y sus interiores se llaman *caras*. Cada intersección entre dos caras se llama *arista*. Los puntos donde se intersecan tres o más aristas se llaman *vértices*.

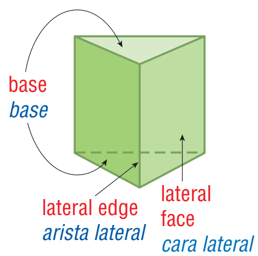
postulate (p. 89) A statement that describes a fundamental relationship between the basic terms of geometry. Postulates are accepted as true without proof.

postulado Enunciado que describe una relación fundamental entre los términos primitivos de geometría. Los postulados se aceptan como verdaderos sin necesidad de demostración.

precision (p. 14) The precision of any measurement depends on the smallest unit available on the measuring tool.

precisión La precisión de una medida depende de la unidad de medida más pequeña del instrumento de medición.

prism (p. 637) A solid with the following characteristics.
1. Two faces, called *bases*, are formed by congruent polygons that lie in parallel planes.
2. The faces that are not bases, called *lateral faces*, are formed by parallelograms.
3. The intersections of two adjacent lateral faces are called *lateral edges* and are parallel segments.



triangular prism
prisma triangular

prisma Sólido que posee las siguientes características:
1. Tiene dos caras llamadas *bases*, formadas por polígonos congruentes que yacen en planos paralelos.
2. Las caras que no son las bases, llamadas *caras laterales*, son formadas por paralelogramos.
3. Las intersecciones de dos aristas laterales adyacentes se llaman *aristas laterales* y son segmentos paralelos.

proof (p. 90) A logical argument in which each statement you make is supported by a statement that is accepted as true.

demostración Argumento lógico en que cada enunciado está basado en un enunciado que se acepta como verdadero.

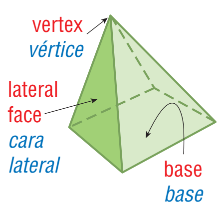
proof by contradiction (p. 255) An indirect proof in which one assumes that the statement to be proved is false. One then uses logical reasoning to deduce a statement that contradicts a postulate, theorem, or one of the assumptions. Once a contradiction is obtained, one concludes that the statement assumed false must in fact be true.

demostración por contradicción Demostración indirecta en que se asume que el enunciado que se va a demostrar es falso. Después, se razona lógicamente para deducir un enunciado que contradiga un postulado, un teorema o una de las conjeturas. Una vez que se obtiene una contradicción, se concluye que el enunciado que se supuso falso es, en realidad, verdadero.

proportion (p. 283) An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that two ratios are equal.

proporción Ecuación de la forma $\frac{a}{b} = \frac{c}{d}$ que establece que dos razones son iguales.

pyramid (p. 637) A solid with the following characteristics.
1. All of the faces, except one face, intersect at a point called the *vertex*.
2. The face that does not contain the vertex is called the *base* and is a polygonal region.
3. The faces meeting at the vertex are called *lateral faces* and are triangular regions.



rectangular pyramid
pirámide rectangular

pirámide Sólido con las siguientes características:
1. Todas, excepto una de las caras, se intersecan en un punto llamado *vértice*.
2. La cara que no contiene el vértice se llama *base* y es una región poligonal.
3. Las caras que se encuentran en los vértices se llaman *caras laterales* y son regiones triangulares.

Glossary/Glosario

Pythagorean identity (p. 391) The identity $\cos^2\theta + \sin^2\theta = 1$.

identidad pitagórica La identidad $\cos^2\theta + \sin^2\theta = 1$.

Pythagorean triple (p. 352) A group of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the greatest number.

triplete de Pitágoras Grupo de tres números enteros que satisfacen la ecuación $a^2 + b^2 = c^2$, donde c es el número más grande.

R

radius **1.** (p. 522) In a circle, any segment with endpoints that are the center of the circle and a point on the circle. **2.** (p. 671) In a sphere, any segment with endpoints that are the center and a point on the sphere.

radio **1.** Cualquier segmento cuyos extremos están en el centro de un círculo y en un punto cualquiera del mismo. **2.** Cualquier segmento cuyos extremos forman el centro y en punto de una esfera.

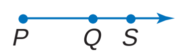
rate of change (p. 140) Describes how a quantity is changing over time.

tasa de cambio Describe cómo cambia una cantidad a través del tiempo.

ratio (p. 282) A comparison of two quantities.

razón Comparación entre dos cantidades.

ray (p. 29) $\overrightarrow{PQ}$ is a ray if it is the set of points consisting of $\overline{PQ}$ and all points S for which Q is between P and S.

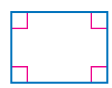


semirrecta $\overrightarrow{PQ}$ es una semirrecta si consta del conjunto de puntos formado por $\overline{PQ}$ y todos los S puntos S para los que Q se localiza entre P y S.

reciprocal identity (p. 391) Each of the three trigonometric ratios called *cosecant*, *secant*, and *cotangent*, that are the reciprocals of sine, cosine, and tangent, respectively.

identidad recíproca Cada una de las tres razones trigonométricas llamadas *cosecante*, *secante* y *tangente* y que son los recíprocos del seno, el coseno y la tangente, respectivamente

rectangle (p. 424) A quadrilateral with four right angles.



rectángulo Cuadrilátero que tiene cuatro ángulos rectos.

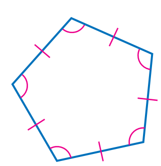
reflection (p. 463) A transformation representing a flip of the figure over a point, line, or plane.

reflexión Transformación que se obtiene cuando se "voltea" una imagen sobre un punto, una línea o un plano.

reflection matrix (p. 507) A matrix that can be multiplied by the vertex matrix of a figure to find the coordinates of the reflected image.

matriz de reflexión Matriz que al ser multiplicada por la matriz de vértices de una figura permite hallar las coordenadas de la imagen reflejada.

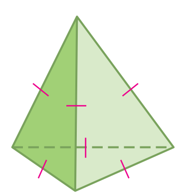
regular polygon (p. 46) A convex polygon in which all of the sides are congruent and all of the angles are congruent.



regular pentagon
pentágono regular

polígono regular Polígono convexo en el que todos los lados y todos los ángulos son congruentes entre sí.

regular polyhedron (p. 637) A polyhedron in which all of the faces are regular congruent polygons.



poliedro regular Poliedro cuyas caras son polígonos regulares congruentes.

regular prism (p. 637) A right prism with bases that are regular polygons.

prisma regular Prisma recto cuyas bases son polígonos regulares.

regular tessellation (p. 484) A tessellation formed by only one type of regular polygon.

teselado regular Teselado formado por un solo tipo de polígono regular.

related conditionals (p. 77) Statements such as the converse, inverse, and contrapositive that are based on a given conditional statement.

enunciados condicionales relacionados Enunciados tales como el recíproco, la inversa y la antítesis que están basados en un enunciado condicional dado.

relative error (p. 19) The ratio of the half-unit difference in precision to the entire measure, expressed as a percent.

error relativo La razón entre la mitad de la unidad más precisa de la medición y la medición completa, expresada en forma de porcentaje.

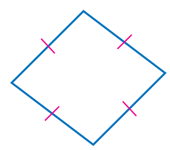
remote interior angles (p. 186) The angles of a triangle that are not adjacent to a given exterior angle.

ángulos internos no adyacentes Ángulos de un triángulo que no son adyacentes a un ángulo exterior dado.

resultant (p. 500) The sum of two vectors.

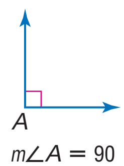
resultante La suma de dos vectores.

rhombus (p. 431) A quadrilateral with all four sides congruent.



rombo Cuadrilátero cuyos cuatro lados son congruentes.

right angle (p. 30) An angle with a degree measure of 90.



ángulo recto Ángulo cuya medida en grados es 90.

right cone (p. 666) A cone with an axis that is also an altitude.

cono recto Cono cuyo eje es también su altura.

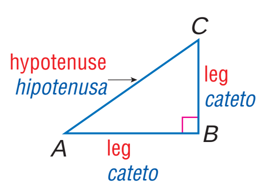
right cylinder (p. 655) A cylinder with an axis that is also an altitude.

cilindro recto Cilindro cuyo eje es también su altura.

right prism (p. 649) A prism with lateral edges that are also altitudes.

prisma recto Prisma cuyas aristas laterales también son su altura.

right triangle (p. 178) A triangle with a right angle. The side opposite the right angle is called the *hypotenuse*. The other two sides are called legs.



triángulo rectángulo Triángulo con un ángulo recto. El lado opuesto al ángulo recto se conoce como *hipotenusa*. Los otros dos lados se llaman catetos.

rotation (p. 476) A transformation that turns every point of a preimage through a specified angle and direction about a fixed point, called the *center of rotation*.

rotación Transformación en que se hace girar cada punto de la preimagen a través de un ángulo y una dirección determinadas alrededor de un punto, conocido como *centro de rotación*.

rotation matrix (p. 507) A matrix that can be multiplied by the vertex matrix of a figure to find the coordinates of the rotated image.

matriz de rotación Matriz que al ser multiplicada por la matriz de vértices de la figura permite calcular las coordenadas de la imagen rotada.

rotational symmetry (p. 478) If a figure can be rotated less than 360° about a point so that the image and the preimage are indistinguishable, the figure has rotational symmetry.

simetría de rotación Si se puede rotar una imagen menos de 360° alrededor de un punto y la imagen y la preimagen son idénticas, entonces la figura presenta simetría de rotación.

S

scalar (p. 501) A constant multiplied by a vector.

escalar Una constante multiplicada por un vector.

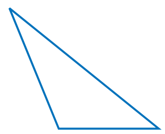
scalar multiplication (p. 501) Multiplication of a vector by a scalar.

multiplicación escalar Multiplicación de un vector por una escalar.

scale factor (p. 290) The ratio of the lengths of two corresponding sides of two similar polygons or two similar solids.

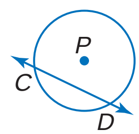
factor de escala La razón entre las longitudes de dos lados correspondientes de dos polígonos o sólidos semejantes.

scalene triangle (p. 179) A triangle with no two sides congruent.



triángulo escaleno Triángulo cuyos lados no son congruentes.

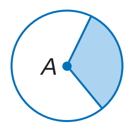
secant (p. 561) Any line that intersects a circle in exactly two points.



$\overleftrightarrow{CD}$ is a secant of $\odot P$.
$\overleftrightarrow{CD}$ *es una secante de* $\odot P$.

secante Cualquier recta que interseca un círculo exactamente en dos puntos.

sector of a circle (p. 623) A region of a circle bounded by a central angle and its intercepted arc.



The shaded region is a sector of $\odot A$.
La región sombreada es un sector de $\odot A$.

sector de un círculo Región de un círculo que está limitada por un ángulo central y el arco que interseca.

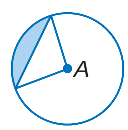
segment (p. 13) *See* line segment.

segmento *Ver* segmento de recta.

segment bisector (p. 24) A segment, line, or plane that intersects a segment at its midpoint.

bisectriz de segmento Segmento, recta o plano que interseca un segmento en su punto medio.

segment of a circle (p. 624) The region of a circle bounded by an arc and a chord.



The shaded region is a segment of $\odot A$.
La región sombreada es un segmento de $\odot A$.

segmento de un círculo Región de un círculo limitada por un arco y una cuerda.

self-similar (p. 325) If any parts of a fractal image are replicas of the entire image, the image is self-similar.

autosemejante Si cualquier parte de una imagen fractal es una réplica de la imagen completa, entonces la imagen es autosemejante.

semicircle (p. 530) An arc that measures 180.

semicírculo Arco que mide 180°.

semi-regular tessellation (p. 484) A uniform tessellation formed using two or more regular polygons.

teselado semirregular Teselado uniforme compuesto por dos o más polígonos regulares.

similar polygons (p. 289) Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

polígonos semejantes Dos polígonos son semejantes si y sólo si sus ángulos correspondientes son congruentes y las medidas de sus lados correspondientes son proporcionales.

similar solids (p. 707) Solids that have exactly the same shape, but not necessarily the same size.

sólidos semejantes Sólidos que tienen exactamente la misma forma, pero no necesariamente el mismo tamaño.

similarity transformation (p. 491) When a figure and its transformation image are similar.

transformación de semejanza Aquélla en que la figura y su imagen transformada son semejantes.

sine (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the hypotenuse.

seno Es la razón entre la medida del cateto opuesto al ángulo agudo y la medida de la hipotenusa de un triángulo rectángulo.

skew lines (p. 127) Lines that do not intersect and are not coplanar.

rectas alabeadas Rectas que no se intersecan y que no son coplanares.

slope (p. 139) For a (nonvertical) line containing two points (x_1, y_1) and (x_2, y_2), the number m given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_2 \neq x_1$.

pendiente Para una recta (no vertical) que contiene dos puntos (x_1, y_1) y (x_2, y_2), el número m dado por la fórmula $m = \frac{y_2 - y_1}{x_2 - x_1}$ donde $x_2 \neq x_1$.

slope-intercept form (p. 145) A linear equation of the form $y = mx + b$. The graph of such an equation has slope m and y-intercept b.

forma pendiente-intersección Ecuación lineal de la forma $y = mx + b$. En la gráfica de tal ecuación, la pendiente es m y la intersección y es b.

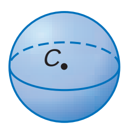
solving a triangle (p. 378) Finding the measures of all of the angles and sides of a triangle.

resolver un triángulo Calcular las medidas de todos los ángulos y todos los lados de un triángulo.

space (p. 8) A boundless three-dimensional set of all points.

espacio Conjunto tridimensional no acotado de todos los puntos.

sphere (p. 638) In space, the set of all points that are a given distance from a given point, called the *center*.



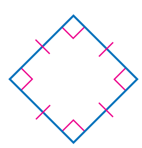
C is the center of the sphere.
C es el centro de la esfera.

esfera El conjunto de todos los puntos en el espacio que se encuentran a cierta distancia de un punto dado llamado *centro*.

spherical geometry (p. 165) The branch of geometry that deals with a system of points, greatcircles (lines), and spheres (planes).

geometría esférica Rama de la geometría que estudia los sistemas de puntos, círculos máximos (rectas) y esferas (planos).

square (p. 432) A quadrilateral with four right angles and four congruent sides.



cuadrado Cuadrilátero con cuatro ángulos rectos y cuatro lados congruentes.

standard position (p. 498) When the initial point of a vector is at the origin.

posición estándar Ocurre cuando la posición inicial de un vector es el origen.

statement (p. 67) Any sentence that is either true or false, but not both.

enunciado Una oración que puede ser falsa o verdadera, pero no ambas.

strictly self-similar (p. 325) A figure is strictly self-similar if any of its parts, no matter where they are located or what size is selected, contain the same figure as the whole.

estrictamente autosemejante Una figura es estrictamente autosemejante si cualquiera de sus partes, sin importar su localización o su tamaño, contiene la figura completa.

supplementary angles (p. 39) Two angles with measures that have a sum of 180.

surface area (p. 644) The sum of the areas of all faces and side surfaces of a three-dimensional figure.

T

tangent **1.** (p. 364) For an acute angle of a right triangle, the ratio of the measure of the leg opposite the acute angle to the measure of the leg adjacent to the acute angle. **2.** (p. 552) A line in the plane of a circle that intersects the circle in exactly one point. The point of intersection is called the *point of tangency*. **3.** (p. 671) A line that intersects a sphere in exactly one point.

tessellation (p. 483) A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces.

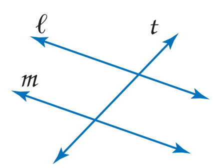
theorem (p. 90) A statement or conjecture that can be proven true by undefined terms, definitions, and postulates.

transformation (p. 462) In a plane, a mapping for which each point has exactly one image point and each image point has exactly one preimage point.

translation (p. 470) A transformation that moves all points of a figure the same distance in the same direction.

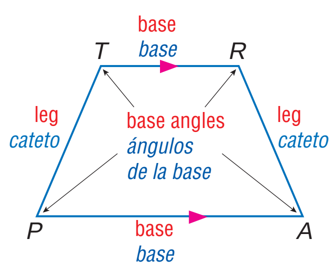
translation matrix (p. 506) A matrix that can be added to the vertex matrix of a figure to find the coordinates of the translated image.

transversal (p. 127) A line that intersects two or more lines in a plane at different points.



Line *t* is a transversal.
La recta t es una transversal.

trapezoid (p. 439) A quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called *bases*. The nonparallel sides are called *legs*. The pairs of angles with their vertices at the endpoints of the same base are called *base angles*.



ángulos suplementarios Dos ángulos cuya suma es igual a 180°.

área de superficie La suma de las áreas de todas las caras y superficies laterales de una figura tridimensional.

tangente **1.** La razón entre la medida del cateto opuesto al ángulo agudo y la medida del cateto adyacente al ángulo agudo de un triángulo rectángulo. **2.** La recta situada en el mismo plano de un círculo y que interseca dicho círculo en un sólo punto. El punto de intersección se conoce como *punto de tangencia*. **3.** Recta que interseca una esfera en un sólo punto.

teselado Patrón que cubre un plano y que se obtiene transformando la misma figura o conjunto de figuras, sin que haya traslapes ni espacios vacíos.

teorema Enunciado o conjetura que se puede demostrar como verdadera mediante el uso de términos primitivos, definiciones y postulados.

transformación La relación en el plano en que cada punto tiene un único punto imagen y cada punto imagen tiene un único punto preimagen.

traslación Transformación en que todos los puntos de una figura se trasladan la misma distancia, en la misma dirección.

matriz de traslación Matriz que al sumarse a la matriz de vértices de una figura permite calcular las coordenadas de la imagen trasladada.

transversal Recta que interseca en diferentes puntos dos o más rectas en el mismo plano.

trapecio Cuadrilátero con un sólo par de lados paralelos. Los lados paralelos del trapecio se llaman *bases*. Los lados no paralelos se llaman *catetos*. Los ángulos cuyos vértices se encuentran en los extremos de la misma base se llaman *ángulos de la base*.

trigonometric identity (p. 391) An equation involving a trigonometric ratio that is true for all values of the angle measure.

identidad trigonométrica Ecuación que contiene una razón trigonométrica que es verdadera para todos los valores de la medida del ángulo.

trigonometric ratio (p. 364) A ratio of the lengths of sides of a right triangle.

razón trigonométrica Razón de las longitudes de los lados de un triángulo rectángulo.

trigonometry (p. 364) The study of the properties of triangles and trigonometric functions and their applications.

trigonometría Estudio de las propiedades de los triángulos y de las funciones trigonométricas y sus aplicaciones.

truth table (p. 70) A table used as a convenient method for organizing the truth values of statements.

tabla verdadera Tabla que se utiliza para organizar de una manera conveniente los valores de verdad de los enunciados.

truth value (p. 67) The truth or falsity of a statement.

valor verdadero La condición de un enunciado de ser verdadero o falso.

two-column proof (p. 95) A formal proof that contains statements and reasons organized in two columns. Each step is called a *statement*, and the properties that justify each step are called *reasons*.

demostración a dos columnas Aquélla que contiene enunciados y razones organizadas en dos columnas. Cada paso se llama *enunciado* y las propiedades que lo justifican son las *razones*.

U

undefined terms (p. 7) Words, usually readily understood, that are not formally explained by means of more basic words and concepts. The basic undefined terms of geometry are point, line, and plane.

términos primitivos Palabras que por lo general se entienden fácilmente y que no se explican formalmente mediante palabras o conceptos más básicos. Los términos básicos primitivos de la geometría son el punto, la recta y el plano.

uniform tessellations (p. 484) Tessellations containing the same arrangement of shapes and angles at each vertex.

teselado uniforme Teselados que contienen el mismo patrón de formas y ángulos en cada vértice.

V

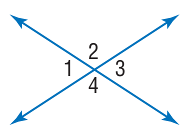
vector (p. 498) A directed segment representing a quantity that has both magnitude, or length, and direction.

vector Segmento dirigido que representa una cantidad que posee tanto magnitud, o longitud, como dirección.

vertex matrix (p. 506) A matrix that represents a polygon by placing all of the column matrices of the coordinates of the vertices into one matrix.

matriz del vértice Matriz que representa un polígono al colocar todas las matrices columna de las coordenadas de los vértices en una matriz.

vertical angles (p. 37) Two nonadjacent angles formed by two intersecting lines.



$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.
$\angle 1$ y $\angle 3$ son ángulos opuestos por el vértice.
$\angle 2$ y $\angle 4$ son ángulos opuestos por el vértice.

ángulos opuestos por el vértice Dos ángulos no adyacentes formados por dos rectas que se intersecan.

volume (p. 688) A measure of the amount of space enclosed by a three-dimensional figure.

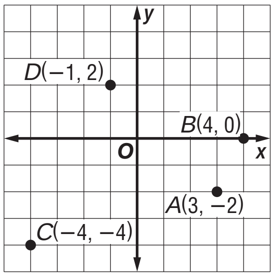
volumen La medida de la cantidad de espacio dentro de una figura tridimensional.

Selected Answers

Chapter 1 Points, Lines, Planes, and Angles

Page 5 Chapter 1 Getting Started

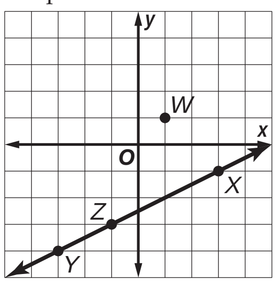
1–4.



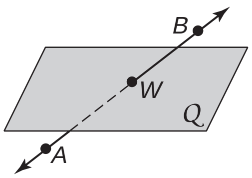
5. $1\frac{1}{8}$ **7.** $\frac{5}{16}$ **9.** −15 **11.** 25 **13.** 20 in. **15.** 24.6 m

Pages 9–11 Lesson 1-1

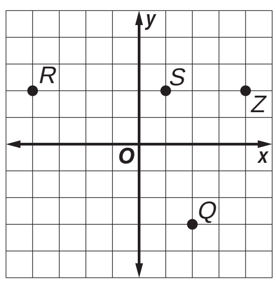
1. point, line, plane **3.** Micha; the points must be noncollinear to determine a plane.
5. Sample answer:



7. 6 **9.** No; A, C, and J lie in plane ABC, but D does not. **11.** point **13.** n **15.** $\mathcal{R}$ **17.** Sample answer: $\overleftrightarrow{PR}$ **19.** (D, 9)
21.



23. Sample answer:



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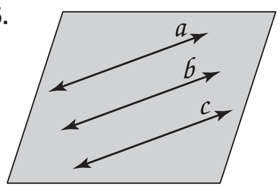

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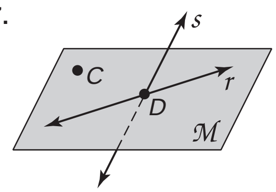
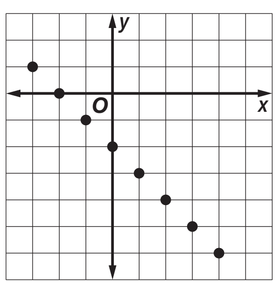
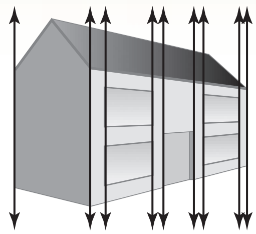


29. points that seem collinear; sample answer: (0, −2), (1, −3), (2, −4), (3, −5)

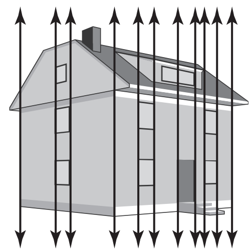


31. 1 **33.** anywhere on $\overleftrightarrow{AB}$ **35.** A, B, C, D or E, F, C, B
37. $\overleftrightarrow{AC}$ **39.** lines **41.** plane **43.** point **45.** point
47.



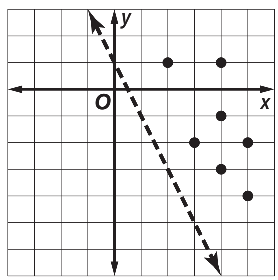
49. See students' work.

51. Sample answer:



53. vertical **55.** Sample answer: Chairs wobble because all four legs do not touch the floor at the same time. Answers should include the following.
- The ends of the legs represent points. If all points lie in the same plane, the chair will not wobble.
- Because it only takes three points to determine a plane, a chair with three legs will never wobble.

57. B
59. part of the coordinate plane above the line $y = -2x + 1$.

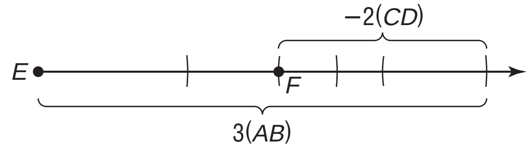


61. = **63.** = **65.** <

Pages 16–19 Lesson 1-2

1. Align the 0 point on the ruler with the leftmost endpoint of the segment. Align the edge of the ruler along the segment. Note where the rightmost endpoint falls on the scale and read the closest eighth of an inch measurement.
3. $1\frac{3}{4}$ in. **5.** 0.5 m; 14 m could be 13.5 to 14.5 m **7.** 3.7 cm
9. $x = 3$; $LM = 9$ **11.** $\overline{BC} \cong \overline{CD}$, $\overline{BE} \cong \overline{ED}$, $\overline{BA} \cong \overline{DA}$
13. 4.5 cm or 45 mm **15.** $1\frac{1}{4}$ in. **17.** 0.5 cm; 21.5 to 22.5 mm
19. 0.5 cm; 307.5 to 308.5 cm **21.** $\frac{1}{8}$ ft.; $3\frac{1}{8}$ to $3\frac{3}{8}$ ft.
23. $1\frac{1}{4}$ in. **25.** 2.8 cm **27.** $1\frac{1}{4}$ in. **29.** $x = 11$; $ST = 22$
31. $x = 2$; $ST = 4$ **33.** $y = 2$; $ST = 3$ **35.** no **37.** yes
39. yes **41.** $\overline{CF} \cong \overline{DG}$, $\overline{AB} \cong \overline{HI}$, $\overline{CE} \cong \overline{ED} \cong \overline{EF} \cong \overline{EG}$
43. 50,000 visitors **45.** No; the number of visitors to Washington state parks could be as low as 46.35 million or as high as 46.45 million. The visitors to Illinois state parks could be as low as 44.45 million or as high as 44.55 million visitors. The difference in visitors could be as high as 2.0 million.

47. 15.5 cm; Each measurement is accurate within 0.5 cm, so the greatest perimeter is 3.5 cm + 5.5 cm + 6.5 cm.
49.



51. Sample answer: Units of measure are used to differentiate between size and distance, as well as for accuracy. Answers should include the following.

- When a measurement is stated, you do not know the precision of the instrument used to make the measure. Therefore, the actual measure could be greater or less than that stated.
- You can assume equal measures when segments are shown to be congruent.

53. 1.7% **55.** 0.08% **57.** D **59.** Sample answer: planes *ABC* and *BCD* **61.** 5 **63.** 22 **65.** 1

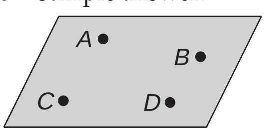
Page 19 Practice Quiz 1

1. $\overrightarrow{PR}$ **3.** $\overrightarrow{PR}$ **5.** 8.35

Pages 25–27 Lesson 1-3

1. Sample answers: (1) Use one of the Midpoint Formulas if you know the coordinates of the endpoints. (2) Draw a segment and fold the paper so that the endpoints match to locate the middle of the segment. (3) Use a compass and straightedge to construct the bisector of the segment.
3. 8 **5.** 10 **7.** −6 **9.** (−2.5, 4) **11.** (3, 5) **13.** 2 **15.** 3 **17.** 11 **19.** 10 **21.** 13 **23.** 15 **25.** $\sqrt{90} \approx 9.5$ **27.** $\sqrt{61} \approx 7.8$ **29.** 17.3 units **31.** −3 **33.** 2.5 **35.** 1 **37.** (10, 3) **39.** (−10, −3) **41.** (5.6, 2.85) **43.** $R(2, 7)$ **45.** $T\left(\frac{8}{3}, 11\right)$ **47.** LaFayette, LA **49a.** 111.8 **49b.** 212.0 **49c.** 353.4 **49d.** 420.3 **49e.** 37.4 **49f.** 2092.9 **51.** ≈ 72.1

53. Sample answer: The perimeter increases by the same factor. **55.** (−1, −3) **57.** B **59.** $4\frac{1}{4}$ in.
61. Sample answer:



63. 10 **65.** 9 **67.** $\frac{13}{3}$

Pages 33–36 Lesson 1-4

1. Yes; they all have the same measure. **3.** $m\angle A = m\angle Z$ **5.** $\overrightarrow{BA}, \overrightarrow{BC}$ **7.** 135°, obtuse **9.** 47 **11.** ∠1, right; ∠2, acute; ∠3, obtuse **13.** *B* **15.** *A* **17.** $\overrightarrow{AB}, \overrightarrow{AD}$ **19.** $\overrightarrow{AD}, \overrightarrow{AE}$ **21.** ∠*FEA*, ∠4 **23.** ∠*AED*, ∠*DEA*, ∠*AEB*, ∠*BEA*, ∠*AEC*, ∠*CEA* **25.** ∠2 **27.** 30, 30 **29.** 60°, acute **31.** 90°, right **33.** 120°, obtuse **35.** 65 **37.** 4 **39.** 4 **41.** Sample answer: *Acute* can mean something that is sharp or having a very fine tip like a pen, a knife, or a needle. *Obtuse* means not pointed or blunt, so something that is obtuse would be wide. **43.** 31; 59 **45.** 1, 3, 6, 10, 15 **47.** 21, 45 **49.** Sample answer: A degree is $\frac{1}{360}$ of a circle. Answers should include the following.

- Place one side of the angle to coincide with 0 on the protractor and the vertex of the angle at the center point of the protractor. Observe the point at which the other side of the angle intersects the scale of the protractor.
- See students' work.

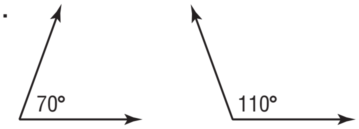
51. C **53.** $\sqrt{80} \approx 8.9$; (2, 2) **55.** $9\frac{2}{3}$ in. **57.** 13 **59.** *F*, *L*, *J* **61.** 5 **63.** −45 **65.** 8

Page 36 Practice Quiz 2

1. $\left(-\frac{1}{2}, 1\right)$; $\sqrt{65} \approx 8.1$ **3.** (0, 0); $\sqrt{2000} \approx 44.7$ **5.** 34; 135

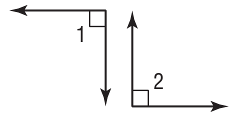
Pages 41–62 Lesson 1-5

1.



3. Sample answer: The noncommon sides of a linear pair of angles form a straight line.

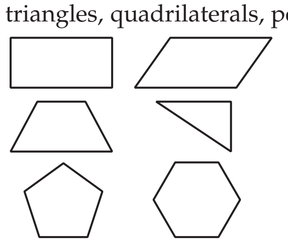
5. Sample answer: ∠*ABC*, ∠*CBE* **7.** $x = 24, y = -20$
9. Yes; they share a common side and vertex, so they are adjacent. Since $\overline{PR}$ falls between $\overline{PQ}$ and $\overline{PS}$, $m\angle QPR < 90$, so the two angles cannot be complementary or supplementary.
11. ∠*WUT*, ∠*VUX* **13.** ∠*UWT*, ∠*TWY* **15.** ∠*WTY*, ∠*WTU* **17.** 53, 37 **19.** 148 **21.** 84, 96 **23.** always **25.** sometimes **27.** 3.75 **29.** 114 **31.** Yes; the symbol denotes that ∠*DAB* is a right angle. **33.** Yes; their sum of their measures is $m\angle ADC$, which 90. **35.** No; we do not know $m\angle ABC$.
37. Sample answer:



39. Because ∠*WUT* and ∠*TUV* are supplementary, let $m\angle WUT = x$ and $m\angle TUV = 180 - x$. A bisector creates measures that are half of the original angle, so $m\angle YUT = \frac{1}{2}m\angle WUT$ or $\frac{x}{2}$ and $m\angle TUZ = \frac{1}{2}m\angle TUV$ or $\frac{180 - x}{2}$. Then $m\angle YUZ = m\angle YUT + m\angle TUZ$ or $\frac{x}{2} + \frac{180 - x}{2}$. This sum simplifies to $\frac{180}{2}$ or 90. Because $m\angle YUZ = 90$, $\overline{YU} \perp \overline{UZ}$. **41.** A **43.** $\ell \perp \overleftrightarrow{AB}$, $m \perp \overleftrightarrow{AB}$, $n \perp \overleftrightarrow{AB}$ **45.** obtuse **47.** right **49.** obtuse **51.** 8 **53.** $\sqrt{173} \approx 13.2$ **55.** $\sqrt{20} \approx 4.5$ **57.** $n = 3$, $QR = 20$ **59.** 24 **61.** 40

Pages 48–50 Lesson 1-6

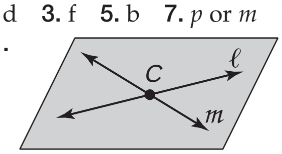
1. Divide the perimeter by 10. **3.** $P = 3s$ **5.** pentagon; concave; irregular **7.** 33 ft **9.** 16 units **11.** 4605 ft **13.** octagon; convex; regular **15.** pentagon **17.** triangle **19.** 82 ft **21.** 40 units **23.** The perimeter is tripled. **25.** 125 m **27.** 30 units **29.** All are 15 cm. **31.** 13 units, 13 units, 5 units **33.** 4 in., 4 in., 17 in., 17 in. **35.** 52 units
37. Sample answer: Some toys use pieces to form polygons. Others have polygon-shaped pieces that connect together. Answers should include the following.

- triangles, quadrilaterals, pentagons
- 

39. D **41.** sometimes **43.** 63

Pages 53–56 Chapter 1 Study Guide and Review

1. d **3.** f **5.** b **7.** *p* or *m* **9.** *F*
11.



13. $x = 6$, $PB = 18$ **15.** $s = 3$, $PB = 12$ **17.** yes **19.** not enough information **21.** $\sqrt{101} \approx 10.0$
23. $\sqrt{13} \approx 3.6$ **25.** (3, −5) **27.** (0.6, −6.35) **29.** $\overrightarrow{FE}, \overrightarrow{FG}$ **31.** 70°, acute **33.** 50°, acute **35.** 36 **37.** 40 **39.** ∠*TWY*, ∠*XWY* **41.** 9 **43.** not a polygon **45.** ≈ 22.5 units

Chapter 2 Reasoning and Proof

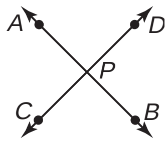
Page 61 Chapter 2 Getting Started

1. 10 **3.** 0 **5.** 50 **7.** 21 **9.** −9 **11.** $-\frac{18}{5}$ **13.** 16

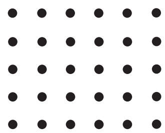
Pages 63–66 Lesson 2-1

1. Sample answer: After the news is over, it's time for dinner. **3.** Sample answer: When it's cloudy, it rains. Counterexample: It is often cloudy and it does not rain.

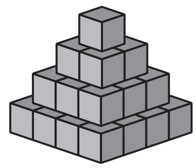
5. 7 **7.** *A*, *B*, *C*, and *D* are noncollinear.



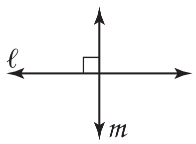
9. true **11.**



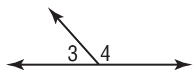
13. 32 **15.** $\frac{11}{3}$ **17.** 162 **19.** 30



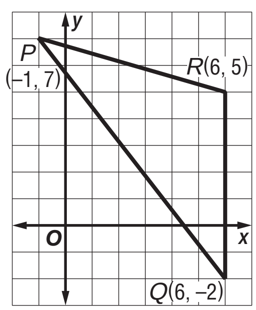
21. Lines ℓ and m form four right angles.



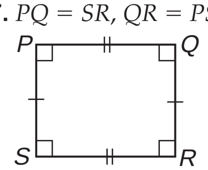
23. ∠3 and∠4 are supplementary.



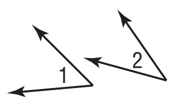
25. △*PQR* is a scalene triangle.



27. *PQ* = *SR*, *QR* = *PS*



29. false;



31. false;

33. true **35.** False; *JKLM* may not have a right angle. **37.** trial and error, a process of inductive reasoning **39.** C_7H_{16} **41.** false; $n = 41$ **43.** C **45.** hexagon, convex, irregular **47.** heptagon, concave, irregular **49.** No; we do not know anything about the angle measures. **51.** Yes; they form a linear pair. **53.** (2, −1) **55.** (1, −12) **57.** (5.5, 2.2) **59.** 8; 56 **61.** 4; 16 **63.** 10; 43 **65.** 4, 5 **67.** 5, 6, 7

Pages 71–74 Lesson 2-2

1. The conjunction (*p* and *q*) is represented by the intersection of the two circles. **3.** A conjunction is a compound statement using the word *and*, while a disjunction is a compound statement using the word *or*.
5. 9 + 5 = 14 and a square has four sides; true.
7. 9 + 5 = 14 or February does not have 30 days; true.
9. 9 + 5 ≠ 14 or a square does not have four sides; false.

11. Sample answer:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

13. Sample answer:

p	r	$\sim p$	$\sim p \wedge r$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

15. 14 **17.** 3 **19.** $\sqrt{-64} = 8$ or an equilateral triangle has three congruent sides; true. **21.** 0 < 0 and an obtuse angle measures greater than 90° and less than 180°; false. **23.** An equilateral triangle has three congruent sides and an obtuse angle measures greater than 90° and less than 180°; true. **25.** An equilateral triangle has three congruent sides and 0 < 0; false. **27.** An obtuse angle measures greater than 90° and less than 180° or an equilateral triangle has three congruent sides; true. **29.** An obtuse angle measures greater than 90° and less than 180°, or an equilateral triangle has three congruent sides and 0 < 0; true.

31.

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

33. Sample answer:

q	r	q and r
T	T	T
T	F	F
F	T	F
F	F	F

35. Sample answer:

p	r	p or r
T	T	T
T	F	T
F	T	T
F	F	F

37. Sample answer:

q	r	$\sim r$	$q \wedge \sim r$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

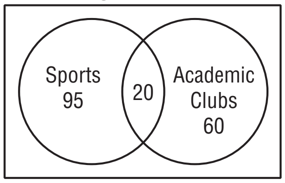
39. Sample answer:

p	q	r	$\sim p$	$\sim r$	$q \wedge \sim r$	$\sim p \vee (q \wedge \sim r)$
T	T	T	F	F	F	F
T	T	F	F	T	T	T
T	F	T	F	F	F	F
T	F	F	F	T	F	F
F	T	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	F	F	T
F	F	F	T	T	F	T

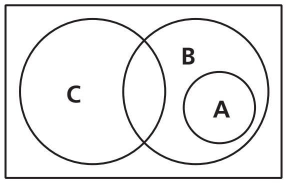
Selected Answers

41. 42 **43.** 25

45. Level of Participation Among 310 Students



47. 135 **49.** true
51.

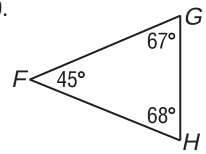


53. Sample answer: Logic can be used to eliminate false choices on a multiple choice test. Answers should include the following.
- Math is my favorite subject and drama club is my favorite activity.
- See students' work.

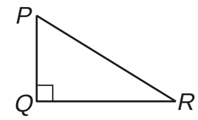
55. C **57.** 81 **59.** 1 **61.** 405 **63.** 34.4 **65.** 29.5 **67.** 55°, acute **69.** 222 feet **71.** 44 **73.** 184

Pages 78–80 Lesson 2-3

1. Writing a conditional in if-then form is helpful so that the hypothesis and conclusion are easily recognizable.
3. In the inverse, you negate both the hypothesis and the conclusion of the conditional. In the contrapositive, you negate the hypothesis and the conclusion of the converse.
5. H: $x - 3 = 7$; C: $x = 10$ **7.** If a pitcher is a 32-ounce pitcher, then it holds a quart of liquid. **9.** If an angle is formed by perpendicular lines, then it is a right angle.
11. true **13.** Converse: If plants grow, then they have water; true. Inverse: If plants do not have water, then they will not grow; true. Contrapositive: If plants do not grow, then they do not have water. False; they may have been killed by overwatering. **15.** Sample answer: If you are in Colorado, then aspen trees cover high areas of the mountains. If you are in Florida, then cypress trees rise from the swamps. If you are in Vermont, then maple trees are prevalent. **17.** H: you are a teenager; C: you are at least 13 years old **19.** H: three points lie on a line; C: the points are collinear **21.** H: the measure of an is between 0 and 90; C: the angle is acute **23.** If you are a math teacher, then you love to solve problems. **25.** Sample answer: If two angles are adjacent, then they have a common side.
27. Sample answer: If two triangles are equiangular, then they are equilateral. **29.** true **31.** true **33.** false
35. true **37.** false **39.** true **41.** Converse: If you are in good shape, then you exercise regularly; true. Inverse: If you do not exercise regularly, then you are not in good shape; true. Contrapositive: If you are not in good shape, then you do not exercise regularly. False; an ill person may exercise a lot, but still not be in good shape.
43. Converse: If a figure is a quadrilateral, then it is a rectangle; false, rhombus. Inverse: If a figure is not a rectangle, then it is not a quadrilateral; false, rhombus. Contrapositive: If a figure is not a quadrilateral, then it is not a rectangle; true. **45.** Converse: If an angle has measure less than 90, then it is acute; true. Inverse: If an angle is not acute, then its measure is not less than 90; true. Contrapositive: If an angle's measure is not less than 90, then it is not acute; true. **47.** Sample answer: In Alaska, if there are more hours of daylight than darkness, then it is summer. In Alaska, if there are more hours of darkness than daylight, then it is winter. **49.** Conditional statements can be used to describe how to get a discount, rebate, or refund. Sample answers should include the following. If you are not 100% satisfied, then return the product for a full refund. Wearing a seatbelt reduces the risk of injuries. **51.** B
53. A hexagon has five sides or $60 \times 3 = 18$.; false
55. A hexagon doesn't have five sides or $60 \times 3 = 18$.; true
57. George Washington was not the first president of the United States and $60 \times 3 \neq 18$.; false
59. The sum of the measures of the angles in a triangle is 180.



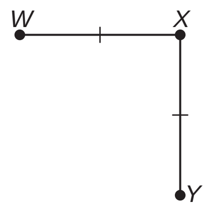
61. $\angle PQR$ is a right angle.



63. $\sqrt{41}$ or 6.4 **65.** $\sqrt{125}$ or 11.2
67. Multiply each side by 2.

Page 80 Practice Quiz 1

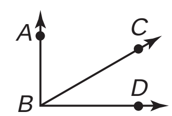
1. false



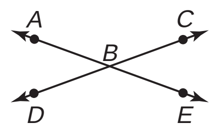
3. Sample answer:

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

5. Converse: If two angles have a common vertex, then the angles are adjacent. False; $\angle ABD$ is not adjacent to $\angle ABC$.



