Introduction
In this unit students learn to calculate measures in two and three dimensions: area, surface area, and volume. They find the areas of triangles and several types of quadrilaterals in addition to regular polygons, circles, and irregular figures. Students make models of three-dimensional figures and find surface area. Geometric probability is also explored.

Three-dimensional figures are investigated further as students learn to find volume. They also identify congruent or similar solids and graph solids in space.

About the Photographs The large photograph is of the Korean War Veterans Memorial located in Washington, D.C., across the reflecting pool from the Vietnam Veterans Memorial. The smaller photo is a sketch of the National World War II Memorial also being constructed in Washington, D.C., scheduled to open in spring 2004.

Assessment Options

Unit 4 Test
Pages 773–774 of the Chapter 13 Resource Masters may be used as a test or review for Unit 4. This assessment contains both multiple-choice and short answer items.

ExamView® Pro
This CD-ROM can be used to create additional unit tests and review worksheets.

Area and Volume
Area and volume can be used to analyze real-world situations. In this unit, you will learn about formulas used to find the areas of two-dimensional figures and the surface areas and volumes of three-dimensional figures.

Chapter 11
Areas of Polygons and Circles
Chapter 12
Surface Area
Chapter 13
Volume

An online, research-based, instructional, assessment, and intervention tool that provides specific feedback on student mastery of state and national standards, instant remediation, and a data management system to track performance. For more information, contact mhdigitallearning.com.

Real-Life Geometry Videos
What’s Math Got to Do With It? Real-Life Geometry Videos engage students, showing them how math is used in everyday situations. Use Video 4 with this unit.
“BEDFORD, Va. For years, World War II was a sore subject that many families in this small farming community avoided. ‘We lost so many men,’ said Boyd Wilson, 79, who joined Virginia’s 116th National Guard before it was sent to war. ‘It was just painful.’ The war hit Bedford harder than perhaps any other small town in America, taking 19 of its sons, fathers and brothers in the opening moments of the Allied invasion of Normandy. Within a week, 23 of Bedford’s 35 soldiers were dead. It was the highest per capita loss for any U.S. community.” In this project, you will use scale drawings, surface area, and volume to design a memorial to honor war veterans.

Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Continue working on your WebQuest as you study Unit 4.

Have students study the USA TODAY Snapshot.
• Ask them which two wars cost about the same. **World War I and the Vietnam War**
• Discuss with students why World War II cost so much more than the other U.S. conflicts.
• Point out to students that there is often a contest to choose a design for a memorial. The choice of Maya Ying Lin’s design for the Vietnam Veterans Memorial Wall surprised many people because the design was so simple. Today it is one of the most admired and visited memorials in the nation.

**Additional USA TODAY Snapshots** appearing in Unit 4:
**Chapter 11** Who’s licensed to fly (p. 614)
**Chapter 12** College kids plug in for fun (p. 653)
**Chapter 13** Every day is mom’s day (p. 705)
# Areas of Polygons and Circles

## Chapter Overview and Pacing

### LESSON OBJECTIVES

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<th>Areas of Parallelograms (pp. 595–600)</th>
<th>Areas of Triangles, Trapezoids, and Rhombi (pp. 601–609)</th>
<th>Areas of Regular Polygons and Circles (pp. 610–616)</th>
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<tr>
<td>11-1</td>
<td>• Find perimeters and areas of parallelograms.</td>
<td>• Find areas of triangles.</td>
<td>• Find areas of regular polygons.</td>
<td>• Find areas of irregular figures.</td>
<td>• Solve problems involving geometric probability.</td>
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<tr>
<td>11-2</td>
<td>• Determine whether points on a coordinate plane define a parallelogram.</td>
<td>• Find areas of trapezoids and rhombi.</td>
<td>• Find areas of circles.</td>
<td>• Find areas of irregular figures on the coordinate plane.</td>
<td>• Solve problems involving sectors and segments of circles.</td>
</tr>
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<td>Study Guide and Practice Test (pp. 628–631)</td>
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### TOTAL

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Year-long pacing: pages T20–T21.

An electronic version of this chapter is available on StudentWorks™. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.
### Chapter Resource Manager

#### CHAPTER 11 RESOURCE MASTERS

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| 641–654, 658–660 |                     |                             |                      |            |            |            |                      |                             |

#### Materials

- **Straightedge, grid paper**
- **Calculator, grid paper, straightedge**
- **Grid paper, straightedge**
- **Graphing Calculator and Computer Masters**
- **School-to-Career Masters**

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*Key to Abbreviations: GCC = Graphing Calculator and Computer Masters  
SC = School-to-Career Masters*
Areas of Parallelograms

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, there is a corresponding altitude that is perpendicular to the base. The altitude corresponds to the height of the parallelogram. If a parallelogram has an area of $A$ square units, a base of $b$ units, and a height of $h$ units, then $A = bh$.

To find the area of a quadrilateral on the coordinate plane, you must first determine whether the figure is a parallelogram. You can use the formula for slope to determine whether opposite sides are parallel. Then you find the measures of the base and height and use those to calculate the area.

Areas of Triangles, Trapezoids, and Rhombi

The formula for the area of a triangle is related to the formula for the area of a parallelogram or rectangle. It is $A = \frac{1}{2}bh$. This formula, in turn, yields the formulas for the areas of trapezoids and rhombi.

If a trapezoid has an area of $A$ square units, bases of $b_1$ units and $b_2$ units, and a height of $h$ units, then $A = \frac{1}{2}h(b_1 + b_2)$. If a rhombus has an area of $A$ square units and diagonals of $d_1$ and $d_2$ units, then $A = \frac{1}{2}d_1d_2$.

Areas of Regular Polygons and Circles

A regular polygon can be divided into congruent isosceles triangles by drawing a line from each vertex to the center of the polygon. The altitude of one of these triangles is called an apothem. The area of the polygon can be determined by adding the areas of the triangles. If a polygon has a side of $s$ units and an apothem of $a$ units, then the area of one of these triangles is $\frac{1}{2}sa$. By multiplying this formula by the number of sides and substituting $P$ for the formula for perimeter contained in the result, you will find that the formula for the area of a regular polygon is $A = \frac{1}{2}Pa$.

The area of a circle cannot be found without the value known as $\pi$. If a circle has an area of $A$ square units and a radius of $r$ units, then $A = \pi r^2$. You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.
Areas of Irregular Figures

An irregular figure is a figure that cannot be classified into the specific shapes that the student has studied. To find the areas of irregular figures, separate the figures into shapes for which you can find the area. The area of the irregular figure is the sum of the areas of these separate shapes.

The formula for the area of a regular polygon does not apply to an irregular polygon. To find the area of an irregular polygon, separate the polygon into figures which have areas that can be calculated easily.

Geometric Probability

Probability that involves a geometric measure such as length or area is called geometric probability. You can find the probability that a point lies in part of a figure by comparing the area of the part to the area of the whole figure. If a point in region $A$ is chosen at random, then the probability $P(B)$ that the point is in region $B$, which is in the interior of region $A$, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}.$$  

When determining the geometric probability with targets, assume that the object lands within the target area. You should also assume that it is equally likely that the object will land anywhere in the region.

Sometimes you need to know the area of a sector of a circle to find a geometric probability. A sector is a region of a circle bounded by a central angle and its intercepted arc. If a sector of a circle has an area of $A$ square units, a central angle measuring $N^\circ$, and a radius of $r$ units, then

$$A = \frac{N}{360} \pi r^2.$$  

The region of a circle bounded by an arc and a chord is called a segment of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.
Chapter 11

Areas of Polygons and Circles

Key to Abbreviations:
TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

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<th>Technology/Internet</th>
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<td>Cumulative Review, CRM p. 658</td>
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ExamView® Pro

Use the networkable ExamView® Pro to:
• Create multiple versions of tests.
• Create modified tests for Inclusion students.
• Edit existing questions and add your own questions.
• Use built-in state curriculum correlations to create tests aligned with state standards.
• Apply art to your test from a program bank of artwork.

For more information on Yearly ProgressPro, see p. 590.

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<td>11-4</td>
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</tr>
<tr>
<td>11-5</td>
<td>Geometric Probability</td>
</tr>
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For more information on Intervention and Assessment, see pp. T8–T11.
Reading and Writing in Mathematics

Glencoe Geometry provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

**Student Edition**
- Foldables Study Organizer, p. 593
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 598, 605, 613, 619, 625)
- Reading Mathematics, p. 594
- Writing in Math questions in every lesson, pp. 600, 608, 616, 620, 627
- Reading Study Tip, p. 617
- WebQuest, p. 618

**Teacher Wraparound Edition**
- Foldables Study Organizer, pp. 592, 628
- Study Notebook suggestions, pp. 594, 598, 605, 613, 619, 624
- Modeling activities, pp. 616, 627
- Speaking activities, p. 600
- Writing activities, pp. 609, 621
- ELL Resources, pp. 592, 594, 599, 606, 614, 620, 626, 628

**Additional Resources**
- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 11 Resource Masters*, pp. vii–viii)
- Proof Builder helps students learn and understand theorems and postulates from the chapter. (*Chapter 11 Resource Masters*, pp. ix–x)
- Reading to Learn Mathematics master for each lesson (*Chapter 11 Resource Masters*, pp. 615, 621, 627, 633, 639)
- Vocabulary PuzzleMaker software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading Strategies for the Mathematics Classroom*
- *WebQuest and Project Resources*

*For more information on Reading and Writing in Mathematics, see pp. T6–T7.*

**Study Skill**

Study cards can be a helpful study aid for students learning definitions and formulas. The cards at the right show some common shapes and the formulas to find the areas of each. A name and sketch is on the front of the card and the area formula is on the back of the card.

As students work through the chapter, have them make study cards for other concepts and topics. They can use the cards to quiz themselves or each other to help prepare for tests.
Have students read over the list of objectives and make a list of any words with which they are not familiar.

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

**Key Vocabulary**
- apothem (p. 610)
- irregular figure (p. 617)
- geometric probability (p. 622)
- sector (p. 623)
- segment (p. 624)

Skydivers use geometric probability when they attempt to land on a target marked on the ground. They can determine the chances of landing in the center of the target. You will learn about skydiving in Lesson 11-5.

**Vocabulary Builder**
The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the Chapter 11 Resource Masters. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 11 test.
This section provides a review of the basic concepts needed before beginning Chapter 11. Page references are included for additional student help. Additional review is provided in the Prerequisite Skills Workbook, pages 27–28, 43–44, 107–108.

**Prerequisite Skills** in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

### For Lesson 11-1
**Area of a Rectangle**
The area and width of a rectangle are given. Find the length of the rectangle.

1. \(A = 150, w = 15\)
2. \(A = 83, w = 19\)
3. \(A = 21.16, w = 4.6\)
4. \(A = 1000, w = 32\)
5. \(A = 38, w = 19\)
6. \(A = 450, w = 25\)

### For Lessons 11-2 and 11-4
**Evaluate a Given Expression**
Evaluate each expression if \(a = 6, b = 8, c = 10,\) and \(d = 11\).

7. \(\frac{1}{2}(a + b)\)
8. \(\frac{1}{2}ab\)
9. \(\frac{1}{2}(2b + c)\)
10. \(\frac{1}{2}(a + c)\)
11. \(\frac{1}{2}(b + c)\)
12. \(\frac{1}{2}cd\)

### For Lesson 11-3
**Height of a Triangle**
Find \(h\) in each triangle.

13. \(h = 15\)
14. \(h = 15\)
15. \(h = 15\)

---

**Foldables Study Organizer**

**Areas of Polygons and Circles** Make this Foldable to help you organize your notes. Begin with five sheets of notebook paper.

**Step 1** Stack
Stack 4 of the 5 sheets of notebook paper as illustrated.

**Step 2** Cut
Cut in about 1 inch along the heading line on the top sheet of paper.

**Step 3** Cut
Cut the margins off along the right edge.

**Step 4** Stack
Stack in order of cuts, placing the uncut fifth sheet at the back. Label the tabs as shown.

**Reading and Writing** As you read and study the chapter, take notes and record examples of areas of polygons and circles.

**Summarizing** Use this Foldable for student writing about polygons and area. After students make their Foldable, have them label the side tabs to correspond to the five lessons in this chapter. Students use their Foldable to take notes, define terms, record concepts, solve problems, and explain how to find areas. At the end of each lesson, ask students to write a summary of the lesson, or write in their own words what the lesson was about. Summaries are useful for condensing data.

---

**For Prerequisite Lesson Skill**

- 11-2 Evaluating expressions, p. 600
- 11-3 Trigonometric ratios in right triangles, p. 609
- 11-4 Special right triangles, p. 616
Many of the words used in mathematics use the same prefixes as other everyday words. Understanding the meaning of the prefixes can help you understand the terminology better.

### Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Meaning</th>
<th>Everyday Words</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>bi-</td>
<td>2</td>
<td>bicycle</td>
<td>a 2-wheeled vehicle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bipartisan</td>
<td>involving members of 2 political parties</td>
</tr>
<tr>
<td>tri-</td>
<td>3</td>
<td>triangle</td>
<td>closed figure with 3 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>tricycle</td>
<td>a 3-wheeled vehicle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>triplet</td>
<td>one of 3 children born at the same time</td>
</tr>
<tr>
<td>quad-</td>
<td>4</td>
<td>quadrilateral</td>
<td>closed figure with 4 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>quadriceps</td>
<td>muscles with 4 parts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>quadruple</td>
<td>four times as many</td>
</tr>
<tr>
<td>penta-</td>
<td>5</td>
<td>pentagon</td>
<td>closed figure with 5 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>pentathlon</td>
<td>athletic contest with 5 events</td>
</tr>
<tr>
<td>hexa-</td>
<td>6</td>
<td>hexagon</td>
<td>closed figure with 6 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>heptagon</td>
<td>closed figure with 7 sides</td>
</tr>
<tr>
<td>oct-</td>
<td>8</td>
<td>octagon</td>
<td>closed figure with 8 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>octopus</td>
<td>animal with 8 legs</td>
</tr>
<tr>
<td>dec-</td>
<td>10</td>
<td>decagon</td>
<td>closed figure with 10 sides</td>
</tr>
<tr>
<td></td>
<td></td>
<td>decade</td>
<td>a period of 10 years</td>
</tr>
<tr>
<td></td>
<td></td>
<td>decathlon</td>
<td>athletic contest with 10 events</td>
</tr>
</tbody>
</table>

Several pairs of words in the chart have different prefixes, but the same root word. *Pentathlon* and *decathlon* are both athletic contests. *Heptagon* and *octagon* are both closed figures. Knowing the meaning of the root of the term as well as the prefix can help you learn vocabulary.

### Reading to Learn

Use a dictionary to find the meanings of the prefix and root for each term. Then write a definition of the term.

1. *bisector* 2. *polygon* 3. *equilateral*
7. **RESEARCH** Use a dictionary to find the meanings of the prefix and root of *circum-* around, about; *ferre-* to carry
8. **RESEARCH** Use a dictionary or the Internet to find as many words as you can with the prefix *poly-* and the definition of each.

**Answers**

1. *bi-* 2, *sector-* a subdivision or region; divide into 2 regions
2. *poly-* many, *gon-* closed figure; closed figure with many sides
3. *equi-* equal, *lateral-* sides; having sides of equal length
4. *co-* together, *centr-* center; circles with a common center
5. *circum-* around, *scribe-* write; to write around (a geometrical figure)
6. *co-* together, *linear-* line; together on the same line
8. Sample answers: *polychromatic-* multicolored, *polymer-* a chemical compound composed of a repeating structural unit, *polysyllabic-* a word with more than three syllables
**Areas of Parallelograms**

**What You’ll Learn**
- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

**How is area related to garden design?**
This composition of square-cut granite and moss was designed by Shigemori Mirei in Kyoto, Japan. How could you determine how much granite was used in this garden?

**AREAS OF PARALLELOGRAMS** Recall that a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, there is a corresponding altitude that is perpendicular to the base.

In \(\square MNPR\), if \(\overline{MN}\) is the base, \(\overline{RN}\) and \(\overline{PQ}\) are altitudes. The length of an altitude is called the height of the parallelogram. If \(MR\) is the base, then the altitudes are \(\overline{PT}\) and \(\overline{NS}\).

**Geometry Activity**

**Area of a Parallelogram**

**Model**
- Draw parallelogram \(ABCD\) on grid paper. Label the vertices on the interior of the angles with letters \(A, B, C,\) and \(D\).
- Fold \(\square ABCD\) so that \(A\) lies on \(B\) and \(C\) lies on \(D\), forming a rectangle.

**Analyze**
1. What is the area of the rectangle? \(20\) units\(^2\)
2. How many rectangles form the parallelogram? \(2\)
3. What is the area of the parallelogram? \(40\) units\(^2\)
4. How do the base and altitude of the parallelogram relate to the length and width of the rectangle?
5. **Make a conjecture** Use what you observed to write a formula for the area of a parallelogram. \(A = bh\)

4. The base of the parallelogram is twice the length of the rectangle. The altitude of the parallelogram is the same length as the width of the rectangle.

**Resource Manager**

- **Workbook and Reproducible Masters**
  - **Chapter 11 Resource Masters**
    - Study Guide and Intervention, pp. 611–612
    - Skills Practice, p. 613
    - Practice, p. 614
    - Reading to Learn Mathematics, p. 615
    - Enrichment, p. 616

- **Graphing Calculator and Computer Masters**, pp. 37, 38
- **School-to-Career Masters**, p. 21
- **Teaching Geometry With Manipulatives Masters**, pp. 1, 180

- **Transparencies**
  - 5-Minute Check Transparency 11-1
  - Answer Key Transparencies

- **Technology**
  - Interactive Chalkboard
### Areas of Parallelograms

**Intervention**

Point out that many different parallelograms can be drawn with the same altitude, with their bases congruent, and thus with the same area. Use a geoboard or similar modeling device to show different parallelograms with these same characteristics, but different slants.

### In-Class Examples

**Teaching Tip** Point out that each parallelogram has two altitudes. Ask students to sketch the other altitude in this figure.

1. **Example 1** Find the area and perimeter of $\square RSTU$.
   
   Area: $384\sqrt{3}$ or about 665.1 in\(^2\);
   Perimeter: 112 in.

   **Teaching Tip** In Example 2, point out that another way to find the square yardage of the rooms they need to recarpet is to find the area of the noncarpeted section.

2. **Example 2** The Kanes are planning to sod some parts of their yard. Find the number of square yards of grass needed. About 2111 yd\(^2\)

### Example 1 Perimeter and Area of a Parallelogram

**Find the perimeter and area of $\square TRVW$.**

**Base and Side:** Each pair of opposite sides of a parallelogram has the same measure. Each base is 18 inches long, and each side is 12 inches long.

**Perimeter:** The perimeter of a parallelogram is the sum of the measures of its sides. So, the perimeter of $\square TRVW$ is $2(18) + 2(12)$ or 60 inches.

**Height:** Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is $x$, then the length of the hypotenuse is $2x$, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$12 = 2x$ Substitute 12 for the hypotenuse.

$6 = x$ Divide each side by 2.

So, the height of the parallelogram is $x\sqrt{3}$ or 6 inches.

**Area:**

\[
A = bh
\]

\[
= 18(6\sqrt{3})
\]

\[
b = 18, h = 6\sqrt{3}
\]

\[
= 108\sqrt{3}\text{ or about }187.1
\]

The perimeter of $\square TRVW$ is 60 inches, and the area is about 187.1 square inches.

### Example 2 Use Area to Solve a Real-World Problem

**INTERIOR DESIGN** The Waroners are planning to recarpet part of the first floor of their house. Find the amount of carpeting needed to cover the living room, den, and hall.

To estimate how much they can spend on carpeting, they need to find the square yardage of each room.

**Living Room:** $w = 13$ ft, $\ell = 15$ ft

**Den:** $w = 9$ ft, $\ell = 15$ ft

**Hall:** It is the same width as the living room, so $w = 13$. The total length of the house is 35 feet. So, $\ell = 35 - 15 = 13$ or 5 feet.

**Area**

\[
\text{Living Room: } A = \ell w
\]

\[
A = 13 \cdot 15
\]

\[
= 195 \text{ ft}^2
\]

\[
\text{Den: } A = \ell w
\]

\[
A = 9 \cdot 15
\]

\[
= 135 \text{ ft}^2
\]

\[
\text{Hall: } A = \ell w
\]

\[
A = 5 \cdot 13
\]

\[
= 65 \text{ ft}^2
\]

### Geometry Activity

**Materials:** grid paper

To help students see the relationship between the area of a parallelogram and the area of a rectangle, have students cut out several different rectangles. Ask them to cut the rectangle on the diagonal to form two triangles. Then reposition the triangles to form a parallelogram. The area of the parallelogram is the same as the area of the rectangle.
The total area is 195 + 135 + 65 or 395 square feet. There are 9 square feet in one square yard, so divide by 9 to convert from square feet to square yards.

\[
395 \text{ ft}^2 \div \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 395 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2} = 43.9 \text{ yd}^2
\]

Therefore, 44 square yards of carpeting are needed to cover these areas.

PARALLELOGRAMS ON THE COORDINATE PLANE  Recall the properties of quadrilaterals that you studied in Chapter 8. Using these properties as well as the formula for slope and the Distance Formula, you can find the areas of quadrilaterals on the coordinate plane.

Example 3  Area on the Coordinate Plane

COORDINATE GEOMETRY  The vertices of a quadrilateral are \( A(-4, -3) \), \( B(2, -3) \), \( C(4, -6) \), and \( D(-2, -6) \).

a. Determine whether the quadrilateral is a square, a rectangle, or a parallelogram.

First graph each point and draw the quadrilateral. Then determine the slope of each side.

- slope of \( AB = \frac{-3 - (-3)}{-4 - 2} = \frac{0}{-6} \) or 0
- slope of \( CD = \frac{-6 - (-6)}{4 - (-2)} = \frac{0}{6} \) or 0
- slope of \( BC = \frac{-3 - (-6)}{2 - (-4)} = \frac{3}{2} \)
- slope of \( AD = \frac{-3 - (-6)}{-4 - (-2)} = \frac{3}{2} \)

Opposite sides have the same slope, so they are parallel. \( ABCD \) is a parallelogram. The slopes of the consecutive sides are not negative reciprocals of each other, so the sides are not perpendicular. Thus, the parallelogram is neither a square nor a rectangle.

b. Find the area of quadrilateral \( ABCD \).

- Base:  \( \overline{CD} \) is parallel to the \( x \)-axis, so subtract the \( x \)-coordinates of the endpoints to find the length: \( CD = |4 - (-2)| \) or 6.
- Height:  Since \( \overline{AB} \) and \( \overline{CD} \) are horizontal segments, the distance between them, or the height, can be measured on any vertical segment. Reading from the graph, the height is 3.

\[
A = bh \quad \text{Area formula}
\]

\[
= 6(3) \quad b = 6, \; h = 3
\]

\[
= 18 \quad \text{Simplify.}
\]

The area of \( \square ABCD \) is 18 square units.
Practice/Apply

**Study Notebook**

Have students—

• add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.

• include the formula for the areas of a rectangle, square, and parallelogram with base b and height h.

• include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

• Areas of Parallelograms: 9–17, 27, 28, 31

• Parallelograms on the Coordinate Plane: 20–25

Odd/Even Assignments

Exercises 9–31 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 9–17 odd, 21–29 odd, 35–53

Average: 9–35 odd, 36–53

Advanced: 10–34 even, 35–49 (optional: 50–53)

Answer

1. The area of a rectangle is the product of the length and the width. The area of a parallelogram is the product of the base and the height. For both quadrilaterals, the measure of the length of one side is multiplied by the length of the altitude.
Find the area of each figure.

27. 150 units²

28. 77 units²

**ART** For Exercises 29 and 30, use the following information.

A triptych painting is a series of three pieces with a similar theme displayed together. Suppose the center panel is a 12-inch square and the panels on either side are 12 inches by 5 inches. The panels are 2 inches apart with a 3 inch wide border around the edges.

29. Determine whether the triptych will fit a 45-inch by 20-inch frame. Explain.

30. Find the area of the artwork. 576 in²

31. CROSSWALKS A crosswalk with two stripes each 52 feet long is at a 60° angle to the curb. The width of the crosswalk at the curb is 16 feet. Find the perpendicular distance between the stripes of the crosswalk. \( \approx 13.9 \) ft

**VARYING DIMENSIONS** For Exercises 32–34, use the following information.

A parallelogram has a base of 8 meters, sides of 11 meters, and a height of 10 meters.

32. Find the perimeter and area of the parallelogram. 38 m, 80 m²

33. Suppose the dimensions of the parallelogram were divided in half. Find the perimeter and the area.

34. Compare the perimeter and area of the parallelogram in Exercise 33 with the original.

35. CRITICAL THINKING A piece of twine 48 inches long is cut into two lengths. Each length is then used to form a square. The sum of the areas of the two squares is 74 square inches. Find the length of each side of the smaller square and the larger square. 5 in., 7 in.
36. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

How is area related to garden design?

Include the following in your answer:
- how to determine the total area of granite squares, and
- other uses for area.

37. What is the area of \( \square ABCD \)?

(A) 24 m\(^2\)  
(B) 30 m\(^2\)  
(C) 48 m\(^2\)  
(D) 60 m\(^2\)

38. **ALGEBRA** Which statement is correct? D

- (A) \( x^2 > (x - 1)^2 \)
- (B) \( x^2 < (x - 1)^2 \)
- (C) \( x^2 = (x - 1)^2 \)
- (D) The relationship cannot be determined.

