UNIT 4 Notes

Introduction

In this unit students learn to calculate measures in two and three dimensions: area, surface area, and volume. They find the areas of triangles and several types of quadrilaterals in addition to regular polygons, circles, and irregular figures. Students make models of three-dimensional figures and find surface area. Geometric probability is also explored.

Three-dimensional figures are investigated further as students learn to find volume. They also identify congruent or similar solids and graph solids in space.

About the Photographs The large photograph is of the Korean War Veterans Memorial located in Washington, D.C., across the reflecting pool from the Vietnam Veterans Memorial. The smaller photo is a sketch of the National World War II Memorial also being constructed in Washington, D.C., scheduled to open in spring 2004.

Assessment Options

Unit 4 Test Pages 773–774 of the *Chapter 13 Resource Masters* may be used as a test or review for Unit 4. This assessment contains both multiple-choice and short answer items.

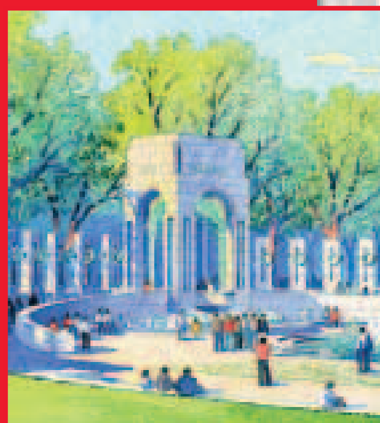
ExamView® Pro

This CD-ROM can be used to create additional unit tests and review worksheets.

UNIT 4

Area and Volume

Area and volume can be used to analyze real-world situations. In this unit, you will learn about formulas used to find the areas of two-dimensional figures and the surface areas and volumes of three-dimensional figures.



Yearly Progress Pro

An online, research-based, instructional, assessment, and intervention tool that provides specific feedback on student mastery of state and national standards, instant remediation, and a data management system to track performance. For more information, contact mhdigitallearning.com.

What's MATH Got To Do With It?

Real-Life Geometry Videos

What's Math Got to Do With It? Real-Life Geometry Videos engage students, showing them how math is used in everyday situations. Use Video 4 with this unit.

WebQuest Internet Project

Town With Major D-Day Losses Gets Memorial

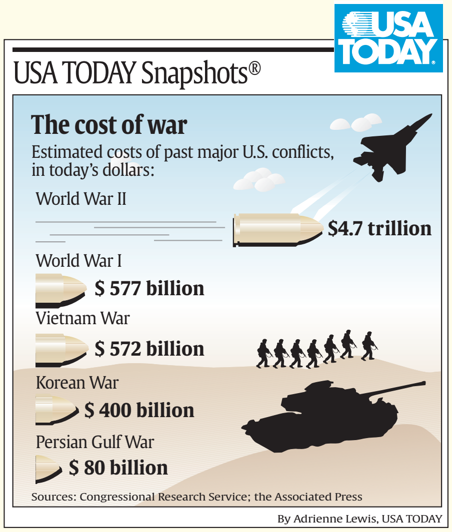
Source: *USA TODAY,* May 27, 2001

"BEDFORD, Va. For years, World War II was a sore subject that many families in this small farming community avoided. 'We lost so many men,' said Boyd Wilson, 79, who joined Virginia's 116th National Guard before it was sent to war. 'It was just painful.' The war hit Bedford harder than perhaps any other small town in America, taking 19 of its sons, fathers and brothers in the opening moments of the Allied invasion of Normandy. Within a week, 23 of Bedford's 35 soldiers were dead. It was the highest per capita loss for any U.S. community." In this project, you will use scale drawings, surface area, and volume to design a memorial to honor war veterans.

Log on to www.geometryonline.com/webquest. Begin your WebQuest by reading the Task.

Continue working on your WebQuest as you study Unit 4.

Lesson	11-4	12-5	13-3
Page	618	662	703



USA TODAY Teaching Suggestions

Have students study the USA TODAY Snapshot.