Inverse: If two angles are not adjacent, then they do not have a common vertex. False; $\angle ABC$ and $\angle DBE$ have a common vertex and are not adjacent.



Contrapositive: If two angles do not have a common vertex, then they are not adjacent; true.

Pages 84–87 Lesson 2-4

1. Sample answer: a: If it is rainy, the game will be cancelled; b: It is rainy; c: The game will be cancelled.
3. Lakeisha; if you are dizzy, that does not necessarily mean that you are seasick and thus have an upset stomach.
5. Invalid; congruent angles do not have to be vertical.
7. The midpoint of a segment divides it into two segments with equal measures. **9.** invalid **11.** No; Terry could be a man or a woman. She could be 45 and have purchased \$30,000 of life insurance. **13.** Valid; since 5 and 7 are odd, the Law of Detachment indicates that their sum is even.
15. Invalid; the sum is even. **17.** Invalid; *E*, *F*, and *G* are not necessarily noncollinear. **19.** Valid; the vertices of a triangle are noncollinear, and therefore determine a plane.
21. If the measure of an angle is less than 90, then it is not obtuse. **23.** no conclusion **25.** yes; Law of Detachment
27. yes; Law of Detachment **29.** invalid **31.** If Catriona Le May Doan skated her second 500 meters in 37.45 seconds, then she would win the race. **33.** Sample answer: Doctors and nurses use charts to assist in determining medications and their doses for patients. Answers should include the following.

- Doctors need to note a patient's symptoms to determine which medication to prescribe, then determine how much to prescribe based on weight, age, severity of the illness, and so on.
- Doctors use what is known to be true about diseases and when symptoms appear, then deduce that the patient has a particular illness.

35. B **37.** They are a fast, easy way to add fun to your family's menu.

39. Sample answer:

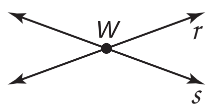
q	r	$q \wedge r$
T	T	T
T	F	F
F	T	F
F	F	F

41. Sample answer:

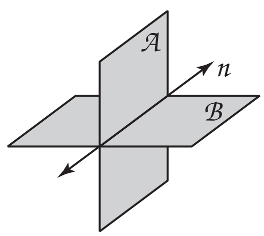
p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

43. $\angle HDG$ **45.** Sample answer: $\angle JHK$ and $\angle DHK$ **47.** Yes, slashes on the segments indicate that they are congruent. **49.** 10 **51.** $\sqrt{130} \approx 11.4$

53.



55.



57. Sample answer: $\angle 1$ and $\angle 2$ are complementary, $m\angle 1 + m\angle 2 = 90$.

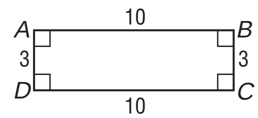
Pages 91–93 Lesson 2-5

1. Deductive reasoning is used to support claims that are made in a proof. **3.** postulates, theorems, algebraic properties, definitions **5.** 15 **7.** definition of collinear **9.** Through any two points, there is exactly one line. **11.** 15 ribbons **13.** 10 **15.** 21 **17.** Always; if two points lie in a plane, then the entire line containing those points lies in that plane. **19.** Sometimes; the three points cannot be on the same line. **21.** Sometimes; ℓ and m could be skew so they would not lie in the same plane $\mathcal{R}$. **23.** If two points lie in a plane, then the entire line containing those points lies in that plane. **25.** If two points lie in a plane, then the entire line containing those points lies in the plane. **27.** Through any three points not on the same line, there is exactly one plane. **29.** She will have 4 different planes and 6 lines. **31.** one, ten **33.** C **35.** yes; Law of Detachment **37.** Converse: If $\triangle ABC$ has an angle with measure greater than 90, then $\triangle ABC$ is a right triangle. False; the triangle would be obtuse. Inverse: If $\triangle ABC$ is not a right triangle, none of its angle measures are greater than 90. False; it could be an obtuse triangle. Contrapositive: If $\triangle ABC$ does not have an angle measure greater than 90, $\triangle ABC$ is not a right triangle. False; $m\angle ABC$ could still be 90 and $\triangle ABC$ be a right triangle. **39.** $\sqrt{17} \approx 4.1$ **41.** $\sqrt{106} \approx 10.3$ **43.** 25 **45.** 12 **47.** 10

Pages 97–100 Lesson 2-6

1. Sample answer: If $x = 2$ and $x + y = 6$, then $2 + y = 6$. **3.** hypothesis; conclusion **5.** Multiplication Property **7.** Addition Property **9a.** $5 - \frac{2}{3}x = 1$ **9b.** Mult. Prop. **9c.** Dist. Prop. **9d.** $-2x = -12$ **9e.** Div. Prop.

11. Given: Rectangle $ABCD$, $AD = 3$, $AB = 10$
Prove: $AC = BD$



Proof:

Statement	Reasons
1. Rectangle $ABCD$, $AD = 3$, $AB = 10$	**1.** Given
2. Draw segments AC and DB.	**2.** Two points determine a line.
3. $\triangle ABC$ and $\triangle BCD$ are right triangles.	**3.** Def. of rt $\triangle$
4. $AC = \sqrt{3^2 + 10^2}$, $DB = \sqrt{3^2 + 10^2}$	**4.** Pythagorean Th.
5. $AC = BD$	**5.** Substitution

13. C **15.** Subt. Prop. **17.** Substitution **19.** Reflexive Property **21.** Substitution **23.** Transitive Prop. **25a.** $2x - 7 = \frac{1}{3}x - 2$ **25b.** $3(2x - 7) = 3\left(\frac{1}{3}x - 2\right)$ **25c.** Dist. Prop. **25d.** $5x - 21 = -6$ **25e.** Add. Prop. **25f.** $x = 3$

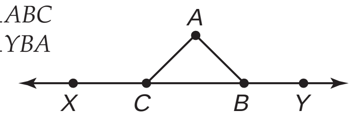
27. Given: $-2y + \frac{3}{2} = 8$
Prove: $y = -\frac{13}{4}$
Proof:

Statement	Reasons
1. $-2y + \frac{3}{2} = 8$	**1.** Given
2. $2\left(-2y + \frac{3}{2}\right) = 2(8)$	**2.** Mult. Prop.
3. $-4y + 3 = 16$	**3.** Dist. Prop.
4. $-4y = 13$	**4.** Subt. Prop.
5. $y = -\frac{13}{4}$	**5.** Div. Prop.

29. Given: $5 - \frac{2}{3}z = 1$
Prove: $z = 6$
Proof:

Statement	Reasons
1. $5 - \frac{2}{3}z = 1$	**1.** Given
2. $3\left(5 - \frac{2}{3}z\right) = 3(1)$	**2.** Mult. Prop.
3. $15 - 2x = 3$	**3.** Dist. Prop.
4. $15 - 2x - 15 = 3 - 15$	**4.** Subt. Prop.
5. $-2x = -12$	**5.** Substitution
6. $\frac{-2x}{-2} = \frac{-12}{-2}$	**6.** Div. Prop.
7. $x = 6$	**7.** Substitution

31. Given: $m\angle ACB = m\angle ABC$
Prove: $m\angle XCA = m\angle YBA$



Proof:

Statement	**Reasons**
1. $m\angle ACB = m\angle ABC$	1. Given
2. $m\angle XCA + m\angle ACB = 180$ $m\angle YBA + m\angle ABC = 180$	2. Def. of supp. $\measuredangle$s
3. $m\angle XCA + m\angle ACB =$ $m\angle YBA + m\angle ABC$	3. Substitution
4. $m\angle XCA + m\angle ACB =$ $m\angle YBA + m\angle ACB$	4. Substitution
5. $m\angle XCA = m\angle YBA$	5. Subt. Prop.

33. All of the angle measures would be equal. **35.** See students' work. **37.** B **39.** 6 **41.** Invalid; $27 \div 6 = 4.5$, which is not an integer. **43.** Sample answer: If people are happy, then they rarely correct their faults. **45.** Sample answer: If a person is a champion, then the person is afraid of losing. **47.** $\frac{1}{2}$ ft **49.** 0.5 in. **51.** 11 **53.** 47

Page 100 Practice Quiz 2

1. invalid **3.** If two lines intersect, then their intersection is exactly one point.

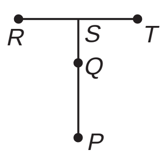
5. Given: $2(n - 3) + 5 = 3(n - 1)$
Prove: $n = 2$
Proof:

Statement	**Reasons**
1. $2(n - 3) + 5 = 3(n - 1)$	1. Given
2. $2n - 6 + 5 = 3n - 3$	2. Dist. Prop.
3. $2n - 1 = 3n - 3$	3. Substitution
4. $2n - 1 - 2n = 3n - 3 - 2n$	4. Subt. Prop.
5. $-1 = n - 3$	5. Substitution
6. $-1 + 3 = n - 3 + 3$	6. Add. Prop.
7. $2 = n$	7. Substitution
8. $n = 2$	8. Symmetric Prop.

Pages 103–106 Lesson 2-7

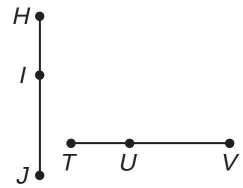
1. Sample answer: The distance from Cleveland to Chicago is the same as the distance from Cleveland to Chicago. **3.** If A, B, and C are collinear and $AB + BC = AC$, then B is between A and C. **5.** Symmetric

7. Given: $\overline{PQ} \cong \overline{RS}$, $\overline{QS} \cong \overline{ST}$
Prove: $\overline{PS} \cong \overline{RT}$
Proof:



Statements	**Reasons**
a. $\overline{PQ} \cong \overline{RS}$, $\overline{QS} \cong \overline{ST}$	a. Given
b. $PQ = RS$, $QS = ST$	b. Def. of $\cong$ segments
c. $PS = PQ + QS$, $RT = RS + ST$	c. Segment Addition Post.
d. $PQ + QS = RS + ST$	d. Addition Property
e. $PS = RT$	e. Substitution
f. $\overline{PS} \cong \overline{RT}$	f. Def. of $\cong$ segments

9. Given: $\overline{HI} \cong \overline{TU}$, $\overline{HJ} \cong \overline{TV}$
Prove: $\overline{IJ} \cong \overline{UV}$

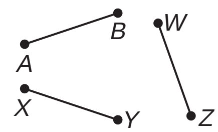


Proof:

Statements	**Reasons**
1. $\overline{HI} \cong \overline{TU}$, $\overline{HJ} \cong \overline{TV}$	1. Given
2. $HI = TU$, $HJ = TV$	2. Def. of $\cong$ segs.
3. $HI + IJ = HJ$	3. Seg. Add. Post.
4. $TU + IJ = TV$	4. Substitution
5. $TU + UV = TV$	5. Seg. Add. Post.
6. $TU + IJ = TU + UV$	6. Substitution
7. $TU = TU$	7. Reflexive Prop.
8. $IJ = UV$	8. Subt. Prop.
9. $\overline{IJ} \cong \overline{UV}$	9. Def. of $\cong$ segs.

11. Helena is between Missoula and Miles City. **13.** Substitution **15.** Transitive **17.** Subtraction

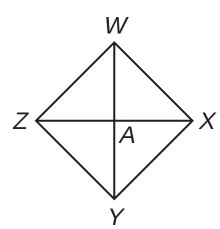
19. Given: $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$
Prove: $\overline{XY} \cong \overline{AB}$



Proof:

Statements	**Reasons**
1. $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{AB}$	1. Given
2. $XY = WZ$ and $WZ = AB$	2. Def. of $\cong$ segs.
3. $XY = AB$	3. Transitive Prop.
4. $\overline{XY} \cong \overline{AB}$	4. Def. of $\cong$ segs.

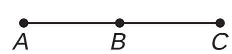
21. Given: $\overline{WY} \cong \overline{ZX}$
A is the midpoint of $\overline{WY}$.
A is the midpoint of $\overline{ZX}$.
Prove: $\overline{WA} \cong \overline{ZA}$



Proof:

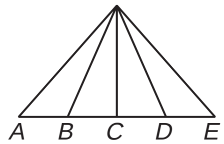
Statements:	**Reasons:**
a. $\overline{WY} \cong \overline{ZX}$ A is the midpoint of $\overline{WY}$. A is the midpoint of $\overline{ZX}$.	a. Given
b. $WY = ZX$	b. Def. of $\cong$ segs.
c. $WA = AY$, $ZA = AX$	c. Definition of midpoint
d. $WY = WA + AY$, $ZX = ZA + AX$	d. Segment Addition Post.
e. $WA + AY = ZA + AX$	e. Substitution
f. $WA + WA = ZA + ZA$	f. Substitution
g. $2WA = 2ZA$	g. Substitution
h. $WA = ZA$	h. Division Property
i. $\overline{WA} \cong \overline{ZA}$	i. Def. of $\cong$ segs.

23. Given: $AB = BC$
Prove: $AC = 2BC$
Proof:



Statements	**Reasons**
1. $AB = BC$	1. Given
2. $AC = AB + BC$	2. Seg. Add. Post.
3. $AC = BC + BC$	3. Substitution
4. $AC = 2BC$	4. Substitution

25. Given: $\overline{AB} \cong \overline{DE}$, C is the midpoint of $\overline{BD}$.
Prove: $\overline{AC} \cong \overline{CE}$



Proof:

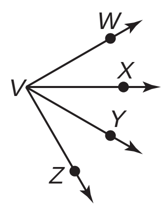
Statements	Reasons
1. $\overline{AB} \cong \overline{DE}$, C is the midpoint of $\overline{BD}$.	**1.** Given
2. $BC = CD$	**2.** Def. of midpoint
3. $AB = DE$	**3.** Def. of $\cong$ segs.
4. $AB + BC = CD + DE$	**4.** Add. Prop.
5. $AB + BC = AC$ $CD + DE = CE$	**5.** Seg. Add. Post.
6. $AC = CE$	**6.** Substitution
7. $\overline{AC} \cong \overline{CE}$	**7.** Def. of $\cong$ segs.

27. Sample answers: $\overline{LN} \cong \overline{QO}$ and $\overline{LM} \cong \overline{MN} \cong \overline{RS} \cong \overline{ST} \cong \overline{QP} \cong \overline{PO}$ **29.** B **31.** Substitution **33.** Addition Property **35.** Never; the midpoint of a segment divides it into two congruent segments. **37.** Always; if two planes intersect, they intersect in a line. **39.** 3; 9 cm by 13 cm **41.** 15 **43.** 45 **45.** 25

Pages 111–114 Lesson 2-8

1. Tomas; Jacob's answer left out the part of $\angle ABC$ represented by $\angle EBF$. **3.** $m\angle 2 = 65$ **5.** $m\angle 11 = 59$, $m\angle 12 = 121$

7. Given: $\overrightarrow{VX}$ bisects $\angle WVY$. $\overrightarrow{VY}$ bisects $\angle XVZ$.
Prove: $\angle WVX \cong \angle YVZ$

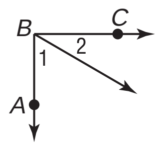


Proof:

Statements	Reasons
1. $\overrightarrow{VX}$ bisects $\angle WVY$, $\overrightarrow{VY}$ bisects $\angle XVZ$.	**1.** Given
2. $\angle WVX \cong \angle XVY$	**2.** Def. of $\angle$ bisector
3. $\angle XVY \cong \angle YVZ$	**3.** Def. of $\angle$ bisector
4. $\angle WVX \cong \angle YVZ$	**4.** Trans. Prop.

9. sometimes

11. Given: $\angle ABC$ is a right angle.
Prove: $\angle 1$ and $\angle 2$ are complementary angles.

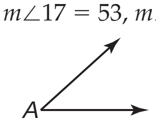


Proof:

Statements	Reasons
1. $\angle ABC$ is a right angle.	**1.** Given
2. $m\angle ABC = 90$	**2.** Def. of rt. $\angle$
3. $m\angle ABC = m\angle 1 + m\angle 2$	**3.** Angle Add. Post.
4. $m\angle 1 + m\angle 2 = 90$	**4.** Substitution
5. $\angle 1$ and $\angle 2$ are complementary angles.	**5.** Def. of complementary $\angle$s

13. 62 **15.** 28 **17.** $m\angle 4 = 52$ **19.** $m\angle 9 = 86$, $m\angle 10 = 94$ **21.** $m\angle 13 = 112$, $m\angle 14 = 112$ **23.** $m\angle 17 = 53$, $m\angle 18 = 53$

25. Given: $\angle A$
Prove: $\angle A \cong \angle A$

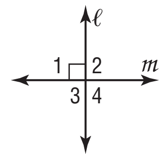


Proof:

Statements	Reasons
1. $\angle A$ is an angle.	**1.** Given
2. $m\angle A = m\angle A$	**2.** Reflexive Prop.
3. $\angle A \cong \angle A$	**3.** Def. of $\cong$ angles

27. sometimes **29.** always **31.** sometimes

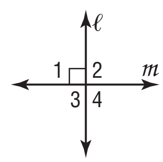
33. Given: $\ell \perp m$
Prove: $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s.



Proof:

Statements	Reasons
1. $\ell \perp m$	**1.** Given
2. $\angle 1$ is a right angle.	**2.** Def. of $\perp$ lines
3. $m\angle 1 = 90$	**3.** Def. of rt. $\angle$s
4. $\angle 1 \cong \angle 4$	**4.** Vert. $\angle$s are $\cong$.
5. $m\angle 1 = m\angle 4$	**5.** Def. of $\cong$ $\angle$s
6. $m\angle 4 = 90$	**6.** Substitution
7. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	**7.** Def. of linear pair
8. $m\angle 1 + m\angle 2 = 180$, $m\angle 4 + m\angle 3 = 180$	**8.** Linear pairs are supplementary.
9. $90 + m\angle 2 = 180$, $90 + m\angle 3 = 180$	**9.** Substitution
10. $m\angle 2 = 90$, $m\angle 3 = 90$	**10.** Subt. Prop.
11. $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s.	**11.** Def. of rt. $\angle$s (steps 6, 10)

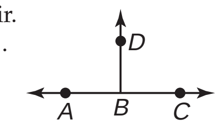
35. Given: $\ell \perp m$
Prove: $\angle 1 \cong \angle 2$



Proof:

Statements	Reasons
1. $\ell \perp m$	**1.** Given
2. $\angle 1$ and $\angle 2$ rt. $\angle$s	**2.** $\perp$ lines intersect to form 4 rt. $\angle$s.
3. $\angle 1 \cong \angle 2$	**3.** All rt. $\angle$s $\cong$.

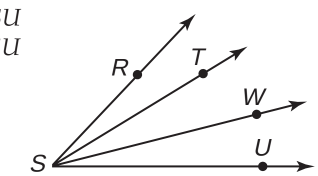
37. Given: $\angle ABD \cong \angle CBD$, $\angle ABD$ and $\angle DBC$ form a linear pair.
Prove: $\angle ABD$ and $\angle CBD$ are rt. $\angle$s.



Proof:

Statements	Reasons
1. $\angle ABD \cong \angle CBD$, $\angle ABD$ and $\angle CBD$ form a linear pair.	**1.** Given
2. $\angle ABD$ and $\angle CBD$ are supplementary.	**2.** Linear pairs are supplementary.
3. $\angle ABD$ and $\angle CBD$ are rt. $\angle$s.	**3.** If $\angle$s are $\cong$ and suppl., they are rt. $\angle$s.

39. Given: $m\angle RSW = m\angle TSU$
Prove: $m\angle RST = m\angle WSU$



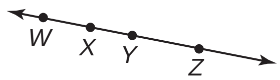
Proof:

Statements	Reasons
1. $m\angle RSW = m\angle TSU$	**1.** Given
2. $m\angle RSW = m\angle RST + m\angle TSW$, $m\angle TSU = m\angle TSW + m\angle WSU$	**2.** Angle Addition Postulate
3. $m\angle RST + m\angle TSW = m\angle TSW + m\angle WSU$	**3.** Substitution
4. $m\angle TSW = m\angle TSW$	**4.** Reflexive Prop.
5. $m\angle RST = m\angle WSU$	**5.** Subt. Prop.

41. Because the lines are perpendicular, the angles formed are right angles. All right angles are congruent. Therefore, $\angle 1$ is congruent to $\angle 2$. **43.** Two angles that are supplementary to the same angle are congruent. Answers should include the following.

- $\angle 1$ and $\angle 2$ are supplementary; $\angle 2$ and $\angle 3$ are supplementary.
- $\angle 1$ and $\angle 3$ are vertical angles, and are therefore congruent.
- If two angles are complementary to the same angle, then the angles are congruent. **45.** B

47. Given: X is the midpoint of $\overline{WY}$.
Prove: $WX + YZ = XZ$



Proof:

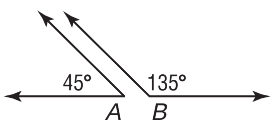
Statements	Reasons
1. X is the midpoint of $\overline{WY}$.	1. Given
2. $WX = XY$	2. Def. of midpoint
3. $XY + YZ = XZ$	3. Segment Addition Postulate
4. $WX + YZ = XZ$	4. Substitution

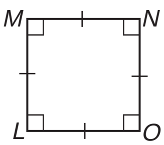
49. $\angle ONM$, $\angle MNR$ **51.** N or R **53.** obtuse

55. $\angle NML$, $\angle NMP$, $\angle NMO$, $\angle RNM$, $\angle ONM$

Pages 115–120 Chapter 2 Study Guide and Review

1. conjecture **3.** compound **5.** hypothesis **7.** Postulates
9. $m\angle A + m\angle B = 180$ **11.** $LMNO$ is a square.



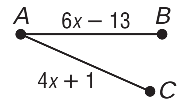


13. In a right triangle with right angle C, $a^2 + b^2 = c^2$ or the sum of the measures of two supplementary angles is 180; true. **15.** $-1 > 0$, and in a right triangle with right angle C, $a^2 + b^2 = c^2$, or the sum of the measures of two supplementary angles is 180; false. **17.** In a right triangle with right angle C, $a^2 + b^2 = c^2$ and the sum of the measures of two supplementary angles is 180, and $-1 > 0$; false. **19.** Converse: If a month has 31 days, then it is March. False; July has 31 days. Inverse: If a month is not March, then it does not have 31 days. False; July has 31 days. Contrapositive: If a month does not have 31 days, then it is not March; true. **21.** true **23.** false **25.** Valid; by definition, adjacent angles have a common vertex. **27.** yes; Law of Detachment **29.** yes; Law of Syllogism **31.** Always; if P is the midpoint of $\overline{XY}$, then $\overline{XP} \cong \overline{PY}$. By definition of congruent segments, $XP = PY$. **33.** Sometimes; if the points are collinear. **35.** Sometimes; if the right angles form a linear pair. **37.** Never; adjacent angles must share a common side, and vertical angles do not. **39.** Distributive Property **41.** Subtraction Property

43. Given: $5 = 2 - \frac{1}{2}x$
Prove: $x = -6$
Proof:

Statements	Reasons
1. $5 = 2 - \frac{1}{2}x$	1. Given
2. $5 - 2 = 2 - \frac{1}{2}x - 2$	2. Subt. Prop.
3. $3 = -\frac{1}{2}x$	3. Substitution
4. $-2(3) = -2\left(-\frac{1}{2}x\right)$	4. Mult. Prop
5. $-6 = x$	5. Substitution
6. $x = -6$	6. Symmetric Prop.

45. Given: $AC = AB$, $AC = 4x + 1$, $AB = 6x - 13$
Prove: $x = 7$

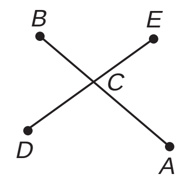


Proof:

Statements	Reasons
1. $AC = AB$, $AC = 4x + 1$, $AB = 6x - 13$	1. Given
2. $4x + 1 = 6x - 13$	2. Substitution
3. $4x + 1 - 1 = 6x - 13 - 1$	3. Subt. Prop.
4. $4x = 6x - 14$	4. Substitution
5. $4x - 6x = 6x - 14 - 6x$	5. Subt. Prop.
6. $-2x = -14$	6. Substitution
7. $\frac{-2x}{-2} = \frac{-14}{-2}$	7. Div. Prop.
8. $x = 7$	8. Substitution

47. Reflexive Property **49.** Addition Property
51. Division or Multiplication Property

53. Given: $BC = EC$, $CA = CD$
Prove: $BA = DE$



Proof:

Statements	Reasons
1. $BC = EC$, $CA = CD$	1. Given
2. $BC + CA = EC + CA$	2. Add. Prop.
3. $BC + CA = EC + CD$	3. Substitution
4. $BC + CA = BA$ $EC + CD = DE$	4. Seg. Add. Post.
5. $BA = DE$	5. Substitution

55. 145 **57.** 90

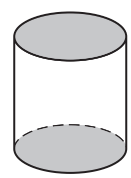
Chapter 3 Parallel and Perpendicular Lines

Page 125 Chapter 3 Getting Started

1. $\overleftrightarrow{PQ}$ **3.** $\overleftrightarrow{ST}$ **5.** $\angle 4, \angle 6, \angle 8$ **7.** $\angle 1, \angle 5, \angle 7$ **9.** 9 **11.** $-\frac{3}{2}$

Pages 128–131 Lesson 3-1

1. Sample answer: The bottom and top of a cylinder are contained in parallel planes.



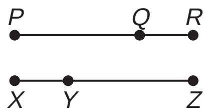
3. Sample answer: looking down railroad tracks **5.** $\overline{AB}$, $\overline{JK}$, $\overline{LM}$ **7.** q and r, q and t, r and t **9.** p and r, p and t, r and t **11.** alternate interior **13.** consecutive interior **15.** p; consecutive interior **17.** q; alternate interior **19.** Sample answer: The roof and the floor are parallel planes. **21.** Sample answer: The top of the memorial "cuts" the pillars. **23.** ABC, ABQ, PQR, CDS, APU, DET **25.** $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$, $\overline{FU}$, $\overline{PU}$, $\overline{QR}$, $\overline{RS}$, $\overline{TU}$ **27.** $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{QR}$, $\overline{RS}$, $\overline{ST}$, $\overline{TU}$ **29.** a and c, a and r, r and c **31.** a and b, a and c, b and c **33.** alternate exterior **35.** corresponding **37.** alternate interior **39.** consecutive interior **41.** p; alternate interior **43.** ℓ; alternate exterior **45.** q; alternate interior **47.** m; consecutive interior **49.** $\overline{CG}$, $\overline{DH}$, $\overline{EI}$ **51.** No; plane ADE will intersect all the planes if they are extended. **53.** infinite number

55. Sample answer: Parallel lines and planes are used in architecture to make structures that will be stable. Answers should include the following.

- Opposite walls should form parallel planes; the floor may be parallel to the ceiling.
- The plane that forms a stairway will not be parallel to some of the walls.

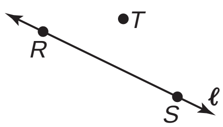
57. 16, 20, or 28

59. **Given:** $\overline{PQ} \cong \overline{ZY}$, $\overline{QR} \cong \overline{XY}$
Prove: $\overline{PR} \cong \overline{XZ}$



Proof: Since $\overline{PQ} \cong \overline{ZY}$ and $\overline{QR} \cong \overline{XY}$, $PQ = ZY$ and $QR = XY$ by the definition of congruent segments. By the Addition Property, $PQ + QR = ZY + XY$. Using the Segment Addition Postulate, $PR = PQ + QR$ and $XZ = XY + YZ$. By substitution, $PR = XZ$. Because the measures are equal, $\overline{PR} \cong \overline{XZ}$ by the definition of congruent segments.

61. $m\angle EFG$ is less than 90; Detachment. 63. 8.25
65. 15.81 67. 10.20
69.

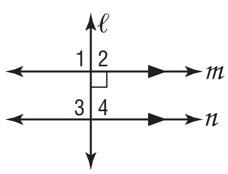


71. 90, 90 73. 72, 108
75. 76, 104

Pages 136–138 Lesson 3-2

1. Sometimes; if the transversal is perpendicular to the parallel lines, then $\angle 1$ and $\angle 2$ are right angles and are congruent. 3. 1 5. 110 7. 70 9. 55 11. $x = 13, y = 6$ 13. 67 15. 75 17. 105 19. 105 21. 43 23. 43 25. 137 27. 60 29. 70 31. 120 33. $x = 34, y = \pm 5$ 35. 113 37. $x = 14, y = 11, z = 73$ 39. (1) Given (2) Corresponding Angles Postulate (3) Vertical Angles Theorem (4) Transitive Property

41. **Given:** $\ell \perp m$, $m \parallel n$
Prove: $\ell \perp n$



Proof: Since $\ell \perp m$, we know that $\angle 1 \cong \angle 2$, because perpendicular lines form congruent right angles. Then by the Corresponding Angles Postulate, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. By the definition of congruent angles, $m\angle 1 = m\angle 2$, $m\angle 1 = m\angle 3$, and $m\angle 2 = m\angle 4$. By substitution, $m\angle 3 = m\angle 4$. Because $\angle 3$ and $\angle 4$ form a congruent linear pair, they are right angles. By definition, $\ell \perp n$.

43. $\angle 2$ and $\angle 6$ are consecutive interior angles for the same transversal, which makes them supplementary because $\overline{WX} \parallel \overline{YZ}$. $\angle 4$ and $\angle 6$ are not necessarily supplementary because $\overline{XY}$ may not be parallel to $\overline{WZ}$. 45. C 47. $\overline{FG}$ 49. CDH 51. $m\angle 1 = 56$ 53. H: it rains this evening; C: I will mow the lawn tomorrow 55. $-\frac{2}{3}$ 57. $\frac{3}{8}$ 59. $-\frac{4}{5}$

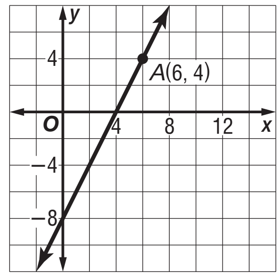
Page 138 Practice Quiz 1

1. p; alternate exterior 3. q; alternate interior 5. 75

Pages 142–144 Lesson 3-3

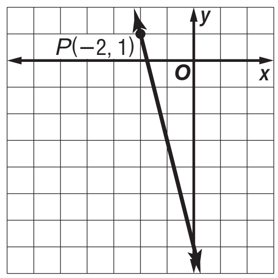
1. horizontal; vertical 3. horizontal line, vertical line 5. $-\frac{1}{2}$ 7. 2 9. parallel

11.

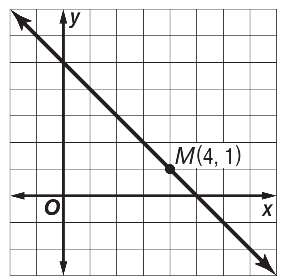


13. (1500, −120) or (−1500, −120)
15. $\frac{1}{7}$ 17. −5
19. perpendicular
21. neither 23. parallel
25. −3 27. 6 29. 6
31. undefined

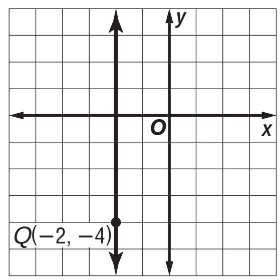
33.



35.

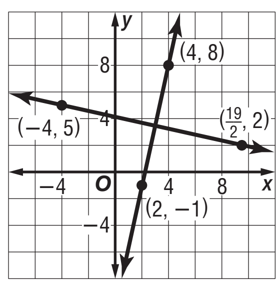


37.



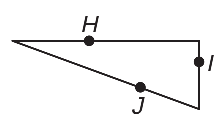
39. Sample answer: 0.24
41. 2016

43. $\frac{19}{2}$;

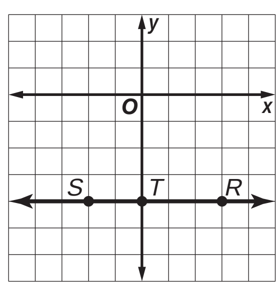


45. 2001
47. $y = \frac{1}{2}x - \frac{11}{2}$
49. C 51. 131 53. 49
55. 49 57. ℓ; alternate exterior
59. p; alternate interior
61. m; alternate interior

63. H, I, and J are noncollinear.



65. R, S, and T are collinear.

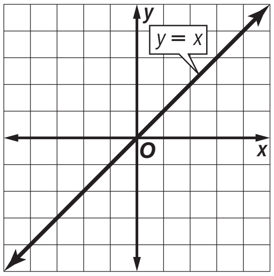


67. obtuse 69. obtuse
71. $y = -\frac{1}{2}x - \frac{5}{4}$

Pages 147–150 Lesson 3-4

1. Sample answer: Use the point-slope form where $(x_1, y_1) = (-2, 8)$ and $m = -\frac{2}{5}$.

3. Sample answer: $y = x$



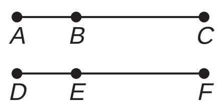
5. $y = -\frac{3}{5}x - 2$
7. $y + 1 = \frac{3}{2}(x - 4)$
9. $y - 137.5 = 1.25(x - 20)$
11. $y = -x + 2$
13. $y = 39.95$, $y = 0.95x + 4.95$
15. $y = \frac{1}{6}x - 4$
17. $y = \frac{5}{8}x - 6$
19. $y = -x - 3$

21. $y - 1 = 2(x - 3)$ **23.** $y + 5 = -\frac{4}{5}(x + 12)$ **25.** $y - 17.12 = 0.48(x - 5)$ **27.** $y = -3x - 2$ **29.** $y = 2x - 4$ **31.** $y = -x + 5$ **33.** $y = -\frac{1}{8}x$ **35.** $y = -3x + 5$ **37.** $y = -\frac{3}{5}x + 3$ **39.** $y = -\frac{1}{5}x - 4$ **41.** no slope-intercept form, $x = -6$ **43.** $y = \frac{2}{5}x - \frac{24}{5}$ **45.** $y = 0.05x + 750$, where x = total price of appliances sold **47.** $y = -750x + 10{,}800$ **49.** in 10 days **51.** $y = x - 180$ **53.** Sample answer: In the equation of a line, the b value indicates the fixed rate, while the mx value indicates charges based on usage. Answers should include the following.

- The fee for air time can be considered the slope of the equation.
- We can find where the equations intersect to see where the plans would be equal.

55. B **57.** undefined **59.** 58 **61.** 75 **63.** 73

65. Given: $AC = DF$, $AB = DE$
Prove: $BC = EF$



Proof:

Statements	Reasons
1. $AC = DF$, $AB = DE$	1. Given
2. $AC = AB + BC$ $DF = DE + EF$	2. Segment Addition Postulate
3. $AB + BC = DE + EF$	3. Substitution Property
4. $BC = EF$	4. Subtraction Property

67. 26.69 **69.** $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$, $\angle 3$ and $\angle 7$ **71.** $\angle 2$ and $\angle 8$, $\angle 3$ and $\angle 5$

Page 150 Practice Quiz 2

1. neither **3.** $\frac{7}{2}$ **5.** $\frac{5}{4}$ **7.** $y = -\frac{4}{5}x + \frac{16}{5}$ **9.** $y + 8 = -\frac{1}{4}(x - 5)$

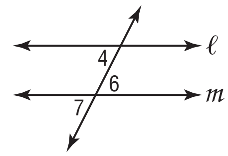
Pages 154–157 Lesson 3-5

1. Sample answer: Use a pair of alternate exterior $\angle$s that are $\cong$ and cut by a transversal; show that a pair of consecutive interior $\angle$s are suppl.; show that alternate interior $\angle$s are $\cong$; show two lines are $\perp$ to same line; show corresponding $\angle$s are $\cong$. **3.** Sample answer: A basketball court has parallel lines, as does a newspaper. The edges should be equidistant along the entire line. **5.** $\ell \parallel m$; $\cong$ alt. int. $\angle$s **7.** $p \parallel q$; $\cong$ alt. ext. $\angle$s **9.** 11.375 **11.** The slope of $\overleftrightarrow{CD}$ is $\frac{1}{8}$, and the slope of line $\overleftrightarrow{AB}$ is $\frac{1}{7}$. The slopes are not equal, so the lines are not parallel. **13.** $a \parallel b$; $\cong$ alt. int. $\angle$s **15.** $\ell \parallel m$; $\cong$ corr. $\angle$s **17.** $\overline{AE} \parallel \overline{BF}$; $\cong$ corr. $\angle$s **19.** $\overline{AC} \parallel \overline{EG}$; $\cong$ alt. int. $\angle$s **21.** $\overleftrightarrow{HS} \parallel \overleftrightarrow{JT}$; $\cong$ corr. $\angle$s **23.** $\overleftrightarrow{KN} \parallel \overleftrightarrow{PR}$; suppl. cons. int. $\angle$s

25. **1.** Given
2. Definition of perpendicular
3. All rt. $\angle$s are $\cong$.
4. If corresponding $\angle$s are $\cong$, then lines are $\parallel$.

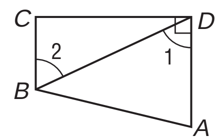
27. 15 **29.** -8 **31.** 21.6

33. Given: $\angle 4 \cong \angle 6$
Prove: $\ell \parallel m$



Proof: We know that $\angle 4 \cong \angle 6$. Because $\angle 6$ and $\angle 7$ are vertical angles they are congruent. By the Transitive Property of Congruence, $\angle 4 \cong \angle 7$. Since $\angle 4$ and $\angle 7$ are corresponding angles, and they are congruent, $\ell \parallel m$.

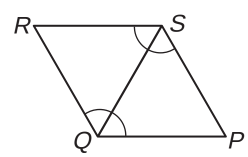
35. Given: $\overline{AD} \perp \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\overline{BC} \perp \overline{CD}$



Proof:

Statements	Reasons
1. $\overline{AD} \perp \overline{CD}$, $\angle 1 \cong \angle 2$	1. Given
2. $\overline{AD} \parallel \overline{BC}$	2. If alternate interior $\angle$s are $\cong$, lines are $\parallel$.
3. $\overline{BC} \perp \overline{CD}$	3. Perpendicular Transversal Th.

37. Given: $\angle RSP \cong \angle PQR$
$\angle QRS$ and $\angle PQR$ are supplementary.
Prove: $\overline{PS} \parallel \overline{QR}$



Proof:

Statements	Reasons
1. $\angle RSP \cong \angle PQR$ $\angle QRS$ and $\angle PQR$ are supplementary.	1. Given
2. $m\angle RSP = m\angle PQR$	2. Def. of $\cong$ $\angle$s
3. $m\angle QRS + m\angle PQR = 180$	3. Def. of suppl. $\angle$s
4. $m\angle QRS + m\angle RSP = 180$	4. Substitution
5. $\angle QRS$ and $\angle RSP$ are supplementary.	5. Def. of suppl. $\angle$s
6. $\overline{PS} \parallel \overline{QR}$	6. If consecutive interior $\angle$s are suppl., lines $\parallel$.

39. No, the slopes are not the same. **41.** The 10-yard lines will be parallel because they are all perpendicular to the sideline and two or more lines perpendicular to the same line are parallel. **43.** See students' work. **45.** B **47.** $y = 0.3x - 6$ **49.** $y = -\frac{1}{2}x + \frac{19}{2}$ **51.** $-\frac{5}{4}$ **53.** 1 **55.** undefined

57.

p	q	p and q
T	T	T
T	F	F
F	T	F
F	F	F

59.

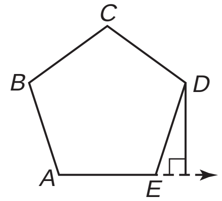
p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

61. complementary angles **63.** $\sqrt{85} \approx 9.22$

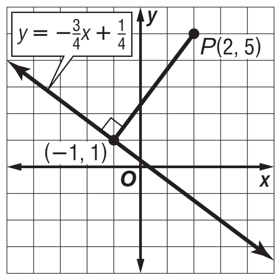
Pages 162–164 Lesson 3-6

1. Construct a perpendicular line between them.
3. Sample answer: Measure distances at different parts; compare slopes; measure angles. Finding slopes is the most readily available method.

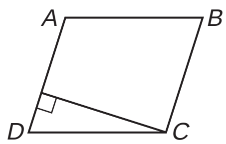
5.



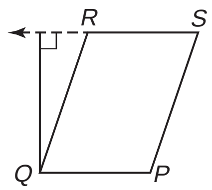
7. 0.9
9. 5 units;



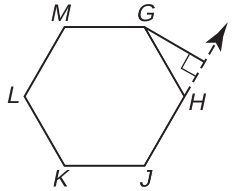
11.



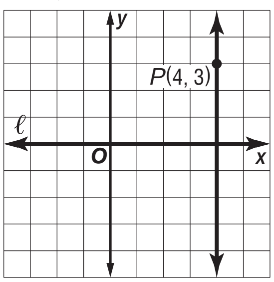
13.



15.

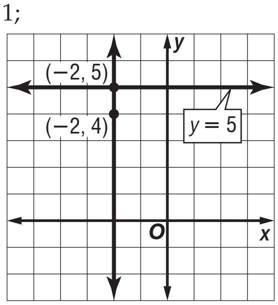


17. $d = 3$;

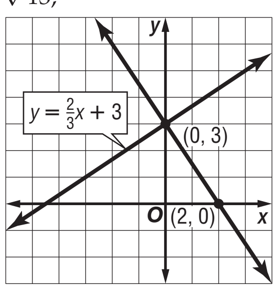


19. 4 **21.** $\sqrt{5}$ **23.** $\frac{7\sqrt{5}}{5}$

25. 1;



27. $\sqrt{13}$;



29. It is everywhere equidistant from the ceiling. **31.** 6
33. Sample answer: We want new shelves to be parallel so they will line up. Answers should include the following.

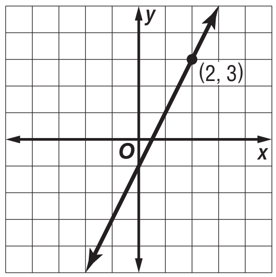
- After marking several points, a slope can be calculated, which should be the same slope as the original brace.
- Building walls requires parallel lines.

35. D **37.** $\overleftrightarrow{DA} \parallel \overleftrightarrow{EF}$; corresponding ∡ **39.** $y = \frac{1}{2}x + 3$
41. $y = \frac{2}{3}x - 2$ **43.** $y = \frac{2}{3}x + \frac{11}{3}$

Pages 167–170 Chapter 3 Study Guide and Review

1. alternate **3.** parallel **5.** alternate exterior
7. consecutive **9.** alternate exterior **11.** corresponding
13. consecutive. interior **15.** alternate interior **17.** 53
19. 127 **21.** 127 **23.** neither **25.** perpendicular

27.