### Maintain Your Skills

#### Mixed Review

- **Problem 39.** \((x - 5)^2 + (y - 2)^2 = 49\), \(r = 7\)

- **Problem 40.** \((x + 3)^2 + (y + 9)^2 - 81 = 0\)

- **Problem 41.** \(x + 2 \times 9 - 4 = 0\)

- **Problem 42.** \((x - 2.8)^2 + (y + 7.6)^2 = 34.81\)

Find \(x\). Assume that segments that appear to be tangent are tangent. **(Lesson 10-7)**

- **Problem 43.**

- **Problem 44.**

- **Problem 45.**

#### COORDINATE GEOMETRY

- **Problem 46.** \(\triangle ABC\) with vertices \(A(-1, 3), B(-4, 6),\) and \(C(-5, 1)\), reflected in the \(y\)-axis and then the \(x\)-axis \(A'(-1, -3), B'(-4, -6), C'(5, -1)\); 180°

- **Problem 47.** \(\triangle FGH\) with vertices \(F(0, 4), G(-2, 2),\) and \(H(2, 2)\), reflected in \(y = x\) and then the \(y\)-axis \(F'(-4, 0), G'(-2, -2), H'(-2, 2)\); 90° counterclockwise

- **Problem 48.** \(\triangle LMN\) with vertices \(L(2, 0), M(3, -3),\) and \(N(1, -4)\), reflected in the \(y\)-axis and then the line \(y = -x\) \(L''(0, 2), M''(3, 3), N''(4, 1)\); 90° counterclockwise

- **Problem 49.** **BIKES** Nate is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 5 feet. What length of plywood does Nate need for the ramp? **(Lesson 7-2)** 13 ft

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Evaluate each expression if \(w = 8, x = 4, y = 2,\) and \(z = 5\). **(To review evaluating expressions, see page 736.)**

- **Problem 50.** \(\frac{1}{2}(7y)\)  
- **Problem 51.** \(\frac{1}{2}(wx)\)  
- **Problem 52.** \(\frac{1}{2}(z(x + y))\)  
- **Problem 53.** \(\frac{1}{2}(y + w)\)

600 Chapter 11 Areas of Polygons and Circles
Areas of Triangles, Trapezoids, and Rhombi

What You’ll Learn

• Find areas of triangles.
• Find areas of trapezoids and rhombi.

How is the area of a triangle related to beach umbrellas?

Umbrellas can protect you from rain, wind, and sun. The umbrella shown at the right is made of triangular panels. To cover the umbrella frame with canvas panels, you need to know the area of each panel.

AREAS OF TRIANGLES You have learned how to find the areas of squares, rectangles, and parallelograms. The formula for the area of a triangle is related to these formulas.

How is the area of a triangle related to beach umbrellas?

Ask students:

• What is the shape of the umbrella fabric when it is flat? a many-sided polygon
• Why are triangles the shape used to make umbrellas? Sample answer: They fit together at one vertex.
• Give another example of an item made from a triangular piece of fabric. Sample answer: a sail

Geometry Activity

Area of a Triangle

Model
You can determine the area of a triangle by using the area of a rectangle.
• Draw a triangle on grid paper so that one edge is along a horizontal line. Label the vertices on the interior of the angles of the triangle as A, B, and C.
• Draw a line perpendicular to AC through A.
• Draw a line perpendicular to AC through C.
• Draw a line parallel to AC through B.
• Label the points of intersection of the lines drawn as D and E as shown.
• Find the area of rectangle ACDE in square units.
• Cut out rectangle ACDE. Then cut out \( \triangle ABC \). Place the two smaller pieces over \( \triangle ABC \) to completely cover the triangle.

Analyze

1. Together the two smaller triangles are the same size as \( \triangle ABC \).
2. What fraction of rectangle ACDE is \( \triangle ABC \)? \( \frac{1}{2} \)
3. Derive a formula that could be used to find the area of \( \triangle ABC \). \( A = \frac{1}{2}bh \)
**Key Concept**

**Area of a Triangle**

If a triangle has an area of $A$ square units, a base of $b$ units, and a corresponding height of $h$ units, then $A = \frac{1}{2}bh$.

---

**Example 1 Areas of Triangles**

Find the area of quadrilateral $XYZW$ if $XZ = 39$, $HW = 20$, and $YG = 21$.

The area of the quadrilateral is equal to the sum of the areas of $\triangle XYZ$ and $\triangle XWZ$.

\[
\text{area of } XYZW = \text{area of } \triangle XYZ + \text{area of } \triangle XWZ \\
= \frac{1}{2}bh_1 + \frac{1}{2}bh_2 \\
= \frac{1}{2}(39)(21) + \frac{1}{2}(39)(20) \quad \text{Substitution} \\
= 409.5 + 390 \quad \text{Simplify} \\
= 799.5
\]

The area of quadrilateral $XYZW$ is 799.5 square units.

---

**AREAS OF TRAPEZOIDS AND RHOMBI**

The formulas for the areas of trapezoids and rhombi are related to the formula for the area of a triangle.

Trapezoid $MNQP$ has diagonal $QN$ with parallel bases $MN$ and $PQ$. Therefore, the altitude $h$ from vertex $Q$ to the extension of base $MN$ is the same length as the altitude from vertex $N$ to the base $QP$.

Since the area of the trapezoid is the area of two nonoverlapping parts, we can write the following equation.

\[
\text{area of } \text{trapezoid } MNQP = \text{area of } \triangle MNQ + \text{area of } \triangle NPQ \\
A = \frac{1}{2}(b_1)h + \frac{1}{2}(b_2)h \quad \text{Let the area be } A, \text{ MN be } b_1, \text{ and } PQ \text{ be } b_2. \\
= \frac{1}{2}(b_1 + b_2)h \quad \text{Factor.} \\
= \frac{1}{2}h(b_1 + b_2) \quad \text{Commutative Property}
\]

This is the formula for the area of any trapezoid.

---

**Study Tip**

To review the height and altitude of a triangle, see Lesson 5-1.

---

**In-Class Example**

Find the area of quadrilateral $ABCD$ if $AC = 35$, $BF = 18$, and $DE = 10$.

\[
\text{490 units}^2 
\]

---

**Geometry Activity**

**Materials:** grid paper

- Ask students to recall what they know about perpendiculars and the altitudes of triangles before you do this activity.
- Some students may seem uninterested because they already know the formula for the area of a triangle. Ask them if they can demonstrate why the area formula works. In this activity, they will explore why the formula works and derive the formula for themselves.
**Example 2** Area of a Trapezoid on the Coordinate Plane

**COORDINATE GEOMETRY** Find the area of trapezoid TVWZ with vertices T(−3, 4), V(3, 4), W(6, −1), and Z(−5, −1).

**Bases:** Since TV and ZW are horizontal, find their length by subtracting the x-coordinates of their endpoints.

\[ TV = |−3 − 3| \quad ZW = |−5 − 6| \]

\[ = 6 \quad or \quad 6 \quad = 11 \quad or \quad 11 \]

**Height:** Because the bases are horizontal segments, the distance between them can be measured on a vertical line. That is, subtract the y-coordinates.

\[ h = |4 − (−1)| = 5 \]

**Area:**

\[ A = \frac{1}{2}h(b_1 + b_2) \quad \text{Area of a trapezoid} \]

\[ = \frac{1}{2}(6 + 11) \quad h = 5, \quad b_1 = 6, \quad b_2 = 11 \]

\[ = 42.5 \quad \text{Simplify.} \]

The area of trapezoid TVWZ is 42.5 square units.

**Key Concept** Area of a Rhombus

If a rhombus has an area of \( A \) square units and diagonals of \( d_1 \) and \( d_2 \) units, then \[ A = \frac{1}{2}d_1d_2. \]

**Example:** \[ A = \frac{1}{2}(4)(2) \]

You will derive this formula in Exercise 46.

**Example 3** Area of a Rhombus on the Coordinate Plane

**COORDINATE GEOMETRY** Find the area of rhombus EFGH with vertices at \( E(−1, 3), \ F(2, 7), \ G(5, 3), \) and \( H(2, −1) \).

**Explore** To find the area of the rhombus, we need to know the lengths of each diagonal.

**Plan** Use coordinate geometry to find the length of each diagonal. Use the formula to find the area of rhombus EFGH.

**Solve** Let \( EG \) be \( d_1 \) and \( FH \) be \( d_2 \).

Subtract the x-coordinates of \( E \) and \( G \) to find that \( d_1 \) is 6.

Subtract the y-coordinates of \( F \) and \( H \) to find that \( d_2 \) is 8.

\[ A = \frac{1}{2}d_1d_2 \quad \text{Area of a rhombus} \]

\[ = \frac{1}{2}(6)(8) \quad or \quad 24 \quad d_1 = 6, \quad d_2 = 8 \]

**Examine** The area of rhombus EFGH is 24 square units.

If you know all but one measure in a quadrilateral, you can solve for the missing measure using the appropriate area formula.

**Teaching Tip** Make sure students understand that the formula for the area of a trapezoid is derived from adding the areas of the two nonoverlapping triangles formed by one diagonal of the trapezoid. The height of the trapezoid is the same as the height of each triangle.

2 Find the area of trapezoid \( RSTU \) with vertices \( R(4, 2), S(6, −1), T(−2, −1), \) and \( U(−1, 2) \). 19.5 units²

3 Find the area of rhombus \( MNPR \) with vertices at \( M(0, 1), N(4, 2), P(3, −2), \) and \( R(−1, −3) \). 15 units²

**Building on Prior Knowledge**

In Chapter 8, students learned that if a parallelogram has all sides congruent, it is a rhombus. In this lesson, emphasize that the diagonals of a rhombus are used to find its area instead of the base and height.
In-Class Examples

4. a. Rhombus $RSTU$ has an area of 64 square inches. Find $US$ if $RT = 8$ inches.

16 in.

b. Trapezoid $DEFG$ has an area of 120 square feet. Find the height of $DEFG$.

8 ft

5. STAINED GLASS This stained glass window is composed of 8 congruent trapezoidal shapes. The total area of the design is 72 square feet. Each trapezoid has bases of 3 and 6 feet. Find the height of each trapezoid.

2 ft

Example 4 Algebra: Find Missing Measures

a. Rhombus $WXYZ$ has an area of 100 square meters. Find $WY$ if $XZ = 10$ meters.

Use the formula for the area of a rhombus and solve for $d_2$.

$A = \frac{1}{2}d_1d_2$

$100 = \frac{1}{2}(10)(d_2)$

$100 = 5d_2$

$20 = d_2$

$WY$ is 20 meters long.

b. Trapezoid $PQRS$ has an area of 250 square inches. Find the height of $PQRS$.

Use the formula for the area of a trapezoid and solve for $h$.

$A = \frac{1}{2}h(b_1 + b_2)$

$250 = \frac{1}{2}h(20 + 30)$

$250 = \frac{1}{2}(50)h$

$25 = 5h$

$10 = h$

The height of trapezoid $PQRS$ is 10 inches.

Since the dimensions of congruent figures are equal, the areas of congruent figures are also equal.

Postulate 11.1 Congruent figures have equal areas.

Example 5 Area of Congruent Figures

QUILTING This quilt block is composed of twelve congruent rhombi arranged in a regular hexagon. The height of the hexagon is 8 inches. If the total area of the rhombi is 48 square inches, find the lengths of each diagonal and the area of one rhombus.

First, find the area of one rhombus. From Postulate 11.1, the area of each rhombus is the same. So, the area of each rhombus is $\frac{48}{12}$ or 4 square inches.

Next, find the length of one diagonal. The height of the hexagon is equal to the sum of the long diagonals of two rhombi. Since the rhombi are congruent, the long diagonals must be congruent. So, the long diagonal is equal to $8 \div 2$, or 4 inches.

Use the area formula to find the length of the other diagonal.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$4 = \frac{1}{2}(4) d_2$ $A = 4, d_1 = 4$

$2 = d_2$ Solve for $d_2$.

Each rhombus in the pattern has an area of 4 square inches and diagonals 4 inches and 2 inches long.
Concept Check

1. OPEN ENDED Draw an isosceles trapezoid that contains at least one isosceles triangle. See margin.

2. Kiku; she simplified the formula properly by adding the terms in the parentheses before multiplying.

2. FIND THE ERROR Robert and Kiku are finding the area of trapezoid JKLM.

Robert

$$A = \frac{1}{2}(8)(14 + 9)$$

$$= \frac{1}{2}(8)(23)$$

$$= 4(23)$$

$$= 92 \text{ cm}^2$$

Kiku

$$A = \frac{1}{2}(8)(14 + 9)$$

$$= \frac{1}{2}(8)(23)$$

$$= 65 \text{ cm}^2$$

Who is correct? Explain your reasoning.

3. Determine whether it is always, sometimes, or never true that rhombi with the same area have the same diagonal lengths. Explain your reasoning. Sometimes; two rhombi can have different corresponding diagonal lengths and have the same area.

Find the area of each quadrilateral.

4. 240 m$^2$

5. 499.5 in$^2$

6. 240 yd$^2$

COORDINATE GEOMETRY Find the area of each figure given the coordinates of the vertices.