- Ask them which two wars cost about the same. **World War I and the Vietnam War**
- Discuss with students why World War II cost so much more than the other U.S. conflicts.
- Point out to students that there is often a contest to choose a design for a memorial. The choice of Maya Ying Lin's design for the Vietnam Veterans Memorial Wall surprised many people because the design was so simple. Today it is one of the most admired and visited memorials in the nation.

Additional USA TODAY Snapshots appearing in Unit 4:

Chapter 11 Who's licensed to fly (p. 614)
Chapter 12 College kids plug in for fun (p. 653)
Chapter 13 Every day is mom's day (p. 705)

WebQuest Internet Project

Problem-Based Learning A WebQuest is an online project in which students do research on the Internet, gather data, and make presentations using word processing, graphing, page-making, or presentation software. In each chapter, students advance to the next step in their WebQuest. At the end of Chapter 13, the project culminates with a presentation of their findings.

Teaching notes and sample answers are available in the *WebQuest and Project Resources.*

Chapter 11

Areas of Polygons and Circles

Chapter Overview and Pacing

Year-long pacing: pages T20–T21.

LESSON OBJECTIVES

	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
11-1 **Areas of Parallelograms** *(pp. 595–600)* • Find perimeters and areas of parallelograms. • Determine whether points on a coordinate plane define a parallelogram.	1	1	0.5	0.5
11-2 **Areas of Triangles, Trapezoids, and Rhombi** *(pp. 601–609)* • Find areas of triangles. • Find areas of trapezoids and rhombi.	2	2	1	1
11-3 **Areas of Regular Polygons and Circles** *(pp. 610–616)* • Find areas of regular polygons. • Find areas of circles.	2	2	1	1
11-4 **Areas of Irregular Figures** *(pp. 617–621)* • Find areas of irregular figures. • Find areas of irregular figures on the coordinate plane.	2	2	1	1
11-5 **Geometric Probability** *(pp. 622–627)* • Solve problems involving geometric probability. • Solve problems involving sectors and segments of circles.	1	1	0.5	0.5
Study Guide and Practice Test *(pp. 628–631)* **Standardized Test Practice** *(p. 632–633)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**10**	**10**	**5**	**5**

An electronic version of this chapter is available on ***StudentWorks™****. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.*

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T5 and T21.

Chapter Resource Manager

Chapter 11 Resource Masters

	Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Prerequisite Skills Workbook	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	GeomPASS: Tutorial Plus (lessons)	Materials
	611–612	613–614	615	616			GCC 37, 38 SC 21	11-1	11-1		straightedge, grid paper
	617–618	619–620	621	622	655			11-2	11-2		calculator, grid paper, straightedge
	623–624	625–626	627	628	655, 657	43–44		11-3	11-3		
	629–630	631–632	633	634	656		SC 22	11-4	11-4	20	grid paper, straightedge
	635–636	637–638	639	640	656	27–28, 107–108		11-5	11-5		
					641–654, 658–660						

**Key to Abbreviations:* GCC = Graphing Calculator and Computer Masters
SC = School-to-Career Masters

Chapter 11

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students found the area of a rectangle and evaluated expressions containing variables in previous courses. In Chapter 7, they worked with triangles, finding the height of a triangle and solving for the missing side of a right triangle. They also used trigonometric ratios in that chapter to solve triangles.

This Chapter

In this chapter, students find areas of parallelograms, rhombi, trapezoids, and triangles. They identify the apothem of a regular polygon and use that measure to find the areas of regular polygons. They also find the areas of irregular figures, circles, and sectors and segments of circles. Students determine geometric probability, which is a probability that involves a geometric measure.

Future Connections

In Chapter 12, students use the formulas for finding area to calculate the surface area of prisms and other three-dimensional figures. In Chapter 13, students will extend their understanding of area when they find the volumes of prisms, spheres, and other figures.

11-1 Areas of Parallelograms

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, there is a corresponding altitude that is perpendicular to the base. The altitude corresponds to the height of the parallelogram. If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.

To find the area of a quadrilateral on the coordinate plane, you must first determine whether the figure is a parallelogram. You can use the formula for slope to determine whether opposite sides are parallel. Then you find the measures of the base and height and use those to calculate the area.