29. $y = 2x - 7$
31. $y = -\frac{2}{7}x + 4$
33. $y = 5x - 3$
35. $\overleftrightarrow{AL}$ and $\overleftrightarrow{BJ}$, alternate exterior ∡ ≅
37. $\overleftrightarrow{CF}$ and $\overleftrightarrow{GK}$, 2 lines ⊥ same line
39. $\overleftrightarrow{CF}$ and $\overleftrightarrow{GK}$, consecutive interior ∡ suppl. **41.** $\sqrt{5}$

Chapter 4 Congruent Triangles

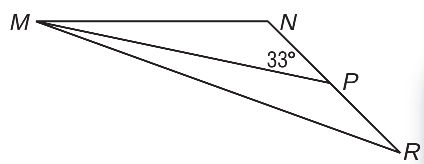
Pages 177 Chapter 4 Getting Started

1. $-6\frac{1}{2}$ **3.** 1 **5.** $2\frac{3}{4}$ **7.** ∠2, ∠12, ∠15, ∠6, ∠9, ∠3, ∠13
9. ∠6, ∠9, ∠3, ∠13, ∠2, ∠8, ∠12, ∠15 **11.** ∠11.2
13. ∠14.6

Pages 180–183 Lesson 4-1

1. Triangles are classified by sides and angles. For example, a triangle can have a right angle and have no two sides congruent. **3.** Always; equiangular triangles have three acute angles. **5.** obtuse **7.** △*MJK*, △*KLM*, △*JKN*, △*LMN*
9. $x = 4$, $JM = 3$, $MN = 3$, $JN = 2$ **11.** $TW = \sqrt{125}$, $WZ = \sqrt{74}$, $TZ = \sqrt{61}$; scalene **13.** right **15.** acute
17. obtuse **19.** equilateral, equiangular **21.** isosceles, acute **23.** △*BAC*, △*CDB* **25.** △*ABD*, △*ACD*, △*BAC*, △*CDB* **27.** $x = 5$, $MN = 9$, $MP = 9$, $NP = 9$
29. $x = 8$, $JL = 11$, $JK = 11$, $KL = 7$ **31.** Scalene; it is 184 miles from Lexington to Nashville, 265 miles from Cairo to Lexington, and 144 miles from Cairo to Nashville.
33. $AB = \sqrt{106}$, $BC = \sqrt{233}$, $AC = \sqrt{65}$; scalene
35. $AB = \sqrt{29}$, $BC = 4$, $AC = \sqrt{29}$; isosceles
37. $AB = \sqrt{124}$, $BC = \sqrt{124}$, $AC = 8$; isosceles

39. Given:
$m\angle NPM = 33$
Prove:
△*RPM* is obtuse.



Proof: ∠*NPM* and ∠*RPM* form a linear pair. ∠*NPM* and ∠*RPM* are supplementary because if two angles form a linear pair, then they are supplementary. So, $m\angle NPM + m\angle RPM = 180$. It is given that $m\angle NPM = 33$. By substitution, $33 + m\angle RPM = 180$. Subtract to find that $m\angle RPM = 147$. ∠*RPM* is obtuse by definition. △*RPM* is obtuse by definition.

41. $AD = \sqrt{\left(0 - \frac{a}{2}\right)^2 + (0 - b)^2}$ $\quad CD = \sqrt{\left(a - \frac{a}{2}\right)^2 + (0 - b)^2}$

$= \sqrt{\left(-\frac{a}{2}\right)^2 + (-b)^2} \qquad = \sqrt{\left(\frac{a}{2}\right)^2 + (-b)^2}$

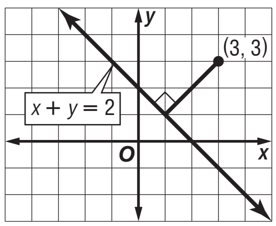
$= \sqrt{\frac{a^2}{4} + b^2} \qquad = \sqrt{\frac{a^2}{4} + b^2}$

$AD = CD$, so $\overline{AD} \cong \overline{CD}$. $\triangle ADC$ is isosceles by definition.

43. Sample answer: Triangles are used in construction as structural support. Answers should include the following.

- Triangles can be classified by sides and angles. If the measure of each angle is less than 90, the triangle is acute. If the measure of one angle is greater than 90, the triangle is obtuse. If one angle equals 90°, the triangle is right. If each angle has the same measure, the triangle is equiangular. If no two sides are congruent, the triangle is scalene. If at least two sides are congruent, it is isosceles. If all of the sides are congruent, the triangle is equilateral.
- Isosceles triangles seem to be used more often in architecture and construction.

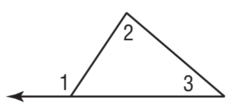
45. B **47.** $\sqrt{8}$;



49. 15 **51.** 44 **53.** any three: ∠2 and ∠11, ∠3 and ∠6, ∠4 and ∠7, ∠3 and ∠12, ∠7 and ∠10, ∠8 and ∠11 **55.** ∠6, ∠9, and ∠12 **57.** ∠2, ∠5, and ∠8

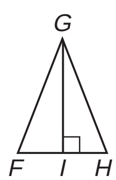
Pages 188–191 Lesson 4-2

1. Sample answer: ∠2 and ∠3 are the remote interior angles of exterior ∠1.



3. 43 **5.** 55 **7.** 147 **9.** 25 **11.** 93 **13.** 65, 65 **15.** 76 **17.** 49 **19.** 53 **21.** 32 **23.** 44 **25.** 123 **27.** 14 **29.** 53 **31.** 103 **33.** 50 **35.** 40 **37.** 129

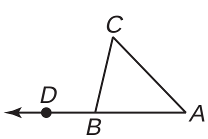
39. Given: $\angle FGI \cong \angle IGH$, $\overline{GI} \perp \overline{FH}$
Prove: $\angle F \cong \angle H$
Proof:



$\boxed{\overline{GI} \perp \overline{FH}}$
Given
↓
$\boxed{\angle GIF \text{ and } \angle GIH \text{ are right angles.}}$
⊥ lines form rt. ∠s.
↓
$\boxed{\angle GIF \cong \angle GIH}$ → $\boxed{\angle F \cong \angle H}$
All rt. ∠s are ≅. Third Angle Theorem

$\boxed{\angle FGI \cong \angle IGH}$
Given
↓ (to $\angle F \cong \angle H$)

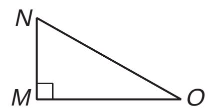
41. Given: $\triangle ABC$
Prove: $m\angle CBD = m\angle A + m\angle C$



Proof:

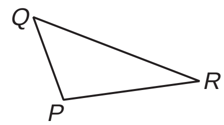
Statements	Reasons
1. $\triangle ABC$	1. Given
2. $\angle CBD$ and $\angle ABC$ form a linear pair.	2. Def. of linear pair
3. $\angle CBD$ and $\angle ABC$ are supplementary.	3. If 2 ∠s form a linear pair,they are suppl.
4. $m\angle CBD + m\angle ABC = 180$	4. Def. of suppl.
5. $m\angle A + m\angle ABC + m\angle C = 180$	5. Angle Sum Theorem
6. $m\angle A + m\angle ABC + m\angle C = m\angle CBD + m\angle ABC$	6. Substitution
7. $m\angle A + m\angle C = m\angle CBD$	7. Subtraction Property

43. Given: $\triangle MNO$
$\angle M$ is a right angle.
Prove: There can be at most one right angle in a triangle.



Proof:
In $\triangle MNO$, $\angle M$ is a right angle. $m\angle M + m\angle N + m\angle O = 180$. $m\angle M = 90$, so $m\angle N + m\angle O = 90$. If $\angle N$ were a right angle, then $m\angle O = 0$. But that is impossible, so there cannot be two right angles in a triangle.

Given: $\triangle PQR$
$\angle P$ is obtuse.
Prove: There can be at most one obtuse angle in a triangle.



Proof:
In $\triangle PQR$, $\angle P$ is obtuse. So $m\angle P > 90$. $m\angle P + m\angle Q + m\angle R = 180$. It must be that $m\angle Q + m\angle R < 90$. So, $\angle Q$ and $\angle R$ must be acute.

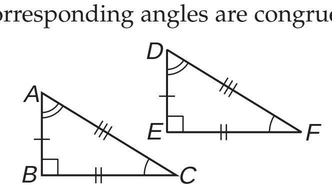
45. $m\angle 1 = 48$, $m\angle 2 = 60$, $m\angle 3 = 72$ **47.** A **49.** $\triangle AED$ **51.** $\triangle BEC$ **53.** $\sqrt{20}$ units **55.** $\frac{\sqrt{117}}{13}$ units **57.** $x = 112$, $y = 28$, $z = 22$ **59.** reflexive **61.** symmetric **63.** transitive

Pages 195–198 Lesson 4-3

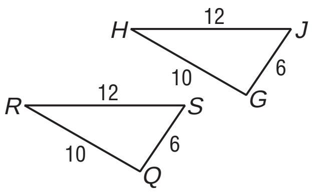
1. The sides and the angles of the triangle are not affected by a congruence transformation, so congruence is preserved. **3.** $\triangle AFC \cong \triangle DFB$ **5.** $\angle W \cong \angle S$, $\angle X \cong \angle T$, $\angle Z \cong \angle J$, $\overline{WX} \cong \overline{ST}$, $\overline{XZ} \cong \overline{TJ}$, $\overline{WZ} \cong \overline{SJ}$ **7.** $QR = 5$, $Q'R' = 5$, $RT = 3$, $R'T' = 3$, $QT = \sqrt{34}$, and $Q'T' = \sqrt{34}$. Use a protractor to confirm that the corresponding angles are congruent; flip. **9.** $\triangle CFH \cong \triangle JKL$ **11.** $\triangle WPZ \cong \triangle QVS$ **13.** $\angle T \cong \angle X$, $\angle U \cong \angle Y$, $\angle V \cong \angle Z$, $\overline{TU} \cong \overline{XY}$, $\overline{UV} \cong \overline{YZ}$, $\overline{TV} \cong \overline{XZ}$ **15.** $\angle B \cong \angle D$, $\angle C \cong \angle G$, $\angle F \cong \angle H$, $\overline{BC} \cong \overline{DG}$, $\overline{CF} \cong \overline{GH}$, $\overline{BF} \cong \overline{DH}$ **17.** $\triangle 1 \cong \triangle 10$, $\triangle 2 \cong \triangle 9$, $\triangle 3 \cong \triangle 8$, $\triangle 4 \cong \triangle 7$, $\triangle 5 \cong \triangle 6$ **19.** △s 1, 5, 6, and 11, △s 3, 8, 10, and 12, △s 2, 4, 7, and 9 **21.** We need to know that all of the angles are congruent and that the other corrresponding sides are congruent. **23.** Flip; $MN = 8$, $M'N' = 8$, $NP = 2$, $N'P' = 2$, $MP = \sqrt{68}$, and $M'P' = \sqrt{68}$. Use a protractor to confirm that the corresponding angles are congruent.

25. Turn; $JK = \sqrt{40}$, $J'K' = \sqrt{40}$, $KL = \sqrt{29}$, $K'L' = \sqrt{29}$, $JL = \sqrt{17}$, and $J'L' = \sqrt{17}$. Use a protractor to confirm that the corresponding angles are congruent.

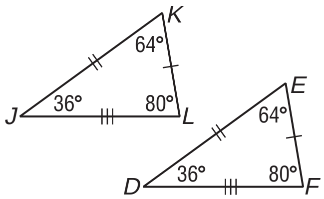
27. True;



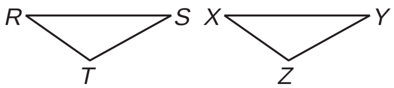
29.



31.



33. Given: $\triangle RST \cong \triangle XYZ$



Prove: $\triangle XYZ \cong \triangle RST$

Proof:

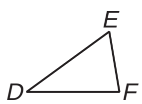
$\triangle RST \cong \triangle XYZ$
Given

$\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$
CPCTC

$\angle X \cong \angle R$, $\angle Y \cong \angle S$, $\angle Z \cong \angle T$, $\overline{XY} \cong \overline{RS}$, $\overline{YZ} \cong \overline{ST}$, $\overline{XZ} \cong \overline{RT}$
Congruence of ∠s and segments is symmetric.

$\triangle XYZ \cong \triangle RST$
Def. of ≅ △s

35. Given: $\triangle DEF$
Prove: $\triangle DEF \cong \triangle DEF$



Proof:

$\triangle DEF$
Given

$\overline{DE} \cong \overline{DE}$, $\overline{EF} \cong \overline{EF}$, $\overline{DF} \cong \overline{DF}$
Congruence of segments is reflexive.

$\angle D \cong \angle D$, $\angle E \cong \angle E$, $\angle F \cong \angle F$
Congruence of ∠s is reflexive.

$\triangle DEF \cong \triangle DEF$
Def. of ≅ △s

37. Sample answer: Triangles are used in bridge design for structure and support. Answers should include the following.
- The shape of the triangle does not matter.
- Some of the triangles used in the bridge supports seem to be congruent.

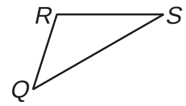
39. D **41.** 58 **43.** $x = 3$, $BC = 10$, $CD = 10$, $BD = 5$
45. $y = -\frac{3}{2}x + 3$ **47.** $y = -4x - 11$ **49.** $\sqrt{5}$ **51.** $\sqrt{13}$

Page 198 Chapter 4 Practice Quiz 1

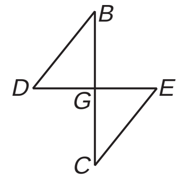
1. $\triangle DFJ$, $\triangle GJF$, $\triangle HJG$, $\triangle DJH$ **3.** $AB = BC = AC = 7$
5. $\angle M \cong \angle J$, $\angle N \cong \angle K$, $\angle P \cong \angle L$; $\overline{MN} \cong \overline{JK}$, $\overline{NP} \cong \overline{KL}$, and $\overline{MP} \cong \overline{JL}$

Pages 203–206 Lesson 4-4

1. Sample answer: In $\triangle QRS$, $\angle R$ is the included angle of the sides $\overline{QR}$ and $\overline{RS}$.



3. $EG = 2$, $MP = 2$, $FG = 4$, $NP = 4$, $EF = \sqrt{20}$, and $MN = \sqrt{20}$. The corresponding sides have the same measure and are congruent. $\triangle EFG \cong \triangle MNP$ by SSS.
5. Given: $\overline{DE}$ and $\overline{BC}$ bisect each other
Prove: $\triangle DGB \cong \triangle EGC$



Proof:

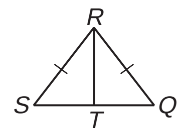
$\overline{DE}$ and $\overline{BC}$ bisect each other.
Given

$\overline{DG} \cong \overline{GE}$, $\overline{BG} \cong \overline{GC}$
Def. of bisector of segments

$\angle DGB \cong \angle EGC$
Vertical ∠s are ≅.

$\triangle DGB \cong \triangle EGC$
SAS

7. SAS
9. Given: T is the midpoint of $\overline{SQ}$. $\overline{SR} \cong \overline{QR}$
Prove: $\triangle SRT \cong \triangle QRT$

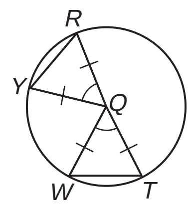


Proof:

Statements	Reasons
1. T is the midpoint of $\overline{SQ}$.	1. Given
2. $\overline{ST} \cong \overline{TQ}$	2. Midpoint Theorem
3. $\overline{SR} \cong \overline{QR}$	3. Given
4. $\overline{RT} \cong \overline{RT}$	4. Reflexive Property
5. $\triangle SRT \cong \triangle QRT$	5. SSS

11. $JK = \sqrt{10}$, $KL = \sqrt{10}$, $JL = \sqrt{20}$, $FG = \sqrt{2}$, $GH = \sqrt{50}$, and $FH = 6$. The corresponding sides are not congruent so $\triangle JKL$ is not congruent to $\triangle FGH$. **13.** $JK = \sqrt{10}$, $KL = \sqrt{10}$, $JL = \sqrt{20}$, $FG = \sqrt{10}$, $GH = \sqrt{10}$, and $FH = \sqrt{20}$. Each pair of corresponding sides have the same measure so they are congruent. $\triangle JKL \cong \triangle FGH$ by SSS.

15. Given: $\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$, $\angle RQY \cong \angle WQT$
Prove: $\triangle QWT \cong \triangle QYR$



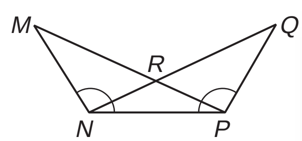
Proof:

$\overline{RQ} \cong \overline{TQ} \cong \overline{YQ} \cong \overline{WQ}$
Given

$\angle RQY \cong \angle WQT$
Given

$\triangle QWT \cong \triangle QYR$
SAS

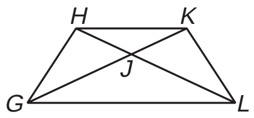
17. Given: $\triangle MRN \cong \triangle QRP$, $\angle MNP \cong \angle QPN$
Prove: $\triangle MNP \cong \triangle QPN$



Proof:

Statement	Reason
1. $\triangle MRN \cong \triangle QRP$, $\angle MNP \cong \angle QPN$	1. Given
2. $\overline{MN} \cong \overline{QP}$	2. CPCTC
3. $\overline{NP} \cong \overline{NP}$	3. Reflexive Property
4. $\triangle MNP \cong \triangle QPN$	4. SAS

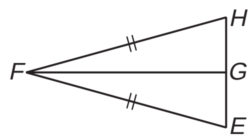
19. Given: $\triangle GHJ \cong \triangle LKJ$
Prove: $\triangle GHL \cong \triangle LKG$



Proof:

Statement	Reason
1. $\triangle GHJ \cong \triangle LKJ$	1. Given
2. $\overline{HJ} \cong \overline{KJ}$, $\overline{GJ} \cong \overline{LJ}$, $\overline{GH} \cong \overline{LK}$,	2. CPCTC
3. $HJ = KJ$, $GJ = LJ$	3. Def. of $\cong$ segments
4. $HJ + LJ = KJ + JG$	4. Addition Property
5. $KJ + GJ = KG$; $HJ + LJ = HL$	5. Segment Addition
6. $KG = HL$	6. Substitution
7. $\overline{KG} \cong \overline{HL}$	7. Def. of $\cong$ segments
8. $\overline{GL} \cong \overline{GL}$	8. Reflexive Property
9. $\triangle GHL \cong \triangle LKG$	9. SSS

21. Given: $\overline{EF} \cong \overline{HF}$
G is the midpoint of $\overline{EH}$.
Prove: $\triangle EFG \cong \triangle HFG$

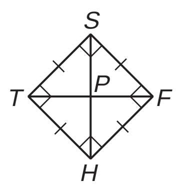


Proof:

Statements	Reasons
1. $\overline{EF} \cong \overline{HF}$; G is the midpoint of $\overline{EH}$.	1. Given
2. $\overline{EG} \cong \overline{GH}$	2. Midpoint Theorem
3. $\overline{FG} \cong \overline{FG}$	3. Reflexive Property
4. $\triangle EFG \cong \triangle HFG$	4. SSS

23. not possible **25.** SSS or SAS

27. Given: $\overline{TS} \cong \overline{SF} \cong \overline{FH} \cong \overline{HT}$
$\angle TSF$, $\angle SFH$, $\angle FHT$, and $\angle HTS$ are right angles.
Prove: $\triangle SHT \cong \triangle SHF$



Proof:

Statements	Reasons
1. $\overline{TS} \cong \overline{SF} \cong \overline{FH} \cong \overline{HT}$	1. Given
2. $\angle TSF$, $\angle SFH$, $\angle FHT$, and $\angle HTS$ are right angles.	2. Given
3. $\angle STH \cong \angle SFH$	3. All rt. $\angle$s are $\cong$.
4. $\triangle STH \cong \triangle SFH$	4. SAS
5. $\angle SHT \cong \angle SHF$	5. CPCTC

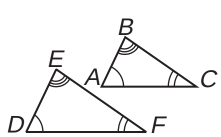
29. Sample answer: The properties of congruent triangles help land surveyors double check measurements. Answers should include the following.

- If each pair of corresponding angles and sides are congruent, the triangles are congruent by definition. If two pairs of corresponding sides and the included angle are congruent, the triangles are congruent by SAS. If each pair of corresponding sides are congruent, the triangles are congruent by SSS.
- Sample answer: Architects also use congruent triangles when designing buildings.

31. B **33.** $\triangle WXZ \cong \triangle YXZ$ **35.** 78 **37.** 68 **39.** 59 **41.** -1 **43.** There is a steeper rate of decline from the second quarter to the third. **45.** $\angle CBD$ **47.** $\overline{CD}$

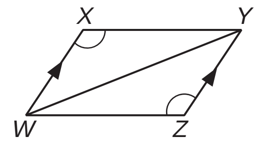
Pages 210–213 Lesson 4-5

1. Two triangles can have corresponding congruent angles without corresponding congruent sides. $\angle A \cong \angle D$, $\angle B \cong \angle E$, and

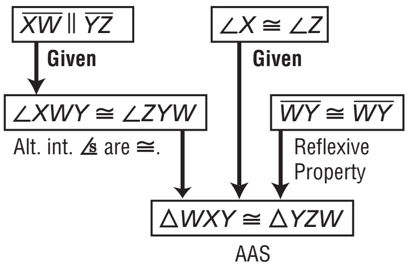


$\angle C \cong \angle F$. However, $\overline{AB} \ncong \overline{DE}$, so $\triangle ABC \ncong \triangle DEF$.
3. AAS can be proven using the Third Angle Theorem. Postulates are accepted as true without proof.

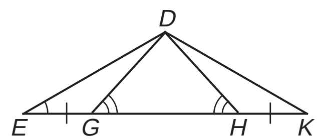
5. Given: $\overline{XW} \parallel \overline{YZ}$, $\angle X \cong \angle Z$
Prove: $\triangle WXY \cong \triangle YZW$



Proof:

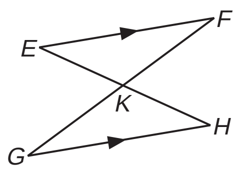


7. Given: $\angle E \cong \angle K$,
$\angle DGH \cong \angle DHG$,
$\overline{EG} \cong \overline{KH}$
Prove: $\triangle EGD \cong \triangle KHD$

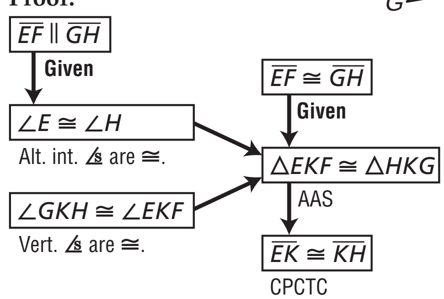


Proof:
Since $\angle EGD$ and $\angle DGH$ are a linear pair, the angles are supplementary. Likewise, $\angle KHD$ and $\angle DHG$ are supplementary. We are given that $\angle DGH \cong \angle DHG$. Angles supplementary to congruent angles are congruent so $\angle EGD \cong \angle KHD$. Since we are given that $\angle E \cong \angle K$ and $\overline{EG} \cong \overline{KH}$, $\triangle EGD \cong \triangle KHD$ by ASA.

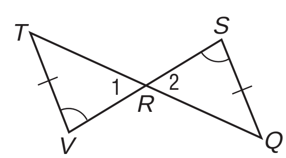
9. Given: $\overline{EF} \parallel \overline{GH}$, $\overline{EF} \cong \overline{GH}$
Prove: $\overline{EK} \cong \overline{KH}$



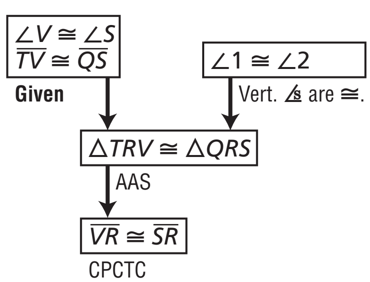
Proof:



11. Given: $\angle V \cong \angle S$,
$\overline{TV} \cong \overline{QS}$
Prove: $\overline{VR} \cong \overline{SR}$

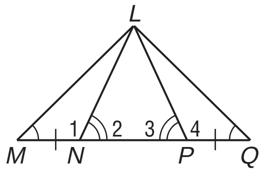


Proof:

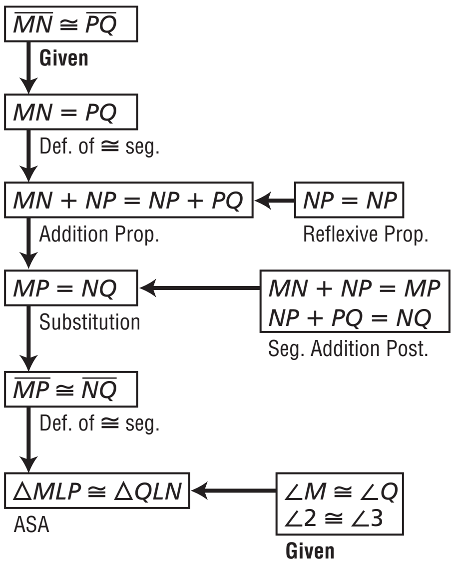


Selected Answers

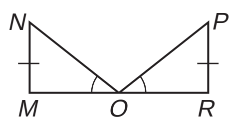
13. Given: $\overline{MN} \cong \overline{PQ}$, $\angle M \cong \angle Q$
$\angle 2 \cong \angle 3$
Prove: $\triangle MLP \cong \triangle QLN$



Proof:

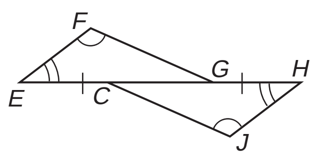


15. Given: $\angle NOM \cong \angle POR$,
$\overline{NM} \perp \overline{MR}$,
$\overline{PR} \perp \overline{MR}$,
$\overline{NM} \cong \overline{PR}$
Prove: $\overline{MO} \cong \overline{OR}$



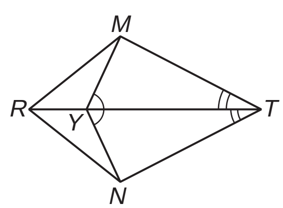
Proof: Since $\overline{NM} \perp \overline{MR}$ and $\overline{PR} \perp \overline{MR}$, $\angle M$ and $\angle R$ are right angles. $\angle M \cong \angle R$ because all right angles are congruent. We know that $\angle NOM \cong \angle POR$ and $\overline{NM} \cong \overline{PR}$. By AAS, $\triangle NMO \cong \triangle PRO$. $\overline{MO} \cong \overline{OR}$ by CPCTC.

17. Given: $\angle F \cong \angle J$,
$\angle E \cong \angle H$,
$\overline{EC} \cong \overline{GH}$
Prove: $\overline{EF} \cong \overline{HJ}$



Proof: We are given that $\angle F \cong \angle J$, $\angle E \cong \angle H$, and $\overline{EC} \cong \overline{GH}$. By the Reflexive Property, $\overline{CG} \cong \overline{CG}$. Segment addition results in $EG = EC + CG$ and $CH = CG + GH$. By the definition of congruence, $EC = GH$ and $CG = CG$. Substitute to find $EG = CH$. By AAS, $\triangle EFG \cong \triangle HJC$. By CPCTC, $\overline{EF} \cong \overline{HJ}$.

19. Given: $\angle MYT \cong \angle NYT$
$\angle MTY \cong \angle NTY$
Prove: $\triangle RYM \cong \triangle RYN$



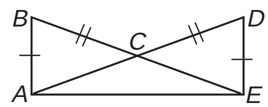
Proof:

Statement	Reason
1. $\angle MYT \cong \angle NYT$, $\angle MTY \cong \angle NTY$	1. Given
2. $\overline{YT} \cong \overline{YT}$, $\overline{RY} \cong \overline{RY}$	2. Reflexive Property
3. $\triangle MYT \cong \triangle NYT$	3. ASA
4. $\overline{MY} \cong \overline{NY}$	4. CPCTC
5. $\angle RYM$ and $\angle MYT$ are a linear pair; $\angle RYN$ and $\angle NYT$ are a linear pair	5. Def. of linear pair
6. $\angle RYM$ and $\angle MYT$ are supplementary and $\angle RYN$ and $\angle NYT$ are supplementary.	6. Supplement Theorem
7. $\angle RYM \cong \angle RYN$	7. ∡ suppl. to ≅ ∡ are ≅.
8. $\triangle RYM \cong \triangle RYN$	8. SAS

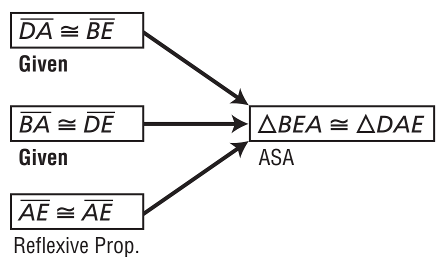
21. $\overline{CD} \cong \overline{GH}$, because the segments have the same measure. $\angle CFD \cong \angle HFG$ because vertical angles are congruent. Since F is the midpoint of $\overline{DG}$, $\overline{DF} \cong \overline{FG}$. It cannot be determined whether $\triangle CFD \cong \triangle HFG$. The information given does not lead to a unique triangle.

23. Since N is the midpoint of $\overline{JL}$, $\overline{JN} \cong \overline{NL}$. $\angle JNK \cong \angle LNK$ because perpendicular lines form right angles and right angles are congruent. By the Reflexive Property, $\overline{KN} \cong \overline{KN}$. $\triangle JKN \cong \triangle LKN$ by SAS. **25.** $\triangle VNR$, AAS or ASA
27. $\triangle MIN$, SAS **29.** Since Aiko is perpendicular to the ground, two right angles are formed and right angles are congruent. The angles of sight are the same and her height is the same for each triangle. The triangles are congruent by ASA. By CPCTC, the distances are the same. The method is valid. **31.** D

33. Given: $\overline{BA} \cong \overline{DE}$,
$\overline{DA} \cong \overline{BE}$
Prove: $\triangle BEA \cong \triangle DAE$



Proof:

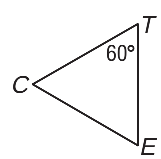


35. Turn; $RS = \sqrt{2}$, $R'S' = \sqrt{2}$, $ST = 1$, $S'T' = 1$, $RT = 1$, $R'T' = 1$. Use a protractor to confirm that the corresponding angles are congruent. **37.** If people are happy, then they rarely correct their faults. **39.** isosceles
41. isosceles

Pages 219–221 Lesson 4-6

1. The measure of only one angle must be given in an isosceles triangle to determine the measures of the other two angles. **3.** Sample answer: Draw a line segment. Set your compass to the length of the line segment and draw an arc from each endpoint. Draw segments from the intersection of the arcs to each endpoint. **5.** $\overline{BH} \cong \overline{BD}$

7. Given: $\triangle CTE$ is isosceles with vertex $\angle C$.
$m\angle T = 60$
Prove: $\triangle CTE$ is equilateral.



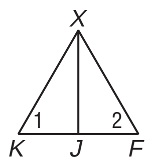
Proof:

Statements	Reasons
1. $\triangle CTE$ is isosceles with vertex $\angle C$.	1. Given
2. $\overline{CT} \cong \overline{CE}$	2. Def. of isosceles triangle
3. $\angle E \cong \angle T$	3. Isosceles Triangle Theorem
4. $m\angle E = m\angle T$	4. Def. of ≅ ∡

5. $m\angle T = 60$	**5.** Given
6. $m\angle E = 60$	**6.** Substitution
7. $m\angle C + m\angle E + m\angle T = 180$	**7.** Angle Sum Theorem
8. $m\angle C + 60 + 60 = 180$	**8.** Substitution
9. $m\angle C = 60$	**9.** Subtraction
10. $\triangle CTE$ is equiangular.	**10.** Def. of equiangular $\triangle$
11. $\triangle CTE$ is equilateral.	**11.** Equiangular $\triangle$s are equilateral.

9. $\angle LTR \cong \angle LRT$ **11.** $\angle LSQ \cong \angle LQS$ **13.** $\overline{LS} \cong \overline{LR}$
15. 20 **17.** 81 **19.** 28 **21.** 56 **23.** 36.5 **25.** 38
27. $x = 3$; $y = 18$

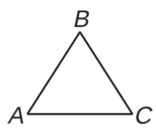
29. Given: $\triangle XKF$ is equilateral.
$\overline{XJ}$ bisects $\angle KXF$.
Prove: J is the midpoint of $\overline{KF}$.



Proof:

Statements	Reasons
1. $\triangle XKF$ is equilateral.	**1.** Given
2. $\overline{KX} \cong \overline{FX}$	**2.** Definition of equilateral $\triangle$
3. $\angle 1 \cong \angle 2$	**3.** Isosceles Triangle Theorem
4. $\overline{XJ}$ bisects $\angle X$	**4.** Given
5. $\angle KXJ \cong \angle FXJ$	**5.** Def. of $\angle$ bisector
6. $\triangle KXJ \cong \triangle FXJ$	**6.** ASA
7. $\overline{KJ} \cong \overline{JF}$	**7.** CPCTC
8. J is the midpoint of $\overline{KF}$.	**8.** Def. of midpoint

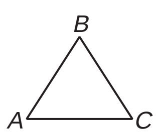
31. Case I:
Given: $\triangle ABC$ is an equilateral triangle.
Prove: $\triangle ABC$ is an equiangular triangle.



Proof:

Statements	Reasons
1. $\triangle ABC$ is an equilateral triangle.	**1.** Given
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$	**2.** Def. of equilateral $\triangle$
3. $\angle A \cong \angle B$, $\angle B \cong \angle C$, $\angle A \cong \angle C$	**3.** Isosceles Triangle Theorem
4. $\angle A \cong \angle B \cong \angle C$	**4.** Substitution
5. $\triangle ABC$ is an equiangular $\triangle$.	**5.** Def. of equiangular $\triangle$

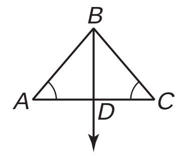
Case II:
Given: $\triangle ABC$ is an equiangular triangle.
Prove: $\triangle ABC$ is an equilateral triangle.



Proof:

Statements	Reasons
1. $\triangle ABC$ is an equiangular triangle.	**1.** Given
2. $\angle A \cong \angle B \cong \angle C$	**2.** Def. of equiangular $\triangle$
3. $\overline{AB} \cong \overline{AC}$, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{BC}$	**3.** Conv. of Isos. $\triangle$ Th.
4. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$	**4.** Substitution
5. $\triangle ABC$ is an equilateral $\triangle$.	**5.** Def. of equilateral $\triangle$

33. Given: $\triangle ABC$
$\angle A \cong \angle C$
Prove: $\overline{AB} \cong \overline{CB}$

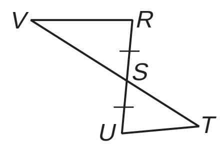


Proof:

Statements	Reasons
1. Let $\overrightarrow{BD}$ bisect $\angle ABC$.	**1.** Protractor Postulate
2. $\angle ABD \cong \angle CBD$	**2.** Def. of $\angle$ bisector
3. $\angle A \cong \angle C$	**3.** Given
4. $\overline{BD} \cong \overline{BD}$	**4.** Reflexive Property
5. $\triangle ABD \cong \triangle CBD$	**5.** AAS
6. $\overline{AB} \cong \overline{CB}$	**6.** CPCTC

35. 18 **37.** 30 **39.** The triangles in each set appear to be acute. **41.** Sample answer: Artists use angles, lines, and shapes to create visual images. Answers should include the following.
- Rectangle, squares, rhombi, and other polygons are used in many works of art.
- There are two rows of isosceles triangles in the painting. One row has three congruent isosceles triangles. The other row has six congruent isosceles triangles.

43. D
45. Given: $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$,
$\overline{RS} \cong \overline{US}$
Prove: $\triangle VRS \cong \triangle TUS$



Proof: We are given that $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, and $\overline{RS} \cong \overline{US}$. Perpendicular lines form four right angles so $\angle R$ and $\angle U$ are right angles. $\angle R \cong \angle U$ because all right angles are congruent. $\angle RSV \cong \angle UST$ since vertical angles are congruent. Therefore, $\triangle VRS \cong \triangle TUS$ by ASA.

47. $QR = \sqrt{52}$, $RS = \sqrt{2}$, $QS = \sqrt{34}$, $EG = \sqrt{34}$, $GH = \sqrt{10}$, and $EH = \sqrt{52}$. The corresponding sides are not congruent so $\triangle QRS$ is not congruent to $\triangle EGH$.

49.

p	q	$\sim p$	$\sim q$	$\sim p$ or $\sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

51.

y	z	$\sim y$	$\sim y$ or z
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

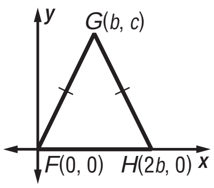
53. $(-1, -3)$

Page 221 Chapter 4 Practice Quiz 2

1. $JM = \sqrt{5}$, $ML = \sqrt{26}$, $JL = 5$, $BD = \sqrt{5}$, $DG = \sqrt{26}$, and $BG = 5$. Each pair of corresponding sides have the same measure so they are congruent. $\triangle JML \cong \triangle BDG$ by SSS. **3.** 52 **5.** 26

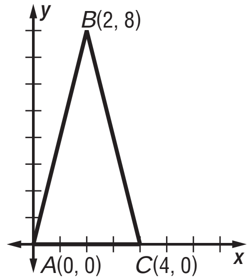
Pages 224–226 Lesson 4-7

1. Place one vertex at the origin, place one side of the triangle on the positive x-axis. Label the coordinates with expressions that will simplify the computations.

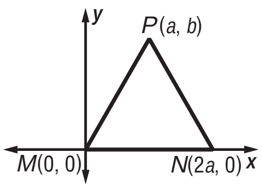
3. 

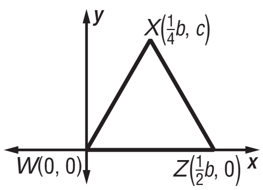
5. $P(0, b)$ **7.** $N(0, b)$, $Q(a, 0)$

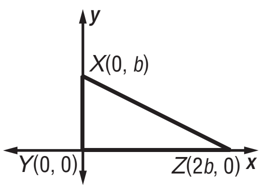
9. Given: $\triangle ABC$
Prove: $\triangle ABC$ is isosceles.



Proof: Use the Distance Formula to find AB and BC.
$AB = \sqrt{(2-0)^2 + (8-0)^2} = \sqrt{4+64}$ or $\sqrt{68}$
$BC = \sqrt{(4-2)^2 + (0-8)^2} = \sqrt{4+64}$ or $\sqrt{68}$
Since $AB = BC$, $\overline{AB} \cong \overline{BC}$. Since the legs are congruent, $\triangle ABC$ is isosceles.

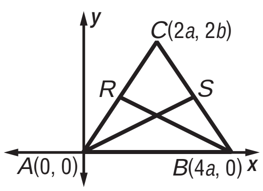
11. 

13. 

15. 

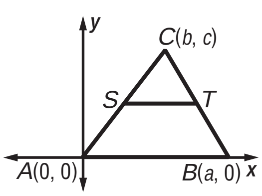
17. $Q(a, a)$, $P(a, 0)$
19. $D(2b, 0)$ **21.** $P(0, c)$, $N(2b, 0)$ **23.** $J(c, b)$

25. Given: isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$
R and S are midpoints of legs $\overline{AC}$ and $\overline{BC}$.
Prove: $\overline{AS} \cong \overline{BR}$



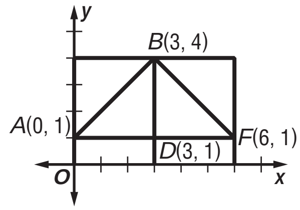
Proof:
The coordinates of R are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).
The coordinates of S are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.
$BR = \sqrt{(4a-a)^2 + (0-b)^2} = \sqrt{(3a)^2 + (-b)^2}$ or $\sqrt{9a^2+b^2}$
$AS = \sqrt{(3a-0)^2 + (b-0)^2} = \sqrt{(3a)^2 + (b)^2}$ or $\sqrt{9a^2+b^2}$
Since $BR = AS$, $\overline{AS} \cong \overline{BR}$.

27. Given: $\triangle ABC$
S is the midpoint of $\overline{AC}$.
T is the midpoint of $\overline{BC}$.
Prove: $\overline{ST} \parallel \overline{AB}$



Proof:
Midpoint S is $\left(\frac{b+0}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{b}{2}, \frac{c}{2}\right)$.
Midpoint T is $\left(\frac{a+b}{2}, \frac{c+0}{2}\right)$ or $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.
Slope of $\overline{ST} = \dfrac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}} = \dfrac{0}{\frac{a}{2}}$ or 0.
Slope of $\overline{AB} = \dfrac{0-0}{a-0} = \dfrac{0}{a}$ or 0.
$\overline{ST}$ and $\overline{AB}$ have the same slope so $\overline{ST} \parallel \overline{AB}$.

29. Given: $\triangle ABD$, $\triangle FBD$
$AF = 6$, $BD = 3$
Prove: $\triangle ABD \cong \triangle FBD$

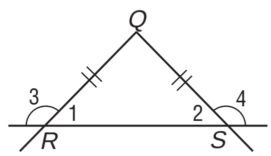


Proof: $\overline{BD} \cong \overline{BD}$ by the Reflexive Property.
$AD = \sqrt{(3-0)^2 + (1-1)^2} = \sqrt{9+0}$ or 3
$DF = \sqrt{(6-3)^2 + (1-1)^2} = \sqrt{9+0}$ or 3
Since $AD = DF$, $\overline{AD} \cong \overline{DF}$.
$AB = \sqrt{(3-0)^2 + (4-1)^2} = \sqrt{9+9}$ or $3\sqrt{2}$
$BF = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{9+9}$ or $3\sqrt{2}$
Since $AB = BF$, $\overline{AB} \cong \overline{BF}$.
$\triangle ABD \cong \triangle FBD$ by SSS.

31. Given: $\triangle BPR$, $\triangle BAR$
$PR = 800$, $BR = 800$, $RA = 800$
Prove: $\overline{PB} \cong \overline{BA}$
Proof:
$PB = \sqrt{(800-0)^2 + (800-0)^2}$ or $\sqrt{1{,}280{,}000}$
$BA = \sqrt{(800-1600)^2 + (800-0)^2}$ or $\sqrt{1{,}280{,}000}$
$PB = BA$, so $\overline{PB} \cong \overline{BA}$.