7. $\triangle ABC$ with $A(2, -3), B(-5, -3),$ and $C(-1, 3)$ 21 units$^2$

8. trapezoid $FGHJ$ with $F(-1, 8), G(5, 8), H(3, 4),$ and $J(1, 4)$ 16 units$^2$

9. rhombus $LMPQ$ with $L(-4, 3), M(-2, 4),$ $P(0, 3),$ and $Q(-2, 2)$ 4 units$^2$

ALGEBRA Find the missing measure for each quadrilateral.

10. Trapezoid $NOPQ$ has an area of 250 square inches. Find the height of $NOPQ$. 10 in.

11. Rhombus $RSTU$ has an area of 675 square meters. Find $SU$. 45 m

Application

12. INTERIOR DESIGN Jacques is designing a window hanging composed of 13 congruent rhombi. The total width of the window hanging is 15 inches, and the total area is 82\(\frac{7}{8}\) square inches. Find the length of each diagonal and the area of one rhombus.

$$4\frac{1}{4} \text{ in.}, \ 3 \text{ in.}, \ 6\frac{3}{8} \text{ in}^2$$

### Lesson 11-2 Areas of Triangles, Trapezoids, and Rhombi 605

**Study Notebook**

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include the formulas and examples of the area of a triangle, rhombus, and trapezoid.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

### DAILY INTERVENTION FIND THE ERROR

In Exercise 2, caution students that the order of operations applies to all mathematical expressions, including area formulas. Kiku did the problem correctly by adding the terms in parentheses before multiplying by the other terms.

### About the Exercises...

**Organization by Objective**

- Areas of Triangles: 13, 14, 19–21
- Areas of Trapezoids and Rhombi: 15–18, 22–44

**Odd/Even Assignments**

Exercises 13–44 are structured so that students practice the same concepts whether they are assigned odd or even problems.

### Assignment Guide

**Basic**: 13–43 odd, 47–57 odd, 58–60, 65–76 (optional: 61–64)

**Average**: 13–57 odd, 58–60, 65–76 (optional: 61–64)

**Advanced**: 14–56 even, 57–73 (optional: 74–76)

**All**: Quiz 1 (1–5)
Find the area of each figure. Round to the nearest tenth if necessary.

13. \[ \text{12.4 cm}^2 \]
14. \[ \text{35.7 ft}^2 \]
15. \[ \text{8 km} \]
16. \[ \text{96.5 yd}^2 \]
17. \[ \text{1200 ft}^2 \]
18. \[ \text{408 cm}^2 \]
19. \[ \text{50 m}^2 \]
20. \[ \text{99 in}^2 \]
21. \[ \text{19 mm} \]
22. \[ \text{129.9 mm}^2 \]

**COORDINATE GEOMETRY**

Find the area of trapezoid \( PQRT \) given the coordinates of the vertices. \[ \text{23. 55 units}^2 \]

22. \( P(0, 3), Q(3, 7), R(5, 7), T(6, 3) \)
23. \( P(−4, −5), Q(−2, −5), R(4, 6), T(−4, 6) \)
24. \( P(−3, 8), Q(6, 8), R(6, 2), T(1, 2) \)
25. \( P(−6, 3), Q(1, 3), R(−2, −2), T(−4, −2) \)

**COORDINATE GEOMETRY**

Find the area of rhombus \( JKLM \) given the coordinates of the vertices. \[ \text{26. 30 units}^2 \]
27. \[ \text{20 units}^2 \]

26. \( J(1, 2), K(7, 4), L(12, 1), M(7, −2) \)
27. \( J(−1, 2), K(1, 7), L(3, 2), M(1, −3) \)
28. \( J(−1, −4), K(2, 2), L(5, −4), M(2, −10) \)
29. \( J(2, 4), K(6, 6), L(10, 4), M(6, 2) \)

**ALGEBRA**

Find the missing measure for each figure.

30. Trapezoid \( ABCD \) has an area of 750 square meters. Find the height of \( ABCD \). \[ 25 m \]
31. Trapezoid \( GHJK \) has an area of 188.35 square feet. If \( HJ \) is 16.5 feet, find \( GK \). \[ 26.8 ft \]
32. Rhombus \( MNQP \) has an area of 375 square inches. If \( MP \) is 25 inches, find \( NQ \). \[ 30 in. \]

33. Rhombus \( QRST \) has an area of 137.9 square meters. If \( RT \) is 12.2 meters, find \( QS \). \[ 22.6 m \]
34. Triangle \( WXY \) has an area of 248 square inches. Find the length of the base. \[ 31 in. \]
35. Triangle \( PQS \) has an area of 300 square centimeters. Find the height. \[ 20 cm \]

**Enrichment**, p. 622

Areas of Similar Triangles

You have learned that if two triangles are similar, the ratio of the lengths of corresponding sides is constant. The ratio of the areas of the two triangles is the square of the ratio of their corresponding sides. Thus, the area of a smaller triangle is always smaller than the area of a larger triangle.

**Theorem:** If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

\[ \frac{\text{area of Triangle II}}{\text{area of Triangle I}} = \left( \frac{\text{side of Triangle II}}{\text{side of Triangle I}} \right)^2 \]

Sample problem:

Triangle \( I \) has an area of 16 square units. If the side of triangle \( II \) is twice the side of triangle \( I \), find the area of triangle \( II \).

Solution:

\[ \text{area of Triangle II} = 16 \times 2^2 = 64 \text{ square units} \]
GARDENS  For Exercises 36 and 37, use the following information.
Keisha designed a garden that is shaped like two congruent rhombi. She wants the long diagonals lined with a stone walkway. The total area of the garden is 150 square feet, and the shorter diagonals are each 12 feet long.

36. Find the length of each side of the garden. 12.5 ft
37. Find the length of each side of the garden. about 8.7 ft

REAL ESTATE  For Exercises 38 and 39, use the following information.
The map shows the layout and dimensions of several lot parcels in Linworth Village. Suppose Lots 35 and 12 are trapezoids. The map shows the layout and dimensions of several lot parcels in Linworth Village. Suppose Lots 35 and 12 are trapezoids.

38. If the height of Lot 35 is 122.81 feet, find the area of this lot. 7718.6 ft²
39. If the height of Lot 12 is 199.8 feet, find the area of this lot. 13,326.7 ft²

Online Research Data Update  Use the Internet or other resource to find the median price of homes in the United States. How does this compare to the median price of homes in your community? Visit www.geometryonline.com/data_update to learn more.

Find the area of each figure.
40. rhombus with a perimeter of 20 meters and a diagonal of 8 meters 24 m²
41. rhombus with a perimeter of 52 inches and a diagonal of 24 inches 120 in²
42. isosceles trapezoid with a perimeter of 52 yards; the measure of one base is 10 yards greater than the other base, the measure of each leg is 3 yards less than twice the length of the shorter base 156 yd²
43. equilateral triangle with a perimeter of 15 inches ≈ 10.8 in²
44. scalene triangle with sides that measure 34.0 meters, 81.6 meters, and 88.4 meters. 1387.2 m²
45. Find the area of ΔJKM. 21 ft²
46. Derive the formula for the area of a rhombus using the formula for the area of a triangle. See margin.
47. Determine whether the statement Two triangles that have the same area also have the same perimeter is true or false. Give an example or counterexample. See margin.
48–49. See margin.
Each pair of figures is similar. Find the area and perimeter of each figure. Describe how changing the dimensions affects the perimeter and area.
48.

49.

50. RECREATION  Becky wants to cover a kite frame with decorative paper. If the length of one diagonal is 20 inches and the other diagonal measures 25 inches, find the area of the surface of the kite. 250 in²

Answers
46. A rhombus is made up of two congruent triangles. Using \( d_1 \) and \( d_2 \) instead of \( b \) and \( h \), its area in reference to \( A = \frac{1}{2}bh \) is \( 2\left(\frac{1}{2}d_1\right)\left(\frac{1}{2}d_2\right) \) or \( \frac{1}{2}d_1d_2 \).
47. False; Sample answer:

The area for each of these right triangles is 6 square units. The perimeter of one triangle is 12 and the perimeter of the other is \( 8 + \sqrt{40} \) or about 14.3.
48. area ≈ 6.9, area ≈ 10.8; perimeter = 12, perimeter = 15; scale factor and ratio of perimeters is \( \frac{5}{4} \), ratio of areas is \( \left(\frac{5}{4}\right)^2 \)
49. area = 12, area = 3, perimeter = \( 8\sqrt{13} \), perimeter = \( 4\sqrt{13} \); scale factor and ratio of perimeters = \( \frac{1}{2} \), ratio of areas = \( \left(\frac{1}{2}\right)^2 \)
Answers

58. Sample answer: Umbrellas have triangular panels of fabric or nylon. In order to make the panels to fit the umbrella frame, the area of the triangles is needed. Answers should include the following.
- Find the area of a triangle by multiplying the base and the height and dividing by two.
- Rhombi are composed of two congruent isosceles triangles, and trapezoids are composed of two triangles and a rectangle.

SIMILAR FIGURES For Exercises 51–56, use the following information.
Triangle ABC is similar to triangle DEF.
51. Find the scale factor. \( \frac{2}{1} \)
52. Find the perimeter of each triangle. 22.8; 11.4
53. Compare the ratio of the perimeters of the triangles to the scale factor.
54. Find the area of each triangle. 24, 6
55. Compare the ratio of the areas of the triangles to the scale factor. 45 ft²; The ratio of the areas is 5:9.
56. Compare the ratio of the areas of the triangles to the ratio of the perimeters of the triangles. The ratio of the areas is the square of the ratio of the perimeters.

57. CRITICAL THINKING In the figure, the vertices of quadrilateral ABCD intersect square EFGH and divide its sides into segments with measures that have a ratio of 1:2. Find the area of ABCD. Describe the relationship between the areas of ABCD and EFGH. 45 ft²; The ratio of the areas is 5:9.

58. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.
How is the area of a triangle related to beach umbrellas?
Include the following in your answer:
- how to find the area of a triangle, and
- how the area of a triangle can help you find the areas of rhombi and trapezoids.

59. In the figure, if point B lies on the perpendicular bisector of AC, what is the area of \( \triangle ABC \)? B
   A. 15 units²  B. 30 units²  C. 50 units²  D. 1602 units²

60. ALGEBRA What are the solutions of the equation \( (2x - 7)(x + 10) = 0 \)? D
   A. −3.5 and 10  B. 7 and −10  C. \( \frac{2}{7} \) and −10  D. 3.5 and −10

Extending the Lesson

Trigonometric Ratios and the Areas of Triangles

The area of any triangle can be found given the measures of two sides of the triangle and the measure of the included angle. Suppose we are given \( AC = 15 \), \( BC = 8 \), and \( m \angle C = 60^\circ \). To find the height of the triangle, use the sine ratio, \( \sin C = \frac{h}{BC} \). Then use the value of \( h \) in the formula for the area of a triangle. So, the area is \( \frac{1}{2}(15)(8 \sin 60^\circ) \) or 52.0 square meters.

61. Derive a formula to find the area of any triangle, given the measures of two sides of the triangle and their included angle. \[ \text{area} = \frac{1}{2}ab \sin C \]

Find the area of each triangle. Round to the nearest hundredth.

62. 6.79 in²
63. 6.02 cm²
64. 0.92 ft²

Standardized Test Practice

608 Chapter 11 Areas of Polygons and Circles
### Maintain Your Skills

#### Mixed Review
Find the area of each figure. Round to the nearest tenth. **(Lesson 11-1)**

65. 22 cm
   \[ 374 \text{ cm}^2 \]
66. 10 in.
   \[ 129.9 \text{ in}^2 \]
67. 16 ft
   \[ 231 \text{ ft}^2 \]

Write an equation of circle \( R \) based on the given information. **(Lesson 10-8)**

68. center: \((1, 2)\); radius: 7
   \[ (x - 1)^2 + (y - 2)^2 = 49 \]
69. center: \((-4, \frac{1}{2})\); radius: \(\frac{11}{2} \)
   \[ (x + 4)^2 + \left( y - \frac{1}{2} \right)^2 = \frac{121}{4} \]
70. center: \((1, 2)\); radius: 3.5
   \[ (x - 1)^2 + (y - 2)^2 = 12.25 \]

#### Getting Ready for the Next Lesson

**CRAFTS** Andria created a pattern to appliqué flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? **(Lesson 10-1)**

- Given the magnitude and direction of a vector, find the component form with values rounded to the nearest tenth.
- Magnitude of 136 at a direction of 25 degrees with the positive \( x \)-axis.
- Magnitude of 280 at a direction of 52 degrees with the positive \( x \)-axis.

**PREREQUISITE SKILL** Find \( x \). Round to the nearest tenth. **(Lesson 11-1)**

73. \( 44.0 \)
74. \( 20.1 \)
75. \( 4.8 \)
76. \( 58^\circ \)

### Practice Quiz 1

#### Lessons 11-1 and 11-2

The coordinates of the vertices of quadrilateral \( JKLM \) are \( J(-8, 4), K(-4, 0), L(0, 4), \) and \( M(-4, 8) \). **(Lesson 11-1)**

1. Determine whether \( JKLM \) is a square, a rectangle, or a parallelogram. **square**
2. Find the area of \( JKLM \). **32 units\(^2\)**

Find the area of each trapezoid. **(Lesson 11-2)**

3. \( 54 \text{ units}^2 \)
4. \( 25 \text{ units}^2 \)

5. The area of a rhombus is 546 square yards. If \( d_1 \) is 26 yards long, find the length of \( d_2 \). **(Lesson 11-2)**

### Assessment Options

#### Writing
Ask students to list all the formulas for area they have learned so far in this chapter. Ask them to include a labeled diagram corresponding to each formula.

#### Getting Ready for Lesson 11-3

**Prerequisite Skill** Students will use right triangle trigonometry in Lesson 11-3 to find the altitude of a regular polygon so that they can calculate the area of the polygon. Use Exercises 74–76 to determine your students’ familiarity with trigonometric ratios in right triangles.

#### Assessment Quiz 1

The quiz provides students with a brief review of the concepts and skills in Lessons 11-1 and 11-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 11-1 and 11-2)** is available on p. 655 of the *Chapter 11 Resource Masters*.

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**Teacher to Teacher**

**Sarah L. Waldrop, Forestview High School, Gastonia, NC**

We play MATHO as a review. I give students (on the overhead) about 30–36 answers to be placed on their MATHO sheet. This sheet is arranged like bingo—with a free space. They fill in any 24 answers. After their card is filled in, I ask questions with the given answer. When they have MATHO they get a prize (candy, points on a quiz, etc.).
**Areas of Regular Polygons and Circles**

**What You’ll Learn**
- Find areas of regular polygons.
- Find areas of circles.

**How can you find the area of a polygon?**

The foundations of most gazebos are shaped like regular hexagons. Suppose the owners of this gazebo would like to install tile on the floor. If tiles are sold in square feet, how can they find out the actual area of tiles needed to cover the floor?

**Areas of Regular Polygons**

In regular hexagon ABCDEF inscribed in circle G, GA and GF are radii from the center of the regular polygon perpendicular to a side of the polygon. This segment is called an **apothem**.

Triangle GFA is an isosceles triangle, since the radii are congruent. If all of the radii were drawn, they would separate the hexagon into 6 nonoverlapping congruent isosceles triangles.

The area of the hexagon can be determined by adding the areas of the triangles. Since GH is perpendicular to AF, it is an altitude of \( \triangle AGF \). Let \( a \) represent the length of GH and let \( s \) represent the length of a side of the hexagon.

**Area of \( \triangle AGF \)**

\[
\text{Area of } \triangle AGF = \frac{1}{2}bh = \frac{1}{2}as
\]

The area of one triangle is \( \frac{1}{2}as \) square units. So the area of the hexagon is \( 6\left(\frac{1}{2}as\right) \) square units. Notice that the perimeter \( P \) of the hexagon is \( 6s \) units. We can substitute \( P \) for \( 6s \) in the area formula. So, \( A = 6\left(\frac{1}{2}as\right) \) becomes \( A = \frac{1}{2}Pa \). This formula can be used for the area of any regular polygon.

**Key Concept**

If a regular polygon has an area of \( A \) square units, a perimeter of \( P \) units, and an apothem of \( a \) units, then \( A = \frac{1}{2}Pa \).
Example 1 Area of a Regular Polygon

Find the area of a regular pentagon with a perimeter of 40 centimeters.

Apothem: The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is \( \frac{360}{5} \) or 72. \( PQ \) is an apothem of pentagon \( JKLMN \).

It bisects \( \angle NPM \) and is a perpendicular bisector of \( NM \). So, \( m \angle MPQ = \frac{1}{2}(72) \) or 36. Since the perimeter is 40 centimeters, each side is 8 centimeters and \( QM = 4 \) centimeters.

Write a trigonometric ratio to find the length of \( PQ \).

\[
\tan \angle MPQ = \frac{QM}{PQ} \quad \tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}
\]

\[
\tan 36^\circ = \frac{4}{PQ} \quad m \angle MPQ = 36, \; QM = 4
\]

\[
(PQ) \tan 36^\circ = 4 \quad \text{Multiply each side by } PQ.
\]

\[
PQ = \frac{4}{\tan 36^\circ} \quad \text{Divide each side by } \tan 36^\circ.
\]

\[
PQ = 5.5 \quad \text{Use a calculator.}
\]

Area:

\[
A = \frac{1}{2}Pa 
\]

\[
= \frac{1}{2}(40)(5.5) \quad P = 40, \; a = 5.5
\]

\[
= 110 \quad \text{Simplify.}
\]

So, the area of the pentagon is about 110 square centimeters.

AREAS OF CIRCLES You can use a calculator to help derive the formula for the area of a circle from the areas of regular polygons.

Geometry Activity

Area of a Circle

Collect Data

Suppose each regular polygon is inscribed in a circle of radius \( r \).

1. Copy and complete the following table. Round to the nearest hundredth.

<table>
<thead>
<tr>
<th>Inscribed Polygon</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sides</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure of a Side</td>
<td>1.73r</td>
<td>1.18r</td>
<td>0.77r</td>
<td>0.62r</td>
<td>0.31r</td>
<td>0.126r</td>
</tr>
<tr>
<td>Measure of Apothem</td>
<td>0.5r</td>
<td>0.81r</td>
<td>0.92r</td>
<td>0.95r</td>
<td>0.99r</td>
<td>0.998r</td>
</tr>
<tr>
<td>Area</td>
<td>1.30r²</td>
<td>2.39r²</td>
<td>2.83r²</td>
<td>2.95r²</td>
<td>3.07r²</td>
<td>3.14r²</td>
</tr>
</tbody>
</table>

Analyze the Data

2. What happens to the appearance of the polygon as the number of sides increases? The polygon appears to be a circle.

3. What happens to the areas as the number of sides increases?

4. Make a conjecture about the formula for the area of a circle.

Geometry Activity

Materials: calculator

• Ask why the radius is left as a variable in this data. (The table can be developed for a circle with any radius \( r \).

• Also point out that the measure of a side and measure of the apothem of an equilateral triangle is found using 30°-60°-90° triangle relationships. Trigonometric ratios are used to determine the measures of a side and an apothem for each polygon.
In-Class Examples

**Example 2** Use Area of a Circle to Solve a Real-World Problem

**SEWING** A caterer has a 48-inch diameter table that is 34 inches tall. She wants a tablecloth that will touch the floor. Find the area of the tablecloth in square yards.

The diameter of the table is 48 inches, and the tablecloth must extend 34 inches in each direction. So the diameter of the tablecloth is 34 + 48 + 34 or 116 inches. Divide by 2 to find that the radius is 58 inches.

\[
A = 
\pi r^2 \quad \text{Area of a circle}
\]

\[
= \pi(58)^2 \quad \text{Substitution}
\]

\[
= 10,568.3 \quad \text{Use a calculator.}
\]

The area of the tablecloth is 10,568.3 square inches. To convert to square yards, divide by 1296. The area of the tablecloth is 8.2 square yards to the nearest tenth.

**Example 3** Area of an Inscribed Polygon

Find the area of the shaded region. Assume that the triangle is equilateral.

The area of the shaded region is the difference between the area of the circle and the area of the triangle. First, find the area of the circle.

\[
A = \pi r^2 \quad \text{Area of a circle}
\]

\[
= \pi(4)^2 \quad \text{Substitution}
\]

\[
= 50.3 \quad \text{Use a calculator.}
\]

To find the area of the triangle, use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of \(\triangle ABC\) is 4, so \(BC\) is \(2\sqrt{3}\). Since \(EC = 2(BC)\), \(EC = 4\sqrt{3}\).

Next, find the height of the triangle, \(DB\). Since \(\angle DCB\) is 60, \(DB = 2\sqrt{3}\sqrt{3}\) or 6.

Use the formula to find the area of the triangle.

\[
A = \frac{1}{2}bh \quad \text{Area of a triangle}
\]

\[
= \frac{1}{2}(4\sqrt{3})(6) \quad b = 4\sqrt{3}, h = 6
\]

\[
= 20.8 \quad \text{Use a calculator.}
\]

The area of the shaded region is 50.3 - 20.8 or 29.5 square meters to the nearest tenth.

**Study Tip**

**Square Yards**

A square yard measures 36 inches by 36 inches or 1296 square inches.

**Key Concept**

If a circle has an area of \(A\) square units and a radius of \(r\) units, then \(A = \pi r^2\).
Practice and Apply

1. **Concept Check**
   - Explain how to derive the formula for the area of a regular polygon. See margin.
   - OPEN ENDED Describe a method for finding the base or height of a right triangle given one acute angle and the length of one side. See margin.

**Guided Practice**

Find the area of each polygon. Round to the nearest tenth.

3. a regular hexagon with a perimeter of 42 yards \( \text{127.3 yd}^2 \)
4. a regular nonagon with a perimeter of 108 meters \( \text{890.2 m}^2 \)

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

5. \( \text{10.6 cm}^2 \)
6. \( \text{18.5 in}^2 \)
7. **UPHOLSTERY** Tyra wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 12 inches, around a center cushion with a diameter of 20 inches, find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion. \( \text{about 3.6 yd}^2 \)

**Application**

8. a regular octagon with a perimeter of 72 inches \( \text{391.1 in}^2 \)
9. a square with a perimeter of \( 84\sqrt{2} \) meters \( \text{882 m}^2 \)
10. a square with apothem length of 12 centimeters \( \text{576 cm}^2 \)
11. a regular hexagon with apothem length of 24 inches \( \text{1995.3 in}^2 \)
12. a regular triangle with side length of 15.5 inches \( \text{104.0 in}^2 \)
13. a regular octagon with side length of 10 kilometers \( \text{482.8 km}^2 \)

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

14. \( \text{114.2 units}^2 \)
15. \( \text{30.4 units}^2 \)
16. \( \text{4.1 units}^2 \)
17. \( \text{26.6 units}^2 \)
18. \( \text{56.9 units}^2 \)
19. \( \text{4.1 units}^2 \)
20. \( \text{54.4 in}^2 \)
21. \( \text{271.2 units}^2 \)
22. \( \text{168.2 units}^2 \)

Answers

1. Sample answer: Separate a hexagon inscribed in a circle into six congruent nonoverlapping isosceles triangles. The area of one triangle is one-half the product of one side of the hexagon and the apothem of the hexagon. The area of the hexagon is \( \frac{1}{2}sa \). The perimeter of the hexagon is \( 6s \), so the formula is \( \frac{1}{2}Pa \).
2. Sample answer: Use the given angle measure, the given side length, and trigonometric ratios to find the missing lengths.
Reading the Lesson

How can you find the area of a regular polygon? Read the introduction to Lesson 11-3 at the top of page 610 in your textbook. Here you can find the area of a regular hexagon without a new area formula.

Enrichment

The area of a circular pool is approximately 7,850 square feet. The owner wants to replace the tiling at the edge of the pool. The edging is 6 inches wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase? 629 tiles

38. Sample answer: Multiply the total area by 40%.

Critical Thinking

Make a conjecture about how you could determine the area of the region representing the pilots who are certified to fly private airplanes.

39. a. 34π units²

40. ± 17π units²

41. ± 54.8 units²

42. ± 91.4 units²

43. ± 907.9 units²

44. ± 227.0 units²

45. ± 239.0 units²

46. ± 664.8 units²

47. ± 45.3 units²

48. ± 625,581 pilot certificates in 2000. More than 19% of the new pilots were licensed for recreational flying. See the Common Certificate Types and Pilot Populations charts in the Snapshot for more data.

See Lesson 12-2 for an exercise involving area, a property of solids. For more on area see page 640.
48. No; the areas of the floors will increase by the squares of 1, 3, 5, and 7, or 1, 9, 25, and 49. The ratio of area is the square of the scale factor.

**GARDENS** For Exercises 45–47, use the following information. The Elizabeth Park Rose Garden in Hartford, Connecticut, was designed with a gazebo surrounded by two concentric rose garden plots. Wide paths emanate from the center, dividing the garden into square and circular sections.

- **Exercise 45.** Find the area and perimeter of the entire Rose Garden. Round to the nearest tenth. 54,677.8 ft²; 899.8 ft
- **Exercise 46.** What is the total of the circumferences of the three concentric circles formed by the gazebo and the two circular rose garden plots? (Ignore the width of the rose plots and the width of the paths.) 120π ≈ 377.0 ft
- **Exercise 47.** Each rose plot has a width of 5 feet. What is the area of the path between the outer two complete circles of rose garden plots? 225π ≈ 706.9 ft²

**ARCHITECTURE** The Anraku-ji Temple in Japan is composed of four octagonal floors of different sizes that are separated by four octagonal roofs of different sizes. Refer to the information at the left. Determine whether the areas of each of the four floors are in the same ratio as their sizes. Explain. See margin.