33. $\sqrt{680{,}000}$ or about 824.6 ft **35.** $(2a, 0)$ **37.** $AB = 4a$; $AC = \sqrt{(0-(-2a))^2 + (2a-0)^2} = \sqrt{4a^2+4a^2}$ or $\sqrt{8a^2}$; $CB = \sqrt{(0-2a)^2 + (2a-0)^2} = \sqrt{4a^2+4a^2}$ or $\sqrt{8a^2}$; Slope of $\overline{AC} = \frac{2a-0}{0-(-2a)}$ or 1; slope of $\overline{CB} = \frac{2a-0}{0-2a}$ or -1. $\overline{AC} \perp \overline{CB}$ and $\overline{AC} \cong \overline{CB}$, so $\triangle ABC$ is a right isosceles triangle. **39.** C

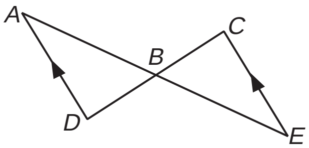
41. Given: $\angle 3 \cong \angle 4$
Prove: $\overline{QR} \cong \overline{QS}$



Proof:

Statements	Reasons
1. $\angle 3 \cong \angle 4$	1. Given
2. $\angle 2$ and $\angle 4$ form a linear pair. $\angle 1$ and $\angle 3$ form a linear pair.	2. Def. of linear pair
3. $\angle 2$ and $\angle 4$ are supplementary. $\angle 1$ and $\angle 3$ are supplementary.	3. If 2 ∠s form a linear pair, then they are suppl.
4. $\angle 2 \cong \angle 1$	4. Angles that are suppl. to ≅ ∠s are ≅.
5. $\overline{QR} \cong \overline{QS}$	5. Conv. of Isos. △ Th.

43. Given: $\overline{AD} \cong \overline{CE}$, $\overline{AD} \parallel \overline{CE}$
Prove: $\triangle ABD \cong \triangle EBC$



Proof:

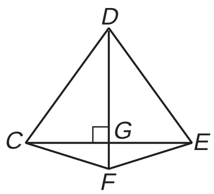
Statements	Reasons
1. $\overline{AD} \parallel \overline{CE}$	1. Given
2. $\angle A \cong \angle E$, $\angle D \cong \angle C$	2. Alt. int. $\measuredangle$ are $\cong$.
3. $\overline{AD} \cong \overline{CE}$	3. Given
4. $\triangle ABD \cong \triangle EBC$	4. ASA

45. $\overline{BC} \parallel \overline{AD}$; if alt. int. $\measuredangle$ are $\cong$, lines are $\parallel$. **47.** $\ell \parallel m$; if 2 lines are $\perp$ to the same line, they are $\parallel$.

Pages 227–230 Chapter 4 Study Guide and Review

1. h **3.** d **5.** a **7.** b **9.** obtuse, isosceles **11.** equiangular, equilateral **13.** 25 **15.** $\angle E \cong \angle D$, $\angle F \cong \angle C$, $\angle G \cong \angle B$, $\overline{EF} \cong \overline{DC}$, $\overline{FG} \cong \overline{CB}$, $\overline{GE} \cong \overline{BD}$ **17.** $\angle KNC \cong \angle RKE$, $\angle NCK \cong \angle KER$, $\angle CKN \cong \angle ERK$, $\overline{NC} \cong \overline{KE}$, $\overline{CK} \cong \overline{ER}$, $\overline{KN} \cong \overline{RK}$ **19.** $MN = \sqrt{20}$, $NP = \sqrt{5}$, $MP = 5$, $QR = \sqrt{20}$, $RS = \sqrt{5}$, and $QS = 5$. Each pair of corresponding sides has the same measure. Therefore, $\triangle MNP \cong \triangle QRS$ by SSS.

21. Given: $\triangle DGC \cong \triangle DGE$, $\triangle GCF \cong \triangle GEF$
Proof: $\triangle DFC \cong \triangle DFE$

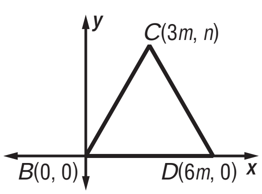


Proof:

Statement	Reason
1. $\triangle DGC \cong \triangle DGE$, $\triangle GCF \cong \triangle GEF$	1. Given
2. $\angle CDG \cong \angle EDG$, $\overline{CD} \cong \overline{ED}$, and $\angle CFD \cong \angle EFD$	2. CPCTC
3. $\triangle DFC \cong \triangle DFE$	3. AAS

23. 40 **25.** 80

27.



Chapter 5 Relationships in Triangles

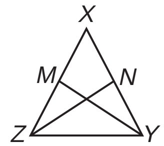
Page 235 Chapter 5 Getting Started

1. $(-4, 5)$ **3.** $(-0.5, -5)$ **5.** 68 **7.** 40 **9.** 26 **11.** 14 **13.** The sum of the measures of the angles is 180.

Pages 242–245 Lesson 5-1

1. Sample answer: Both pass through the midpoint of a side. A perpendicular bisector is perpendicular to the side of a triangle, and does not necessarily pass through the vertex opposite the side, while a median does pass through the vertex and is not necessarily perpendicular to the side. **3.** Sample answer: An altitude and angle bisector of a triangle are the same segment in an equilateral triangle.

5. Given: $\overline{XY} \cong \overline{XZ}$, $\overline{YM}$ and $\overline{ZN}$ are medians.
Prove: $\overline{YM} \cong \overline{ZN}$

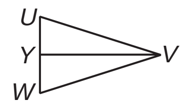


Proof:

Statements	Reasons
1. $\overline{XY} \cong \overline{XZ}$, $\overline{YM}$ and $\overline{ZN}$ are medians.	1. Given
2. M is the midpoint of $\overline{XZ}$. N is the midpoint of $\overline{XY}$.	2. Def. of median
3. $XY = XZ$	3. Def. of $\cong$ segs.
4. $\overline{XM} \cong \overline{MZ}$, $\overline{XN} \cong \overline{NY}$	4. Def. of median
5. $XM = MZ$, $XN = NY$	5. Def. of $\cong$ segs.
6. $XM + MZ = XZ$, $XN + NY = XY$	6. Segment Addition Postulate
7. $XM + MZ = XN + NY$	7. Substitution
8. $MZ + MZ = NY + NY$	8. Substitution
9. $2MZ = 2NY$	9. Addition Property
10. $MZ = NY$	10. Division Property
11. $\overline{MZ} \cong \overline{NY}$	11. Def. of $\cong$ segs.
12. $\angle XZY \cong \angle XYZ$	12. Isosceles Triangle Theorem
13. $\overline{YZ} \cong \overline{YZ}$	13. Reflexive Property
14. $\triangle MYZ \cong \triangle NZY$	14. SAS
15. $\overline{YM} \cong \overline{ZN}$	15. CPCTC

7. $\left(\frac{2}{3}, 3\frac{1}{3}\right)$ **9.** $\left(1\frac{2}{5}, 2\frac{3}{5}\right)$

11. Given: $\triangle UVW$ is isosceles with vertex angle UVW. $\overline{YV}$ is the bisector of $\angle UVW$.
Prove: $\overline{YV}$ is a median.

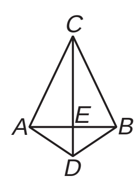


Proof:

Statements	Reasons
1. $\triangle UVW$ is an isosceles triangle with vertex angle UVW, $\overline{YV}$ is the bisector of $\angle UVW$.	1. Given
2. $\overline{UV} \cong \overline{WV}$	2. Def. of isosceles $\triangle$
3. $\angle UVY \cong \angle WVY$	3. Def. of angle bisector
4. $\overline{YV} \cong \overline{YV}$	4. Reflexive Property
5. $\triangle UVY \cong \triangle WVY$	5. SAS
6. $\overline{UY} \cong \overline{WY}$	6. CPCTC
7. Y is the midpoint of $\overline{UW}$.	7. Def. of midpoint
8. $\overline{YV}$ is a median.	8. Def. of median

13. $x = 7$, $m\angle 2 = 58$ **15.** $x = 20$, $y = 4$; yes; because $m\angle WPA = 90$ **17.** always **19.** never **21.** 2 **23.** 40 **25.** $PR = 18$ **27.** $(0, 7)$ **29.** $-\frac{4}{3}$

31. Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$
Prove: C and D are on the perpendicular bisector of $\overline{AB}$.

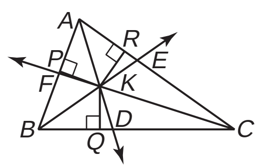


Proof:

Statements	Reasons
1. $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$	1. Given
2. $\overline{CD} \cong \overline{CD}$	2. Reflexive Property
3. $\triangle ACD \cong \triangle BCD$	3. SSS
4. $\angle ACD \cong \angle BCD$	4. CPCTC
5. $\overline{CE} \cong \overline{CE}$	5. Reflexive Property
6. $\triangle CEA \cong \triangle CEB$	6. SAS

7. $\overline{AE} \cong \overline{BE}$	7. CPCTC
8. E is the midpoint of $\overline{AB}$.	8. Def. of midpoint
9. $\angle CEA \cong \angle CEB$	9. CPCTC
10. $\angle CEA$ and $\angle CEB$ form a linear pair.	10. Def. of linear pair
11. $\angle CEA$ and $\angle CEB$ are supplementary.	11. Supplement Theorem
12. $m\angle CEA + m\angle CEB = 180$	12. Def. of suppl. $\angle$s
13. $m\angle CEA + m\angle CEA = 180$	13. Substitution
14. $2(m\angle CEA) = 180$	14. Substitution
15. $m\angle CEA = 90$	15. Division Property
16. $\angle CEA$ and $\angle CEB$ are rt. $\angle$s.	16. Def. of rt. $\angle$
17. $\overline{CD} \perp \overline{AB}$	17. Def. of $\perp$
18. $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.	18. Def. of $\perp$ bisector
19. C and D are on the perpendicular bisector of $\overline{AB}$.	19. Def. of points on a line

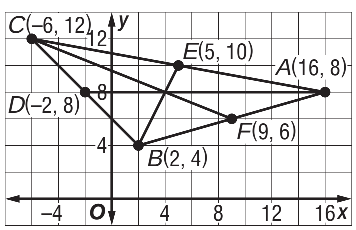
33. Given: $\triangle ABC$, $\overrightarrow{AD}$, $\overrightarrow{BE}$, $\overrightarrow{CF}$, $\overline{KP} \perp \overline{AB}$, $\overline{KQ} \perp \overline{BC}$, $\overline{KR} \perp \overline{AC}$
Prove: $KP = KQ = KR$



Proof:

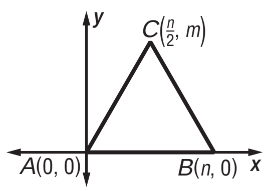
Statements	Reasons
1. $\triangle ABC$, $\overrightarrow{AD}$, $\overrightarrow{BE}$, $\overrightarrow{CF}$, $\overline{KP} \perp \overline{AB}$, $\overline{KQ} \perp \overline{BC}$, $\overline{KR} \perp \overline{AC}$	1. Given
2. $KP = KQ$, $KQ = KR$, $KP = KR$	2. Any point on the $\angle$ bisector is equidistant from the sides of the angle.
3. $KP = KQ = KR$	3. Transitive Property

35. 4
37.

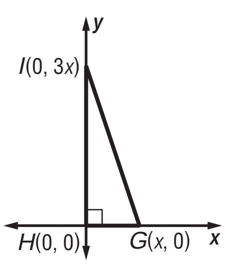


39. The altitude will be the same for both triangles, and the bases will be congruent, so the areas will be equal. **41.** C

43. Sample answer:



45. Sample answer:

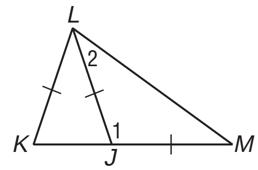


47. $\angle 5 \cong \angle 11$ **49.** $\overline{ML} \cong \overline{MN}$ **51.** $>$ **53.** $>$

Pages 251–254 Lesson 5-2

1. never **3.** Grace; she placed the shorter side with the smaller angle, and the longer side with the larger angle. **5.** $\angle 3$ **7.** $\angle 4, \angle 5, \angle 6$ **9.** $\angle 2, \angle 3, \angle 5, \angle 6$ **11.** $m\angle XZY < m\angle XYZ$ **13.** $AE < EB$ **15.** $BC = EC$ **17.** $\angle 1$ **19.** $\angle 7$ **21.** $\angle 7$ **23.** $\angle 2, \angle 7, \angle 8, \angle 10$ **25.** $\angle 3, \angle 5$ **27.** $\angle 8, \angle 7, \angle 3, \angle 1$ **29.** $m\angle KAJ < m\angle AJK$ **31.** $m\angle SMJ > m\angle MJS$ **33.** $m\angle MYJ < m\angle JMY$

35. Given: $\overline{JM} \cong \overline{JL}$
$\overline{JL} \cong \overline{KL}$
Prove: $m\angle 1 > m\angle 2$



Proof:

Statements	Reasons
1. $\overline{JM} \cong \overline{JL}$, $\overline{JL} \cong \overline{KL}$	1. Given
2. $\angle LKJ \cong \angle LJK$	2. Isosceles $\triangle$ Theorem
3. $m\angle LKJ = m\angle LJK$	3. Def. of $\cong$ $\angle$s
4. $m\angle 1 > m\angle LKJ$	4. Ext. $\angle$ Inequality Theorem
5. $m\angle 1 > m\angle LJK$	5. Substitution
6. $m\angle LJK > m\angle 2$	6. Ext. $\angle$ Inequality Theorem
7. $m\angle 1 > m\angle 2$	7. Trans. Prop. of Inequality

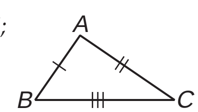
37. $ZY > YR$ **39.** $RZ > SR$ **41.** $TY < ZY$ **43.** $\angle M, \angle L, \angle K$ **45.** Phoenix to Atlanta, Des Moines to Phoenix, Atlanta to Des Moines **47.** 5; $\overline{PR}$, $\overline{QR}$, $\overline{PQ}$ **49.** 12; $\overline{QR}$, $\overline{PR}$, $\overline{PQ}$ **51.** $2(y+1) > \frac{x}{3}$, $y > \frac{x-6}{6}$ **53.** $3x + 15 > 4x + 7 > 0$, $-\frac{7}{4} < x < 8$ **55.** A **57.** $(15, -6)$ **59.** Yes; $\frac{1}{3}(-3) = -1$, and F is the midpoint of $\overline{BD}$. **61.** Label the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ as E, F, and G respectively. Then the coordinates of E, F, and G are $\left(\frac{a}{2}, 0\right)$, $\left(\frac{a+b}{2}, \frac{c}{2}\right)$, and $\left(\frac{b}{2}, \frac{c}{2}\right)$ respectively. The slope of $\overline{AF} = \frac{c}{a+b}$, and the slope of $\overline{AD} = \frac{c}{a+b}$, so D is on $\overline{AF}$. The slope of $\overline{BG} = \frac{c}{b-2a}$ and the slope of $\overline{BD} = \frac{c}{b-2a}$, so D is on $\overline{BG}$. The slope of $\overline{CE} = \frac{2c}{2b-a}$ and the slope of $\overline{CD} = \frac{2c}{2b-a}$, so D is on $\overline{CE}$. Since D is on $\overline{AF}$, $\overline{BG}$, and $\overline{CE}$, it is the intersection point of the three segments. **63.** $\angle C \cong \angle R$, $\angle D \cong \angle S$, $\angle G \cong \angle W$, $\overline{CD} \cong \overline{RS}$, $\overline{DG} \cong \overline{SW}$, $\overline{CG} \cong \overline{RW}$ **65.** 9.5 **67.** false

Page 254 Practice Quiz 1

1. 5 **3.** never **5.** sometimes **7.** no triangle **9.** $m\angle Q = 56$, $m\angle R = 61$, $m\angle S = 63$

Pages 257–260 Lesson 5-3

1. If a statement is shown to be false, then its opposite must be true.
3. Sample answer: $\triangle ABC$ is scalene.
Given: $\triangle ABC$; $AB \neq BC$; $BC \neq AC$; $AB \neq AC$
Prove: $\triangle ABC$ is scalene.



Proof:
Step 1: Assume $\triangle ABC$ is not scalene.
Case 1: $\triangle ABC$ is isosceles.
If $\triangle ABC$ is isosceles, then $AB = BC$, $BC = AC$, or $AB = AC$. This contradicts the given information, so $\triangle ABC$ is not isosceles.
Case 2: $\triangle ABC$ is equilateral.
In order for a triangle to be equilateral, it must also be isosceles, and Case 1 proved that $\triangle ABC$ is not isosceles. Thus, $\triangle ABC$ is not equilateral. Therefore, $\triangle ABC$ is scalene.
5. The lines are not parallel.

7. Given: $a > 0$

Prove: $\frac{1}{a} > 0$

Proof:

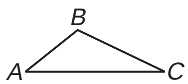
Step 1: Assume $\frac{1}{a} \le 0$.

Step 2: $\frac{1}{a} \le 0$; $a \cdot \frac{1}{a} \le 0 \cdot a$, $1 \le 0$

Step 3: The conclusion that $1 \le 0$ is false, so the assumption that $\frac{1}{a} \le 0$ must be false. Therefore, $\frac{1}{a} > 0$.

9. Given: $\triangle ABC$

Prove: There can be no more than one obtuse angle in $\triangle ABC$.



Proof:

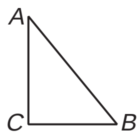
Step 1: Assume that there can be more than one obtuse angle in $\triangle ABC$.

Step 2: The measure of an obtuse angle is greater than 90, $x > 90$, so the measure of two obtuse angles is greater than 180, $2x > 180$.

Step 3: The conclusion contradicts the fact that the sum of the angles of a triangle equals 180. Thus, there can be at most one obtuse angle in $\triangle ABC$.

11. Given: $\triangle ABC$ is a right triangle; $\angle C$ is a right angle.

Prove: $AB > BC$ and $AB > AC$



Proof:

Step 1: Assume that the hypotenuse of a right triangle is not the longest side. That is, $AB < BC$ or $AB < AC$.

Step 2: If $AB < BC$, then $m\angle C < m\angle A$. Since $m\angle C = 90$, $m\angle A > 90$.
So, $m\angle C + m\angle A > 180$. By the same reasoning, if $AB < BC$, then $m\angle C + m\angle B > 180$.

Step 3: Both relationships contradict the fact that the sum of the measures of the angles of a triangle equals 180. Therefore, the hypotenuse must be the longest side of a right triangle.

13. $\overline{PQ} \ncong \overline{ST}$ **15.** A number cannot be expressed as $\frac{a}{b}$.

17. Points P, Q, and R are noncollinear.

19. Given: $\frac{1}{a} < 0$

Prove: a is negative.

Proof:

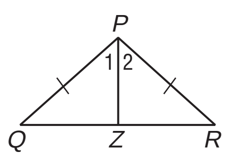
Step 1: Assume $a > 0$. $a \ne 0$ since that would make $\frac{1}{a}$ undefined.

Step 2:
$$\frac{1}{a} < 0$$
$$a\left(\frac{1}{a}\right) < 0 \cdot a$$
$$1 < 0$$

Step 3: $1 > 0$, so the assumption must be false. Thus, a must be negative.

21. Given: $\overline{PQ} \cong \overline{PR}$
$\angle 1 \ncong \angle 2$

Prove: $\overline{PZ}$ is not a median of $\triangle PQR$.



Proof:

Step 1: Assume $\overline{PZ}$ is a median of $\triangle PQR$.

Step 2: If $\overline{PZ}$ is a median of $\triangle PQR$, then Z is the midpoint of $\overline{QR}$, and $\overline{QZ} \cong \overline{RZ}$. $\overline{PZ} \cong \overline{PZ}$ by the Reflexive Property. $\triangle PZQ \cong \triangle PZR$ by SSS. $\angle 1 \cong \angle 2$ by CPCTC.

Step 3: This conclusion contradicts the given fact $\angle 1 \ncong \angle 2$. Thus, $\overline{PZ}$ is not a median of $\triangle PQR$.

23. Given: $a > 0$, $b > 0$, and $a > b$

Prove: $\frac{a}{b} > 1$

Proof:

Step 1: Assume that $\frac{a}{b} \le 1$.

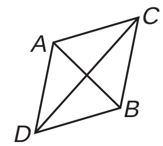
Step 2:

Case 1	Case 2
$\frac{a}{b} < 1$	$\frac{a}{b} = 1$
$a < b$	$a = b$

Step 3: The conclusion of both cases contradicts the given fact $a > b$. Thus, $\frac{a}{b} > 1$.

25. Given: $\triangle ABC$ and $\triangle ABD$ are equilateral.
$\triangle ACD$ is not equilateral.

Prove: $\triangle BCD$ is not equilateral.



Proof:

Step 1: Assume that $\triangle BCD$ is an equilateral triangle.

Step 2: If $\triangle BCD$ is an equilateral triangle, then $\overline{BC} \cong \overline{CD} \cong \overline{DB}$. Since $\triangle ABC$ and $\triangle ABD$ are equilateral triangles, $\overline{AC} \cong \overline{AB} \cong \overline{BC}$ and $\overline{AD} \cong \overline{AB} \cong \overline{DB}$. By the Transitive Property, $\overline{AC} \cong \overline{AD} \cong \overline{CD}$. Therefore, $\triangle ACD$ is an equilateral triangle.

Step 3: This conclusion contradicts the given information. Thus, the assumption is false. Therefore, $\triangle BCD$ is not an equilateral triangle.

27. Use $r = \frac{d}{t}$, $t = 3$, and $d = 175$.

Proof:

Step 1: Assume that Ramon's average speed was greater than or equal to 60 miles per hour, $r \ge 60$.

Step 2:

Case 1	Case 2
$r = 60$	$r > 60$
$60 \stackrel{?}{=} \frac{175}{3}$	$\frac{175}{3} \stackrel{?}{>} 60$
$60 \ne 58.3$	$58.3 \not> 60$

Step 3: The conclusions are false, so the assumption must be false. Therefore, Ramon's average speed was less than 60 miles per hour.

29.
$$1500 \cdot 15\% \stackrel{?}{=} 225$$
$$1500 \cdot 0.15 \stackrel{?}{=} 225$$
$$225 = 225$$

31. Yes; if you assume the client was at the scene of the crime, it is contradicted by his presence in Chicago at that time. Thus, the assumption that he was present at the crime is false.

33. Proof:

Step 1: Assume that $\sqrt{2}$ is a rational number.

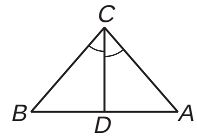
Step 2: If $\sqrt{2}$ is a rational number, it can be written as $\frac{a}{b}$, where a and b are integers with no common factors, and $b \ne 0$. If $\sqrt{2} = \frac{a}{b}$, then $2 = \frac{a^2}{b^2}$, and $2b^2 = a^2$. Thus a^2 is an even number, as is a. Because a is even it can be written as $2n$.
$$2b^2 = a^2$$
$$2b^2 = (2n)^2$$
$$2b^2 = 4n^2$$
$$b^2 = 2n^2$$
Thus, b^2 is an even number. So, b is also an even number.

Step 3: Because b and a are both even numbers, they have a common factor of 2. This contradicts the definition of rational numbers. Therefore, $\sqrt{2}$ is not rational.

Selected Answers

35. *D* **37.** $\angle P$

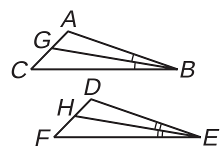
39. Given: $\overline{CD}$ is an angle bisector.
$\overline{CD}$ is an altitude.
Prove: $\triangle ABC$ is isosceles.



Proof:

Statements	Reasons
1. $\overline{CD}$ is an angle bisector. $\overline{CD}$ is an altitude.	1. Given
2. $\angle ACD \cong \angle BCD$	2. Def. of $\angle$ bisector
3. $\overline{CD} \perp \overline{AB}$	3. Def. of altitude
4. $\angle CDA$ and $\angle CDB$ are rt. $\angle$s	4. $\perp$ lines form 4 rt. $\angle$s.
5. $\angle CDA \cong \angle CDB$	5. All rt. $\angle$s are $\cong$.
6. $\overline{CD} \cong \overline{CD}$	6. Reflexive Prop.
7. $\triangle ACD \cong \triangle BCD$	7. ASA
8. $\overline{AC} \cong \overline{BC}$	8. CPCTC
9. $\triangle ABC$ is isosceles.	9. Def. of isosceles $\triangle$

41. Given: $\triangle ABC \cong \triangle DEF$; $\overline{BG}$ is an angle bisector of $\angle ABC$. $\overline{EH}$ is an angle bisector of $\angle DEF$.
Prove: $\overline{BG} \cong \overline{EH}$



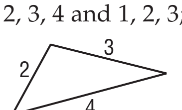
Proof:

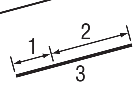
Statements	Reasons
1. $\triangle ABC \cong \triangle DEF$	1. Given
2. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\angle ABC \cong \angle DEF$	2. CPCTC
3. $\overline{BG}$ is an angle bisector of $\angle ABC$. $\overline{EH}$ is an angle bisector of $\angle DEF$.	3. Given
4. $\angle ABG \cong \angle GBC$, $\angle DEH \cong \angle HEF$	4. Def. of $\angle$ bisector
5. $m\angle ABC = m\angle DEF$	5. Def. of $\cong$ $\angle$s
6. $m\angle ABG = m\angle GBC$, $m\angle DEH = m\angle HEF$	6. Def. of $\cong$ $\angle$s
7. $m\angle ABC = m\angle ABG + m\angle GBC$, $m\angle DEF = m\angle DEH + m\angle HEF$	7. Angle Addition Property
8. $m\angle ABC = m\angle ABG + m\angle ABG$, $m\angle DEF = m\angle DEH + m\angle DEH$	8. Substitution
9. $m\angle ABG + m\angle ABG = m\angle DEH + m\angle DEH$	9. Substitution
10. $2m\angle ABG = 2m\angle DEH$	10. Addition
11. $m\angle ABG = m\angle DEH$	11. Division
12. $\angle ABG \cong \angle DEH$	12. Def. of $\cong$ $\angle$s
13. $\triangle ABG \cong \triangle DEH$	13. ASA
14. $\overline{BG} \cong \overline{EH}$	14. CPCTC

43. $y - 3 = 2(x - 4)$ **45.** $y + 9 = 11(x + 4)$ **47.** false

Pages 263–266 Lesson 5-4

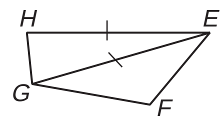
1. Sample answer: If the lines are not horizontal, then the segment connecting their y-intercepts is not perpendicular to either line. Since distance is measured along a perpendicular segment, this segment cannot be used.
3. Sample answer:
2, 3, 4 and 1, 2, 3;





5. no; $5 + 10 \ngtr 15$
7. yes; $5.2 + 5.6 > 10.1$
9. $9 < n < 37$ **11.** $3 < n < 33$
13. B **15.** no; $2 + 6 \ngtr 11$
17. no; $13 + 16 \ngtr 29$ **19.** yes; $9 + 20 > 21$ **21.** yes; $17 + 30 > 30$ **23.** yes; $0.9 + 4 > 4.1$
25. no; $0.18 + 0.21 \ngtr 0.52$ **27.** $2 < n < 16$ **29.** $6 < n < 30$
31. $29 < n < 93$ **33.** $24 < n < 152$ **35.** $0 < n < 150$
37. $97 < n < 101$

39. Given: $\overline{HE} \cong \overline{EG}$
Prove: $HE + FG > EF$



Proof:

Statements	Reasons
1. $\overline{HE} \cong \overline{EG}$	1. Given
2. $HE = EG$	2. Def. of $\cong$ segments
3. $EG + FG > EF$	3. Triangle Inequality
4. $HE + FG > EF$	4. Substitution

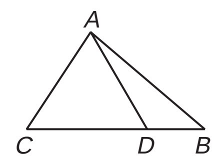
41. yes; $AB + BC > AC$, $AB + AC > BC$, $AC + BC > AB$
43. no; $XY + YZ = XZ$ **45.** 4 **47.** 3 **49.** $\frac{1}{2}$ **51.** Sample answer: You can use the Triangle Inequality Theorem to verify the shortest route between two locations. Answers should include the following.
- A longer route might be better if you want to collect frequent flier miles.
- A straight route might not always be available.

53. A **55.** $\overline{QR}$, $\overline{PQ}$, $\overline{PR}$ **57.** $JK = 5$, $KL = 2$, $JL = \sqrt{29}$, $PQ = 5$, $QR = 2$, and $PR = \sqrt{29}$. The corresponding sides have the same measure and are congruent. $\triangle JKL \cong \triangle PQR$ by SSS. **59.** $JK = \sqrt{113}$, $KL = \sqrt{50}$, $JL = \sqrt{65}$, $PQ = \sqrt{58}$, $QR = \sqrt{61}$, and $PR = \sqrt{65}$. The corresponding sides are not congruent, so the triangles are not congruent. **61.** $x < 6.6$

Page 266 Practice Quiz 2

1. The number 117 is not divisible by 13.
3. Step 1: Assume that $x \leq 8$.
Step 2: $7x > 56$ so $x > 8$
Step 3: The solution of $7x > 56$ contradicts the assumption. Thus, $x \leq 8$ must be false. Therefore, $x > 8$.

5. Given: $m\angle ADC \neq m\angle ADB$
Prove: $\overline{AD}$ is not an altitude of $\triangle ABC$.



Proof:

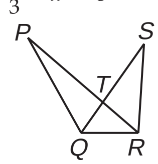
Statements	Reasons
1. $\overline{AD}$ is an altitude of $\triangle ABC$.	1. Assumption
2. $\angle ADC$ and $\angle ADB$ are right angles.	2. Def. of altitude
3. $\angle ADC \cong \angle ADB$	3. All rt $\angle$s are $\cong$.
4. $m\angle ADC = m\angle ADB$	4. Def. of $\cong$ angles

This contradicts the given information that $m\angle ADC \neq m\angle ADB$. Thus, $\overline{AD}$ is not an altitude of $\triangle ABC$.
7. no; $25 + 35 \ngtr 60$ **9.** yes; $5 + 6 > 10$

Pages 270–273 Lesson 5-5

1. Sample answer: A pair of scissors illustrates the SSS inequality. As the distance between the tips of the scissors decreases, the angle between the blades decreases, allowing the blades to cut. **3.** $AB < CD$ **5.** $\frac{7}{3} < x < 6$

7. Given: $\overline{PQ} \cong \overline{SQ}$
Prove: $PR > SR$



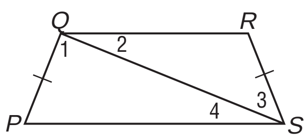
Selected Answers

Proof:

Statements	Reasons
1. $\overline{PQ} \cong \overline{SQ}$	1. Given
2. $\overline{QR} \cong \overline{QR}$	2. Reflexive Property
3. $m\angle PQR = m\angle PQS + m\angle SQR$	3. Angle Addition Postulate
4. $m\angle PQR > m\angle SQR$	4. Def. of inequality
5. $PR > SR$	5. SAS Inequality

9. Sample answer: The pliers are an example of the SAS inequality. As force is applied to the handles, the angle between them decreases causing the distance between the ends of the pliers to decrease. As the distance between the ends of the pliers decreases, more force is applied to a smaller area. **11.** $m\angle BDC < m\angle FDB$ **13.** $AD > DC$ **15.** $m\angle AOD > m\angle AOB$ **17.** $4 < x < 10$ **19.** $7 < x < 20$

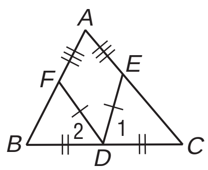
21. Given: $\overline{PQ} \cong \overline{RS}$, $QR < PS$
Prove: $m\angle 3 < m\angle 1$



Proof:

Statements	Reasons
1. $\overline{PQ} \cong \overline{RS}$	1. Given
2. $\overline{QS} \cong \overline{QS}$	2. Reflexive Property
3. $QR < PS$	3. Given
4. $m\angle 3 < m\angle 1$	4. SSS Inequality

23. Given: $\overline{ED} \cong \overline{DF}$; $m\angle 1 > m\angle 2$; D is the midpoint of $\overline{CB}$; $\overline{AE} \cong \overline{AF}$.
Prove: $AC > AB$



Proof:

Statements	Reasons
1. $\overline{ED} \cong \overline{DF}$; D is the midpoint of $\overline{DB}$.	1. Given
2. $CD = BD$	2. Def. of midpoint
3. $\overline{CD} \cong \overline{BD}$	3. Def. of $\cong$ segments
4. $m\angle 1 > m\angle 2$	4. Given
5. $EC > FB$	5. SAS Inequality
6. $\overline{AE} \cong \overline{AF}$	6. Given
7. $AE = AF$	7. Def. of $\cong$ segments
8. $AE + EC > AE + FB$	8. Add. Prop. of Inequality
9. $AE + EC > AF + FB$	9. Substitution Prop. of Inequality
10. $AE + EC = AC$, $AF + FB = AB$	10. Segment Add. Post.
11. $AC > AB$	11. Substitution

25. As the door is opened wider, the angle formed increases and the distance from the end of the door to the door frame increases.

27. As the vertex angle increases, the base angles decrease. Thus, as the base angles decrease, the altitude of the triangle decreases.

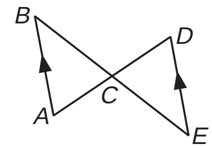
29.

Stride (m)	Velocity (m/s)
0.25	0.07
0.50	0.22
0.75	0.43
1.00	0.70
1.25	1.01
1.50	1.37

31. Sample answer: A backhoe digs when the angle between the two arms decreases and the shovel moves through the dirt. Answers should include the following.
- As the operator digs, the angle between the arms decreases.
- The distance between the ends of the arms increases as the angle between the arms increases, and decreases as the angle decreases.

33. B **35.** yes; $16 + 6 > 19$ **37.** $\overline{AD}$ is a not median of $\triangle ABC$.

39. Given: $\overline{AD}$ bisects $\overline{BE}$; $\overline{AB} \parallel \overline{DE}$.
Prove: $\triangle ABC \cong \triangle DEC$



Proof:

Statements	Reasons
1. $\overline{AD}$ bisects $\overline{BE}$; $\overline{AB} \parallel \overline{DE}$.	1. Given
2. $\overline{BC} \cong \overline{EC}$	2. Def. of seg. bisector
3. $\angle B \cong \angle E$	3. Alt. int. ∠s Thm.
4. $\angle BCA \cong \angle ECD$	4. Vert. ∠s are $\cong$.
5. $\triangle ABC \cong \triangle DEC$	5. ASA

41. $EF = 5$, $FG = 50$, $EG = 5$; isosceles **43.** $EF = \sqrt{145}$, $FG = \sqrt{544}$, $EG = 35$; scalene **45.** yes, by the Law of Detachment

Pages 274–276 Chapter 5 Study Guide and Review

1. incenter **3.** Triangle Inequality Theorem **5.** angle bisector **7.** orthocenter **9.** 72 **11.** $m\angle DEF > m\angle DFE$ **13.** $m\angle DEF > m\angle FDE$ **15.** $DQ < DR$ **17.** $SR > SQ$ **19.** The triangles are not congruent. **21.** no; $7 + 5 \ngtr 20$ **23.** yes; $6 + 18 > 20$ **25.** $BC > MD$ **27.** $x > 7$

Chapter 6 Proportions and Similarity

Page 281 Chapter 6 Getting Started

1. 15 **3.** 10 **5.** 2 **7.** $-\frac{6}{5}$ **9.** yes; $\cong$ alt. int. ∠s **11.** 2, 4, 8, 16 **13.** 1, 7, 25, 79

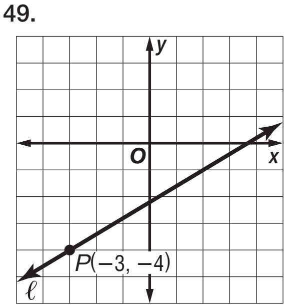
Page 284–287 Lesson 6-1

1. Cross multiply and divide by 28. **3.** Suki; Madeline did not find the cross products correctly. **5.** $\frac{1}{12}$ **7.** 2.1275 **9.** 54, 48, 42 **11.** 320 **13.** 76:89 **15.** 25.3:1 **17.** 18 ft, 24 ft **19.** 43.2, 64.8, 72 **21.** 18 in., 24 in., 30 in. **23.** $\frac{3}{2}$ **25.** 2:19 **27.** 16.4 lb **29.** 1.295 **31.** 14 **33.** 3 **35.** $-1, \frac{-2}{3}$ **37.** 36%

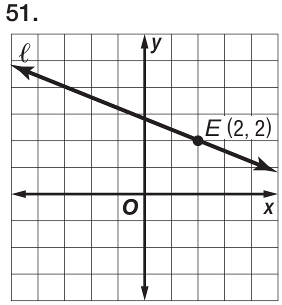
39. Sample answer: It appears that Tiffany used rectangles with areas that were in proportion as a background for this artwork. Answers should include the following.
- The center column pieces are to the third column from the left pieces as the pieces from the third column are to the pieces in the outside column.
- The dimensions are approximately 24 inches by 34 inches.

41. D **43.** always **45.** $15 < x < 47$ **47.** $12 < x < 34$

49.



51.



53. Yes; 100 km and 62 mi are the same length, so $AB = CD$. By the definition of congruent segments, $\overline{AB} \cong \overline{CD}$. **55.** 13.0 **57.** 1.2

Page 292–297 Lesson 6-2

1. Both students are correct. One student has inverted the ratio and reversed the order of the comparison. **3.** If two polygons are congruent, then they are similar. All of the corresponding angles are congruent, and the ratio of measures of the corresponding sides is 1.Two similar figures have congruent angles, and the sides are in proportion, but not always congruent. If the scale factor is 1, then the figures are congruent. **5.** Yes; $\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, $\angle D \cong \angle H$ and $\frac{AD}{EH} = \frac{DC}{HG} = \frac{CB}{GF} = \frac{BA}{FE} = \frac{2}{3}$. So $\square ABCD \sim \square EFGH$. **7.** polygon $ABCD \sim$ polygon $EFGH$; 23; 28; 20; 32; $\frac{1}{2}$ **9.** 60 m **11.** $ABCF$ is similar to $EDCF$ since they are congruent. **13.** $\triangle ABC$ is not similar to $\triangle DEF$. $\angle A \not\cong \angle D$. **15.** $\frac{1}{3}$ **17.** polygon $ABCD \sim$ polygon $EFGH$; $\frac{13}{3}$; $AB = \frac{16}{3}$; $CD = \frac{10}{3}$; $\frac{2}{3}$ **19.** $\triangle ABE \sim \triangle ACD$; 6; $BC = 8$; $ED = 5$; $\frac{5}{9}$ **21.** about 3.9 in. by 6.25 in. **23.** $\frac{25}{16}$

25.

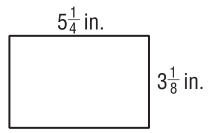
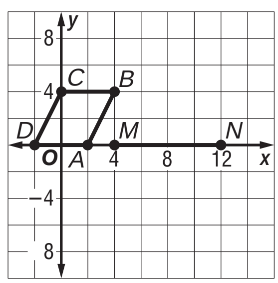


Figure not shown actual size.

27. always **29.** never **31.** sometimes **33.** always **35.** 30; 70 **37.** 27; 14 **39.** 71.05; 48.45 **41.** 7.5 **43.** 108 **45.** 73.2 **47.** $\frac{8}{5}$

49. $L(16, 8)$ and $P(8, 8)$ or $L(16, -8)$ and $P(8, -8)$

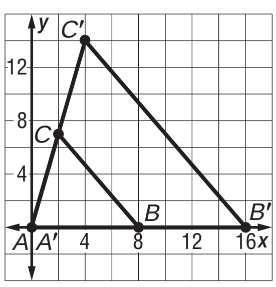


51. 18 ft by 15 ft **53.** 16 : 1 **55.** 16 : 1 **57.** 2 : 1; ratios are the same. **59.** $\frac{a}{3a} = \frac{b}{3b} = \frac{c}{3c} = \frac{a + b + c}{3(a + b + c)} = \frac{1}{3}$

61. Sample answer: Artists use geometric shapes in patterns to create another scene or picture. The included objects have the same shape but are different sizes. Answers should include the following.
- The objects are enclosed within a circle. The objects seem to go on and on
- Each "ring" of figures has images that are approximately the same width, but vary in number and design.

63. D

65.



67. $\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'} = \frac{1}{2}$ **69.** The sides are proportional and the angles are congruent, so the triangles are similar. **71.** −23 **73.** $OC > AO$ **75.** $m\angle ABD > m\angle ADB$ **77.** 91 **79.** $m\angle 1 = m\angle 2 = 111$ **81.** 62 **83.** 118 **85.** 62 **87.** 118

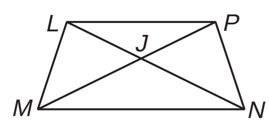
Page 301–306 Lesson 6-3

1. Sample answer: Two triangles are congruent by the SSS, SAS, and ASA Postulates and the AAS Theorem. In these triangles, corresponding parts must be congruent. Two triangles are similar by AA Similarity, SSS Similarity, and SAS Similarity. In similar triangles, the sides are proportional and the angles are congruent. Congruent triangles are always similar triangles. Similar triangles are congruent only when the scale factor for the proportional sides is 1. SSS and SAS are common relationships for both congruence and similarity. **3.** Alicia; while both have corresponding sides in a ratio, Alicia has them in proper order with the numerators from the same triangle.

5. $\triangle ABC \sim \triangle DEF$; $x = 10$; $AB = 10$; $DE = 6$ **7.** yes: $\triangle DEF \sim \triangle ACB$ by SSS Similarity **9.** 135 ft **11.** yes; $\triangle QRS \sim \triangle TVU$ by SSS Similarity **13.** yes; $\triangle RST \sim \triangle JKL$ by AA Similarity **15.** Yes; $\triangle ABC \sim \triangle JKL$ by SAS Similarity **17.** No; sides are not proportional. **19.** $\triangle ABE \sim \triangle ACD$; $x = \frac{8}{5}$; $AB = 3\frac{3}{5}$; AC $= 9\frac{3}{5}$ **21.** $\triangle ABC \sim \triangle ARS$; $x = 8$; 15; 8 **23.** $\frac{3}{2}$ **25.** true **27.** $\triangle EAB \sim \triangle EFC \sim \triangle AFD$: AA Similarity **29.** $KP = 5$, $KM = 15$, $MR = 13\frac{1}{3}$, $ML = 20$, $MN = 12$, $PR = 16\frac{2}{3}$ **31.** $m\angle TUV = 43$, $m\angle R = 43$, $m\angle RSU = 47$, $m\angle SUV = 47$ **33.** $x = y$; if $\overline{BD} \parallel \overline{AE}$, then $\triangle BCD \sim \triangle ACE$ by AA Similarity and $\frac{BC}{AC} = \frac{DC}{EC}$. Thus, $\frac{2}{4} = \frac{x}{x + y}$. Cross multiply and solve for y, yielding $y = x$.

35. Given: $\overline{LP} \parallel \overline{MN}$

Prove: $\frac{LJ}{JN} = \frac{PJ}{JM}$

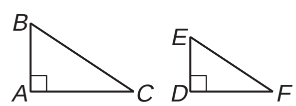


Proof:

Statements	**Reasons**
1. $\overline{LP} \parallel \overline{MN}$	1. Given
2. $\angle PLN \cong \angle LNM$, $\angle LPM \cong \angle PMN$	2. Alt. Int. $\angle$ Theorem
3. $\triangle LPJ \sim \triangle NMJ$	3. AA Similarity
4. $\frac{LJ}{JN} = \frac{PJ}{JM}$	4. Corr. sides of $\sim \triangle$s are proportional.

37. Given: $\triangle BAC$ and $\triangle EDF$ are right triangles. $\frac{AB}{DE} = \frac{AC}{DF}$

Prove: $\triangle ABC \sim \triangle DEF$



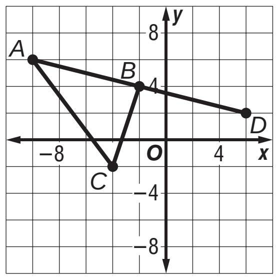
Proof:

Statements	**Reasons**
1. $\triangle BAC$ and $\triangle EDF$ are right triangles.	1. Given
2. $\angle BAC$ and $\angle EDF$ are right angles.	2. Def. of rt. $\triangle$
3. $\angle BAC \cong \angle EDF$	3. All rt. $\angle$s are $\cong$.
4. $\frac{AB}{DE} = \frac{AC}{DF}$	4. Given
5. $\triangle ABC \sim \triangle DEF$	5. SAS Similarity

39. 13.5 ft **41.** about 420.5 m **43.** 10.75 m

Selected Answers

45.



47. $\triangle ABC \sim \triangle ACD$; $\triangle ABC \sim \triangle CBD$; $\triangle ACD \sim \triangle CBD$; they are similar by AA Similarity. **49.** A **51.** $PQRS \sim ABCD$; 1.6; 1.4; 1.1; $\frac{1}{2}$ **53.** 5 **55.** 15 **57.** No; $\overline{AT}$ is not perpendicular to $\overline{BC}$. **59.** (5.5, 13) **61.** (3.5, −2.5)

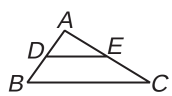
Page 306 Practice Quiz 1

1. yes; $\angle A \cong \angle E$, $\angle B \cong \angle D$, $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$ and $\frac{AB}{ED} = \frac{BC}{DC} = \frac{AF}{EF} = \frac{FC}{FC} = 1$ **3.** $\triangle ADE \sim \triangle CBE$; 2; 8; 4 **5.** 1947 mi

Page 311–315 Lesson 6-4

1. Sample answer: If a line intersects two sides of a triangle and separates sides into corresponding segments of proportional lengths, then it is parallel to the third side. **3.** Given three or more parallel lines intersecting two transversals, Corollary 6.1 states that the parts of the transversals are proportional. Corollary 6.2 states that if the parts of one transversal are congruent, then the parts of every transversal are congruent. **5.** 10 **7.** The slopes of $\overline{DE}$ and $\overline{BC}$ are both 0. So $\overline{DE} \parallel \overline{BC}$. **9.** Yes; $\frac{MN}{NP} = \frac{MR}{RQ} = \frac{9}{16}$, so $\overline{RN} \parallel \overline{QP}$. **11.** $x = 2$; $y = 5$ **13.** 1100 yd **15.** 3 **17.** $x = 6$, ED = 9 **19.** $BC = 10$, $FE = 13\frac{1}{3}$, $CD = 9$, $DE = 15$ **21.** 10 **23.** No; segments are not proportional; $\frac{PQ}{QR} = \frac{3}{7}$ and $\frac{PT}{TS} = 2$. **25.** yes **27.** $\sqrt{52}$ **29.** The endpoints of $\overline{DE}$ are $D\left(3, \frac{1}{2}\right)$ and $E\left(\frac{3}{2}, -4\right)$. Both $\overline{DE}$ and $\overline{AB}$ have slope of 3. **31.** (3, 8) or (4, 4) **33.** $x = 21$, $y = 15$ **35.** 25 ft **37.** 18.75 ft

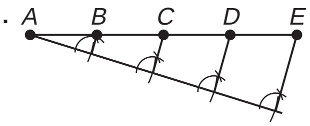
39. Given: D is the midpoint of $\overline{AB}$.
E is the midpoint of $\overline{AC}$.
Prove: $\overline{DE} \parallel \overline{BC}$; $DE = \frac{1}{2}BC$



Proof:

Statements	Reasons
1. D is the midpoint of $\overline{AB}$. E is the midpoint of $\overline{AC}$.	1. Given
2. $\overline{AD} \cong \overline{DB}$, $\overline{AE} \cong \overline{EC}$	2. Midpoint Theorem
3. $AD = DB$, $AE = EC$	3. Def. of ≅ segments
4. $AB = AD + DB$, $AC = AE + AC$	4. Segment Addition Postulate
5. $AB = AD + AD$, $AC = AE + AE$	5. Substitution
6. $AB = 2AD$, $AC = 2AE$	6. Substitution
7. $\frac{AB}{AD} = 2$, $\frac{AC}{AE} = 2$	7. Division Prop.
8. $\frac{AB}{AD} = \frac{AC}{AE}$	8. Transitive Prop.
9. $\angle A \cong \angle A$	9. Reflexive Prop.
10. $\triangle ADE \sim \triangle ABC$	10. SAS Similarity
11. $\angle ADE \cong \angle ABC$	11. Def. of ~ polygons
12. $\overline{DE} \parallel \overline{BC}$	12. If corr. $\angle$s are ≅, the lines are parallel.
13. $\frac{BC}{DE} = \frac{AB}{AD}$	13. Def. of ~ polygons
14. $\frac{BC}{DE} = 2$	14. Substitution
15. $2DE = BC$	15. Mult. Prop.
16. $DE = \frac{1}{2}BC$	16. Division Prop.

41.



43. $u = 24$; $w = 26.4$; $x = 30$; $y = 21.6$; $z = 33.6$

45. Sample answer: City planners use maps in their work. Answers should include the following.

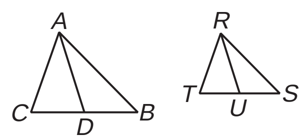
- City planners need to know geometry facts when developing zoning laws.
- A city planner would need to know that the shortest distance between two parallel lines is the perpendicular distance.

47. 4 **49.** yes; AA **51.** no; angles not congruent **53.** $x = 12$, $y = 6$ **55.** $m\angle ABD > m\angle BAD$ **57.** $m\angle CBD > m\angle BCD$ **59.** 18 **61.** false **63.** true **65.** $\angle R \cong \angle X$, $\angle S \cong \angle Y$, $\angle T \cong \angle Z$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$, $\overline{RT} \cong \overline{XZ}$

Page 319–323 Lesson 6-5

1. $\triangle ABC \sim \triangle MNQ$ and $\overline{AD}$ and $\overline{MR}$ are altitudes, angle bisectors, or medians. **3.** 10.8 **5.** 6 **7.** 6.75 **9.** 330 cm or 3.3 m **11.** 63 **13.** 20.25 **15.** 78 **17.** Yes; the perimeters are in the same ratio as the sides, $\frac{300}{600}$ or $\frac{1}{2}$. **19.** $\frac{3}{2}$ **21.** 4 **23.** $11\frac{1}{5}$ **25.** 6 **27.** 5, 13.5 **29.** $xy = z^2$; $\triangle ACD \sim \triangle CBD$ by AA Similarity. Thus, $\frac{CD}{BD} = \frac{AD}{CD}$ or $\frac{z}{y} = \frac{x}{z}$. The cross products yield $xy = z^2$.

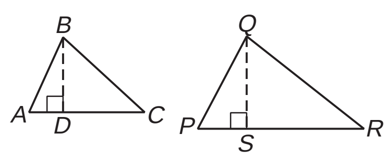
31. Given: $\triangle ABC \sim \triangle RST$, $\overline{AD}$ is a median of $\triangle ABC$.
$\overline{RU}$ is a median of $\triangle RST$.
Prove: $\frac{AD}{RU} = \frac{AB}{RS}$



Proof:

Statements	Reasons
1. $\triangle ABC \sim \triangle RST$ $\overline{AD}$ is a median of $\triangle ABC$. $\overline{RU}$ is a median of $\triangle RST$.	1. Given
2. $CD = DB$; $TU = US$	2. Def. of median
3. $\frac{AB}{RS} = \frac{CB}{TS}$	3. Def. of ~ polygons
4. $CB = CD + DB$; $TS = TU + US$	4. Segment Addition Postulate
5. $\frac{AB}{RS} = \frac{CD + DB}{TU + US}$	5. Substitution
6. $\frac{AB}{RS} = \frac{DB + DB}{US + US}$ or $\frac{2(DB)}{2(US)}$	6. Substitution
7. $\frac{AB}{RS} = \frac{DB}{US}$	7. Substitution
8. $\angle B \cong \angle S$	8. Def. of ~ polygons
9. $\triangle ABD \sim \triangle RSU$	9. SAS Similarity
10. $\frac{AD}{RU} = \frac{AB}{RS}$	10. Def. of ~ polygons

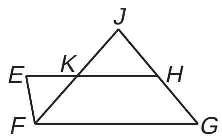
33. Given: $\triangle ABC \sim \triangle PQR$, $\overline{BD}$ is an altitude of $\triangle ABC$.
$\overline{QS}$ is an altitude of $\triangle PQR$.
Prove: $\frac{QP}{BA} = \frac{QS}{BD}$



Selected Answers

Proof:
$\angle A \cong \angle P$ because of the definition of similar polygons. Since $\overline{BD}$ and $\overline{QS}$ are perpendicular to $\overline{AC}$ and $\overline{PR}$, $\angle BDA \cong \angle QSP$. So, $\triangle ABD \sim \triangle PQS$ by AA Similarity and $\frac{QP}{BA} = \frac{QS}{BD}$ by definition of similar polygons.

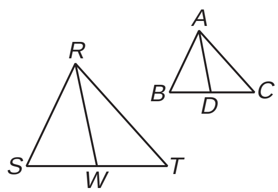
35. Given: $\overline{JF}$ bisects $\angle EFG$.
$\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$
Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



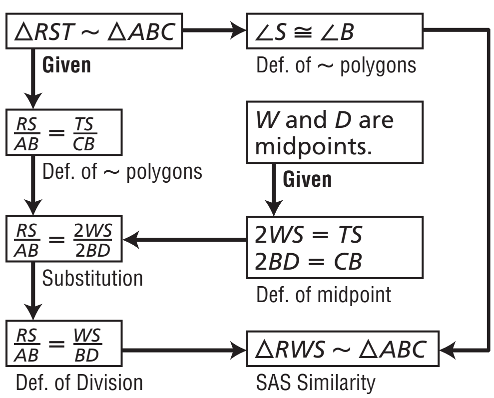
Proof:

Statements	Reasons
1. $\overline{JF}$ bisects $\angle EFG$. $\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$	1. Given
2. $\angle EFK \cong \angle KFG$	2. Def. of $\angle$ bisector
3. $\angle KFG \cong \angle JKH$	3. Corresponding $\angle$s Post.
4. $\angle JKH \cong \angle EKF$	4. Vertical $\angle$s are $\cong$.
5. $\angle EFK \cong \angle EKF$	5. Transitive Prop.
6. $\angle FJH \cong \angle EFK$	6. Alternate Interior $\angle$s Th.
7. $\angle FJH \cong \angle EKF$	7. Transitive Prop.
8. $\triangle EKF \sim \triangle GJF$	8. AA Similarity
9. $\frac{EK}{KF} = \frac{GJ}{JF}$	9. Def. of $\sim \triangle$s

37. Given: $\triangle RST \sim \triangle ABC$, W and D are midpoints of $\overline{TS}$ and $\overline{CB}$, respectively.
Prove: $\triangle RWS \sim \triangle ADB$



Proof:



39. 12.9 **41.** no; sides not proportional **43.** yes; $\frac{LM}{MO} = \frac{LN}{NP}$
45. $\triangle PQT \sim \triangle PRS$, $x = 7$, $PQ = 15$ **47.** $y = 2x + 1$
49. 320, 640 **51.** $-27, -33$

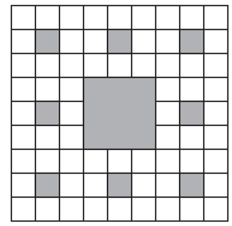
Page 323 Practice Quiz 2

1. 20 **3.** no; sides not proportional **5.** 12.75 **7.** 10.5 **9.** 5

Page 328–331 Lesson 6-6

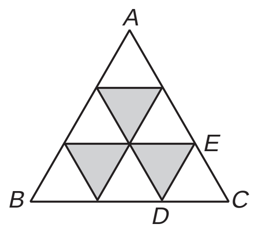
1. Sample answer: irregular shape formed by iteration of self-similar shapes **3.** Sample answer: icebergs, ferns, leaf veins **5.** $A_n = 2(2^n - 1)$ **7.** 1.4142…; 1.1892… **9.** Yes, the procedure is repeated over and over again.

11. 9 holes



13. Yes, any part contains the same figure as the whole, 9 squares with the middle shaded. **15.** 1, 3, 6, 10, 15…; Each difference is 1 more than the preceding difference. **17.** The result is similar to a Stage 3 Sierpinski triangle. **19.** 25

21. Given: $\triangle ABC$ is equilateral.
$CD = \frac{1}{3}CB$ and
$CE = \frac{1}{3}CA$
Prove: $\triangle CED \sim \triangle CAB$



Proof:

Statements	Reasons
1. $\triangle ABC$ is equilateral. $CD = \frac{1}{3}CB$, $CE = \frac{1}{3}CA$	1. Given
2. $\overline{AC} \cong \overline{BC}$	2. Def. of equilateral $\triangle$
3. $AC = BC$	3. Def. of $\cong$ segments
4. $\frac{1}{3}AC = \frac{1}{3}CB$	4. Mult. Prop.
5. $CD = CE$	5. Substitution
6. $\frac{CD}{CB} = \frac{CE}{CB}$	6. Division Prop.
7. $\frac{CD}{CB} = \frac{CE}{CA}$	7. Substitution
8. $\angle C \cong \angle C$	8. Reflexive Prop.
9. $\triangle CED \sim \triangle CAB$	9. AA Similarity

23. Yes; the smaller and smaller details of the shape have the same geometric characteristics as the original form. **25.** $A_n = 4^n$; 65,536 **27.** Stage 0: 3 units, Stage 1: $3 \cdot \frac{4}{3}$ or 4 units, Stage 2: $3\left(\frac{4}{3}\right)\frac{4}{3} = 3\left(\frac{4}{3}\right)^2$ or $5\frac{1}{3}$ units, Stage 3: $3\left(\frac{4}{3}\right)^3$ or $7\frac{1}{9}$ units **29.** The original triangle and the new triangles are equilateral and thus, all of the angles are equal to 60. By AA Similarity, the triangles are similar. **31.** 0.2, 5, 0.2, 5, 0.2; the numbers alternate between 0.2 and 5.0. **33.** 1, 2, 4, 16, 65,536; the numbers approach positive infinity. **35.** 0, -5, -10 **37.** -6, 24, -66 **39.** When $x = 0.00$: 0.64, 0.9216, 0.2890…, 0.8219…, 0.5854…, 0.9708…, 0.1133…, 0.4019…, 0.9615…, 0.1478…; when $x = 0.201$: 0.6423…, 0.9188…, 0.2981…, 0.8369…, 0.5458…, 0.9916…, 0.0333…, 0.1287…, 0.4487…, 0.9894… . Yes, the initial value affected the tenth value. **41.** The leaves in the tree and the branches of the trees are self-similar. These self-similar shapes are repeated throughout the painting. **43.** See students' work.

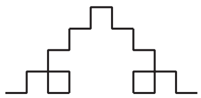
45. Sample answer: Fractal geometry can be found in the repeating patterns of nature. Answers should include the following.

- Broccoli is an example of fractal geometry because the shape of the florets is repeated throughout; one floret looks the same as the stalk.
- Sample answer: Scientists can use fractals to study the human body, rivers, and tributaries, and to model how landscapes change over time.

47. C **49.** $13\frac{3}{5}$ **51.** $\frac{7}{3}$ **53.** $16\frac{1}{4}$ **55.** Miami, Bermuda, San Juan **57.** 10 ft, 10 ft, 17 ft, 17 ft

Page 332–336 Chapter 6 Study Guide and Review

1. true **3.** true **5.** false, iteration **7.** true **9.** false, parallel to **11.** 12 **13.** $\frac{58}{3}$ **15.** $\frac{3}{5}$ **17.** 24 in. and 84 in. **19.** Yes, these are rectangles, so all angles are congruent. Additionally, all sides are in a 3:2 ratio. **21.** $\triangle PQT \sim \triangle RQS$; 0; $PQ = 6$; $QS = 3$; 1 **23.** yes, $\triangle GHI \sim \triangle GJK$ by AA Similarity **25.** $\triangle ABC \sim \triangle DEC$, 4 **27.** no; lengths not proportional **29.** yes; $\frac{HI}{GH} = \frac{IK}{KL}$ **31.** 6 **33.** 9 **35.** 24 **37.** 36 **39.** Stage 2 is not similar to Stage 1. **41.** −8, −20, −56 **43.** −6, −9.6, −9.96



Chapter 7 Right Triangles and Trigonometry

Page 541 Chapter 7 Getting Started

1. $a = 16$ **3.** $e = 24, f = 12$ **5.** 13 **7.** 21.21 **9.** $2\sqrt{2}$ **11.** 15 **13.** 98 **15.** 23

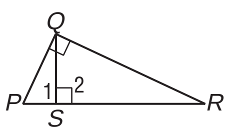
Pages 345–348 Lesson 7-1

1. Sample answer: 2 and 72 **3.** Ian; his proportion shows that the altitude is the geometric mean of the two segments of the hypotenuse. **5.** 42 **7.** $2\sqrt{3} \approx 3.5$ **9.** $4\sqrt{3} \approx 6.9$ **11.** $x = 6; y = 4\sqrt{3}$ **13.** $\sqrt{30} \approx 5.5$ **15.** $2\sqrt{15} \approx 7.7$ **17.** $\frac{\sqrt{15}}{5} \approx 0.8$ **19.** $\frac{\sqrt{5}}{3} \approx 0.7$ **21.** $3\sqrt{5} \approx 6.7$ **23.** $8\sqrt{2} \approx 11.3$ **25.** $\sqrt{26} \approx 5.1$ **27.** $x = 2\sqrt{15} \approx 9.4$; $y = \sqrt{33} \approx 5.7$; $z = 2\sqrt{6} \approx 4.9$ **29.** $x = \frac{40}{3}$; $y = \frac{5}{3}$; $z = 10\sqrt{2} \approx 14.1$ **31.** $x = 6\sqrt{6} \approx 14.7$; $y = 6\sqrt{42} \approx 38.9$; $z = 36\sqrt{7} \approx 95.2$ **33.** $\frac{17}{7}$ **35.** never **37.** sometimes

39. $\triangle FGH$ is a right triangle. $\overline{OG}$ is the altitude from the vertex of the right angle to the hypotenuse of that triangle. So, by Theorem 7.2, OG is the geometric mean between OF and OH, and so on. **41.** 2.4 yd **43.** yes; Indiana and Virginia

45. Given: $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$.

Prove: $\triangle PSQ \sim \triangle PQR$
$\triangle PQR \sim \triangle QSR$
$\triangle PSQ \sim \triangle QSR$

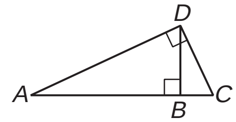


Proof:

Statements	Reasons
1. $\angle PQR$ is a right angle. $\overline{QS}$ is an altitude of $\triangle PQR$.	1. Given
2. $\overline{QS} \perp \overline{RP}$	2. Definition of altitude
3. $\angle 1$ and $\angle 2$ are right angles.	3. Definition of perpendicular lines
4. $\angle 1 \cong \angle PQR$ $\angle 2 \cong \angle PQR$	4. All right $\angle$s are $\cong$.
5. $\angle P \cong \angle P$ $\angle R \cong \angle R$	5. Congruence of angles is reflexive.
6. $\triangle PSQ \sim \triangle PQR$ $\triangle PQR \sim \triangle QSR$	6. AA Similarity Statements 4 and 5
7. $\triangle PSQ \sim \triangle QSR$	7. Similarity of triangles is transitive.

47. Given: $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.

Prove: $\frac{AB}{AD} = \frac{AD}{AC}$
$\frac{BC}{DC} = \frac{DC}{AC}$



Proof:

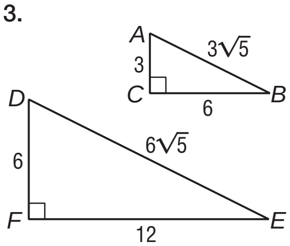
Statements	Reasons
1. $\angle ADC$ is a right angle. $\overline{DB}$ is an altitude of $\triangle ADC$.	1. Given
2. $\triangle ADC$ is a right triangle.	2. Definition of right triangle
3. $\triangle ABD \sim \triangle ADC$ $\triangle DBC \sim \triangle ADC$	3. If the altitude is drawn from the vertex of the rt. $\angle$ to the hypotenuse of a rt. $\triangle$, then the 2 $\triangle$s formed are similar to the given $\triangle$ and to each other.
4. $\frac{AB}{AD} = \frac{AD}{AC}, \frac{BC}{DC} = \frac{DC}{AC}$	4. Definition of similar polygons

49. C **51.** 15, 18, 21 **53.** 7, 47, 2207 **55.** $8\frac{8}{9}$, $11\frac{1}{9}$ **57.** $\angle 5, \angle 7$ **59.** $\angle 2, \angle 7, \angle 8$ **61.** $y = 4x - 8$ **63.** $y = -4x - 11$ **65.** 13 ft

Pages 353–356 Lesson 7-2

1. Maria; Colin does not have the longest side as the value of c.

3.

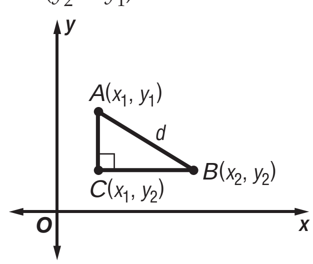


Sample answer : $\triangle ABC \sim \triangle DEF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$, $\overline{AB}$ corresponds to $\overline{DE}$, $\overline{BC}$ corresponds to $\overline{EF}$, $\overline{AC}$ corresponds to $\overline{DF}$. The scale factor is $\frac{2}{1}$. No; the measures do not form a Pythagorean triple since $6\sqrt{5}$ and $3\sqrt{5}$ are not whole numbers.

5. $\frac{3}{7}$ **7.** yes; $JK = \sqrt{17}$, $KL = \sqrt{17}$, $JL = \sqrt{34}$; $(\sqrt{17})^2 + (\sqrt{17})^2 = (\sqrt{34})^2$ **9.** no, no **11.** about 15.1 in. **13.** $4\sqrt{3} \approx 6.9$ **15.** $8\sqrt{41} \approx 51.2$ **17.** 20 **19.** no; $QR = 5$, $RS = 6$, $QS, = 5$; $5^2 + 5^2 \neq 6^2$ **21.** yes; $QR = \sqrt{29}$, $RS = \sqrt{29}$, $QS = \sqrt{58}$; $(\sqrt{29})^2 + (\sqrt{29})^2 = (\sqrt{58})^2$ **23.** yes, yes **25.** no, no **27.** no, no **29.** yes, no **31.** 5-12-13 **33.** Sample answer: They consist of any number of similar triangles. **35a.** 16-30-34; 24-45-51 **35b.** 18-80-82; 27-120-123 **35c.** 14-48-50; 21-72-75 **37.** 10.8 degrees

39. Given: $\triangle ABC$ with right angle at C, $AB = d$

Prove: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Proof:

Statements	Reasons
1. $\triangle ABC$ with right angle at C, $AB = d$	1. Given

2. $(CB)^2 + (AC)^2 = (AB)^2$	2. Pythagorean Theorem
3. $\lvert x_2 - x_1 \rvert = CB$; $\lvert y_2 - y_1 \rvert = AC$	3. Distance on a number line
4. $\lvert x_2 - x_1 \rvert^2 + \lvert y_2 - y_1 \rvert^2 = d^2$	4. Substitution
5. $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$	5. Substitution
6. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$	6. Take square root of each side.
7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	7. Reflexive Property

41. about 76.53 ft **43.** about 13.4 mi **45.** Sample answer: The road, the tower that is perpendicular to the road, and the cables form the right triangles. Answers should include the following.

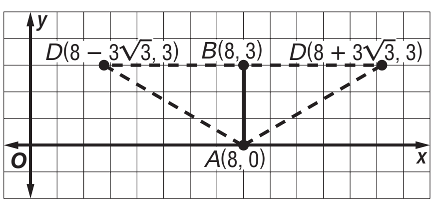
- Right triangles are formed by the bridge, the towers, and the cables.
- The cable is the hypotenuse in each triangle.

47. C **49.** yes **51.** $6\sqrt{3} \approx 10.4$ **53.** $3\sqrt{6} \approx 7.3$ **55.** $\sqrt{10} \approx 3.2$ **57.** 3; approaches positive infinity. **59.** 0.25; alternates between 0.25 and 4. **61.** $\frac{7\sqrt{3}}{3}$ **63.** $\sqrt{7}$ **65.** $12\sqrt{2}$ **67.** $2\sqrt{2}$ **69.** $\frac{\sqrt{2}}{2}$

Pages 360–363 Lesson 7-3

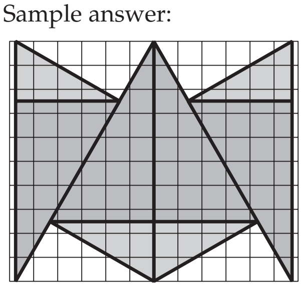
1. Sample answer: Construct two perpendicular lines. Use a ruler to measure 3 cm from the point of intersection on the one ray. Use the compass to copy the 3 cm segment. Connect the two endpoints to form a 45°-45°-90° triangle with sides of 3 cm and a hypotenuse of $3\sqrt{2}$cm. **3.** The length of the rectangle is $\sqrt{3}$ times the width; $\ell = \sqrt{3}w$. **5.** $x = 5\sqrt{2}$; $y = 5\sqrt{2}$ **7.** $a = 4$; $b = 4\sqrt{3}$

9.



11. $90\sqrt{2}$ or 127.28 ft **13.** $x = \frac{17\sqrt{2}}{2}$; $y = 45$ **15.** $x = 8\sqrt{3}$; $y = 8\sqrt{3}$ **17.** $x = 5\sqrt{2}$; $y = \frac{5\sqrt{2}}{2}$ **19.** $a = 14\sqrt{3}$; $CE = 21$; $y = 21\sqrt{3}$; $b = 42$ **21.** $7.5\sqrt{3}$ cm ≈ 12.99 cm **23.** $14.8\sqrt{3}$ m ≈ 25.63 m **25.** $8\sqrt{2} \approx 11.31$ **27.** (4, 8) **29.** $\left(-3 - \frac{13\sqrt{3}}{3}, -6\right)$ or about $(-10.51, -6)$ **31.** $a = 3\sqrt{3}$, $b = 9$, $c = 3\sqrt{3}$, $d = 9$ **33.** 30° angle

35. Sample answer:



37. $BH = 16$ **39.** $12\sqrt{3} \approx 20.78$ cm **41.** $52 + 4\sqrt{3} + 4\sqrt{6}$ units **43.** C **45.** yes, yes **47.** no, no **49.** yes, yes **51.** $2\sqrt{21} \approx 9.2$; 21; 25 **53.** $\frac{40}{3}$; $\frac{5}{3}$; $10\sqrt{2} \approx 14.1$

55. $m\angle ALK < m\angle NLO$ **57.** $m\angle KLO = m\angle ALN$ **59.** 15 **61.** 20 **63.** 28 **65.** 60

Page 363 Chapter 7 Practice Quiz 1

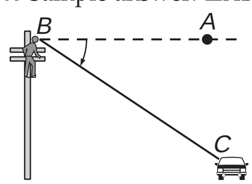
1. $7\sqrt{3} \approx 12.1$ **3.** yes; $AB = \sqrt{5}$, $BC = \sqrt{50}$, $AC = \sqrt{45}$; $\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$ **5.** $x = 12$; $y = 6\sqrt{3}$

Pages 367–370 Lesson 7-4

1. The triangles are similar, so the ratios remain the same. **3.** All three ratios involve two sides of a right triangle. The sine ratio is the measure of the opposite leg divided by the measure of the hypotenuse. The cosine ratio is the measure of the adjacent leg divided by the measure of the hypotenuse. The tangent ratio is the measure of the opposite leg divided by the measure of the adjacent leg. **5.** $\frac{14}{50} = 0.28$; $\frac{48}{50} = 0.96$; $\frac{14}{48} \approx 0.29$; $\frac{48}{50} = 0.96$; $\frac{14}{50} = 0.28$; $\frac{48}{14} \approx 3.43$ **7.** 0.8387 **9.** 0.8387 **11.** 1.0000 **13.** $m\angle A \approx 54.8$ **15.** $m\angle A \cong 33.7$ **17.** 2997 ft **19.** $\frac{\sqrt{3}}{3} \approx 0.58$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{3}}{3} \approx 0.58$; $\sqrt{2} \approx 1.41$ **21.** $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{3} \approx 0.75$; $\frac{2\sqrt{5}}{5} \approx 0.89$; $\frac{\sqrt{5}}{3} \approx 0.75$; $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{2} \approx 1.12$ **23.** 0.9260 **25.** 0.9974 **27.** 0.9239 **29.** $\frac{5}{1} = 5.0000$ **31.** $\frac{5\sqrt{26}}{6} \approx 0.9806$ **33.** $\frac{1}{5} = 0.2000$ **35.** $\frac{\sqrt{26}}{26} \approx 0.1961$ **37.** 46.4 **39.** 84.0 **41.** 83.0 **43.** $x \approx 8.5$ **45.** $x \approx 28.2$ **47.** $x \approx 22.6$ **49.** 4.1 mi **51.** about 5.18 ft **53.** about 54.5 **55.** about 47.9 in. **57.** $x = 17.1$; $y = 23.4$ **59.** about 272,837 astronomical units **61.** $\frac{2\sqrt{2}}{5}$ **63.** C **65.** $\csc A = \frac{5}{3}$; $\sec A = \frac{5}{4}$; $\cot A = \frac{4}{3}$; $\csc B = \frac{5}{4}$; $\sec B = \frac{5}{3}$; $\cot B = \frac{3}{4}$ **67.** $\csc A = 2$; $\sec A = \frac{2\sqrt{3}}{3}$; $\cot A = \sqrt{3}$; $\csc B = \frac{2\sqrt{3}}{3}$; $\sec B = 2$; $\cot B = \frac{\sqrt{3}}{3}$ **69.** $b = 4\sqrt{3}$, $c = 8$ **71.** $a = 2.5$, $b = 2.5\sqrt{3}$ **73.** yes, yes **75.** no, no **77.** 117 **79.** 150 **81.** 63

Pages 373–376 Lesson 7-5

1. Sample answer: $\angle ABC$



3. The angle of depression is $\angle FPB$ and the angle of elevation is $\angle TBP$. **5.** 22.7° **7.** 706 ft **9.** about 173.2 yd **11.** about 5.3° **13.** about 118.2 yd **15.** about 4° **17.** about 40.2° **19.** 100 ft, 300 ft **21.** about 8.3 in. **23.** no **25.** About 5.1 mi **27.** Answers should include the following.

- Pilots use angles of elevation when they are ascending and angles of depression when descending.
- Angles of elevation are formed when a person looks upward and angles of depression are formed when a person looks downward.

29. A **31.** 30.8 **33.** 70.0 **35.** 19.5 **37.** $14\sqrt{3}$; 28 **39.** 31.2 cm **41.** 5 **43.** 34 **45.** 52 **47.** 3.75

Pages 380–383 Lesson 7-6

1. Felipe; Makayla is using the definition of the sine ratio for a right triangle, but this is not a right triangle. **3.** In one case you need the measures of two sides and the measure of an angle opposite one of the sides. In the other case you need the measures of two angles and the measure of a side. **5.** 13.1 **7.** 55 **9.** $m\angle R \approx 19$, $m\angle Q \approx 56$, $q \approx 27.5$ **11.** $m\angle Q \approx 43$, $m\angle R \approx 17$, $r \approx 9.5$ **13.** $m\angle P \approx 37$, $p \approx 11.1$, $m\angle R \approx 32$ **15.** about 237.8 feet **17.** 2.7 **19.** 29 **21.** 29 **23.** $m\angle X \approx 25.6$, $m\angle W \approx 58.4$, $w \approx 20.3$ **25.** $m\angle X \approx 19.3$, $m\angle W \approx 48.7$, $w \approx 45.4$ **27.** $m\angle X = 82$, $x \approx 5.2$, $y \approx 4.7$ **29.** $m\angle X \approx 49.6$, $m\angle Y \approx 42.4$, $y \approx 14.2$ **31.** 56.9 units **33.** about 14.9 mi, about 13.6 mi **35.** about 536 ft **37.** about 1000.7 m **39.** about 13.6 mi **41.** Sample answer: Triangles are used to determine distances in space. Answers should include the following.

- The VLA is one of the world's premier astronomical radio observatories. It is used to make pictures from the radio waves emitted by astronomical objects.
- Triangles are used in the construction of the antennas.

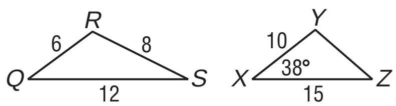
43. A **45.** about 5.97 ft **47.** $\frac{20}{29} \approx 0.69$; $\frac{21}{29} \approx 0.72$; $\frac{20}{21} \approx 0.95$; $\frac{21}{29} \approx 0.72$; $\frac{20}{29} \approx 0.69$; $\frac{21}{20} = 1.05$ **49.** $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00 **51.** 54 **53.** $\frac{13}{112}$ **55.** $-\frac{11}{80}$ **57.** $\frac{7}{15}$

Page 383 Chapter 7 Practice Quiz 2

1. 58.0 **3.** 53.2 **5.** $m\angle D \approx 41$, $m\angle E \approx 57$, $e \approx 10.2$

Pages 387–390 Lesson 7-7

1. Sample answer: Use the Law of Cosines when you have all three sides given (SSS) or two sides and the included angle (SAS).

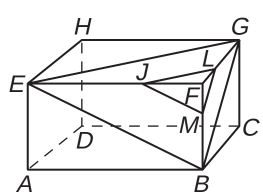


3. If two angles and one side are given, then the Law of Cosines cannot be used. **5.** 159.7 **7.** 98 **9.** $\ell \approx 17.9$; $m\angle K \approx 55$; $m\angle M \approx 78$ **11.** $u \approx 4.9$ **13.** $t \approx 22.5$ **15.** 16 **17.** 36 **19.** $m\angle H \approx 31$; $m\angle G \approx 109$; $g \approx 14.7$ **21.** $m\angle B \approx 86$; $m\angle C \approx 56$; $m\angle D \approx 38$ **23.** $c \approx 6.3$; $m\angle A \approx 80$; $m\angle B \approx 63$ **25.** $m\angle B = 99$; $b \approx 31.3$; $a \approx 25.3$ **27.** $m\angle M \approx 18.6$; $m\angle N \approx 138.4$; $n \approx 91.8$ **29.** $\ell \approx 21.1$; $m\angle M \approx 42.8$; $m\angle N \approx 88.2$ **31.** $m\angle L \approx 101.9$; $m\angle M \approx 36.3$; $m\angle N \approx 41.8$ **33.** $m \approx 6.0$; $m\angle L \approx 22.2$; $m\angle N \approx 130.8$ **35.** $m \approx 18.5$; $m\angle L \approx 40.9$; $m\angle N \approx 79.1$ **37.** $m\angle N \approx 42.8$; $m\angle M \approx 86.2$; $m \approx 51.4$ **39.** 561.2 units **41.** 59.8, 63.4, 56.8
43a. Pythagorean Theorem **43b.** Substitution
43c. Pythagorean Theorem **43d.** Substitution
43e. Def. of cosine **43f.** Cross products **43g.** Substitution
43h. Commutative Property
45. Sample answer: Triangles are used to build supports, walls, and foundations. Answers should include the following.

- The triangular building was more efficient with the cells around the edge.
- The Law of Sines requires two angles and a side or two sides and an angle opposite one of those sides.

47. C **49.** 33 **51.** yes **53.** no

55. Given: $\triangle JFM \sim \triangle EFB$
$\triangle LFM \sim \triangle GFB$
Prove: $\triangle JFL \sim \triangle EFG$



Proof:
Since $\triangle JFM \sim \triangle EFB$ and $\triangle LFM \sim \triangle GFB$, then by the definition of similar triangles, $\frac{JF}{EF} = \frac{MF}{BF}$ and $\frac{MF}{BF} = \frac{LF}{GF}$. By the Transitive Property of Equality, $\frac{JF}{EF} = \frac{LF}{GF}$. $\angle F \cong \angle F$ by the Reflexive Property of Congruence. Then, by SAS Similarity, $\triangle JFL \sim \triangle EFG$.
57. $(-1.6, 9.6)$ **59.** $(2.8, 5.2)$

Pages 392–396 Chapter 7 Study Guide and Review

1. true **3.** false; a right **5.** true **7.** false; depression **9.** 18 **11.** $6\sqrt{22} \approx 28.1$ **13.** 25 **15.** $4\sqrt{17} \approx 16.5$ **17.** $x = \frac{13\sqrt{2}}{2}$; $y = \frac{13\sqrt{2}}{2}$ **19.** $z = 18\sqrt{3}$, $a = 36\sqrt{3}$ **21.** $\frac{3}{5} = 0.60$; $\frac{4}{5} = 0.80$; $\frac{3}{4} = 0.75$; $\frac{4}{5} = 0.80$; $\frac{3}{5} = 0.60$; $\frac{4}{3} \approx 1.33$ **23.** 26.9 **25.** 43.0 **27.** $\approx 22.6°$ **29.** ≈ 31.1 yd **31.** 21.3 yd **33.** $m\angle B \approx 41$, $m\angle C \approx 75$, $c \approx 16.1$ **35.** $m\angle B \approx 61$, $m\angle C \approx 90$, $c \approx 9.9$ **37.** $z \approx 5.9$ **39.** $a \approx 17.0$, $m\angle B \approx 43$, $m\angle C \approx 73$

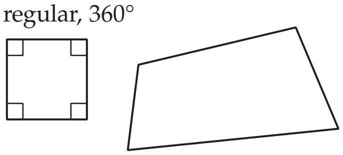
Chapter 8 Quadrilaterals

Page 403 Chapter 8 Getting Started

1. 130 **3.** 120 **5.** $\frac{1}{6}$, -6; perpendicular **7.** $\frac{4}{3}$, $-\frac{3}{4}$; perpendicular **9.** $-\frac{a}{b}$

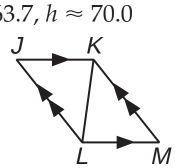
Pages 407–409 Lesson 8-1

1. A concave polygon has at least one obtuse angle, which means the sum will be different from the formula.
3. Sample answer: regular quadrilateral, 360°; quadrilateral that is not regular, 360°



5. 1800 **7.** 4 **9.** $m\angle J = m\angle M = 30$, $m\angle K = m\angle L = m\angle P = m\angle N = 165$ **11.** 20, 160 **13.** 5400 **15.** 3060 **17.** $360(2y - 1)$ **19.** 1080 **21.** 9 **23.** 18 **25.** 16 **27.** $m\angle M = 30$, $m\angle P = 120$, $m\angle Q = 60$, $m\angle R = 150$ **29.** $m\angle M = 60$, $m\angle N = 120$, $m\angle P = 60$, $m\angle Q = 120$ **31.** 105, 110, 120, 130, 135, 140, 160, 170, 180, 190 **33.** Sample answer: 36, 72, 108, 144 **35.** 36, 144 **37.** 40, 140 **39.** 147.3, 32.7 **41.** 150, 30 **43.** 108, 72
45. $\frac{180(n-2)}{n} = \frac{180n - 360}{n} = \frac{180n}{n} - \frac{360}{n} = 180 - \frac{360}{n}$
47. B **49.** 92.1 **51.** 51.0 **53.** $m\angle G \approx 67$, $m\angle H \approx 60$, $h \approx 16.1$ **55.** $m\angle F = 57$, $f \approx 63.7$, $h \approx 70.0$

57. Given: $\overline{JL} \parallel \overline{KM}$, $\overline{JK} \parallel \overline{LM}$
Prove: $\triangle JKL \cong \triangle MLK$



Proof:

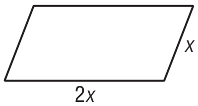
Statements	Reasons
1. $\overline{JL} \parallel \overline{KM}$, $\overline{JK} \parallel \overline{LM}$	1. Given
2. $\angle MKL \cong \angle JLK$, $\angle JKL \cong \angle MLK$	2. Alt. int. $\angle$s are $\cong$.
3. $\overline{KL} \cong \overline{KL}$	3. Reflexive Property
4. $\triangle JKL \cong \triangle MLK$	4. ASA

59. m; cons. int. **61.** n; alt. ext. **63.** $\angle 3$ and $\angle 5$, $\angle 2$ and $\angle 6$ **65.** none

Pages 414–416 Lesson 8-2

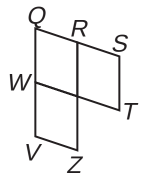
1. Opposite sides are congruent; opposite angles are congruent; consecutive angles are supplementary; and if there is one right angle, there are four right angles.

3. Sample answer:



5. $\triangle VTQ$, SSS; diag. bisect each other and opp. sides of ▱ are ≅. **7.** 100 **9.** 80 **11.** 7

13. Given: ▱$VZRQ$ and ▱$WQST$
Prove: $\angle Z \cong \angle T$

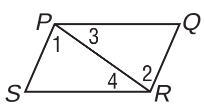


Proof:

Statements	Reasons
1. ▱$VZRQ$ and ▱$WQST$	**1.** Given
2. $\angle Z \cong \angle Q$, $\angle Q \cong \angle T$	**2.** Opp. ∠s of a ▱ are ≅.
3. $\angle Z \cong \angle T$	**3.** Transitive Prop.

15. C **17.** $\angle CDB$, alt. int. ∠s are ≅. **19.** $\overline{GD}$, diag. of ▱ bisect each other. **21.** $\angle BAC$, alt. int. ∠s are ≅. **23.** 33 **25.** 109 **27.** 83 **29.** 6.45 **31.** 6.1 **33.** $y = 5$, $FH = 9$ **35.** $a = 6$, $b = 5$, $DB = 32$ **37.** $EQ = 5$, $QG = 5$, $HQ = \sqrt{13}$, $QF = \sqrt{13}$ **39.** Slope of $\overline{EH}$ is undefined, slope of $\overline{EF} = -\frac{1}{3}$; no, the slopes of the sides are not negative reciprocals of each other.