**SIMILAR FIGURES** For Exercises 49–54, use the following information. Polygons $FGHJK$ and $VWXUZ$ are similar regular pentagons.

- **Exercise 49.** Find the scale factor. \(2:3\)
- **Exercise 50.** Find the perimeter of each pentagon.
- **Exercise 51.** Compare the ratio of the perimeters of the pentagons to the scale factor.
- **Exercise 52.** Find the area of each pentagon.
- **Exercise 53.** Compare the ratio of the areas of the pentagons to the scale factor.
- **Exercise 54.** Compare the ratio of the areas of the pentagons to the ratio of the perimeters of the pentagons. The ratio of the areas is the square of the ratio of the perimeters.
56. **Sample answer:** You can find the areas of regular polygons by finding the product of the perimeter and the apothem and then multiplying by one half. Answers should include the following:

- We need to know the length of each side and the length of the apothem.
- One method is to divide the area of the floor by the area of each tile. Since the floor is hexagonal and not rectangular, tiles of different shapes will need to be ordered to cover the floor.
**11-4 Areas of Irregular Figures**

**What You’ll Learn**
- Find areas of irregular figures.
- Find areas of irregular figures on the coordinate plane.

**Vocabulary**
- irregular figure
- irregular polygon

**How do windsurfers use area?**
The sail for a wind surf board cannot be classified as a triangle or a parallelogram. However, it can be separated into figures that can be identified, such as trapezoids and a triangle.

**IRREGULAR FIGURES** An **irregular figure** is a figure that cannot be classified into the specific shapes that we have studied. To find areas of irregular figures, separate the figure into shapes of which we can find the area. The sum of the areas of each is the area of the figure.

Auxiliary lines are drawn in quadrilateral $ABCD$. $DE$ and $DF$ separate the figure into $\triangle ADE$, $\triangle CDF$, and rectangle $BEDF$.

**Postulate 11.2**
The area of a region is the sum of all of its nonoverlapping parts.

**Example 1 Area of an Irregular Figure**

Find the area of the figure.

The figure can be separated into a rectangle with dimensions 6 units by 19 units, an equilateral triangle with sides each measuring 6 units, and a semicircle with a radius of 3 units.

Use $30^\circ$-$60^\circ$-$90^\circ$ relationships to find that the height of the triangle is $3\sqrt{3}$.

$$\text{area of irregular figure} = \text{area of rectangle} - \text{area of triangle} + \text{area of semicircle}$$

$$= lw - \frac{1}{2}bh + \frac{1}{2}\pi r^2$$

$$= 19 \cdot 6 - \frac{1}{2}(6)(3\sqrt{3}) + \frac{1}{2}\pi(3^2)$$

$$= 114 - 9\sqrt{3} + \frac{9}{2}\pi$$

$$= 112.5$$

The area of the irregular figure is 112.5 square units to the nearest tenth.

**Reading Math**
Irregular figures are also called composite figures because the regions can be separated into smaller regions.

**Study Tip**
Postulate 11.2

The area of a region is the sum of all of its nonoverlapping parts.

**Transparencies**
5-Minute Check Transparency 11-4

**Resource Manager**
Workbook and Reproducible Masters
- Chapter 11 Resource Masters
  - Study Guide and Intervention, pp. 629–630
  - Skills Practice, p. 631
  - Practice, p. 632
  - Reading to Learn Mathematics, p. 633
  - Enrichment, p. 634
  - Assessment, p. 656

- School-to-Career Masters, p. 22
- Teaching Geometry With Manipulatives Masters, p. 1

- 5-Minute Check Transparency 11-4
- Answer Key Transparencies

- Technology
  - GeomPASS: Tutorial Plus, Lesson 20
  - Interactive Chalkboard
  - Multimedia Applications: Virtual Activities
IRREGULAR FIGURES ON THE COORDINATE PLANE

Teaching Tip Watch for students who think that the area formula for a regular polygon applies to any polygon. Stress that to find the area of a polygon that is not regular, you may need to divide the polygon into shapes with known formulas. Show students both types of polygons.

3 Find the area of polygon MNPQR. 44.5 units²

Example 2 Find the Area of an Irregular Figure to Solve a Problem

FURNITURE Melissa’s dining room table has hardwood around the outside. Find the area of wood around the edge of the table.

First, draw auxiliary lines to separate the figure into regions. The table can be separated into four rectangles and four corners.

The four corners of the table form a circle with radius 3 inches.

Area of wood edge = area of rectangles + area of circle

\[ = 2lw + 2lw + \pi r^2 \]

\[ = 2(3)(60) + 2(3)(40) + \pi(3^2) \]

\[ = 360 + 240 + 9\pi \]

\[ = 628.3 \]

Use a calculator.

The area of the wood edge of the table is 628.3 square inches to the nearest tenth.

Example 3 Coordinate Plane

COORDINATE GEOMETRY Find the area of polygon RSTUV.

First, separate the figure into regions. Draw an auxiliary line from S to U. This divides the figure into triangle STU and trapezoid RSVU.

Find the difference between x-coordinates to find the length of the base of the triangle and the lengths of the bases of the trapezoid. Find the difference between y-coordinates to find the heights of the triangle and trapezoid.

area of RSTUV = area of \( \triangle STU \) + area of trapezoid RSVU

\[ = \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2) \]

Area formulas

\[ = \frac{1}{2}(6)(3) + \frac{1}{2}(7)(8 + 6) \]

Substitution

\[ = 58 \]

Simplify.

The area of RSTUV is 58 square units.

Differentiated Instruction

Kinesthetic Have your students use string, masking tape, and a tiled floor to mark off irregular shapes on the floor. Ask them to estimate the area by counting squares and then verify the estimate by calculating the sum of the individual parts.
Practice/Apply

Lesson 11-4 Areas of Irregular Figures 619

Check for Understanding
Concept Check

1. OPEN ENDED Sketch an irregular figure on a coordinate plane and find its area.

2. Describe the difference between an irregular figure and an irregular polygon. 1–2. See margin.

Find the area of each figure. Round to the nearest tenth if necessary.

3. 53.4 units²

4. 612.5 units²

COORDINATE GEOMETRY

Find the area of each figure.

5. 24 units²

6. 52.6 units²

Application

7. GATES The Roths have a series of interlocking gates to form a play area for their baby. Find the area enclosed by the wall and gates. 1247.4 in²

Practice and Apply

Find the area of each figure. Round to the nearest tenth if necessary.

8. 50 units²

9. 70.9 units²

10. 340 units²

11. 4185 units²

12. 310 units²

13. 154.1 units²

WINDOWS For Exercises 14 and 15, use the following information.

Mr. Cortez needs to replace this window in his house. The window panes are rectangles and sectors.

14. Find the perimeter of the window. 188.5 in.

15. Find the area of the window. 2236.9 in²

Answers

1. Sample answer: ≈18.3 units²

2. An irregular polygon is a polygon in which all sides are not congruent. If a shape can be separated into semicircles or smaller circular regions, it is an irregular figure.
Find the area of each figure. Round to the nearest tenth if necessary.

Example

Find the area of the shaded region. The area of the rectangle is 500 ft \( \times \) 100 ft = 50,000 ft

The area of the shaded region is given by the formula:

\[
\text{Area of shaded region} = \text{Area of rectangle} - \text{Area of triangle}
\]

The area of the triangle is \( \frac{1}{2} \times \text{base} \times \text{height} \), where the base is 50 ft and the height is 100 ft.

Therefore:

\[
\text{Area of triangle} = \frac{1}{2} \times 50 \times 100 = 2,500 \text{ ft}^2
\]

Then, subtract the area of the triangle from the area of the rectangle:

\[
\text{Area of shaded region} = 50,000 - 2,500 = 47,500 \text{ ft}^2
\]

The area of the shaded region is approximately 47,500 ft

Skills Practice, p. 631 and Practice, p. 632 (shown)

Find the area of each figure. Round to the nearest tenth if necessary:

- 400 units
- 905.5 units
- 163.2 units
- 625.4 units
- 90 units
- 23 units

LANDSCAPING For Exercises 7 and 8, use the following information:

The figure shows the dimensions of the pond and the walking path.

Find the area of each figure. Round to the nearest tenth if necessary:

- 22 cm
- 18.5 cm
- 22 cm
- 5.2 cm

EXERCISES

29.\( \text{Write the ratio of the area of } \triangle ABC\text{ to the area of square } BCDE. \ \frac{\sqrt{3}}{4}\) : 1

29. Writing in Math

Answer the question that was posed at the beginning of the lesson. See margin.

How do windsurfers use area?

Include the following in your answer:
- describe how to find the area of the sail, and
- another example of an irregular figure.

CALCULUS

For Exercises 25–27, use the following information. 26. See margin.

The irregular region under the curve has been approximated by rectangles of equal width.

25. Use the rectangles to approximate the area of the region. 462

26. Analyze the estimate. Do you think the actual area is larger or smaller than your estimate? Explain.

27. How could the irregular region be separated to give an estimate of the area that is more accurate? Sample answer: Reduce the width of each rectangle.

28. Critical Thinking

Find the ratio of the area of \( \triangle ABC\) to the area of square \( BCDE. \) \( \frac{\sqrt{3}}{4}\) : 1

29. Writing in Math

Answer the question that was posed at the beginning of the lesson. See margin.

More About...
30. In the figure consisting of squares A, B, and C, \( JK = 2KL \) and \( KL = 2LM \). If the perimeter of the figure is 66 units, what is the area? \( B \)

\[ \begin{array}{l}
A: \ 117 \text{ units}^2 \\
B: \ 189 \text{ units}^2 \\
C: \ 224 \text{ units}^2 \\
D: \ 258 \text{ units}^2 \\
\end{array} \]

31. **ALGEBRA** For all integers \( n \), \( \sqrt{n} = n^2 \) if \( n \) is odd and \( n = \sqrt{n} \) if \( n \) is even.

What is the value of \( \sqrt{16} + \sqrt{9} \)? \( C \)

\[ \begin{array}{l}
A: \ 7 \\
B: \ 25 \\
C: \ 85 \\
D: \ 97 \\
\end{array} \]

**Maintain Your Skills**

**Mixed Review**

Find the area of each shaded region. Assume that all polygons are regular unless otherwise stated. Round to the nearest tenth. (Lesson 11-3)

32. \( 42.1 \text{ units}^2 \)

33. \( 154.2 \text{ units}^2 \)

34. \( 139.1 \text{ units}^2 \)

Find the area of each figure. Round to the nearest tenth if necessary. (Lesson 11-2)

35. equilateral triangle with perimeter of 57 feet \( 156.3 \text{ ft}^2 \)

36. rhombus with a perimeter of 40 yards and a diagonal of 12 yards \( 96 \text{ yd}^2 \)

37. isosceles trapezoid with a perimeter of 90 meters if the longer base is 5 meters less than twice as long as the other base and each leg is 3 meters less than the shorter base \( \approx 384.0 \text{ m}^2 \)

38. **COORDINATE GEOMETRY** The point \((6, 0)\) is rotated 45° clockwise about the origin. Find the exact coordinates of its image. (Lesson 9-3) \( (3\sqrt{2}, -3\sqrt{2}) \)

**Getting Ready for the Next Lesson** BASIC SKILL Express each fraction as a decimal to the nearest hundredth.

39. \( \frac{5}{8} = 0.63 \)

40. \( \frac{13}{16} = 0.81 \)

41. \( \frac{9}{47} = 0.19 \)

42. \( \frac{10}{21} = 0.48 \)

**Practice Quiz 2**

Find the area of each polygon. Round to the nearest tenth. (Lesson 11-3)

1. regular hexagon with apothem length of 14 millimeters \( 679.0 \text{ mm}^2 \)

2. regular octagon with a perimeter of 72 inches \( 391.1 \text{ in}^2 \)

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth. (Lesson 11-3)

3. \( 1208.1 \text{ units}^2 \)

4. \( 216.6 \text{ units}^2 \)

5. **COORDINATE GEOMETRY** Find the area of \( CDGHJ \) with vertices \( C(-3, -2), D(1, 3), G(5, 5), H(8, 3), \) and \( J(5, -2) \). (Lesson 11-4) \( 44.5 \text{ units}^2 \)

**Answer**

29. Sample answer: Windsurfers use the area of the sail to catch the wind and stay afloat on the water. Answers should include the following.

- To find the area of the sail, separate it into shapes. Then find the area of each shape. The sum of areas is the area of the sail.

- Sample answer: Surfboards and sailboards are also irregular figures.
5-Minute Check
Transparency 11-5 Use as a quiz or review of Lesson 11-4.

Mathematical Background notes are available for this lesson on p. 592D.

How can geometric probability help you win a game of darts?
Ask students:
• What shapes are used in a dart board? circles and trapezoids
• How can you get the most points playing darts? Throw the darts into the smaller circular regions.
• How are concentric circles used in the real-world? Sample answer: a roof landing pad for a helicopter

Study Tip
Look Back To review probability with line segments, see page 20.