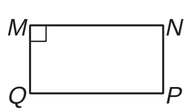
41. Given: ▱$PQRS$
Prove: $\overline{PQ} \cong \overline{RS}$
$\overline{QR} \cong \overline{SP}$



Proof:

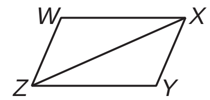
Statements	Reasons
1. ▱$PQRS$	**1.** Given
2. Draw an auxiliary segment $\overline{PR}$ and label angles 1, 2, 3, and 4 as shown.	**2.** Diagonal of ▱$PQRS$
3. $\overline{PQ} \parallel \overline{SR}$, $\overline{PS} \parallel \overline{QR}$	**3.** Opp. sides of ▱ are ∥.
4. $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$	**4.** Alt. int. ∠s are ≅.
5. $\overline{PR} \cong \overline{PR}$	**5.** Reflexive Prop.
6. $\triangle QPR \cong \triangle SRP$	**6.** ASA
7. $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$	**7.** CPCTC

43. Given: ▱$MNPQ$
$\angle M$ is a right angle.
Prove: $\angle N$, $\angle P$ and $\angle Q$ are right angles.



Proof:
By definition of a parallelogram, $\overline{MN} \parallel \overline{QP}$. Since $\angle M$ is a right angle, $\overline{MQ} \perp \overline{MN}$. By the Perpendicular Transversal Theorem, $\overline{MQ} \perp \overline{QP}$. $\angle Q$ is a right angle, because perpendicular lines form a right angle. $\angle N \cong \angle Q$ and $\angle M \cong \angle P$ because opposite angles in a parallelogram are congruent. $\angle P$ and $\angle N$ are right angles, since all right angles are congruent.

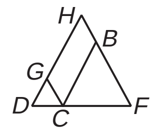
45. Given: ▱$WXYZ$
Prove: $\triangle WXZ \cong \triangle YZX$



Proof:

Statements	Reasons
1. ▱$WXYZ$	**1.** Given
2. $\overline{WX} \cong \overline{ZY}$, $\overline{WZ} \cong \overline{XY}$	**2.** Opp. sides of ▱ are ≅.
3. $\angle ZWX \cong \angle XYZ$	**3.** Opp. ∠s of ▱ are ≅.
4. $\triangle WXZ \cong \triangle YZX$	**4.** SAS

47. Given: ▱$BCGH$, $\overline{HD} \cong \overline{FD}$
Prove: $\angle F \cong \angle GCB$

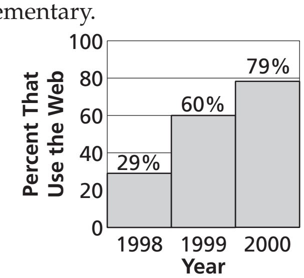


Proof:

Statements	Reasons
1. ▱$BCGH$, $\overline{HD} \cong \overline{FD}$	**1.** Given
2. $\angle F \cong \angle H$	**2.** Isosceles Triangle Th.
3. $\angle H \cong \angle GCB$	**3.** Opp. ∠s of ▱ are ≅.
4. $\angle F \cong \angle GCB$	**4.** Congruence of angles is transitive.

49. The graphic uses the illustration of wedges shaped like parallelograms to display the data. Answers should include the following.

- The opposite sides are parallel and congruent, the opposite angles are congruent, and the consecutive angles are supplementary.
- Sample answer:

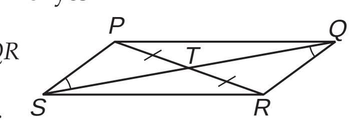


51. B **53.** 3600 **55.** 6120 **57.** Sines; $m\angle C \approx 69.9$, $m\angle A \approx 53.1$, $a \approx 11.9$ **59.** 30 **61.** side, $\frac{7}{3}$ **63.** side, $\frac{7}{3}$

Pages 420–423 Lesson 8-3

1. Both pairs of opposite sides are congruent; both pairs of opposite angles are congruent; diagonals bisect each other; one pair of opposite sides is parallel and congruent.
3. Shaniqua; Carter's description could result in a shape that is not a parallelogram. **5.** Yes; each pair of opp. ∠s is ≅. **7.** $x = 41$, $y = 16$ **9.** yes

11. Given: $\overline{PT} \cong \overline{TR}$
$\angle TSP \cong \angle TQR$
Prove: $PQRS$ is a parallelogram.



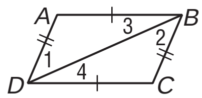
Proof:

Statements	Reasons
1. $\overline{PT} \cong \overline{TR}$, $\angle TSP \cong \angle TQR$	**1.** Given
2. $\angle PTS \cong \angle RTQ$	**2.** Vertical ∠s are ≅.
3. $\triangle PTS \cong \triangle RTQ$	**3.** AAS
4. $\overline{PS} \cong \overline{QR}$	**4.** CPCTC
5. $\overline{PS} \parallel \overline{QR}$	**5.** If alt. int. ∠s are ≅, lines are ∥.
6. $PQRS$ is a parallelogram.	**6.** If one pair of opp. sides is ∥ and ≅, then the quad. is a ▱.

Selected Answers

13. Yes; each pair of opposite angles is congruent. **15.** Yes; opposite angles are congruent. **17.** Yes; one pair of opposite sides is parallel and congruent. **19.** $x = 6, y = 24$ **21.** $x = 1, y = 2$ **23.** $x = 34, y = 44$ **25.** yes **27.** yes **29.** no **31.** yes **33.** Move M to $(-4, 1)$, N to $(-3, 4)$, P to $(0, -9)$, or R to $(-7, 3)$. **35.** $(-2, -2)$, $(4, 10)$, or $(10, 0)$ **37.** Parallelogram; $\overline{KM}$ and $\overline{JL}$ are diagonals that bisect each other.

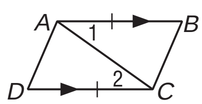
39. Given: $\overline{AD} \cong \overline{BC}$
$\overline{AB} \cong \overline{DC}$
Prove: $ABCD$ is a parallelogram.



Proof:

Statements	Reasons
1. $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$	1. Given
2. Draw $\overline{DB}$.	2. Two points determine a line.
3. $\overline{DB} \cong \overline{DB}$	3. Reflexive Property
4. $\triangle ABD \cong \triangle CDB$	4. SSS
5. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	5. CPCTC
6. $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{DC}$	6. If alt. int. $\angle$s are $\cong$, lines are $\parallel$.
7. $ABCD$ is a parallelogram.	7. Definition of parallelogram

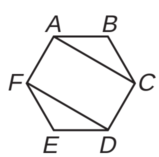
41. Given: $\overline{AB} \cong \overline{DC}$
$\overline{AB} \parallel \overline{DC}$
Prove: $ABCD$ is a parallelogram.



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$	1. Given
2. Draw $\overline{AC}$	2. Two points determine a line.
3. $\angle 1 \cong \angle 2$	3. Alternate Interior Angles Theorem
4. $\overline{AC} \cong \overline{AC}$	4. Reflexive Property
5. $\triangle ABC \cong \triangle CDA$	5. SAS
6. $\overline{AD} \cong \overline{BC}$	6. CPCTC
7. $ABCD$ is a parallelogram.	7. If both pairs of opp. sides are $\cong$, then the quad. is $\square$.

43. Given: $ABCDEF$ is a regular hexagon.
Prove: $FDCA$ is a parallelogram.



Proof:

Statements	Reasons
1. $ABCDEF$ is a regular hexagon.	1. Given
2. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\angle E \cong \angle B$, $\overline{FA} \cong \overline{CD}$	2. Def. of regular hexagon
3. $\triangle ABC \cong \triangle DEF$	3. SAS
4. $\overline{AC} \cong \overline{DF}$	4. CPCTC
5. $FDCA$ is a $\square$.	5. If both pairs of opp. sides are $\cong$, then the quad. is $\square$.

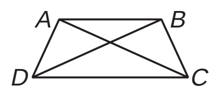
45. B **47.** 12 **49.** 14 units **51.** 8 **53.** 30 **55.** 72 **57.** 45, $12\sqrt{2}$ **59.** $16\sqrt{3}$, 16 **61.** 5, $-\frac{3}{2}$; not $\perp$ **63.** $\frac{2}{3}$, $-\frac{3}{2}$; $\perp$

Page 423 Chapter 8 Practice Quiz 1

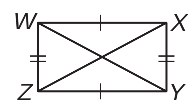
1. 11 **3.** 66 **5.** $x = 8, y = 6$

Pages 427–430 Lesson 8-4

1. If consecutive sides are perpendicular or diagonals are congruent, then the parallelogram is a rectangle. **3.** McKenna; Consuelo's definition is correct if one pair of opposite sides is parallel and congruent. **5.** 40 **7.** 52 or 10 **9.** Make sure that the angles measure 90 or that the diagonals are congruent. **11.** 11 **13.** $29\frac{1}{3}$ **15.** 4 **17.** 60 **19.** 30 **21.** 60 **23.** 30 **25.** Measure the opposite sides and the diagonals to make sure they are congruent. **27.** No; $\overline{DH}$ and $\overline{FG}$ are not parallel. **29.** Yes; opp. sides are $\parallel$, diag. are $\cong$. **31.** $\left(\frac{1}{2}, -\frac{3}{2}\right), \left(\frac{7}{2}, \frac{3}{2}\right)$ **33.** Yes; consec. sides are $\perp$. **35.** Move L and K until the length of the diagonals is the same. **37.** See students' work.
39. Sample answer: $\overline{AC} \cong \overline{BD}$ but $ABCD$ is not a rectangle



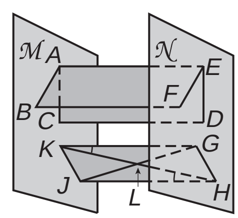
41. Given: $\square WXYZ$ and $\overline{WY} \cong \overline{XZ}$
Prove: $WXYZ$ is a rectangle.



Proof:

Statements	Reasons
1. $\square WXYZ$ and $\overline{WY} \cong \overline{XZ}$	1. Given
2. $\overline{XY} \cong \overline{WZ}$	2. Opp. sides of $\square$ are $\cong$.
3. $\overline{WX} \cong \overline{WX}$	3. Reflexive Property
4. $\triangle WZX \cong \triangle XYW$	4. SSS
5. $\angle ZWX \cong \angle YXW$	5. CPCTC
6. $\angle ZWX$ and $\angle YXW$ are supplementary.	6. Consec. $\angle$s of $\square$ are suppl.
7. $\angle ZWX$ and $\angle YXW$ are right angles.	7. If 2 $\angle$s are $\cong$ and suppl, each $\angle$ is a rt. $\angle$.
8. $\angle WZY$ and $\angle XYZ$ are right angles.	8. If $\square$ has 1 rt. $\angle$, it has 4 rt. $\angle$s.
9. $WXYZ$ is a rectangle.	9. Def. of rectangle

43. Given: $DEAC$ and $FEAB$ are rectangles.
$\angle GKH \cong \angle JHK$;
$\overline{GJ}$ and $\overline{HK}$ intersect at L.
Prove: $GHJK$ is a parallelogram.



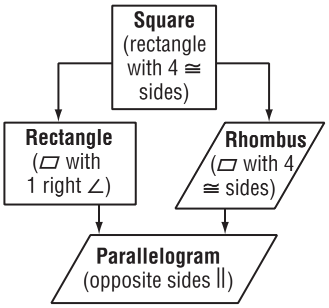
Proof:

Statements	Reasons
1. $DEAC$ and $FEAB$ are rectangles. $\angle GKH \cong \angle JHK$ $\overline{GJ}$ and $\overline{HK}$ intersect at L.	1. Given
2. $\overline{DE} \parallel \overline{AC}$ and $\overline{FE} \parallel \overline{AB}$	2. Def. of parallelogram
3. plane $\mathcal{N} \parallel$ plane $\mathcal{M}$	3. Def. of parallel planes
4. G, J, H, K, L are in the same plane.	4. Def. of intersecting lines
5. $\overline{GH} \parallel \overline{KJ}$	5. Def. of parallel lines
6. $\overline{GK} \parallel \overline{HJ}$	6. If alt. int. $\angle$s are $\cong$, lines are $\parallel$.
7. $GHJK$ is a parallelogram.	7. Def. of parallelogram

45. No; there are no parallel lines in spherical geometry. **47.** No; the sides are not parallel. **49.** A **51.** 31 **53.** 43 **55.** 49 **57.** 5 **59.** $\sqrt{297} \approx 17.2$ **61.** 5 **63.** 29

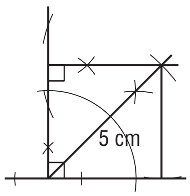
Pages 434–437 Lesson 8-5

1. Sample answer:



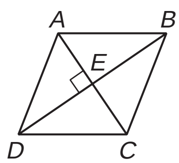
3. A square is a rectangle with all sides congruent. **5.** 5 **7.** 96.8 **9.** None; the diagonals are not congruent or perpendicular. **11.** If the measure of each angle is 90 or if the diagonals are congruent, then the floor is a square. **13.** 120 **15.** 30 **17.** 53 **19.** 5 **21.** Rhombus; the diagonals are perpendicular. **23.** None; the diagonals are not congruent or perpendicular.

25. Sample answer:



27. always **29.** sometimes **31.** always **33.** 40 cm

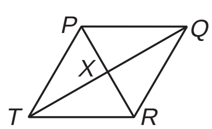
35. Given: *ABCD* is a parallelogram. $\overline{AC} \perp \overline{BD}$
Prove: *ABCD* is a rhombus.



Proof: We are given that *ABCD* is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{AE} \cong \overline{EC}$. $\overline{BE} \cong \overline{BE}$ because congruence of segments is reflexive. We are also given that $\overline{AC} \perp \overline{BD}$. Thus, $\angle AEB$ and $\angle BEC$ are right angles by the definition of perpendicular lines. Then $\angle AEB \cong \angle BEC$ because all right angles are congruent. Therefore, $\triangle AEB \cong \triangle CEB$ by SAS. $\overline{AB} \cong \overline{CB}$ by CPCTC. Opposite sides of parallelograms are congruent, so $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Then since congruence of segments is transitive, $\overline{AB} \cong \overline{CD} \cong \overline{CB} \cong \overline{AD}$. All four sides of *ABCD* are congruent, so *ABCD* is a rhombus by definition.

37. No; it is about 11,662.9 mm. **39.** The flag of Denmark contains four red rectangles. The flag of St. Vincent and the Grenadines contains a blue rectangle, a green rectangle, a yellow rectangle, a blue and yellow rectangle, a yellow and green rectangle, and three green rhombi. The flag of Trinidad and Tobago contains two white parallelograms and one black parallelogram.

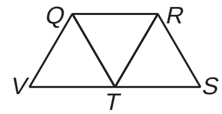
41. Given: $\triangle TPX \cong \triangle QPX \cong \triangle QRX \cong \triangle TRX$
Prove: *TPQR* is a rhombus.



Proof:

Statements	Reasons
1. $\triangle TPX \cong \triangle QPX \cong \triangle QRX \cong \triangle TRX$	1. Given
2. $\overline{TP} \cong \overline{PQ} \cong \overline{QR} \cong \overline{TR}$	2. CPCTC
3. *TPQR* is a rhombus.	3. Def. of rhombus

43. Given: *QRST* and *QRTV* are rhombi.
Prove: $\triangle QRT$ is equilateral.



Proof:

Statements	Reasons
1. *QRST* and *QRTV* are rhombi.	1. Given
2. $\overline{QV} \cong \overline{VT} \cong \overline{TR} \cong \overline{QR}$, $\overline{QT} \cong \overline{TS} \cong \overline{RS} \cong \overline{QR}$	2. Def. of rhombus
3. $\overline{QT} \cong \overline{TR} \cong \overline{QR}$	3. Substitution Property
4. $\triangle QRT$ is equilateral.	4. Def. of equilateral triangle

45. Sample answer: You can ride a bicycle with square wheels over a curved road. Answers should include the following.

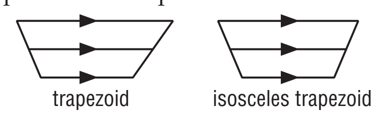
- Rhombi and squares both have all four sides congruent, but the diagonals of a square are congruent. A square has four right angles and rhombi have each pair of opposite angles congruent, but not all angles are necessarily congruent.
- Sample answer: Since the angles of a rhombus are not all congruent, riding over the same road would not be smooth.

47. C **49.** 140 **51.** $x = 2, y = 3$ **53.** yes **55.** no **57.** 13.5 **59.** 20 **61.** $\angle AJH \cong \angle AHJ$ **63.** $\overline{AK} \cong \overline{AB}$ **65.** 2.4 **67.** 5

Pages 442–445 Lesson 8-6

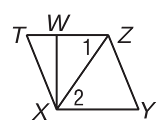
1. Exactly one pair of opposite sides is parallel.

3. Sample answer: The median of a trapezoid is parallel to both bases.

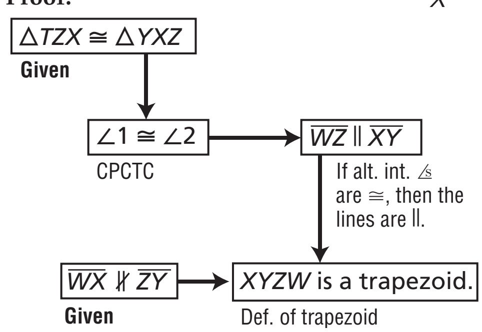


5. isosceles, $QR = \sqrt{20}$, $ST = \sqrt{20}$ **7.** 4 **9a.** $\overline{AD} \parallel \overline{BC}$, $\overline{CD} \nparallel \overline{AB}$ **9b.** not isosceles, $AB = \sqrt{17}$ and $CD = 5$ **11a.** $\overline{DC} \parallel \overline{FE}$, $\overline{DE} \nparallel \overline{FC}$ **11b.** isosceles, $DE = \sqrt{50}$, $CF = \sqrt{50}$ **13.** 8 **15.** 14, 110, 110 **17.** 62 **19.** 15 **21.** Sample answer: triangles, quadrilaterals, trapezoids, hexagons **23.** trapezoid, exactly one pair opp. sides ∥ **25.** square, all sides ≅, consecutive sides ⊥ **27.** $A(-2, 3.5)$, $B(4, -1)$ **29.** $\overline{DG} \parallel \overline{EF}$, not isosceles, $DE \neq GF$, $\overline{DE} \nparallel \overline{GF}$ **31.** $WV = 6$

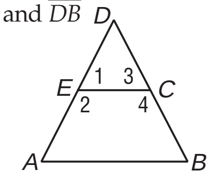
33. Given: $\triangle TZX \cong \triangle YXZ$, $\overline{WX} \nparallel \overline{ZY}$
Prove: *XYZW* is a trapezoid.



Proof:



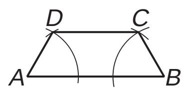
35. Given: E and C are midpoints of $\overline{AD}$ and $\overline{DB}$
Prove: $ABCE$ is an isosceles trapezoid.



Proof:

Statement	Reason
E and C are midpoints of $\overline{AD}$ and $\overline{DB}$.	Given
$\overline{AD} \cong \overline{DB}$	Given
$\overline{EC} \parallel \overline{AB}$	A segment joining the midpoints of two sides of a triangle is parallel to the third side.
$\frac{1}{2}AD = \frac{1}{2}DB$	Def. of Midpt.
$AE = BC$	Substitution
$\overline{AE} \cong \overline{BC}$	Def. of $\cong$
$ABCE$ is an isos. trapezoid.	Def. of isos. trapezoid

37. Sample answer:



39. 4

41. Sample answer: Trapezoids are used in monuments as well as other buildings. Answers should include the following.
- Trapezoids have exactly one pair of opposite sides parallel.
- Trapezoids can be used as window panes.

43. B **45.** 10 **47.** 70 **49.** $RS = 7\sqrt{2}$, $TV = \sqrt{113}$
51. No; opposite sides are not congruent and the diagonals do not bisect each other. **53.** $\frac{17}{5}$ **55.** $\frac{13}{2}$ **57.** 0 **59.** $\frac{2b}{a}$
61. $\frac{c}{b}$

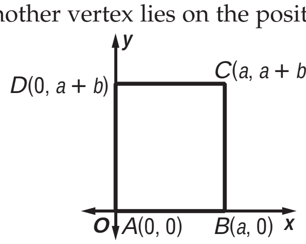
Page 445 Chapter 8 Practice Quiz 2

1. 12 **3.** rhombus, opp. sides $\parallel$, diag. $\perp$, consec. sides not $\perp$ **5.** 18

Pages 449–451 Lesson 8-7

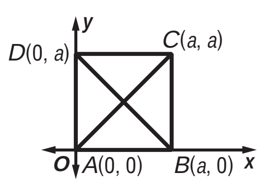
1. Place one vertex at the origin and position the figure so another vertex lies on the positive x-axis.

3.



5. (c, b)

7. Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{DB}$



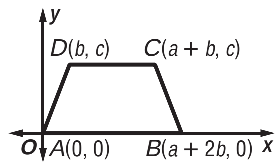
Proof:

Slope of $\overline{DB} = \frac{0-a}{a-0}$ or -1

Slope of $\overline{AC} = \frac{0-a}{0-a}$ or 1

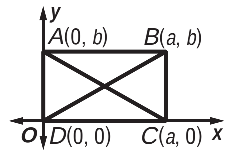
The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{DB}$, so they are perpendicular.

9.



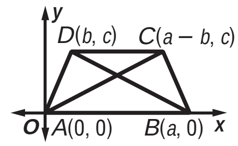
11. $B(-b, c)$
13. $G(a, 0)$, $E(-b, c)$
15. $T(-2a, c)$, $W(-2a, -c)$

17. Given: $ABCD$ is a rectangle.
Prove: $\overline{AC} \cong \overline{DB}$



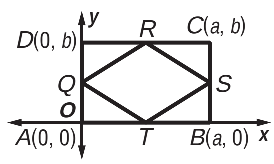
Proof:
Use the Distance Formula to find $AC = \sqrt{a^2 + b^2}$ and $BD = \sqrt{a^2 + b^2}$. $\overline{AC}$ and $\overline{BC}$ have the same length, so they are congruent.

19. Given: isosceles trapezoid $ABCD$ with $\overline{AD} \cong \overline{BC}$
Prove: $\overline{BD} \cong \overline{AC}$



Proof:
$BD = \sqrt{(a-b)^2 + (0-c)^2} = \sqrt{(a-b)^2 + c^2}$
$AC = \sqrt{((a-b)-0)^2 + (c-0)^2} = \sqrt{(a-b)^2 + c^2}$
$BD = AC$ and $\overline{BD} \cong \overline{AC}$

21. Given: $ABCD$ is a rectangle. Q, R, S, and T are midpoints of their respective sides.
Prove: $QRST$ is a rhombus.



Proof:

Midpoint Q is $\left(\frac{0+0}{2}, \frac{b+0}{2}\right)$ or $\left(0, \frac{b}{2}\right)$.

Midpoint R is $\left(\frac{a+0}{2}, \frac{b+b}{2}\right)$ or $\left(\frac{a}{2}, \frac{2b}{2}\right)$ or $\left(\frac{a}{2}, b\right)$

Midpoint S is $\left(\frac{a+a}{2}, \frac{b+0}{2}\right)$ or $\left(\frac{2a}{2}, \frac{b}{2}\right)$ or $\left(a, \frac{b}{2}\right)$.

Midpoint T is $\left(\frac{a+0}{2}, \frac{0+0}{2}\right)$ or $\left(\frac{a}{2}, 0\right)$.

$$QR = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(b - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$RS = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - b\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} \text{ or } \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$ST = \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$QT = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(0 - \frac{b}{2}\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} \text{ or } \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$QR = RS = ST = QT$ so $\overline{QR} \cong \overline{RS} \cong \overline{ST} \cong \overline{QT}$.
$QRST$ is a rhombus.

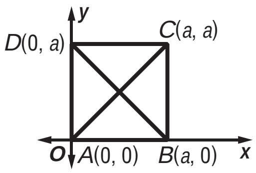
23. Sample answer: $C(a + c, b)$, $D(2a + c, 0)$ **25.** No, there is not enough information given to prove that the sides of the tower are parallel. **27.** Sample answer: The coordinate plane is used in coordinate proofs. The Distance Formula, Midpoint Formula and Slope Formula are used to prove theorems. Answers should include the following.
- Place the figure so one of the vertices is at the origin. Place at least one side of the figure on the positive x-axis. Keep the figure in the first quadrant if possible and use coordinates that will simplify calculations.
- Sample answer: Theorem 8.3 Opposite sides of a parallelogram are congruent.

29. A **31.** 55 **33.** 160 **35.** $\sqrt{60} \approx 7.7$ **37.** $m\angle XVZ = m\angle VXZ$ **39.** $m\angle XZY > m\angle ZXY$

Pages 452–456 Chapter 8 Study Guide and Review

1. true **3.** false, rectangle **5.** false, trapezoid **7.** true **9.** 120 **11.** 90 **13.** $m\angle W = 62$, $m\angle X = 108$, $m\angle Y = 80$, $m\angle Z = 110$ **15.** 52 **17.** 87.9 **19.** 6 **21.** no **23.** yes **25.** 52 **27.** 28 **29.** Yes, opp. sides are parallel and diag. are congruent **31.** 7.5 **33.** 102

35. Given: $ABCD$ is a square.
Prove: $\overline{AC} \perp \overline{BD}$



Proof:

Slope of $\overline{AC} = \frac{a-0}{a-0}$ or 1

Slope of $\overline{BD} = \frac{a-0}{0-a}$ or -1

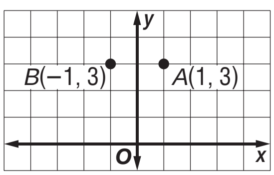
The slope of $\overline{AC}$ is the negative reciprocal of the slope of $\overline{BD}$. Therefore, $\overline{AC} \perp \overline{BD}$.

37. $P(3a, c)$

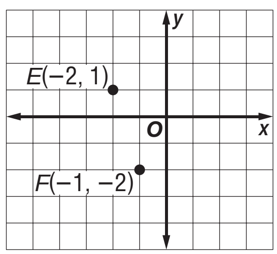
Chapter 9 Transformations

Page 461 Chapter 9 Getting Started

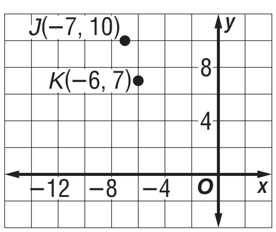
1.



3.



5.



7. 36.9 **9.** 41.8 **11.** 41.4

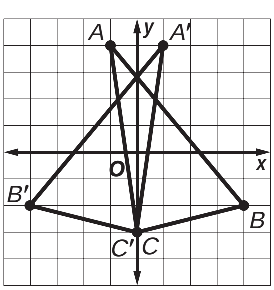
13. $\begin{bmatrix} -5 & -1 \\ 10 & 5 \end{bmatrix}$

15. $\begin{bmatrix} -2 & -5 & 1 \\ 3 & -4 & -5 \end{bmatrix}$

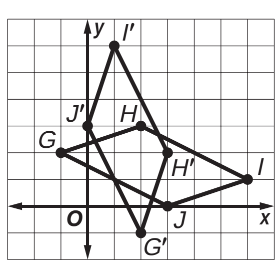
Pages 463–469 Lesson 9-1

1. Sample Answer: The centroid of an equilateral triangle is not a point of symmetry. **3.** angle measure, betweenness of points, collinearity, distance **5.** 4; yes **7.** 6; yes

9.

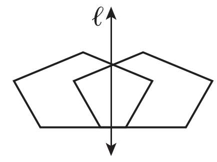


11.

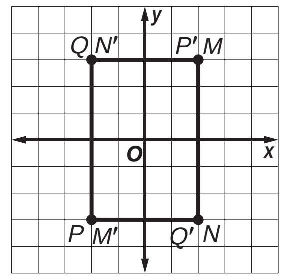


13. 4, yes **15.** $\overline{YX}$ **17.** $\angle XZW$ **19.** $\overline{UV}$ **21.** T **23.** $\triangle WTZ$

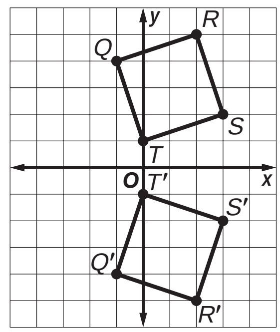
25.



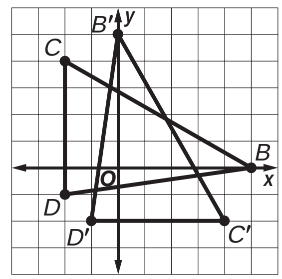
27.



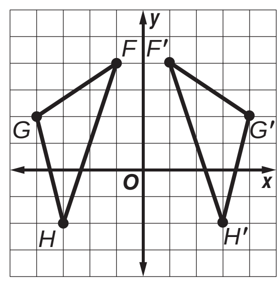
29.



31.

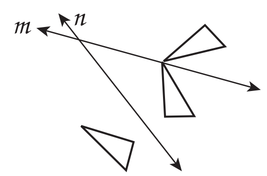


33.



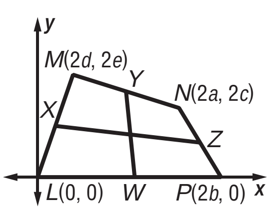
$(x, y) \rightarrow (-x, y)$

35. 2; yes **37.** 1; no **39.** same shape, but turned or rotated



41. $A(4, 7)$, $B(10, -3)$, and $C(-6, -8)$ **43.** Consider point (a, b). Upon reflection in the origin, its image is $(-a, -b)$. Upon reflection in the x-axis and then the y-axis, its image is $(a, -b)$ and then $(-a, -b)$. The images are the same. **45.** vertical line of symmetry **47.** vertical, horizontal lines of symmetry; point of symmetry at the center **49.** D

51. Given: Quadrilateral $LMNP$; X, Y, Z, and W are midpoints of their respective sides.
Prove: $\overline{YW}$ and $\overline{XZ}$ bisect each other.



Proof:

Midpoint Y of $\overline{MN}$ is $\left(\frac{2d+2a}{2}, \frac{2e+2c}{2}\right)$ or $(d+a, e+c)$.

Midpoint Z of $\overline{NP}$ is $\left(\frac{2a+2b}{2}, \frac{2c+0}{2}\right)$ or $(a+b, c)$. Midpoint W of $\overline{PL}$ is $\left(\frac{0+2b}{2}, \frac{0+0}{2}\right)$ or $(b, 0)$.

Midpoint X of $\overline{LM}$ is $\left(\frac{0+2d}{2}, \frac{0+2e}{2}\right)$ or (d, e). Midpoint of $\overline{WY}$ is $\left(\frac{d+a+b}{2}, \frac{e+c+0}{2}\right)$ or $\left(\frac{a+b+d}{2}, \frac{c+e}{2}\right)$.

Midpoint of $\overline{XZ}$ is $\left(\frac{d+a+b}{2}, \frac{e+c}{2}\right)$ or $\left(\frac{a+b+d}{2}, \frac{c+e}{2}\right)$.

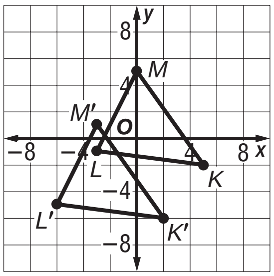
The midpoints of $\overline{XZ}$ and $\overline{WY}$ are the same, so $\overline{XZ}$ and $\overline{WY}$ bisect each other.

53. 40 **55.** 36 **57.** $f \approx 25.5$, $m\angle H = 76$, $h \approx 28.8$ **59.** $\sqrt{2}$ **61.** $\sqrt{5}$

Pages 470–475 Lesson 9-2

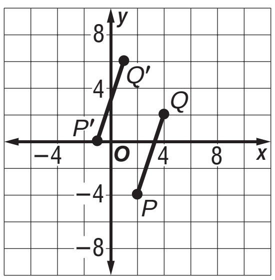
1. Sample answer: $A(3, 5)$ and $B(-4, 7)$; start at 3, count to the left to -4, which is 7 units to the left or -7. Then count up 2 units from 5 to 7 or $+2$. The translation from A to B is $(x, y) \rightarrow (x - 7, y + 2)$. **3.** Allie; counting from the point $(-2, 1)$ to $(1, -1)$ is right 3 and down 2 to the image. The reflections would be too far to the right. The image would be reversed as well. **5.** No; quadrilateral $WXYZ$ is oriented differently than quadrilateral $NPQR$.

7.

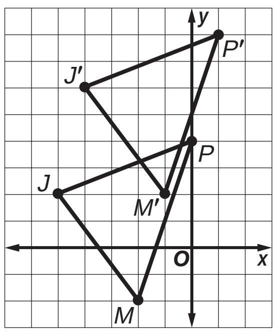


9. Yes; it is one reflection after another with respect to the two parallel lines. **11.** No; it is a reflection followed by a rotation. **13.** Yes; it is one reflection after another with respect to the two parallel lines.

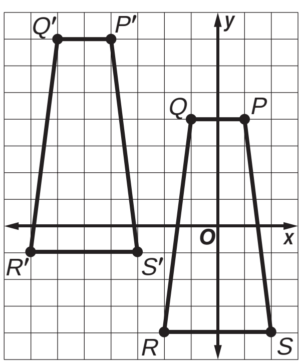
15.



17.

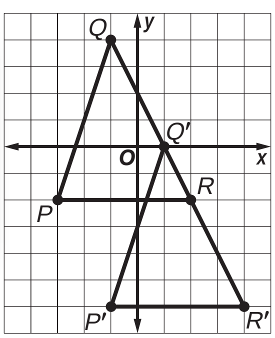


19.



21. left 3 squares and down 7 squares **23.** 48 in. right **25.** 72 in. right, $24\sqrt{3}$ in. down

27.



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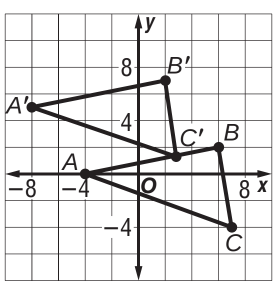
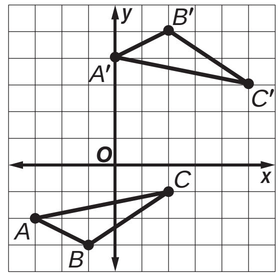


$A(-4, 0)$, $B(6, 2)$, $C(7, -4)$

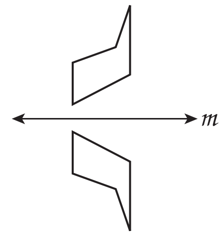
31. more brains; more free time **33.** No; the percent per figure is different in each category. **35.** Translations and reflections preserve the congruences of segments and angles. The composition of the two transformations will preserve both congruences. Therefore, a glide reflection is an isometry.

37.



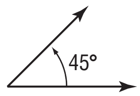
39. A

41.

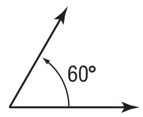


43. $Q(a - b, c)$, $T(0, 0)$ **45.** 23 ft **47.** You did not fill out an application. **49.** The two lines are not parallel. **51.** 5 **53.** $3\sqrt{2}$

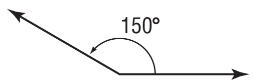
55.



57.

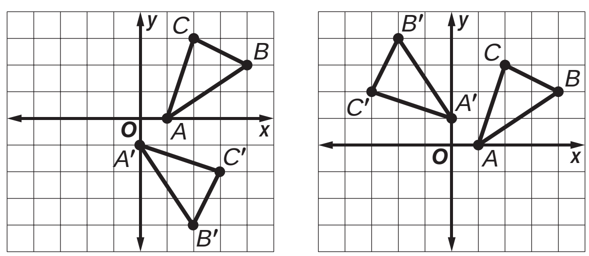


59.



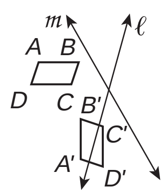
Pages 476–482 Lesson 9-3

1. clockwise $(x, y) \rightarrow (y, -x)$; counterclockwise $(x, y) \rightarrow (-y, x)$

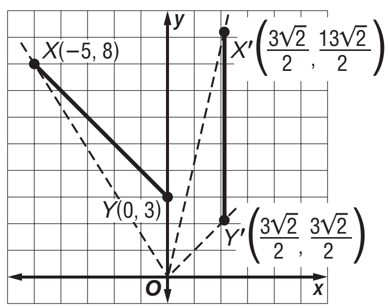


3. Both translations and rotations are made up of two reflections. The difference is that translations reflect across parallel lines and rotations reflect across intersecting lines.

5.

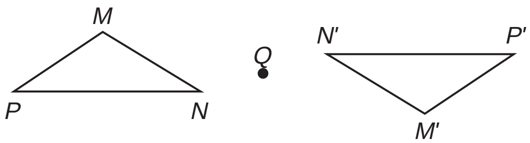


7.

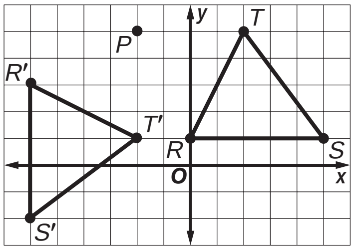


9. order 6; magnitude 60° **11.** order 5 and magnitude 72°; order 4 and magnitude 90°; order 3 and magnitude 120°

13.

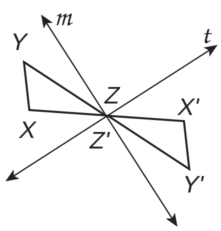


15.

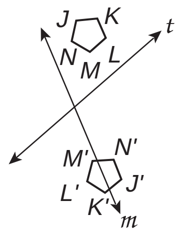


17. 72°

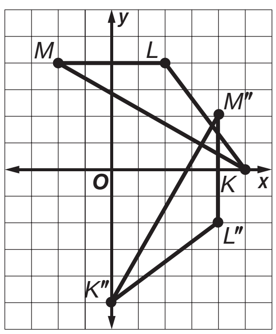
19.



21.



23. $K''(0, -5)$, $L''(4, -2)$, and $M''(4, 2)$; 90° clockwise



25. $(\sqrt{3}, 1)$ **27.** Yes; it is a proper successive reflection with respect to the two intersecting lines. **29.** yes **31.** no **33.** 9 **35.** $(x, y) \rightarrow (y, -x)$ **37.** any point on the line of reflection **39.** no invariant points **41.** B

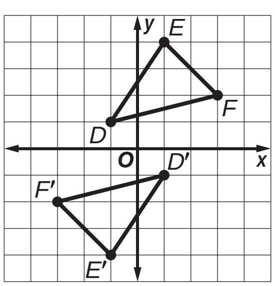
43.

Transformation	angle measure	betweenness of points	orientation	collinearity	distance measure
reflection	yes	yes	no	yes	yes
translation	yes	yes	yes	yes	yes
rotation	yes	yes	yes	yes	yes

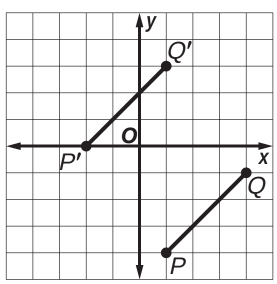
45. direct **47.** Yes; it is one reflection after another with respect to the two parallel lines. **49.** Yes; it is one reflection after another with respect to the two parallel lines. **51.** *C* **53.** $\angle AGF$ **55.** $\overline{TR}$; diagonals bisect each other **57.** $\angle QRS$; opp. ⦞ ≅ **59.** no **61.** yes **63.** (0, 4), (1, 2), (2, 0) **65.** (0, 12), (1, 8), (2, 4), (3, 0) **67.** (0, 12), (1, 6), (2, 0)

Page 482 Chapter 9 Practice Quiz 1

1.



3.



5. order 36; magnitude 10°

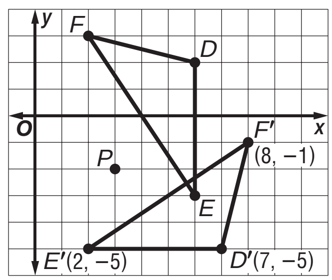
Pages 483–488 Lesson 9-4

1. Semi-regular tessellations contain two or more regular polygons, but uniform tessellations can be any combination of shapes. **3.** The figure used in the tesselation appears to be a trapezoid, which is not a regular polygon. Thus, the tessellation cannot be regular. **5.** no; measure of interior angle = 168 **7.** yes **9.** yes; not uniform **11.** no; measure of interior angle = 140 **13.** yes; measure of interior angle = 60 **15.** no; measure of interior angle ≈ 164.3 **17.** no **19.** yes **21.** yes; uniform **23.** yes; not uniform **25.** yes; not uniform **27.** yes; uniform, regular **29.** semi-regular, uniform **31.** Never; semi-regular tessellations have the same combination of shapes and angles at each vertex like uniform tessellations. The shapes for semi-regular tessellations are just regular. **33.** Always; the sum of the measures of the angles of a quadrilateral is 360°. So if each angle of the quadrilateral is rotated at the vertex, then that equals 360° and the tessellation is possible. **35.** yes **37.** uniform, regular **39.** Sample answer: Tessellations can be used in art to create abstract art. Answers should include the following.

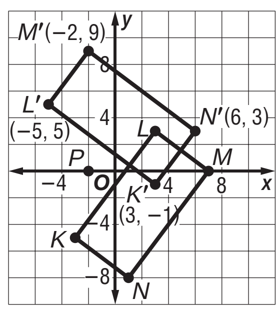
- The equilateral triangles are arranged to form hexagons, which are arranged adjacent to one another.
- Sample answers: kites, trapezoids, isosceles triangles

41. A

43.



45.

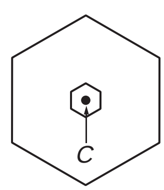


47. $x = 4, y = 1$ **49.** $x = 56, y = 12$ **51.** no, no **53.** yes, no **55.** no, no **57.** $AB = 7$, $BC = 10$, $AC = 9$ **59.** $1(-1) = -1$ and $-1(1) = -1$ **61.** square **63.** 15 **65.** 22.5

Pages 490–497 Lesson 9-5

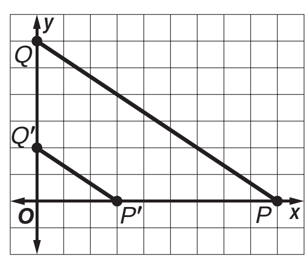
1. Dilations only preserve length if the scale factor is 1 or −1. So for any other scale factor, length is not preserved and the dilation is not an isometry. **3.** Trey; Desiree found the image using a positive scale factor.

5.



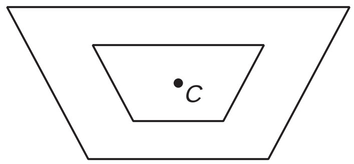
7. $A'B' = 12$

9.

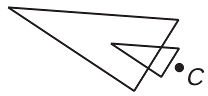


11. $r = 2$; enlargement **13.** C

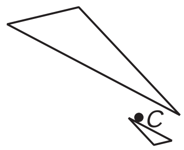
15.



17.

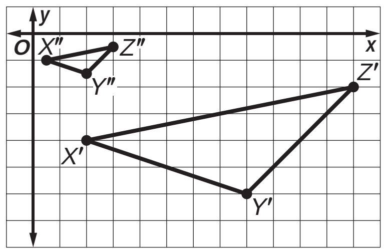


19.