Vocabulary
• geometric probability
• sector
• segment

How can geometric probability help you win a game of darts?
To win at darts, you have to throw a dart at either the center or the part of the dartboard that earns the most points. In games, probability can sometimes be used to determine chances of winning. Probability that involves a geometric measure such as length or area is called geometric probability.

GEOMETRIC PROBABILITY In Chapter 1, you learned that the probability that a point lies on a part of a segment can be found by comparing the length of the part to the length of the whole segment. Similarly, you can find the probability that a point lies in a part of a two-dimensional figure by comparing the area of the part to the area of the whole figure.

Key Concept
Probability and Area

If a point in region $A$ is chosen at random, then the probability $P(B)$ that the point is in region $B$, which is in the interior of region $A$, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}.$$

When determining geometric probability with targets, we assume
• that the object lands within the target area, and
• it is equally likely that the object will land anywhere in the region.

Standardized Test Practice
Example 1 Probability with Area
Grid-In Test Item

A square game board has black and white stripes of equal width as shown. What is the chance that a dart thrown at the board will land on a white stripe?

Read the Test Item
You want to find the probability of landing on a white stripe, not a black stripe.

Resource Manager

Workbook and Reproducible Masters
Chapter 11 Resource Masters
• Study Guide and Intervention, pp. 635–636
• Skills Practice, p. 637
• Practice, p. 638
• Reading to Learn Mathematics, p. 639
• Enrichment, p. 640
• Assessment, p. 656

Prerequisite Skills Workbook, pp. 27–28, 107–108
Teaching Geometry With Manipulatives Masters, pp. 1, 18

Transparencies
5-Minute Check Transparency 11-5
Real-World Transparency 11
Answer Key Transparencies

Technology
Interactive Chalkboard
SECTORS AND SEGMENTS OF CIRCLES

Sometimes you need to know the area of a sector of a circle in order to find a geometric probability. A sector of a circle is a region of a circle bounded by a central angle and its intercepted arc.

Key Concept

If a sector of a circle has an area of \( A \) square units, a central angle measuring \( N \)°, and a radius of \( r \) units, then \( A = \frac{N}{360} \pi r^2 \).

Example 2  Probability with Sectors

a. Find the area of the blue sector.

Use the formula to find the area of the sector.

\[
A = \frac{N}{360} \pi r^2 \quad \text{Area of a sector}
\]

\[
= \frac{46}{360}(6^2) \quad N = 46, r = 6
\]

\[
= 4.6\pi \quad \text{Simplify}
\]

b. Find the probability that a point chosen at random lies in the blue region.

To find the probability, divide the area of the sector by the area of the circle. The area of the circle is \( \pi r^2 \) with a radius of 6.

\[
P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}} \quad \text{Geometric probability formula}
\]

\[
= \frac{4.6\pi}{\pi \cdot 6^2} \quad \text{Area of sector} = 4.6\pi, \text{ area of circle} = \pi \cdot 6^2
\]

\[
= 0.13 \quad \text{Use a calculator}
\]

The probability that a random point is in the blue sector is about 0.13 or 13%.

Solve the Test Item

We need to divide the area of the white stripes by the total area of the game board. Extend the sides of each stripe. This separates the square into 36 small unit squares.

The white stripes have an area of 15 square units.

The total area is 36 square units.

The probability of tossing a chip onto the white stripes is \( \frac{15}{36} \) or \( \frac{5}{12} \).

Fill in the Grid

Write \( \frac{5}{12} \) as 5/12 in the top row of the grid-in.

Then shade in the appropriate bubble under each entry.

1 GRID IN A game board consists of a circle inscribed in a square. What is the chance that a dart thrown at the board will land in the shaded area?

~~0.215~~

SECTORS AND SEGMENTS OF CIRCLES

A regular hexagon is inscribed in a circle with a diameter of 12.

a. Find the area of the shaded regions. \( \approx 9.78 \text{ units}^2 \)

b. Find the probability that a point chosen at random lies in the shaded regions. 0.087 or 8.7%
The region of a circle bounded by an arc and a chord is called a segment of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.

**Example 3 Probability with Segments**

A regular hexagon is inscribed in a circle with a diameter of 14.

a. Find the area of the red segment.

**Area of the sector:**

\[
A = \frac{N}{360} \pi r^2 \quad \text{Area of a sector}
\]

\[
= \frac{60}{360} \pi (7^2) \quad N = 60, r = 7
\]

\[
= \frac{49}{6} \pi \quad \text{Simplify.}
\]

\[
= 25.66 \quad \text{Use a calculator.}
\]

**Area of the triangle:**

Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 7 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of \(x\) is 3.5, the apothem is \(x\sqrt{3}\) or \(3.5\sqrt{3}\), which is approximately 6.06.

Next, use the formula for the area of a triangle.

\[
A = \frac{1}{2}bh \quad \text{Area of a triangle}
\]

\[
= \frac{1}{2}(7)(6.06) \quad b = 7, h = 6.06
\]

\[
= 21.22 \quad \text{Simplify.}
\]

**Area of the segment:**

\[
\text{area of segment} = \text{area of sector} - \text{area of triangle}
\]

\[
= 25.66 - 21.22 \quad \text{Substitution}
\]

\[
= 4.44 \quad \text{Simplify.}
\]

b. Find the probability that a point chosen at random lies in the red region.

Divide the area of the sector by the area of the circle to find the probability. First, find the area of the circle. The radius is 7, so the area is \(\pi(7^2)\) or about 153.94 square units.

\[
P(\text{blue}) = \frac{\text{area of segment}}{\text{area of circle}}
\]

\[
= \frac{4.44}{153.94}
\]

\[
= 0.03
\]

The probability that a random point is on the red segment is about 0.03 or 3%.

---

**Auditory/Musical**

Ask students to name the similarities between probability and geometric probability. The similarity is that the probability is still found by dividing the favorable outcomes by the total number of outcomes. In geometric probability, the favorable outcome is an area or a length, and the total number of outcomes is an area or a length.
Lesson 11-5
Geometric Probability

1. Multiply the measure of the central angle of the sector by the area of the circle and then divide the product by 360°.
2. OPEN ENDED  List three games that involve geometric probability.
3. FIND THE ERROR  Rachel and Taimi are finding the probability that a point chosen at random lies in the green region.

Rachel:  
\[ A = \frac{N}{360} \pi r^2 \]  
\[ = \frac{62}{360} \pi (5^2) \]  
\[ = 26.4 \]  
\[ P(\text{green}) = \frac{26.4}{85} = 0.31 \]

Taimi:  
\[ A = \frac{N}{360} \pi r^2 \]  
\[ = \frac{59}{360} \pi (5^2) + \frac{62}{360} \pi (5^2) \]  
\[ = 13.0 \]  
\[ P(\text{green}) = \frac{13.0}{25} = 0.52 \]

Who is correct? Explain your answer.  **Rachel:** Taimi did not multiply \( \frac{62}{360} \) by the area of the circle.

Find the area of the blue region. Then find the probability that a point chosen at random will be in the blue region.

4. \( 112^\circ \)  
\[ \approx 17.5 \text{ units}^2, \quad 0.22 \]
5. \( 45^\circ \)  
\[ \approx 114.2 \text{ units}^2, \quad 0.36 \]

6. What is the chance that a point chosen at random lies in the shaded region?  \( \frac{3}{5} \) or 0.6

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 15 centimeters.

9.  
\[ \text{pink} \]  
\[ \approx 35.3 \text{ units}^2, \quad 0.20 \]
10.  
\[ \text{blue} \]  
\[ \approx 58.9 \text{ units}^2, \quad 0.3 \]
11.  
\[ \approx 66.3 \text{ units}^2, \quad 0.375 \]
12.  
\[ \text{purple} \]

**About the Exercises...**

**Organization by Objective**
- Geometric Probability: 7–9, 16, 24–30
- Sectors and Segments of Circles: 10–15, 17–23

**Odd/Even Assignments**
Exercises 7–30 are structured so that students practice the same concepts whether they are assigned odd or even problems.

**Assignment Guide**
- **Basic:** 7–31 odd, 32–45
- **Average:** 7–31 odd, 32–45
- **Advanced:** 8–30 even, 31–45

**Unlocking Misconceptions**
A common error is to believe that you can estimate a geometric probability by looking at the figure. Remind students to be careful that they calculate all areas by using formulas, and then compare the calculated probability and the estimate.
Find the area of the indicated sector. Then find the probability of choosing the color indicated if the diameter of each spinner is 15 centimeters.

14. red \approx 19.6 \text{ units}^2, \approx \frac{0.1}{15}
14. green
15. yellow

16. PARACHUTES A skydiver must land on a target of three concentric circles. The diameter of the center circle is 2 yards, and the circles are spaced 1 yard apart. Find the probability that she will land on the shaded area. \approx \frac{25.8 \text{ units}^2}{15}

**SURVEYS** For Exercises 20–23, use the following information.
A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favorite colors. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.

20. Favorite color is red. \approx 0.08
21. Favorite color is blue or green. \approx 0.68
22. Favorite color is not red or blue. \approx 0.51
23. Favorite color is not orange or green. \approx 0.68

**TENNIS** For Exercises 24 and 25, use the following information.
A tennis court has stripes dividing it into rectangular regions. For singles play, the inbounds region is defined by segments \(AB\) and \(CD\). The doubles court is bound by the segments \(EF\) and \(GH\).

24. Find the probability that a ball in a singles game will land inside the court, but out of bounds. \approx 0.31
25. When serving, the ball must land within \(AXYZ\), the service box. Find the probability that a ball will land in the service box, relative to the court. \approx 0.19

**Polygon Probability**
Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you must first find the total area of the polygon. If you wish to find the area, round your answer to the nearest hundredth.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answers to the nearest hundredth.

**Reading to Learn Mathematics**
**ELL**

- **Pre-Activity** How can geometric probability help you win a game of darts?
  - Use the introduction to Section 11.5 at the top of page 636 in your textbook.
  - Find the probability of winning at darts, by using your geometric probability to compute areas or lengths.

**Reading the Lesson**
1. Explain the difference between a sector of a circle and a segment of a circle. Sample answer: A sector of a circle is bounded by a central angle and its intercepted arc, while a segment is bounded by an arc and a chord.
2. Suppose you are playing a game of darts with a target like the one shown at the right. If you fire dart inside circle labeled \(C\), you get a point. Assume that every dart will land within the circle. The radius of the circle is \(r\). Complete the following steps to figure out the probability of getting a point.
   - The area of sector \(A\) is \(\frac{\theta}{360^\circ} \pi r^2\).
   - The area of circle \(C\) is \(\pi r^2\).
   - The probability that the dart will fall inside the sector is \(\frac{\frac{\theta}{360^\circ} \pi r^2}{\pi r^2} = \frac{\theta}{360^\circ}\).
   - Therefore, the probability that the dart will fall inside the triangle is \(\frac{2\theta}{360^\circ} = \frac{\theta}{180^\circ}\) or \(\frac{\theta}{\pi r^2}\).

- **Helping You Remember**
  - Many students find it difficult to remember a large number of geometric formulas. Here is a way you can use the formula for the area of a circle to find the area of a sector of a circle that is not regular by comparing areas or lengths.
  - Sample answer: First use \(A = \pi r^2\) to find the area of the circle. Then use the measure of the central angle to find the area of the sector. Multiply the area of the circle by this fraction and you will have the area of the sector.

**Study Guide and Intervention, p. 635 (shown) and p. 636**

**Geometric Probability**
<table>
<thead>
<tr>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>The probability that a dart lands in a particular region of a dartboard is the ratio of the area of the region to the area of the board.</td>
</tr>
<tr>
<td>Area of a Circle</td>
<td>(A = \pi r^2)</td>
</tr>
<tr>
<td>Area of a Sector</td>
<td>(A = \frac{\theta}{360^\circ} \pi r^2)</td>
</tr>
</tbody>
</table>

**Skills Practice, p. 637 and Practice, p. 638**

- **Find the probability that a point chosen at random lies in the shaded region**.
- **Find the area of the indicated sector**.
- **Calculate the probability that a point chosen at random lies in the shaded region**.

**More About...**

- **Tennis**
  - In tennis, the linesman determines whether the hit ball is in or out. The umpire may only overrule the linesman if he or she immediately thinks the call was wrong without a doubt and never as a result of a player’s protest.
  - Source: www.usatennis.com
DARTS  For Exercises 26–30, use the following information.
Each sector of the dartboard has congruent central angles. Find the probability that the dart will land on the indicated color. The diameter of the center circle is 5 feet.
26. black  = 0.29  27. white  = 0.29  28. red  = 0.43
29. Point values are assigned to each color. Should any of the colors have the same point value? Explain. See margin.  30. Which color should have the lowest point value? Explain. See margin.
31a–b. See margin.