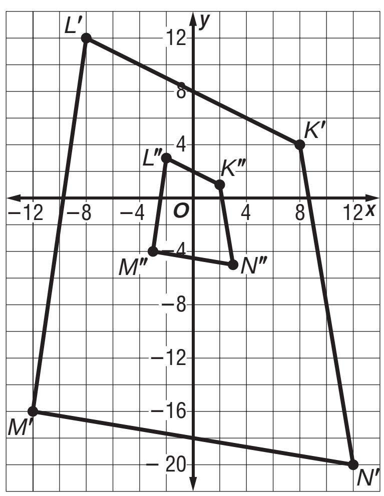


21. $S'T' = \frac{3}{5}$
23. $ST = 4$
25. $S'T' = 0.9$

27.



29.



31. $\frac{1}{2}$; reduction

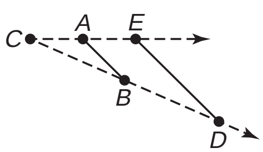
33. $\frac{1}{3}$; reduction

35. -2; enlargement

37. 7.5 by 10.5

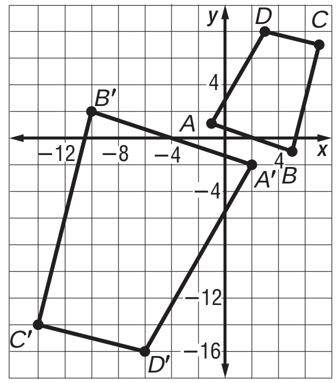
39. The perimeter is four times the original perimeter.

41. Given: dilation with center C and scale factor r
Prove: $ED = r(AB)$
Proof:

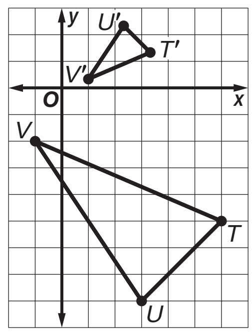


$CE = r(CA)$ and $CD = r(CB)$ by the definition of a dilation. $\frac{CE}{CA} = r$ and $\frac{CD}{CB} = r$.
So, $\frac{CE}{CA} = \frac{CD}{CB}$ by substitution.
$\angle ACB \cong \angle ECD$, since congruence of angles is reflexive. Therefore, by SAS Similarity, $\triangle ACB$ is similar to $\triangle ECD$. The corresponding sides of similar triangles are proportional, so $\frac{ED}{AB} = \frac{CE}{CA}$. We know that $\frac{CE}{CA} = r$, so $\frac{ED}{AB} = r$ by substitution. Therefore, $ED = r(AB)$ by the Multiplication Property of Equality.

43. 2 **45.** $\frac{1}{20}$ **47.** 60% **49.**



51.

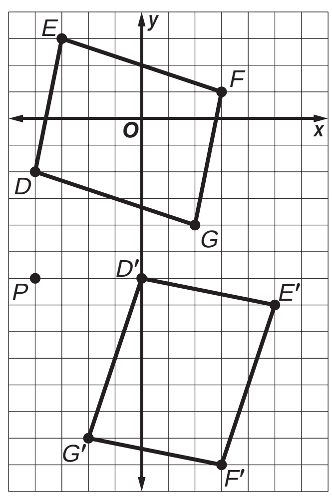


53. Sample answer: Yes; a cut and paste produces an image congruent to the original. Answers should include the following.

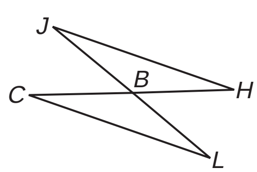
- Congruent figures are similar, so cutting and pasting is a similarity transformation.
- If you scale both horizontally and vertically by the same factor, you are creating a dilation.

55. A **57.** no **59.** no

61.



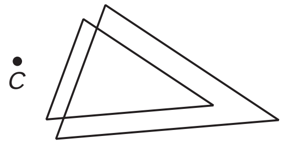
63. Given: $\angle J \cong \angle L$ B is the midpoint of $\overline{JL}$.
Prove: $\triangle JHB \cong \triangle LCB$
Proof: It is known that $\angle J \cong \angle L$. Since B is the midpoint of $\overline{JL}$, $\overline{JB} \cong \overline{LB}$ by the Midpoint Theorem.



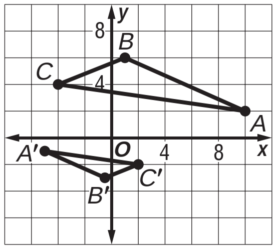
$\angle JBH \cong \angle LBC$ because vertical angles are congruent. Thus, $\triangle JHB \cong \triangle LCB$ by ASA. **65.** 76.0

Page 497 Chapter 9 Practice Quiz 2

1. yes; uniform; semi-regular **3.**

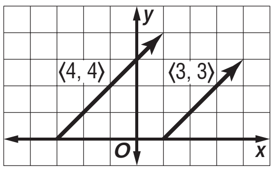


5. $A'(-5, -1)$, $B'\left(-\frac{1}{2}, -3\right)$, $C'(2, -2)$



Pages 498–505 Lesson 9-6

1. Sample answer; $\langle 7, 7\rangle$

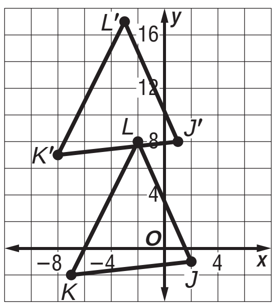


3. Sample answer: Using a vector to translate a figure is the same as using an ordered pair because a vector has horizontal and vertical components, each of which can be represented by one coordinate of an ordered pair.

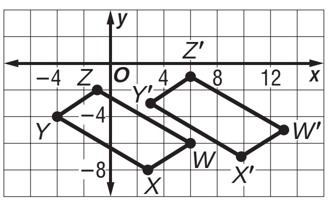
5. $\langle 4, -3\rangle$

7. $2\sqrt{13} \approx 7.2$, $\approx 213.7°$

9.

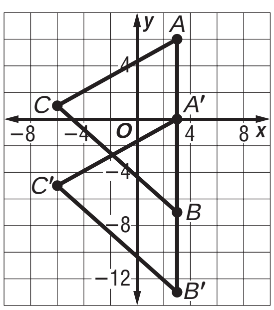


11.

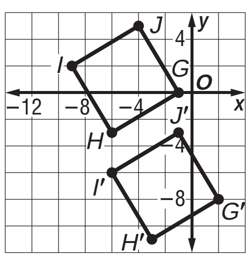


13. $6\sqrt{13} \approx 21.6$, $303.7°$ **15.** $\langle 2, 6\rangle$ **17.** $\langle -7, -4\rangle$ **19.** $\langle -3, 5\rangle$ **21.** 5, 0° **23.** $2\sqrt{5} \approx 4.5$, $296.6°$ **25.** $7\sqrt{5} \approx 15.7$, $26.6°$ **27.** 25, $\approx 73.7°$ **29.** $5\sqrt{41} \approx 32.0$, $\approx 218.7°$ **31.** $6\sqrt{2} \approx 8.5$, $135.0°$ **33.** $4\sqrt{10} \approx 12.6$, $198.4°$ **35.** $2\sqrt{122} \approx 22.1$, $275.2°$

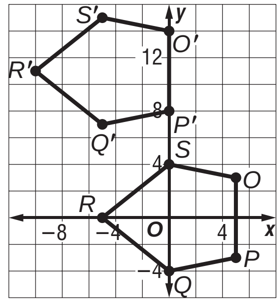
37.



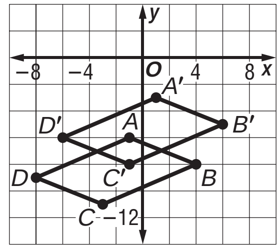
39.



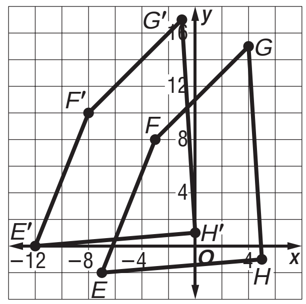
41.



43.



45.



47. 13, $\approx 67.4°$
49. 5, $\approx 306.9°$
51. $2\sqrt{5} \approx 4.5$, $\approx 26.6°$
53. about 44.8 mi; about 38.7° south of due east
55. $\langle -350, 450\rangle$ mph
57. 52.1° north of due west

59. Sample answer: Quantities such as velocity are vectors. The velocity of the wind and the velocity of the plane together factor into the overall flight plan. Answers should include the following.

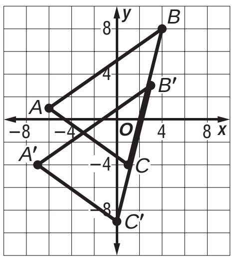
- A wind from the west would add to the velocity contributed by the plane resulting in an overall velocity with a larger magnitude.
- When traveling east, the prevailing winds add to the velocity of the plane. When traveling west, they detract from it.

61. D **63.** $A'B' = 6$ **65.** $AB = 48$ **67.** yes; not uniform **69.** 12 **71.** 30 **73.** $\begin{bmatrix} -4 & -3 \\ -10 & 4 \end{bmatrix}$ **75.** $\begin{bmatrix} -27 & -15 & -3 \\ 27 & 3 & 15 \end{bmatrix}$ **77.** $\begin{bmatrix} 12 & 4 \\ -4 & -12 \end{bmatrix}$

Pages 506–511 Lesson 9-7

1. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **3.** Sample answer: $\begin{bmatrix} -2 & -2 & -2 & -2 \\ -1 & -1 & -1 & -1 \end{bmatrix}$

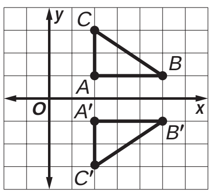
5. $D'(-1, 9)$, $E'(5, 9)$, $F'(3, 6)$, $G'(-3, 6)$ **7.** $A'\left(-\frac{1}{4}, -\frac{1}{2}\right)$, $B'\left(-\frac{3}{4}, -\frac{3}{4}\right)$, $C'\left(-\frac{3}{4}, -\frac{5}{4}\right)$, $D'\left(-\frac{1}{4}, -1\right)$ **9.** $H'(5, 4)$, $I'(1, -1)$, $J'(3, -6)$, $K'(7, -3)$ **11.** $P'(3, -6)$, $Q'(7, -6)$, $R'(7, -2)$ **13.** (1.5, −0.5), (3.5, −1.5), (2.5, −3.5), (0.5, −2.5) **15.** $E'(-6, 6)$, $F'(-3, 8)$ **17.** $M'(1, 1)$, $N'(5, 3)$, $O'(5, 1)$, $P'(1, -1)$ **19.** $A'(12, 10)$, $B'(8, 10)$, $C'(6, 14)$ **21.** $G'(-2, -1)$, $H'(2, -3)$, $I'(3, 4)$, $J'(-3, 5)$ **23.** $X'(-2, 2)$, $Y'(-4, -1)$ **25.** $D'(-4, -5)$, $E'(2, -6)$, $F'(3, -1)$, $G'(-3, 4)$ **27.** $V'(-2, 2)$, $W'\left(\frac{2}{3}, 2\right)$, $X'\left(2, -\frac{4}{3}\right)$ **29.** $V'(-3, -3)$, $W'(-3, 1)$, $X'(2, 3)$ **31.** $P'(2, -3)$, $Q'(-1, -1)$, $R'(1, 2)$, $S'(3, 2)$, $T'(5, -1)$ **33.** $P'(1, -1)$, $Q'(4, 1)$, $R'(2, 4)$, $S'(0, 4)$, $T'(-2, 1)$ **35.** $M'(-1, 12)$, $N'(-10, -3)$ **37.** $S'(-1, 2)$, $T'(-1, 6)$, $U'(3, 5)$, $V'(3, 1)$ **39.** $A'\left(-1, -\frac{1}{3}\right)$, $B'\left(-\frac{2}{3}, -\frac{4}{3}\right)$, $C'\left(\frac{2}{3}, -\frac{4}{3}\right)$, $D'\left(1, -\frac{1}{3}\right)$, $E'\left(\frac{2}{3}, \frac{2}{3}\right)$, $F'\left(-\frac{2}{3}, \frac{2}{3}\right)$ **41.** $A'(2, 1)$, $B'(5, 2)$, $C'(5, 6)$, $D'(2, 7)$, $E'(-1, 6)$, $F'(-1, 2)$ **43.** Each footprint is reflected in the y-axis, then translated up two units. **45.** $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ **47.** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **49.** $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

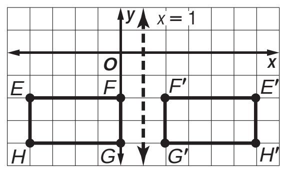
51. 

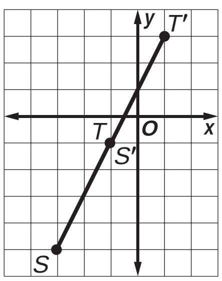
53. $-\frac{1}{2}$; reduction **55.** 60, 120 **57.** 36, 144

Pages 512–516 Chapter 9 Study Guide and Review

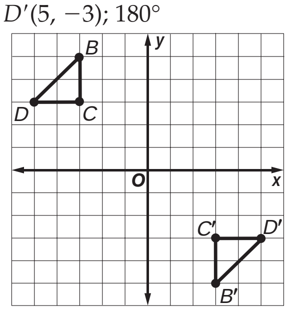
1. false, center **3.** false, component form **5.** false, center of rotation **7.** false, scale factor

9. 

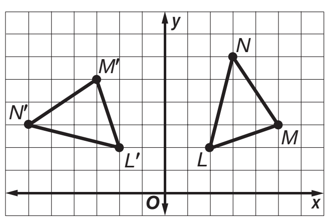
11. 

13. 

15. $B'(3, -5)$, $C'(3, -3)$, $D'(5, -3)$; 180°



17. $L'(-2, 2)$, $M'(-3, 5)$, $N'(-6, 3)$; 90° counterclockwise



19. 200° **21.** yes; not uniform **23.** yes; uniform **25.** Yes; the measure of an interior angle is 60, which is a factor of 360. **27.** $C'D' = 24$ **29.** $CD = 4$

31. $C'D' = 10$ **33.** $P'(2, -6)$, $Q'(-4, -4)$, $R'(-2, 2)$ **35.** $\langle 3, 4\rangle$ **37.** $\langle 0, 8\rangle$ **39.** ≈ 14.8, $\approx 208.3°$ **41.** ≈ 72.9, $\approx 213.3°$ **43.** $D'\left(-\frac{12}{5}, -\frac{8}{5}\right)$, $E'(0, 4)$, $F'\left(\frac{8}{5}, -\frac{16}{5}\right)$ **45.** $D'(-2, 3)$, $E'(5, 0)$, $F'(-4, -2)$ **47.** $W'(-16, 2)$, $X'(-4, 6)$, $Y'(-2, 0)$, $Z'(-12, -6)$

Chapter 10 Circles

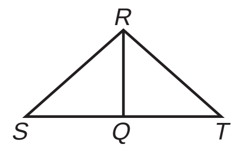
Pages 521 Chapter 10 Getting Started

1. 162 **3.** 2.4 **5.** $r = \frac{C}{2p}$ **7.** 15 **9.** 17.0 **11.** 1.5, −0.9 **13.** 2.5, −3

Pages 522–528 Lesson 10-1

1. Sample answer: The value of π is calculated by dividing the circumference of a circle by the diameter. **3.** Except for a diameter, two radii and a chord of a circle can form a triangle. The Triangle Inequality Theorem states that the sum of two sides has to be greater than the third. So, $2r$ has to be greater than the measure of any chord that is not a diameter, but $2r$ is the measure of the diameter. So the diameter has to be longer than any other chord of the circle. **5.** $\overline{EA}$, $\overline{EB}$, $\overline{EC}$, or $\overline{ED}$ **7.** $\overline{AC}$ or $\overline{BD}$ **9.** 10.4 in. **11.** 6 **13.** 10 m, 31.42 m **15.** B **17.** $\overline{FA}$, $\overline{FB}$, or $\overline{FE}$ **19.** $\overline{BE}$ **21.** $\odot R$ **23.** $\overline{ZV}$, $\overline{TX}$, or $\overline{WZ}$ **25.** $\overline{RU}$, $\overline{RV}$ **27.** 2.5 ft **29.** 64 in. or 5 ft 4 in. **31.** 0.6 m **33.** 3 **35.** 12 **37.** 34 **39.** 20 **41.** 5 **43.** 2.5 **45.** 13.4 cm, 84.19 cm **47.** 24.32 m, 12.16 m **49.** $13\frac{1}{2}$ in., 42.41 in. **51.** $0.33a$, $1.05a$ **53.** 5π ft **55.** 8π cm **57.** 0; The longest chord of a circle is the diameter, which contains the center. **59.** 500–600 ft **61.** 24π units **63.** 27 **65.** 10π, 20π, 30π **67.** 9.8; 66° **69.** 44.7; 27° **71.** 24

73. Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$



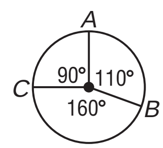
Proof:

Statements	Reasons
1. $\overline{RQ}$ bisects $\angle SRT$.	1. Given
2. $\angle SRQ \cong \angle QRT$	2. Def. of ∠ bisector
3. $m\angle SRQ = m\angle QRT$	3. Def. of ≅ ∠s
4. $m\angle SQR = m\angle T + m\angle QRT$	4. Exterior Angle Theorem
5. $m\angle SQR > m\angle QRT$	5. Def. of Inequality
6. $m\angle SQR > m\angle SRQ$	6. Substitution

75. 60 **77.** 30 **79.** 30

Pages 529–535 Lesson 10-2

1. Sample answer: $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, $\overset{\frown}{AC}$, $\overset{\frown}{ABC}$, $\overset{\frown}{BCA}$, $\overset{\frown}{CAB}$; $m\overset{\frown}{AB} = 110$, $m\overset{\frown}{BC} = 160$, $m\overset{\frown}{AC} = 90$, $m\overset{\frown}{ABC} = 270$, $m\overset{\frown}{BCA} = 250$, $m\overset{\frown}{CAB} = 200$



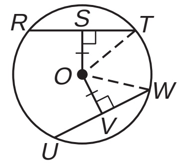
3. Sample answer: Concentric circles have the same center, but different radius measures; congruent circles usually have different centers but the same radius measure. **5.** 137 **7.** 103 **9.** 180 **11.** 138 **13.** Sample answer: 25% = 90°, 23% = 83°, 28% = 101°, 22% = 79°, 2% = 7° **15.** 60 **17.** 30 **19.** 120 **21.** 115 **23.** 65 **25.** 90 **27.** 90 **29.** 135 **31.** 270 **33.** 76 **35.** 52 **37.** 256 **39.** 308 **41.** $24\pi \approx 75.40$ units **43.** $4\pi \approx 12.57$ units **45.** The first category is a major arc, and the other three categories are minor arcs. **47.** always **49.** never **51.** $m\angle 1 = 80$, $m\angle 2 = 120$, $m\angle 3 = 160$ **53.** 56.5 ft **55.** No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent. **57.** B **59.** 20; 62.83 **61.** 28; 14 **63.** 84.9 newtons, 32° north of due east **65.** 36.68 **67.** $\sqrt{24.5}$ **69.** If ABC has three sides, then ABC is a triangle. **71.** 42 **73.** 100 **75.** 36

Pages 536–543 Lesson 10-3

1. Sample answer: An inscribed polygon has all vertices on the circle. A circumscribed circle means the circle is drawn around so that the polygon lies in its interior and all vertices lie on the circle. **3.** Tokei; to bisect the chord, it must be a diameter and be perpendicular. **5.** 30 **7.** $5\sqrt{3}$ **9.** $10\sqrt{5} \approx 22.36$ **11.** 15 **13.** 15 **15.** 40 **17.** 80 **19.** 4 **21.** 5 **23.** $m\overset{\frown}{AB} = m\overset{\frown}{BC} = m\overset{\frown}{CD} = m\overset{\frown}{DE} = m\overset{\frown}{EF} = m\overset{\frown}{FG} = m\overset{\frown}{GH} = m\overset{\frown}{HA} = 45$ **25.** $m\overset{\frown}{NP} = m\overset{\frown}{RQ} = 120$; $m\overset{\frown}{NR} = m\overset{\frown}{PQ} = 60$ **27.** 30 **29.** 15 **31.** 16 **33.** 6 **35.** $\sqrt{2} \approx 1.41$

Selected Answers

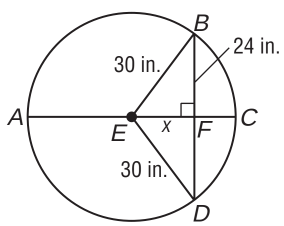
37. Given: $\odot O$, $\overline{OS} \perp \overline{RT}$, $\overline{OV} \perp \overline{UW}$, $\overline{OS} \cong \overline{OV}$
Prove: $\overline{RT} \cong \overline{UW}$



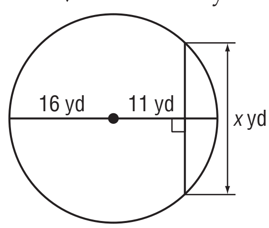
Proof:

Statements	Reasons
1. $\overline{OT} \cong \overline{OW}$	1. All radii of a $\odot$ are $\cong$.
2. $\overline{OS} \perp \overline{RT}$, $\overline{OV} \perp \overline{VW}$, $\overline{OS} \cong \overline{OV}$	2. Given
3. $\angle OST$, $\angle OVW$ are right angles.	3. Definition of $\perp$ lines
4. $\triangle STO \cong \triangle VWO$	4. HL
5. $\overline{ST} \cong \overline{VW}$	5. CPCTC
6. $ST = VW$	6. Definition of $\cong$ segments
7. $2(ST) = 2(VW)$	7. Multiplication Property
8. $\overline{OS}$ bisects $\overline{RT}$; $\overline{OV}$ bisects $\overline{UW}$.	8. Radius $\perp$ to a chord bisects the chord.
9. $RT = 2(ST)$, $UW = 2(VW)$	9. Definition of segment bisector
10. $RT = UW$	10. Substitution
11. $\overline{RT} \cong \overline{UW}$	11. Definition of $\cong$ segments

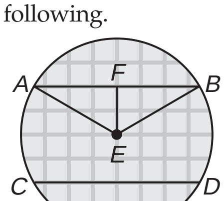
39. 2.82 in.
41. 18 inches



43. $2\sqrt{135} \approx 23.24$ yd



45. Let r be the radius of $\odot P$. Draw radii to points D and E to create triangles. The length DE is $r\sqrt{3}$ and $AB = 2r$; $r\sqrt{3} \neq \frac{1}{2(2r)}$. **47.** Inscribed equilateral triangle; the six arcs making up the circle are congruent because the chords intercepting them were congruent by construction. Each of the three chords drawn intercept two of the congruent chords. Thus, the three larger arcs are congruent. So, the three chords are congruent, making this an equilateral triangle. **49.** No; congruent arcs are must be in the same circle, but these are in concentric circles. **51.** Sample answer: The grooves of a waffle iron are chords of the circle. The ones that pass horizontally and vertically through the center are diameters. Answers should include the following.



- If you know the measure of the radius and the distance the chord is from the center, you can use the Pythagorean Theorem to find the length of half of the chord and then multiply by 2.
- There are four grooves on either side of the diameter, so each groove is about 1 in. from the center. In the figure, $EF = 2$ and $EB = 4$ because the radius is half the diameter. Using the Pythagorean Theorem, you find that $FB \approx 3.464$ in. so $AB \approx 6.93$ in. Approximate lengths for other chords are 5.29 in. and 7.75 in., but exactly 8 in. for the diameter.

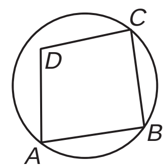
53. 14,400 **55.** 180 **57.** $\overline{SU}$ **59.** $\overline{RM}$, $\overline{AM}$, $\overline{DM}$, $\overline{IM}$ **61.** 50 **63.** 10 **65.** 20

Page 543 Chapter 10 Practice Quiz 1

1. $\overline{BC}$, $\overline{BD}$, $\overline{BA}$ **3.** 95 **5.** 9 **7.** 28 **9.** 21

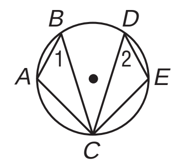
Page 544–551 Lesson 10-4

1. Sample answer:



3. $m\angle 1 = 30$, $m\angle 2 = 60$, $m\angle 3 = 60$, $m\angle 4 = 30$, $m\angle 5 = 30$, $m\angle 6 = 60$, $m\angle 7 = 60$, $m\angle 8 = 30$ **5.** $m\angle 1 = 35$, $m\angle 2 = 55$, $m\angle 3 = 39$, $m\angle 4 = 39$ **7.** 1 **9.** $m\angle 1 = m\angle 2 = 30$, $m\angle 3 = 25$

11. Given: $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$
Prove: $\triangle ABC \cong \triangle EDC$

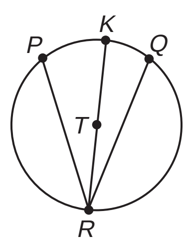


Proof:

Statements	Reasons
1. $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$	1. Given
2. $m\widehat{AB} = m\widehat{DE}$, $m\widehat{AC} = m\widehat{CE}$	2. Def. of $\cong$ arcs
3. $\frac{1}{2}m\widehat{AB} = \frac{1}{2}m\widehat{DE}$, $\frac{1}{2}m\widehat{AC} = \frac{1}{2}m\widehat{CE}$	3. Mult. Prop.
4. $m\angle ACB = \frac{1}{2}m\widehat{AB}$, $m\angle ECD = \frac{1}{2}m\widehat{DE}$, $m\angle 1 = \frac{1}{2}m\widehat{AC}$, $m\angle 2 = \frac{1}{2}m\widehat{CE}$	4. Inscribed Angle Theorem
5. $m\angle ACB = m\angle ECD$, $m\angle 1 = m\angle 2$	5. Substitution
6. $\angle ACB \cong \angle ECD$, $\angle 1 \cong \angle 2$	6. Def. of $\cong$ ∠s
7. $\overline{AB} \cong \overline{DE}$	7. $\cong$ arcs have $\cong$ chords.
8. $\triangle ABC \cong \triangle EDC$	8. AAS

13. $m\angle 1 = m\angle 2 = 13$ **15.** $m\angle 1 = 51$, $m\angle 2 = 90$, $m\angle 3 = 39$ **17.** 45, 30, 120 **19.** $m\angle B = 120$, $m\angle C = 120$, $m\angle D = 60$ **21.** Sample answer: $\overline{EF}$ is a diameter of the circle and a diagonal and angle bisector of $EDFG$. **23.** 72 **25.** 144 **27.** 162 **29.** 9 **31.** $\frac{8}{9}$ **33.** 1

35. Given: T lies inside $\angle PRQ$. $\overline{RK}$ is a diameter of $\odot T$.
Prove: $m\angle PRQ = \frac{1}{2}m\widehat{PKQ}$



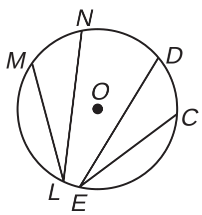
Proof:

Statements	Reasons
1. $m\angle PRQ = m\angle PRK + m\angle KRQ$	1. Angle Addition Theorem
2. $m\widehat{PKQ} = m\widehat{PK} + m\widehat{KQ}$	2. Arc Addition Theorem
3. $\frac{1}{2}m\widehat{PKQ} = \frac{1}{2}m\widehat{PK} + \frac{1}{2}m\widehat{KQ}$	3. Multiplication Property

Selected Answers

4. $m\angle PRK = \frac{1}{2}m\widehat{PK}$, $m\angle KRQ = \frac{1}{2}m\widehat{KQ}$	4. The measure of an inscribed angle whose side is a diameter is half the measure of the intercepted arc (Case 1).
5. $\frac{1}{2}m\widehat{PKQ} = m\angle PRK + m\angle KRQ$	5. Substitution (Steps 3, 4)
6. $\frac{1}{2}m\widehat{PKQ} = m\angle PRQ$	6. Substitution (Steps 5, 1)

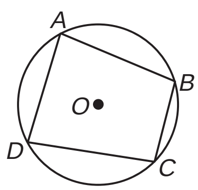
37. Given: inscribed $\angle MLN$ and $\angle CED$, $\overline{CD} \cong \overline{MN}$
Prove: $\angle CED \cong \angle MLN$



Proof:

Statements	Reasons
1. $\angle MLN$ and $\angle CED$ are inscribed; $\overline{CD} \cong \overline{MN}$	1. Given
2. $m\angle MLN = \frac{1}{2}m\widehat{MN}$; $m\angle CED = \frac{1}{2}m\widehat{CD}$	2. Measure of an inscribed $\angle$ = half measure of intercepted arc.
3. $m\widehat{CD} = m\widehat{MN}$	3. Def. of $\cong$ arcs
4. $\frac{1}{2}m\widehat{CD} = \frac{1}{2}m\widehat{MN}$	4. Mult. Prop.
5. $m\angle CED = m\angle MLN$	5. Substitution
6. $\angle CED \cong \angle MLN$	6. Def. of $\cong$ $\angle$s

39. Given: quadrilateral $ABCD$ inscribed in $\odot O$
Prove: $\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.



Proof: By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}$, $m\angle C + m\angle A = \frac{1}{2}(m\widehat{DCB} + m\widehat{DAB})$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 80. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

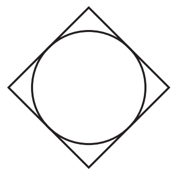
41. Isosceles right triangle because sides are congruent radii making it isosceles and $\angle AOC$ is a central angle for an arc of 90°, making it a right angle. **43.** Square because each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.
45. Sample answer: The socket is similar to an inscribed polygon because the vertices of the hexagon can be placed on a circle that is concentric with the outer circle of the socket. Answers should include the following.
- An inscribed polygon is one in which all of its vertices are points on a circle.
- The side of the regular hexagon inscribed in a circle $\frac{3}{4}$ inch wide is $\frac{3}{8}$ inch.

47. 234 **49.** $\sqrt{135} \approx 11.62$ **51.** 4π units **53.** always **55.** sometimes **57.** no

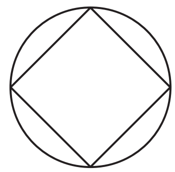
Page 552–558 Lesson 10-5

1a. Two; from any point outside the circle, you can draw only two tangents. **1b.** None; a line containing a point inside the circle would intersect the circle in two points. A tangent can only intersect a circle in one point. **1c.** One; since a tangent intersects a circle in exactly one point, there is one tangent containing a point on the circle.
3. Sample answer:

polygon circumscribed about a circle

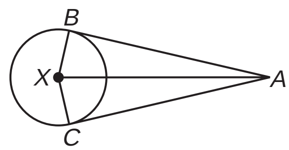


polygon inscribed in a circle



5. Yes; $5^2 + 12^2 = 13^2$ **7.** 576 ft **9.** no **11.** yes **13.** 16 **15.** 12 **17.** 3 **19.** 30 **21.** See students' work. **23.** 60 units **25.** $15\sqrt{3}$ units

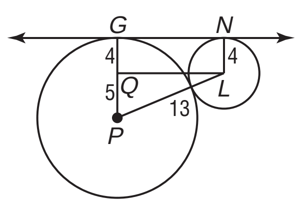
27. Given: $\overline{AB}$ is tangent to $\odot X$ at B. $\overline{AC}$ is tangent to $\odot X$ at C.
Prove: $\overline{AB} \cong \overline{AC}$



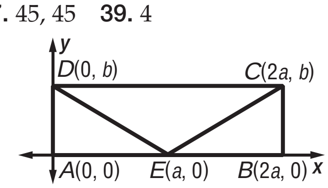
Proof:

Statements	Reasons
1. $\overline{AB}$ is tangent to $\odot X$ at B. $\overline{AC}$ is tangent to $\odot X$ at C.	1. Given
2. Draw $\overline{BX}$, $\overline{CX}$, and $\overline{AX}$.	2. Through any two points, there is one line.
3. $\overline{AB} \perp \overline{BX}$, $\overline{AC} \perp \overline{CX}$	3. Line tangent to a circle is $\perp$ to the radius at the pt. of tangency.
4. $\angle ABX$ and $\angle ACX$ are right angles.	4. Def. of $\perp$ lines
5. $\overline{BX} \cong \overline{CX}$	5. All radii of a circle are $\cong$.
6. $\overline{AX} \cong \overline{AX}$	6. Reflexive Prop.
7. $\triangle ABX \cong \triangle ACX$	7. HL
8. $\overline{AB} \cong \overline{AC}$	8. CPCTC

29. $\overline{AE}$ and $\overline{BF}$
31. 12; Draw $\overline{PG}$, $\overline{NL}$, and $\overline{PL}$. Construct $\overline{LQ} \perp \overline{GP}$, thus $LQGN$ is a rectangle. $GQ = NL = 4$, so $QP = 5$. Using the Pythagorean Theorem, $(QP)^2 + (QL)^2 = (PL)^2$. So, $QL = 12$. Since $GN = QL$, $GN = 12$.



33. 27 **35.** $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ **37.** 45, 45 **39.** 4
41. Sample answer:
Given: $ABCD$ is a rectangle. E is the midpoint of $\overline{AB}$.
Prove: $\triangle CED$ is isosceles.



Selected Answers

Proof: Let the coordinates of E be $(a, 0)$. Since E is the midpoint and is halfway between A and B, the coordinates of B will be $(2a, 0)$. Let the coordinates of D be $(0, b)$. The coordinates of C will be $(2a, b)$ because it is on the same horizontal as D and the same vertical as B.

$ED = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$ $\quad EC = \sqrt{(a-2a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

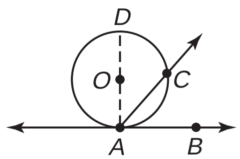
Since $ED = EC$, $\overline{ED} \cong \overline{EC}$. $\triangle DEC$ has two congruent sides, so it is isosceles.

43. 6 **45.** 20.5

Page 561–568 Lesson 10-6

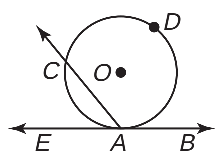
1. Sample answer: A tangent intersects the circle in only one point and no part of the tangent is in the interior of the circle. A secant intersects the circle in two points and some of its points do lie in the interior of the circle. **3.** 138 **5.** 20 **7.** 235 **9.** 55 **11.** 110 **13.** 60 **15.** 110 **17.** 90 **19.** 50 **21.** 30 **23.** 8 **25.** 4 **27.** 25 **29.** 130 **31.** 10 **33.** 141 **35.** 44 **37.** 118 **39.** about 103 ft **41.** 4.6 cm

43a. Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is acute.
Prove: $m\angle CAB = \frac{1}{2}m\widehat{CA}$



Proof: $\angle DAB$ is a right $\angle$ with measure 90, and $\widehat{DCA}$ is a semicircle with measure 180, since if a line is tangent to a $\odot$, it is $\perp$ to the radius at the point of tangency. Since $\angle CAB$ is acute, C is in the interior of $\angle DAB$, so by the Angle and Arc Addition Postulates, $m\angle DAB = m\angle DAC + m\angle CAB$ and $m\widehat{DCA} = m\widehat{DC} + m\widehat{CA}$. By substitution, $90 = m\angle DAC + m\angle CAB$ and $180 = m\widehat{DC} + m\widehat{CA}$. So, $90 = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by Division Prop., and $m\angle DAC + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by substitution. $m\angle DAC = \frac{1}{2}m\widehat{DC}$ since $\angle DAC$ is inscribed, so substitution yields $\frac{1}{2}m\widehat{DC} + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$. By Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CA}$.

43b. Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is obtuse.
Prove: $m\angle CAB = \frac{1}{2}m\widehat{CDA}$



Proof: $\angle CAB$ and $\angle CAE$ form a linear pair, so $m\angle CAB + m\angle CAE = 180$. Since $\angle CAB$ is obtuse, $\angle CAE$ is acute and Case 1 applies, so $m\angle CAE = \frac{1}{2}m\widehat{CA}$. $m\widehat{CA} + m\widehat{CDA} = 360$, so $\frac{1}{2}m\widehat{CA} + \frac{1}{2}m\widehat{CDA} = 180$ by Divison Prop., and $m\angle CAE + \frac{1}{2}m\widehat{CDA} = 180$ by substitution. By the Transitive Prop., $m\angle CAB + m\angle CAE = m\angle CAE + \frac{1}{2}m\widehat{CDA}$, so by Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CDA}$.

45. $\angle 3$, $\angle 1$, $\angle 2$; $m\angle 3 = m\widehat{RQ}$, $m\angle 1 = \frac{1}{2}m\widehat{RQ}$ so $m\angle 3 > m\angle 1$, $m\angle 2 = \frac{1}{2}(m\widehat{RQ} - m\widehat{TP}) = \frac{1}{2}m\widehat{RQ} - \frac{1}{2}m\widehat{TP}$, which is less than $\frac{1}{2}m\widehat{RQ}$, so $m\angle 2 < m\angle 1$. **47.** A **49.** 16 **51.** 33 **53.** 44.5 **55.** 30 in. **57.** 4, −10 **59.** 3, 5

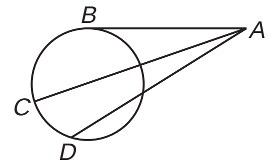
Page 568 Chapter 10 Practice Quiz 2

1. 67.5 **3.** 12 **5.** 115.5

Page 569–574 Lesson 10-7

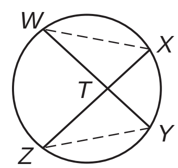
1. Sample answer: The product equation for secant segments equates the product of exterior segment measure and the whole segment measure for each secant. In the case of secant-tangent, the product involving the tangent segment becomes (measure of tangent segment)^2 because the exterior segment and the whole segment are the same segment.

3. Sample answer:



5. 28.1 **7.** ≈7 : 3.54 **9.** 4 **11.** 2 **13.** 6 **15.** 3.2 **17.** 4 **19.** 5.6

21. Given: $\overline{WY}$ and $\overline{ZX}$ intersect at T.
Prove: $WT \cdot TY = ZT \cdot TX$

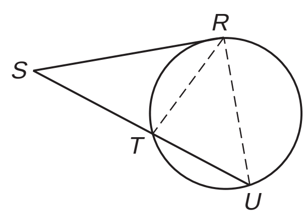


Proof:

Statements	Reasons
a. $\angle W \cong \angle Z$, $\angle X \cong \angle Y$	**a.** Inscribed angles that intercept the same arc are congruent.
b. $\triangle WXT \sim \triangle ZYT$	**b.** AA Similarity
c. $\frac{WT}{ZT} = \frac{TX}{TY}$	**c.** Definition of similar triangles
d. $WT \cdot TY = ZT \cdot TX$	**d.** Cross products

23. 4 **25.** 11 **27.** 14.3 **29.** $113.\overline{3}$ cm

31. Given: tangent $\overline{RS}$ and secant $\overline{US}$
Prove: $(RS)^2 = US \cdot TS$



Proof:

Statements	Reasons
1. tangent $\overline{RS}$ and secant $\overline{US}$	**1.** Given
2. $m\angle RUT = \frac{1}{2}m\widehat{RT}$	**2.** The measure of an inscribed angle equals half the measure of its intercepted arc.
3. $m\angle SRT = \frac{1}{2}m\widehat{RT}$	**3.** The measure of an angle formed by a secant and a tangent equals half the measure of its intercepted arc.
4. $m\angle RUT = m\angle SRT$	**4.** Substitution

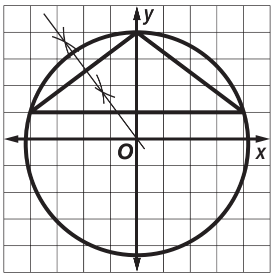
5. $\angle RUT \cong \angle SRT$	5. Definition of $\cong$ $\measuredangle$s
6. $\angle S \cong \angle S$	6. Reflexive Prop.
7. $\triangle SUR \sim \triangle SRT$	7. AA Similarity
8. $\frac{RS}{US} = \frac{TS}{RS}$	8. Definition of $\sim$ $\triangle$s
9. $(RS)^2 = US \cdot TS$	9. Cross products

33. Sample answer: The product of the parts of one intersecting chord equals the product of the parts of the other chord. Answers should include the following.
- $\overline{AF}, \overline{FD}, \overline{EF}, \overline{FB}$
- $AF \cdot FD = EF \cdot FB$

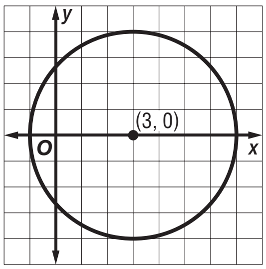
35. C **37.** 157.5 **39.** 7 **41.** 36 **43.** scalene, obtuse **45.** equilateral, acute or equiangular **47.** $\sqrt{13}$

Pages 575–580 Lesson 10-8

1. Sample answer:

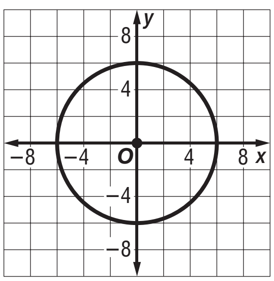


3. $(x + 3)^2 + (y - 5)^2 = 100$
5. $(x + 2)^2 + (y - 11)^2 = 32$
7.

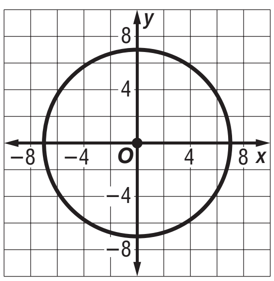


9. $x^2 + y^2 = 1600$ **11.** $(x + 2)^2 + (y + 8)^2 = 25$ **13.** $x^2 + y^2 = 36$ **15.** $x^2 + (y - 5)^2 = 100$ **17.** $(x + 3)^2 + (y + 10)^2 = 144$ **19.** $x^2 + y^2 = 8$ **21.** $(x + 2)^2 + (y - 1)^2 = 10$ **23.** $(x - 7)^2 + (y - 8)^2 = 25$

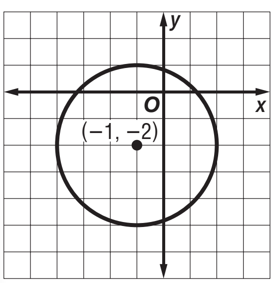
25.



27.



29.

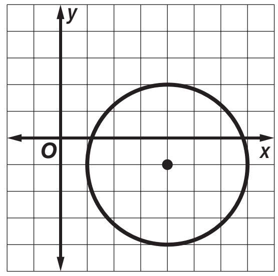


31. $(x + 3)^2 + y^2 = 9$ **33.** 2 **35.** $x^2 + y^2 = 49$ **37.** 13 **39.** $(2, -4)$; $r = 6$ **41.** See students' work **43a.** (0, 3) or (−3, 0) **43b.** none **43c.** (0, 0) **45.** B **47.** 24 **49.** 18 **51.** 59 **53.** 20 **55.** (3, 2), (−4, −1), (0, −4)

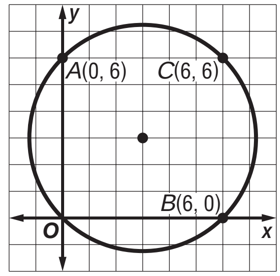
Pages 581–586 Chapter 10 Study Guide and Review

1. a **3.** h **5.** b **7.** d **9.** c **11.** 7.5 in.; 47.12 in. **13.** 10.82 yd; 21.65 yd **15.** 21.96 ft; 43.93 ft **17.** 60 **19.** 117 **21.** 30 **23.** 30 **25.** 150 **27.** $\frac{22}{5}\pi$ **29.** 10 **31.** 10 **33.** 45 **35.** 48 **37.** 32 **39.** $m\angle 1 = m\angle 3 = 30$, $m\angle 2 = 60$ **41.** 9 **43.** 18 **45.** 37 **47.** 17.1 **49.** 7.2 **51.** $(x + 4)^2 = (y - 8)^2 = 9$ **53.** $(x + 1)^2 + (y - 4)^2 = 4$

55.



57.



Chapter 11 Areas of Polygons and Circles

Page 593 Chapter 11 Getting Started

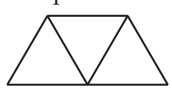
1. 10 **3.** 4.6 **5.** 18 **7.** 54 **9.** 13 **11.** 9 **13.** $6\sqrt{3}$ **15.** $\frac{15\sqrt{2}}{2}$

Pages 598–600 Lesson 11-1

1. The area of a rectangle is the product of the length and the width. The area of a parallelogram is the product of the base and the height. For both quadrilaterals, the measure of the length of one side is multiplied by the length of the altitude. **3.** 28 ft; 39.0 ft^2 **5.** 12.8 m; 10.2 m^2 **7.** rectangle, 170 units^2 **9.** 80 in.; 259.8 in^2 **11.** 21.6 cm; 29.2 cm^2 **13.** 44 m; 103.9 m^2 **15.** 45.7 mm^2 **17.** 108.5 m **19.** h = 40 units, b = 50 units **21.** parallelogram, 56 units^2 **23.** parallelogram, 64 units^2 **25.** square, 13 units^2 **27.** 150 units^2 **29.** Yes; the dimensions are 32 in. by 18 in. **31.** ≈ 13.9 ft **33.** The perimeter is 19 m, half of 38 m. The area is 20 m^2. **35.** 5 in., 7 in. **37.** C **39.** (5, 2), $r = 7$ **41.** $\left(-\frac{2}{3}, \frac{1}{9}\right)$, $r = \frac{2}{3}$ **43.** 32 **45.** 21 **47.** $F''(-4, 0)$, $G''(-2, -2)$, $H''(-2, 2)$; 90° counterclockwise **49.** 13 ft **51.** 16 **53.** 20

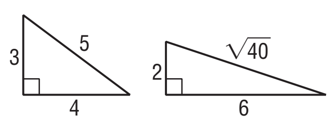
Pages 605–609 Lesson 11-2

1. Sample answer:



3. Sometimes; two rhombi can have different corresponding diagonal lengths and have the same area. **5.** 499.5 in^2

7. 21 units^2 **9.** 4 units^2 **11.** 45 m **13.** 12.4 cm^2 **15.** 95 km^2 **17.** 1200 ft^2 **19.** 50 m^2 **21.** 129.9 mm^2 **23.** 55 units^2 **25.** 22.5 units^2 **27.** 20 units^2 **29.** 16 units^2 **31.** ≈ 26.8 ft **33.** ≈ 22.6 m **35.** 20 cm **37.** about 8.7 ft **39.** 13,326 ft^2 **41.** 120 in^2 **43.** ≈ 10.8 in^2 **45.** 21 ft^2 **47.** False; sample answer: the area for each of these right triangles is 6 square units. The perimeter of one triangle is 12 and the perimeter of the other is $8 + \sqrt{40}$ or about 14.3.



49. area = 12, area = 3; perimeter = $8\sqrt{13}$, perimeter = $4\sqrt{13}$; scale factor and ratio of perimeters = $\frac{1}{2}$, ratio of areas = $\left(\frac{1}{2}\right)^2$ **51.** $\frac{2}{1}$ **53.** The ratio is the same. **55.** 4 : 1; The ratio of the areas is the square of the scale factor. **57.** 45 ft^2; The ratio of the areas is 5 : 9. **59.** B **61.** area = $\frac{1}{2}ab \sin C$ **63.** 6.02 cm^2 **65.** 374 cm^2

67. 231 ft^2 **69.** $(x + 4)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{121}{4}$ **71.** 275 in. **73.** $\langle 172.4, 220.6\rangle$ **75.** 20.1

Page 609 Practice Quiz 1

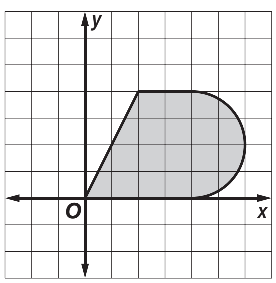
1. square **3.** 54 units^2 **5.** 42 yd

Pages 613–616 Lesson 11-3

1. Sample answer: Separate a hexagon inscribed in a circle into six congruent nonoverlapping isosceles triangles. The area of one triangle is one-half the product of one side of the hexagon and the apothem of the hexagon. The area of the hexagon is $6\left(\frac{1}{2}sa\right)$. The perimeter of the hexagon is $6s$, so the formula is $\frac{1}{2}Pa$. **3.** 127.3 yd^2 **5.** 10.6 cm^2 **7.** about 3.6 yd^2 **9.** 882 m^2 **11.** 1995.3 in^2 **13.** 482.8 km^2 **15.** 30.4 units^2 **17.** 26.6 units^2 **19.** 4.1 units^2 **21.** 271.2 units^2 **23.** 2 : 1 **25.** One 16-inch pizza; the area of the 16-inch pizza is greater than the area of two 8-inch pizzas, so you get more pizza for the same price. **27.** 83.1 units^2 **29.** 48.2 units^2 **31.** 227.0 units^2 **33.** 664.8 units^2 **35.** triangles; 629 tiles **37.** $\approx$ 380.1 in^2 **39.** 34.6 units^2 **41.** 157.1 units^2 **43.** 471.2 units^2 **45.** 54,677.8 ft^2; 899.8 ft **47.** $225\pi \approx 706.9$ ft^2 **49.** 2 : 3 **51.** The ratio is the same. **53.** The ratio of the areas is the square of the scale factor. **55.** 3 to 4 **57.** B **59.** 260 cm^2 **61.** $\approx$ 2829.0 yd^2 **63.** square; 36 units^2 **65.** rectangle; 30 units^2 **67.** 42 **69.** 6 **71.** $4\sqrt{2}$

Pages 619–621 Lesson 11-4

1. Sample answer: $\approx$ 18.3 units^2



3. 53.4 units^2 **5.** 24 units^2 **7.** $\approx$ 1247.4 in^2 **9.** 70.9 units^2 **11.** 4185 units^2 **13.** 154.1 units^2 **15.** $\approx$ 2236.9 in^2 **17.** 23.1 units^2 **19.** 21 units^2 **21.** 33 units^2 **23.** Sample answer: 57,500 mi^2 **25.** 462 **27.** Sample answer: Reduce the width of each rectangle.

29. Sample answer: Windsurfers use the area of the sail to catch the wind and stay afloat on the water. Answers should include the following.

- To find the area of the sail, separate it into shapes. Then find the area of each shape. The sum of areas is the area of the sail.
- Sample answer: Surfboards and sailboards are also irregular figures.

31. C **33.** 154.2 units^2 **35.** 156.3 ft^2 **37.** $\approx$ 384.0 m^2 **39.** 0.63 **41.** 0.19

Page 621 Practice Quiz 2

1. 679.0 mm^2 **3.** 1208.1 units^2 **5.** 44.5 units^2

Pages 625–627 Lesson 11-5

1. Multiply the measure of the central angle of the sector by the area of the circle and then divide the product by 360°. **3.** Rachel; Taimi did not multiply $\frac{62}{360}$ by the area of the circle. **5.** $\approx$ 114.2 units^2, $\approx$ 0.36 **7.** 0.60 **9.** 0.54 **11.** $\approx$ 58.9 units^2, $0.\overline{3}$ **13.** $\approx$ 19.6 units^2, $0.\overline{1}$ **15.** 74.6 units^2, 0.42 **17.** $\approx$ 3.3 units^2, $\approx$ 0.03 **19.** $\approx$ 25.8 units^2, $\approx$ 0.15 **21.** 0.68 **23.** 0.68 **25.** 0.19 **27.** $\approx$ 0.29 **29.** The chances of landing on a black or white sector are the same, so they should have the same point value. **31a.** No; each colored sector has a different central angle. **31b.** No; there is not an equal chance of landing on each color. **33.** C **35.** 1050 units^2 **37.** 110.9 ft^2 **39.** 221.7 in^2 **41.** 123 **43.** 165 **45.** $g = 21.5$

Pages 628–630 Chapter 11 Study Guide and Review

1. c **3.** a **5.** b **7.** 78 ft, $\approx$ 318.7 ft^2 **9.** square; 49 units^2 **11.** parallelogram; 20 units^2 **13.** 28 in. **15.** 688.2 in^2 **17.** 31.1 units^2 **19.** $0.\overline{3}$

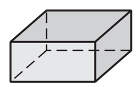
Chapter 12 Surface Area

Page 635 Chapter 12 Getting Started

1. true **3.** cannot be determined **5.** 384 ft^2 **7.** 1.8 m^2 **9.** 7.1 yd^2

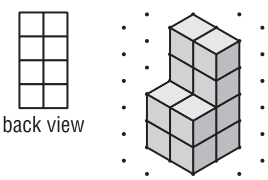
Pages 639–642 Lesson 12-1

1. The Platonic solids are the five regular polyhedra. All of the faces are congruent, regular polygons. In other polyhedra, the bases are congruent parallel polygons, but the faces are not necessarily congruent.
3. Sample answer:

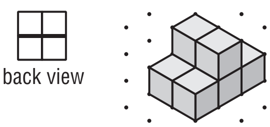


5. Hexagonal pyramid; base: $ABCDEF$; faces: $ABCDEF$, $\triangle AGF$, $\triangle FGE$, $\triangle EGD$, $\triangle DGC$, $\triangle CGB$, $\triangle BGA$; edges: $\overline{AF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DC}$, $\overline{CB}$, $\overline{BA}$, $\overline{AG}$, $\overline{FG}$, $\overline{EG}$, $\overline{DG}$, $\overline{CG}$, and $\overline{BG}$; vertices: A, B, C, D, E, F, and G **7.** cylinder; bases: circles P and Q

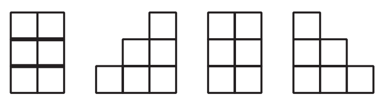
9.



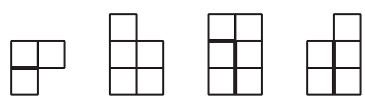
11.



13.

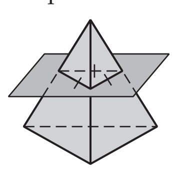


15.

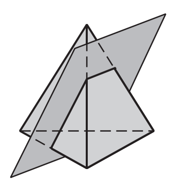


17. rectangular pyramid; base: $\square DEFG$; faces: $\square DEFG$, $\triangle DHG$, $\triangle GHF$, $\triangle FHE$, $\triangle DHE$; edges: $\overline{DG}$, $\overline{GF}$, $\overline{FE}$, $\overline{ED}$, $\overline{DH}$, $\overline{EH}$, $\overline{FH}$, and $\overline{GH}$; vertices: D, E, F, G, and H **19.** cylinder: bases: circles S and T **21.** cone; base: circle B; vertex A **23.** No, not enough information is provided by the top and front views to determine the shape. **25.** parabola **27.** circle **29.** rectangle

31. intersecting three faces and parallel to base;



33. intersecting all four faces, not parallel to any face;



35. cylinder **37.** rectangles, triangles, quadrilaterals

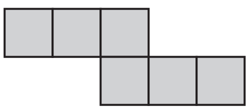
39a. triangular **39b.** cube, rectangular, or hexahedron **39c.** pentagonal **39d.** hexagonal **39e.** hexagonal **41.** No; the number of faces is not enough information to classify a polyhedron. A polyhedron with 6 faces could be a cube, rectangular prism, hexahedron, or a pentagonal pyramid. More information is needed to classify a polyhedron. **43.** Sample answer: Archaeologists use two dimensional drawings to learn more about the structure they are studying. Egyptologists can compare two-dimensional drawings to learn more about the structure they are studying. Egyptologists can compare two-dimensional drawings of the pyramids and note similarities and any differences. Answers should include the following.

- Viewpoint drawings and corner views are types of two-dimensional drawings that show three dimensions.
- To show three dimensions in a drawing, you need to know the views from the front, top, and each side.

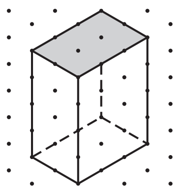
45. D **47.** infinite **49.** 0.242 **51.** 0.611 **53.** 21 units^2 **55.** 11 units^2 **57.** 90 ft, 433.0 ft^2 **59.** 300 cm^2 **61.** 4320 in^2

Pages 645–648 Lesson 12-2

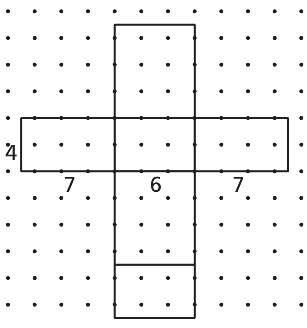
1. Sample answer:



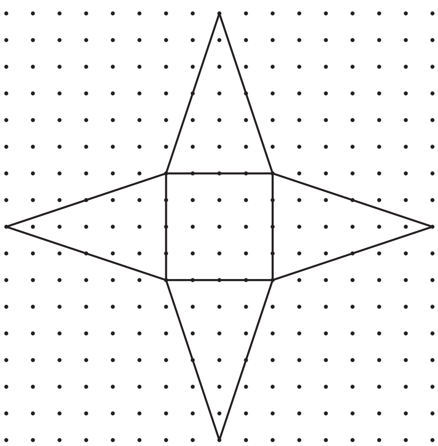
3.



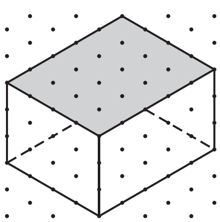
5. 188 in^2;



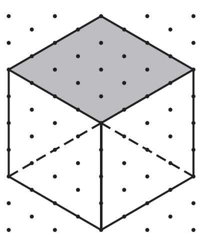
7. 64 cm^2;



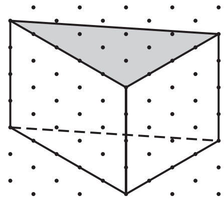
9.



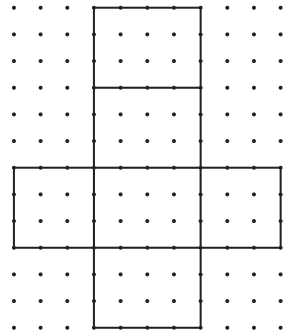
11.



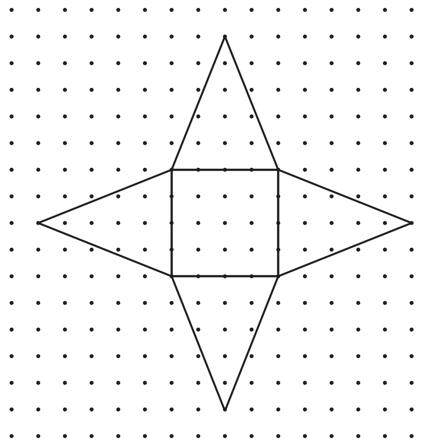
13.



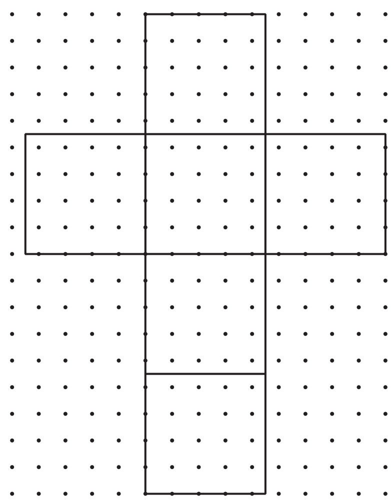
15. 66 units^2;



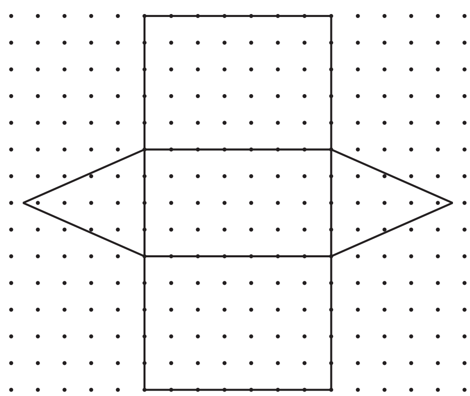
17. 56 units^2;



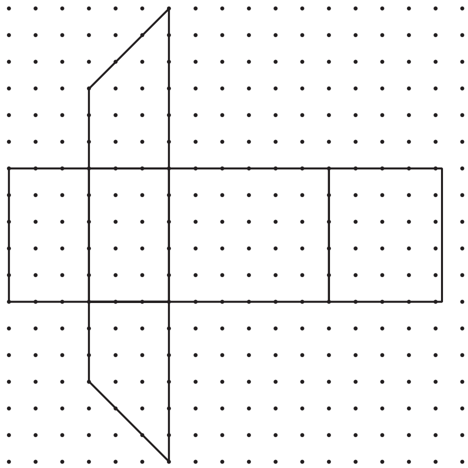
19. 121.5 units^2;



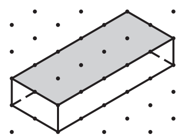
21. 116.3 units^2;



23. 108.2 units^2;



25.



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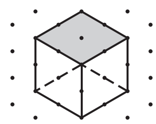

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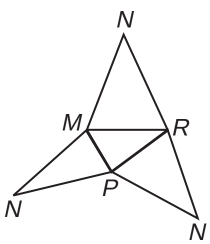
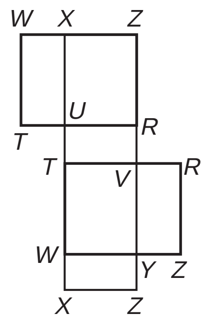
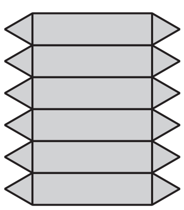


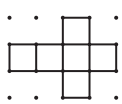
31.



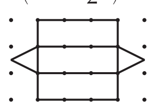
33.



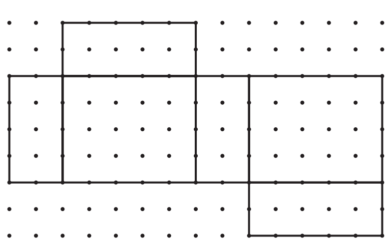
35. **A** 6 units^2;



B $\left(9 + \frac{\sqrt{3}}{2}\right) = 9.87$ units^2;



C 76 units^2;

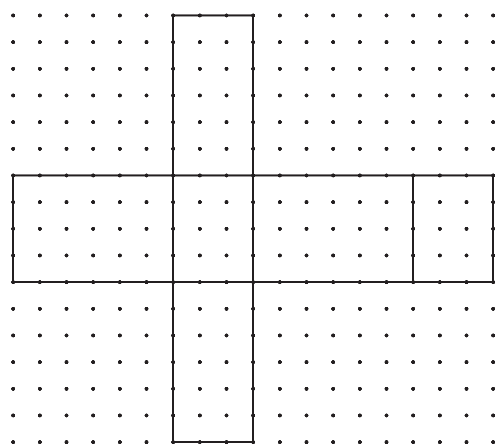


37. The surface area quadruples when the dimensions are doubled. For example, the surface area of the cube is $6(1^2)$ or 6 square units. When the dimensions are doubled the surface area is $6(2^2)$ or 24 square units. **39.** No; 5 and 3 are opposite faces; the sum is 8. **41.** C **43.** rectangle **45.** rectangle **47.** 90 **49.** 120 **51.** 63 cm^2 **53.** 110 cm^2

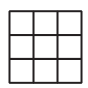
Pages 651–654 Lesson 12-3

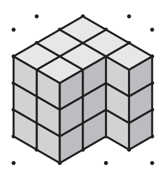
1. In a right prism a lateral edge is also an altitude. In an oblique prism, the lateral edges are not perpendicular to the bases. **3.** 840 units^2, 960 units^2 **5.** 1140 ft^2 **7.** 128 units^2 **9.** 162 units^2 **11.** 160 units^2 (square base), 126 units^2 (rectangular base) **13.** 16 cm **15.** The perimeter of the base must be 24 meters. There are six rectangles with integer values for the dimensions that have a perimeter of 24. The dimensions of the base could be 1×11, 2×10, 3×9, 4×8, 5×7, or 6×6. **17.** 114 units^2 **19.** 522 units^2 **21.** 454.0 units^2 **23.** 3 gallons for 2 coats **25.** 44,550 ft^2 **27.** The actual amount needed will be higher because the area of the curved architectural element appears to be greater than the area of the doors. **29.** base of A $\cong$ base of C; base of A $\sim$ base of B; base of C $\sim$ base of B **31.** A : B = 1 : 4, B : C = 4 : 1, A : C = 1 : 1 **33.** A : B, because the heights of A and B are in the same ratio as perimeters of bases **35.** No, the surface area of the finished product will be the sum of the lateral areas of each prism plus the area of the bases of the TV and DVD prisms. It will also include the area of the overhang between each prism, but not the area of the overlapping prisms. **37.** 198 cm^2 **39.** B **41.** $L = 1416$ cm^2, $T = 2056$ cm^2 **43.** See students' work.

45. 108 units^2;



47.


back view


corner view

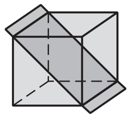
49. 43
51. 35 **53.** $\frac{1}{72}$
55. 1963.50 in^2
57. 21,124.07 mm^2

Pages 657–659 Lesson 12-4

1. Multiply the circumference of the base by the height and add the area of each base. **3.** Jamie; since the cylinder has one base removed, the surface area will be the sum of the lateral area and one base. **5.** 1520.5 m^2 **7.** 5 ft **9.** 2352.4 m^2 **11.** 517.5 in^2 **13.** 251.3 ft^2 **15.** 30.0 cm^2 **17.** 3 cm **19.** 8 m **21.** The lateral areas will be in the ratio 3 : 2 : 1; 45π in^2, 30π in^2, 15π in^2. **23.** The lateral area is tripled. The surface area is increased, but not tripled. **25.** 1.25 m **27.** Sample answer: Extreme sports participants use a semicylinder for a ramp. Answers should include the following.

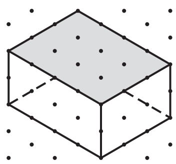
- To find the lateral area of a semicylinder like the half-pipe, multiply the height by the circumference of the base and then divide by 2.
- A half-pipe ramp is half of a cylinder if the ramp is an equal distance from the axis of the cylinder.

29. C **31.** a plane perpendicular to the line containing the opposite vertices of the face of the cube



33. 300 units^2

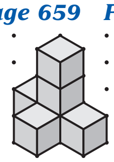
35.



37. 27 **39.** 8
41. $m\angle A = 64$, $b \approx 12.2$, $c \approx 15.6$
43. 54 cm^2

Page 659 Practice Quiz 1

1.

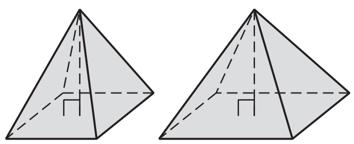


corner view

3. 231.5 m^2 **5.** 5.4 ft

Pages 663–665 Lesson 12-5

1. Sample answer:



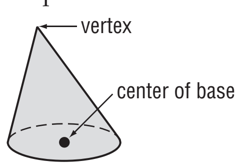
square base (regular) rectangular base (not regular)

3. 74.2 ft^2
5. 340 cm^2
7. 119 cm^2
9. 147.7 ft^2
11. 173.2 yd^2
13. 326.9 in^2

15. 27.7 ft^2 **17.** ≈ 2.3 inches on each side **19.** ≈ 615,335.3 ft^2 **21.** 20 ft **23.** 960 ft^2 **25.** The surface area of the original cube is 6 square inches. The surface area of the truncated cube is approximately 5.37 square inches. Truncating the corner of the cube reduces the surface area by about 0.63 square inch. **27.** D **29.** 967.6 m^2 **31.** 1809.6 yd^2 **33.** 74 ft, 285.8 ft^2 **35.** 98 m, 366 m^2 **37.** $\overline{GF}$ **39.** $\overline{JM}$ **41.** True; each pair of opposite sides are congruent. **43.** 21.3 m

Pages 668–670 Lesson 12-6

1. Sample answer:



3. 848.2 cm^2 **5.** 485.4 in^2
7. 282.7 cm^2 **9.** 614.3 in^2
11. 628.8 m^2 **13.** 679.9 in^2
15. 7.9 m **17.** 5.6 ft
19. 475.2 in^2 **21.** 1509.8 m^2
23. 1613.7 in^2 **25.** ≈ 12 ft
27. 8.1 in.; 101.7876 in^2

29. Using the store feature on the calculator is the most accurate technique to find the lateral area. Rounding the slant height to either the tenths place or hundredths place changes the value of the slant height, which affects the final computation of the lateral area. **31.** Sometimes; only when the heights are in the same ratio as the radii of the bases. **33.** Sample answer: Tepees are conical shaped structures. Lateral area is used because the ground may not always be covered in circular canvas. Answers should include the following.

- We need to know the circumference of the base or the radius of the base and the slant height of the cone.
- The open top reduces the lateral area of canvas needed to cover the sides. To find the actual lateral area, subtract the lateral area of the conical opening from the lateral area of the structure.

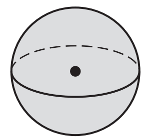
35. D **37.** 5.8 ft **39.** 6.0 yd **41.** 48 **43.** 24 **45.** 45 **47.** 21 **49.** $8\sqrt{11} \approx 26.5$ **51.** 25.1 **53.** 51.5 **55.** 25.8

Page 670 Practice Quiz 2

1. 423.9 cm^2 **3.** 144.9 ft^2 **5.** 3.9 in.

Pages 674–676 Lesson 12-7

1. Sample answer:



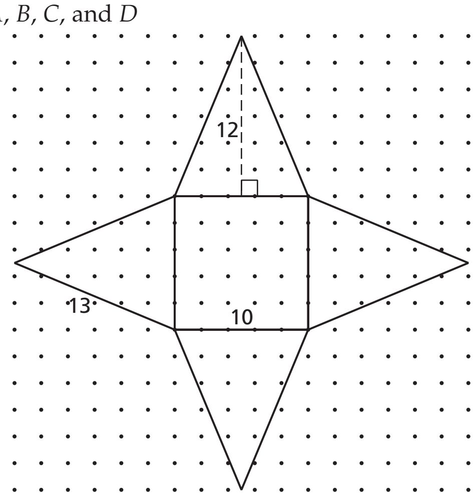
3. 15 **5.** 18 **7.** 150.8 cm^2 **9.** ≈ 283.5 in^2
11. ≈ 8.5 **13.** 8 **15.** 12.8 **17.** 7854.0 in^2
19. 636,172.5 m^2 **21.** 397.4 in^2
23. 3257.2 m^2 **25.** true **27.** true
29. true **31.** ≈ 206,788,161.4 mi^2
33. 398.2 ft^2

35. $\frac{\sqrt{2}}{2} : 1$ **37.** The surface area can range from about 452.4 to about 1256.6 mi^2. **39.** The radius of the sphere is half the side of the cube. **41.** None; every line (great circle) that passes through X will also intersect g. All great circles intersect. **43.** A **45.** 1430.3 in^2 **47.** 254.7 cm^2 **49.** 969 yd^2 **51.** 649 cm^2 **53.** $(x + 2)^2 + (y - 7)^2 = 50$

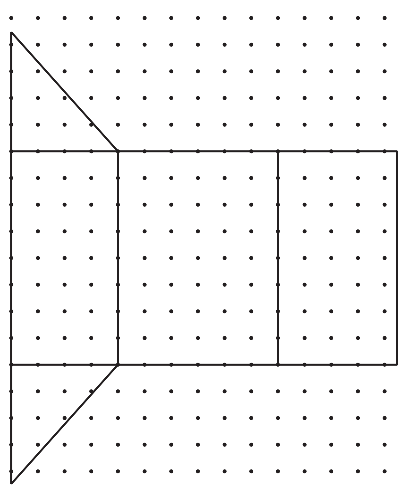
Pages 678–682 Chapter 12 Study Guide and Review

1. d **3.** b **5.** a **7.** e **9.** c **11.** cylinder; bases: $\odot F$ and $\odot G$ **13.** triangular prism; base: $\triangle BCD$; faces: $\triangle ABC$, $\triangle ABD$, $\triangle ACD$, and $\triangle BCD$; edges: $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{CD}$; vertices: A, B, C, and D

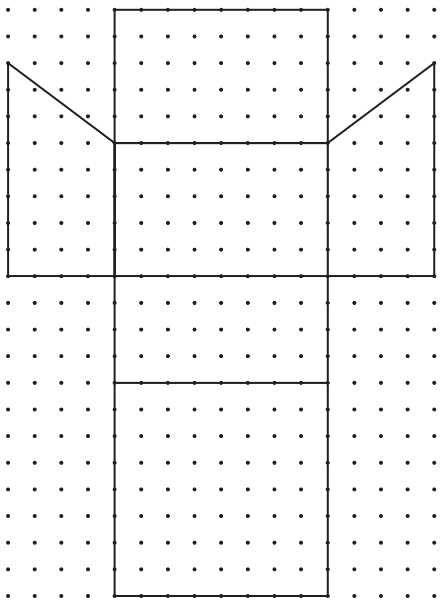
15. 340 units^2;



17. ≈ 133.7 units^2;



19. 228 units^2;



21. 72 units^2 **23.** 175.9 in^2 **25.** 1558.2 mm^2 **27.** 304 units^2 **29.** 33.3 units^2 **31.** 75.4 yd^2 **33.** 1040.6 ft^2 **35.** 363 mm^2 **37.** 2412.7 ft^2 **39.** 880 ft^2

Chapter 13 Volume

Page 687 Chapter 13 Getting Started

1. ± 5 **3.** ± 3 **5.** $\pm\sqrt{305}$ **7.** 134.7 cm^2 **9.** 867.0 mm^2 **11.** $25b^2$ **13.** $\frac{9x^2}{16y^2}$ **15.** $W(-2.5, 1.5)$ **17.** $B(19, 21)$

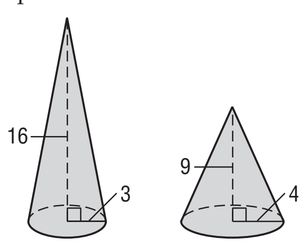
Pages 691–694 Lesson 13-1

1. Sample answers: cans, roll of paper towels, and chalk; boxes, crystals, and buildings **3.** 288 cm^3 **5.** 3180.9 mm^3 **7.** 763.4 cm^3 **9.** 267.0 cm^3 **11.** 750 in^3 **13.** 28 ft^3 **15.** 15,108.0 mm^3 **17.** $\approx$ 14 m **19.** 24 units^3 **21.** 48.5 mm^3 **23.** 173.6 ft^3 **25.** $\approx$ 304.1 cm^3 **27.** about 19.2 ft **29.** $\approx$ 104,411.5 mm^3 **31.** $\approx$ 137.6 ft^3 **33.** A **35.** 452.4 ft^2 **37.** 1017.9 m^2 **39.** 320.4 m^2 **41.** 282.7 in^2 **43.** $\approx$ 0.42 **45.** 186 m^2 **47.** 8.8 **49.** 21.22 in^2 **51.** 61.94 m^2

Pages 698–701 Lesson 13-2

1. Each volume is 8 times as large as the original.

3. Sample answer:



$$V = \frac{1}{3}\pi(3^2)(16) = 48\pi$$
$$V = \frac{1}{3}\pi(4^2)(9) = 48\pi$$

5. 603.2 mm^3 **7.** 975,333.3 ft^3 **9.** 1561.2 ft^3 **11.** 8143.0 mm^3 **13.** 2567.8 m^3 **15.** 188.5 cm^3 **17.** 1982.0 mm^3 **19.** 7640.4 cm^3 **21.** $\approx$ 2247.5 km^3 **23.** $\approx$ 158.8 km^3 **25.** $\approx$ 91,394,008.3 ft^3 **27.** $\approx$ 6,080,266.7 ft^3 **29.** $\approx$ 522.3 units^3 **31.** $\approx$ 203.6 in^3 **33.** B **35.** 1008 in^3 **37.** 1140 ft^3 **39.** 258 yd^2 **41.** 145.27 **43.** 1809.56

Page 701 Practice Quiz 1

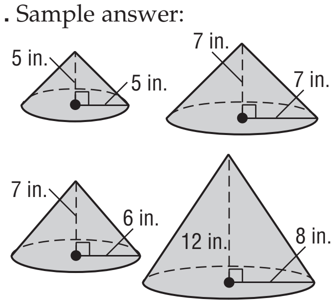
1. 125.7 in^3 **3.** 935.3 cm^3 **5.** 42.3 in^3

Pages 704–706 Lesson 13-3

1. The volume of a sphere was generated by adding the volumes of an infinite number of small pyramids. Each pyramid has its base on the surface of the sphere and its height from the base to the center of the sphere. **3.** 9202.8 in^3 **5.** 268.1 in^3 **7.** 155.2 m^3 **9.** 1853.3 m^3 **11.** 3261.8 ft^3 **13.** 233.4 in^3 **15.** 68.6 m^3 **17.** 7238.2 in^3 **19.** $\approx$ 21,990,642,871 km^3 **21.** No, the volume of the cone is 41.9 cm^3; the volume of the ice cream is about 33.5 cm^3. **23.** $\approx$ 20,579.5 mm^3 **25.** $\approx$ 1162.1 mm^2 **27.** $\frac{2}{3}$ **29.** $\approx$ 587.7 in^3 **31.** 32.7 m^3 **33.** about 184 mm^3 **35.** See students' work. **37.** A **39.** 412.3 m^3 **41.** $(x-2)^2 + (y+1)^2 = 64$ **43.** $(x-2)^2 + (y-1)^2 = 34$ **45.** $27x^3$ **47.** $\frac{8k^3}{125}$

Pages 710–713 Lesson 13-4

1. Sample answer:



3. congruent **5.** $\frac{4}{3}$ **7.** $\frac{64}{27}$ **9.** 1 : 64 **11.** neither **13.** congruent **15.** neither **17.** 130 m high, 245 m wide, and 465 m long **19.** Always; congruent solids have equal dimensions. **21.** Never; different types of solids cannot be similar. **23.** Sometimes; solids that are not similar can have the same surface area. **25.** 1,000,000x cm^2 **27.** $\frac{2}{5}$ **29.** $\frac{8}{125}$ **31.** 18 cm **33.** $\frac{29}{30}$ **35.** $\frac{24{,}389}{27{,}000}$ **37.** $\approx$ 0.004 in^3 **39.** 3 : 4; 3 : 1 **41.** The volume of the cone on the right is equal to the sum of the volumes of the cones inside the cylinder. Justification: Call h the height of both solids. The volume of the cone on the right is $\frac{1}{3}\pi r^2 h$. If the height of one cone inside the cylinder is c, then the height of the other one is $h - c$. Therefore, the sum of the volumes of the two cones is: $\frac{1}{3}\pi r^2 c + \frac{1}{3}\pi r^2(h - c)$ or $\frac{1}{3}\pi r^2(c + h - c)$ or $\frac{1}{3}\pi r^2 h$. **43.** C **45.** 268.1 ft^3 **47.** 14,421.8 cm^3 **49.** 323.3 in^3 **51.** 2741.8 ft^3 **53.** 2.8 yd **55.** 36 ft^2 **57.** yes **59.** no

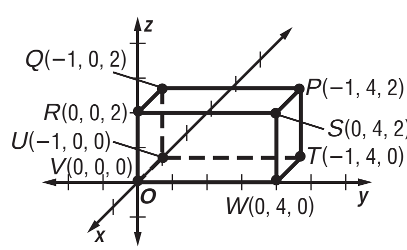
Page 713 Practice Quiz 2

1. 67,834.4 ft^3 **3.** $\frac{7}{5}$ **5.** $\frac{343}{125}$

Pages 717–719 Lesson 13-5

1. The coordinate plane has 4 regions or quadrants with 4 possible combinations of signs for the ordered pairs. Three-dimensional space is the intersection of 3 planes that create 8 regions with 8 possible combinations of signs for the ordered triples. **3.** A dilation of a rectangular prism will provide a similar figure, but not a congruent one unless $r = 1$ or $r = -1$.

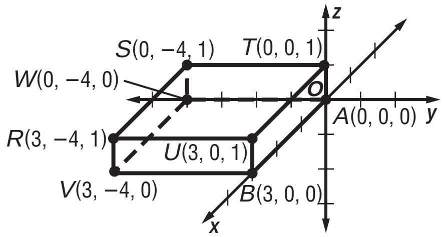
5.



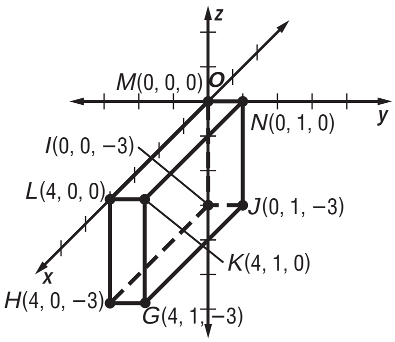
Selected Answers

7. $\sqrt{186}$; $\left(1, -\frac{7}{2}, \frac{1}{2}\right)$ **9.** (12, 8, 8), (12, 0, 8), (0, 0, 8), (0, 8, 8), (12, 8, 0), (12, 0, 0), (0, 0, 0), and (0, 8, 0); (−36, 8, 24), (−36, 0, 24), (−48, 0, 24), (−48, 8, 24) (−36, 8, 16), (−36, 0, 16), (−48, 0, 16), and (−48, 8, 16)

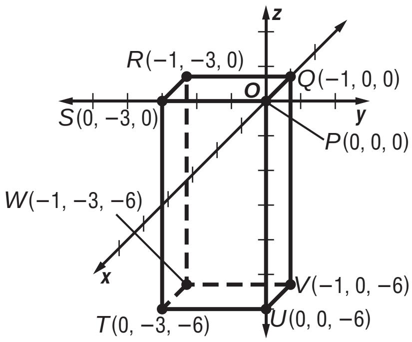
11.



13.

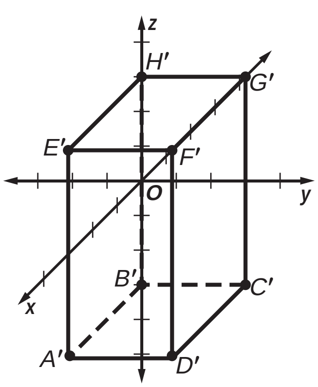


15.



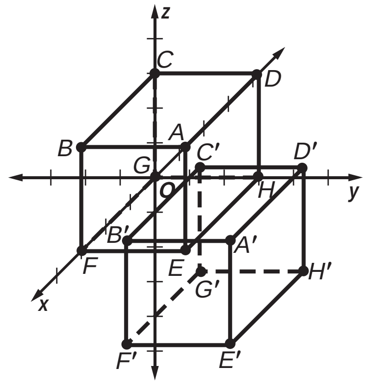
17. $PQ = \sqrt{115}$; $\left(\frac{1}{2}, -\frac{7}{2}, \frac{7}{2}\right)$ **19.** $GH = \sqrt{17}$; $\left(\frac{3}{5}, -\frac{7}{10}, 4\right)$
21. $BC = \sqrt{39}$; $\left(-\frac{\sqrt{3}}{2}, 3, 3\sqrt{2}\right)$

23.

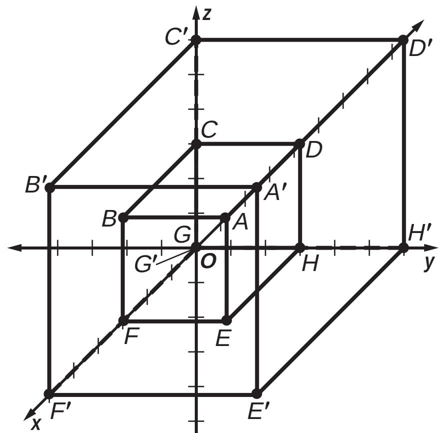


25. $P'(0, 2, -2)$, $Q'(0, 5, -2)$, $R'(2, 5, -2)$, $S'(2, 2, -2)$, $T'(0, 5, -5)$, $U'(0, 2, -5)$, $V'(2, 2, -5)$, and $W'(2, 5, -5)$

27. $A'(4, 5, 1)$, $B'(4, 2, 1)$, $C'(1, 2, 1)$, $D'(1, 5, 1)$ $E'(4, 5, -2)$, $F'(4, 2, -2)$, $G'(1, 2, -2)$, and $H'(1, 5, -2)$;



29. $A'(6, 6, 6)$, $B'(6, 0, 6)$, $C'(0, 0, 6)$, $D'(0, 6, 6)$, $E'(6, 6, 0)$, $F'(6, 0, 0)$, $G'(0, 0, 0)$, and $H'(0, 6, 0)$; $V = 216$ units^3;



31. 8.2 mi **33.** (0, −14, 14) **35.** $(x, y, z) \rightarrow (x + 2, y + 3, z - 5)$ **37.** Sample answer: Three-dimensional graphing is used in computer animation to render images and allow them to move realistically. Answers should include the following.

- Ordered triples are a method of locating and naming points in space. An ordered triple is unique to one point.
- Applying transformations to points in space would allow an animator to create realistic movement in animation.

39. B **41.** The locus of points in space with coordinates that satisfy the equation of $x + z = 4$ is a plane perpendicular to the xz-plane whose intersection with the xz-plane is the graph of $z = -x + 4$ in the xz-plane.
43. similar **45.** 1150.3 yd^3 **47.** 12,770.1 ft^3

Pages 720–722 Chapter 13 Study Guide and Review

1. pyramid **3.** an ordered triple **5.** similar **7.** the Distance Formula in Space **9.** Cavalieri's Principle
11. 504 in^3 **13.** 749.5 ft^3 **15.** 1466.4 ft^3 **17.** 33.5 ft^3
19. 4637.6 mm^3 **21.** 523.6 units^3 **23.** similar **25.** $CD = \sqrt{58}$; (−9, 5.5, 5.5) **27.** $FG = \sqrt{422}$; $\left(1.5\sqrt{2}, 3\sqrt{7}, -3\right)$

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