Critical Thinking  Study each spinner in Exercises 13–15.
a. Are the chances of landing on each color equal? Explain.
b. Would this be considered a fair spinner to use in a game? Explain.

Writing in Math  Answer the question that was posed at the beginning of the lesson. See margin.
How can geometric probability help you win a game of darts?
Include the following in your answer:
• an explanation of how to find the geometric probability of landing on a red sector, and
• an explanation of how to find the geometric probability of landing in the center circle.

Standardized Test Practice
33. One side of a square is a diameter of a circle. The length of one side of the square is 5 feet. To the nearest hundredth, what is the probability that a point chosen at random is in the shaded region? C
A 0.08  B 0.22  C 0.44  D 0.77
34. ALGEBRA  If 4y = 16, then 12 ÷ y = C

Maintain Your Skills
Mixed Review
Find the area of each figure. Round to the nearest tenth, if necessary. (Lesson 11-4)
35. 1050 units²
36. 82.9 ft²

Find the area of each polygon. Round to the nearest tenth, if necessary. (Lesson 11-3)
37. a regular triangle with a perimeter of 48 feet 110.9 ft²
38. a square with a side length of 21 centimeters 441 cm²
39. a regular hexagon with an apothem length of 8 inches 221.7 in²

ALGEBRA  Find the measure of each angle on ∠ABC with diameter AC. (Lesson 10-2)
40. ∠AFB  41. ∠CFD  42. ∠AFD  43. ∠DFB
40. 92°  41. 72°  42. 114°  43. 24°

Find the length of the third side of a triangle given the measures of two sides and the included angle of the triangle. Round to the nearest tenth. (Lesson 7-7)
44. m = 6.8, n = 11.1, m∠P = 57  p = 9.3
45. f = 32, h = 29, m∠G = 41  g = 21.5

Open-Ended Assessment
Modeling  Have students design a gameboard. Ask them to draw a shaded region in polygons or squares whose area can be found. Then calculate the area of the shaded region and the corresponding probability.

Assessment Options
Quiz (Lesson 11-5) is available on p. 656 of the Chapter 11 Resource Masters.

Answers
29. The chances of landing on a black or white sector are the same, so they should have the same point value.
30. Of the three colors, there is the highest probability of landing on red, so red should have a lower point value than white or black.
31a. No; each colored sector has a different central angle.
31b. No; there is not an equal chance of landing on each color.
32. Sample answer: Geometric probability can help you determine the chance of a dart landing on the bullseye or high scoring sector. Answers should include the following.
• Find the area of the circles containing the red sector. Divide the difference by the area of the larger circle.
• Find the area of the center circle and divide by the area of the largest circle on the board.

www.geometryonline.com/self_check_quiz
Lesson 11-5  Geometric Probability 627
Chapter 11
Areas of Polygons and Circles

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 11 includes a page reference where each term was introduced.
- Assessment A vocabulary test/review for Chapter 11 is available on p. 654 of the Chapter 11 Resource Masters.

Lesson-by-Lesson Review

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students’ mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

11-1 Area of Parallelograms

Concept Summary

- The area of a parallelogram is the product of the base and the height.

Example

Find the area of \( \text{\#GHJK} \).

The area of a parallelogram is given by the formula \( A = bh \).

\[
A = bh
\]

\[
A = 14(9) \text{ or } 126
\]

The area of the parallelogram is 126 square units.

Exercises

Find the perimeter and area of each parallelogram.

See Example 1 on page 596.

7. \( 78 \text{ ft, } 318.7 \text{ ft}^2 \)

8. \( 116 \text{ mm, } 396 \text{ mm}^2 \)

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a square, a rectangle, or a parallelogram.

Then find the area of the quadrilateral. See Example 3 on page 597.

9. \( A(−6, 1), B(1, 1), C(1, −6), D(−6, −6) \) square; 49 units²

10. \( E(7, −2), F(1, −2), G(2, 2), H(8, 2) \) parallelogram; 24 units²

11. \( J(−1, −4), K(−5, 0), L(−5, 5), M(−1, 1) \) parallelogram; 20 units²

12. \( P(−7, −1), Q(−3, 3), R(−1, 1), S(−5, −3) \) rectangle; 16 units²

For more information about Foldables, see Teaching Mathematics with Foldables.

Have students look through the chapter to make sure they have included notes and examples in their Foldables for each lesson of Chapter 11.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

For more information about Foldables, see Teaching Mathematics with Foldables.
11-2 Areas of Triangles, Rhombi, and Trapezoids

Concept Summary
- The formula for the area of a triangle can be used to find the areas of many different figures.
- Congruent figures have equal areas.

Example
Trapezoid $MNPQ$ has an area of 360 square feet. Find the length of $MN$.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$360 = \frac{1}{2}(18)(b_1 + 26)$ $A = 360, h = 18, b_2 = 26$

$360 = 9b_1 + 234$ Multiply.

$14 = b_1$ Solve for $b_1$.

The length of $MN$ is 14 feet.

Exercises
Find the missing measure for each quadrilateral. See Example 4 on page 604.
13. Triangle $CDE$ has an area of 336 square inches. Find $CE$. 28 in.
14. Trapezoid $GHJK$ has an area of 75 square meters. Find the height. 5 m

11-3 Areas of Regular Polygons and Circles

Concept Summary
- A regular $n$-gon is made up of $n$ congruent isosceles triangles.
- The area of a circle of radius $r$ units is $\pi r^2$ square units.

Example
Find the area of a regular hexagon with a perimeter of 72 feet.
Since the perimeter is 72 feet, the measure of each side is 12 feet. The central angle of a hexagon is 60°. Use the properties of $30^\circ$-$60^\circ$-$90^\circ$ triangles to find that the apothem is $6\sqrt{3}$ feet.

$A = \frac{1}{2}Pa$ Area of a regular polygon

$= \frac{1}{2}(72)(6\sqrt{3})$ $P = 72, a = 6\sqrt{3}$

$= 216\sqrt{3}$ Simplify.

$= 374.1$ The area of the regular hexagon is 374.1 square feet to the nearest tenth.

Exercises
Find the area of each polygon. Round to the nearest tenth. See Example 1 on page 611.
15. a regular pentagon with perimeter of 100 inches 688.2 in$^2$
16. a regular decagon with side length of 12 millimeters 1108.0 mm$^2$
Areas of Irregular Figures

Concept Summary

- The area of an irregular figure is the sum of the areas of its nonoverlapping parts.

**Example**

Find the area of the figure.

Separate the figure into a rectangle and a triangle.

\[
\text{area of irregular figure} = \text{area of rectangle} - \text{area of semicircle} + \text{area of triangle}
\]

\[
= lw - \frac{1}{2} \pi r^2 + \frac{1}{2}bh
\]

Area formulas

\[
= (6)(8) - \frac{1}{2} \pi (4^2) + \frac{1}{2}(8)(8)
\]

Substitution

\[
= 48 - 8\pi + 32 \text{ or about 54.9} \text{ Simplify.}
\]

The area of the irregular figure is 54.9 square units to the nearest tenth.

**Exercises** Find the area of each figure to the nearest tenth. See Example 1 on page 617.

17. 31.1 units^2
18. 87.5 units^2

Geometric Probability

Concept Summary

- To find a geometric probability, divide the area of a part of a figure by the total area.

**Example**

Find the probability that a point chosen at random will be in the blue sector.

First find the area of the blue sector.

\[
A = \frac{N}{360} \pi r^2 \quad \text{Area of a sector}
\]

\[
= \frac{104}{360} \pi (8^2) \text{ or about 58.08 Substitute and simplify.}
\]

To find the probability, divide the area of the sector by the area of the circle.

\[
P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}} \quad \text{Geometric probability formula}
\]

\[
\approx 0.29 \text{ The probability is about 0.29 or 29%}
\]

**Exercises** Find the probability that a point chosen at random will be in the sector of the given color. See Example 2 on page 623.

19. red 0.3
20. purple or green \approx 0.27
13. The indicated color.

Each spinner has a diameter of 12 inches. Find the probability of spinning a red spinner.

14. orange

15. green

Find the area of each figure. Round to the nearest tenth.

16. 474 units²

17. 91.2 units²

18. 87.5 units²

19. SOCCER BALLS

The surface of a soccer ball is made of a pattern of regular pentagons and hexagons. If each hexagon on a soccer ball has a perimeter of 9 inches, what is the area of a hexagon? 5.8 in²

20. STANDARDIZED TEST PRACTICE

What is the area of a quadrilateral with vertices at (−3, −1), (−1, 4), (7, 4), and (5, −1)?

- A 50 units²
- B 45 units²
- C 8√29 units²
- D 40 units²

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Portfolio Suggestion

Introduction

Areas of irregular figures are used in architecture.

Ask Students

Ask students to design the floor plan of an elaborate house or garden. Challenge them to include rooms of many different shapes. Have them find the area of each room, showing their calculations. Have students add their designs and area calculations to their portfolios.
Part 1  Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Solve \(3 \cdot \frac{2x - 4}{-6} = 18\). (Prerequisite Skill) B
   - A -19
   - B -16
   - C 4
   - D 12

2. Sam rode his bike along the path from the library to baseball practice. What type of angle did he form during the ride? (Lesson 1-5) B
   - A straight
   - B obtuse
   - C acute
   - D right

3. What is the logical conclusion of these statements?
   If you exercise, you will maintain better health.
   If you maintain better health, you will live longer. (Lesson 2-4) A
   - A If you exercise, you will live longer.
   - B If you do not exercise, you will not live longer.
   - C If you do not exercise, you will not maintain better health.
   - D If you maintain better health, you will not live longer.

4. Which segments are parallel? (Lesson 3-5) C
   - A \(\overline{AB}\) and \(\overline{CD}\)
   - B \(\overline{AD}\) and \(\overline{BC}\)
   - C \(\overline{AE}\) and \(\overline{BC}\)

5. The front view of a pup tent resembles an isosceles triangle. The entrance to the tent is an angle bisector. The tent is secured by stakes. What is the distance between the two stakes? (Lesson 5-1) D
   - A 3 ft
   - B 4 ft
   - C 5 ft
   - D 6 ft

6. A carpenter is building steps leading to a hexagonal gazebo. The outside edges of the steps need to be cut at an angle. Find \(x\). (Lesson 8-1) D
   - A 180
   - B 120
   - C 72
   - D 60

7. Which statement is always true? (Lesson 10-4) A
   - A When an angle is inscribed in a circle, the angle's measure equals one-half of the measure of the intercepted arc.
   - B In a circle, an inscribed quadrilateral will have consecutive angles that are supplementary.
   - C In a circle, an inscribed angle that intercepts a semicircle is obtuse.
   - D If two inscribed angles of a circle intercept congruent arcs, then the angles are complementary.

8. The apothem of a regular hexagon is 7.8 centimeters. If the length of each side is 9 centimeters, what is the area of the hexagon? (Lesson 11-3) C
   - A 35.1 cm²
   - B 70.2 cm²
   - C 210.6 cm²
   - D 421.2 cm²

ExamView® Pro

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and state proficiency tests can be found on this CD-ROM.

632 Chapter 11 Areas of Polygons and Circles
**Part 2 Short Response/Grid In**

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The post office is located halfway between the fire station and the library. What are the coordinates of the post office? *(Lesson 1-3)* *(2, 0)*

10. What is the slope of a line perpendicular to the line represented by the equation $3x - 6y = 12$? *(Lesson 3-3)* $-2$

11. $\triangle RST$ is a right triangle. Find $m \angle R$. *(Lesson 4-2)* $60$

12. If $\angle A$ and $\angle E$ are congruent, find $AB$, the distance in feet across the pond. *(Lesson 6-3)* $140$

13. If point $J(6, -3)$ is translated 5 units up and then reflected over the y-axis, what will the new coordinates of $J'$ be? *(Lesson 9-2)* $(-6, 2)$

14. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest.

**Part 3 Extended Response**

Record your answers on a sheet of paper. Show your work.

14. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest.

**Test-Taking Tip**

Question 4
To find the pair of parallel lines, first you need to find the missing angle measures. Use the Angle Sum Theorem to find the measures of the angles in each triangle.

15. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(3, 4)$, and $C(8, 4)$.

a. Find the possible coordinates for $D$. *(Lesson 8-2)* $(5, 0)$

b. Find the area of $ABCD$. *(Lesson 11-1)* $20$ units$^2$

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**Score Criteria**

<table>
<thead>
<tr>
<th>Score</th>
<th>Criteria</th>
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<tbody>
<tr>
<td>4</td>
<td>A correct solution that is supported by well-developed, accurate explanations</td>
</tr>
<tr>
<td>3</td>
<td>A generally correct solution, but may contain minor flaws in reasoning or computation</td>
</tr>
<tr>
<td>2</td>
<td>A partially correct interpretation and/or solution to the problem</td>
</tr>
<tr>
<td>1</td>
<td>A correct solution with no supporting evidence or explanation</td>
</tr>
<tr>
<td>0</td>
<td>An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given</td>
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