11-2 Areas of Triangles, Trapezoids, and Rhombi

The formula for the area of a triangle is related to the formula for the area of a parallelogram or rectangle. It is $A = \frac{1}{2}bh$. This formula, in turn, yields the formulas for the areas of trapezoids and rhombi.

If a trapezoid has an area of A square units, bases of b_1 units and b_2 units, and a height of h units, then $A = \frac{1}{2}h(b_1 + b_2)$. If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.

11-3 Areas of Regular Polygons and Circles

A regular polygon can be divided into congruent isosceles triangles by drawing a line from each vertex to the center of the polygon. The altitude of one of these triangles is called an *apothem*. The area of the polygon can be determined by adding the areas of the triangles. If a polygon has a side of s units and an apothem of a units, then the area of one of these triangles is $\frac{1}{2}sa$. By multiplying this formula by the number of sides and substituting P for the formula for perimeter contained in the result, you will find that the formula for the area of a regular polygon is $A = \frac{1}{2}Pa$.

The area of a circle cannot be found without the value known as π. If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$. You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.

11-4 Areas of Irregular Figures

An irregular figure is a figure that cannot be classified into the specific shapes that the student has studied. To find the areas of irregular figures, separate the figures into shapes for which you can find the area. The area of the irregular figure is the sum of the areas of these separate shapes.

The formula for the area of a regular polygon does not apply to an irregular polygon. To find the area of an irregular polygon, separate the polygon into figures which have areas that can be calculated easily.

11-5 Geometric Probability

Probability that involves a geometric measure such as length or area is called geometric probability. You can find the probability that a point lies in part of a figure by comparing the area of the part to the area of the whole figure. If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is $P(B) = \frac{\text{area of region } B}{\text{area of region } A}$. When determining the geometric probability with targets, assume that the object lands within the target area. You should also assume that it is equally likely that the object will land anywhere in the region.

Sometimes you need to know the area of a sector of a circle to find a geometric probability. A sector is a region of a circle bounded by a central angle and its intercepted arc. If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$. The region of a circle bounded by an arc and a chord is called a *segment* of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.

Chapter 11

DAILY INTERVENTION and Assessment

Key to Abbreviations:
TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

	Type	Student Edition	Teacher Resources	Technology/Internet
INTERVENTION	Ongoing	Prerequisite Skills, pp. 593, 600, 609, 616, 621 Practice Quiz 1, p. 609 Practice Quiz 2, p. 621	5-Minute Check Transparencies *Prerequisite Skills Workbook*, pp. 27–28, 43–44, 107–108 Quizzes, *CRM* pp. 655–656 Mid-Chapter Test, *CRM* p. 657 Study Guide and Intervention, *CRM* pp. 611–612, 617–618, 623–624, 629–630, 635–636	GeomPASS: Tutorial Plus, Lesson 20 www.geometryonline.com/self_check_quiz www.geometryonline.com/extra_examples
	Mixed Review	pp. 600, 609, 616, 621, 627	Cumulative Review, *CRM* p. 658	
	Error Analysis	Find the Error, pp. 605, 625 Common Misconceptions, p. 623	Find the Error, *TWE* pp. 605, 624 Unlocking Misconceptions, *TWE* p. 625 Tips for New Teachers, *TWE* pp. 596, 602	
ASSESSMENT	Standardized Test Practice	pp. 600, 608, 616, 621, 622, 625, 627, 631, 632, 633	*TWE* pp. 632–633 Standardized Test Practice, *CRM* pp. 659–660	Standardized Test Practice CD-ROM www.geometryonline.com/standardized_test
	Open-Ended Assessment	Writing in Math, pp. 600, 608, 616, 620, 627 Open Ended, pp. 598, 605, 613, 619, 625 Standardized Test, p. 633	Modeling: *TWE* pp. 616, 627 Speaking: *TWE* p. 600 Writing: *TWE* pp. 609, 621 Open-Ended Assessment, *CRM* p. 653	
	Chapter Assessment	Study Guide, pp. 628–630 Practice Test, p. 631	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 641–646 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 647–652 Vocabulary Test/Review, *CRM* p. 654	ExamView® Pro (see below) MindJogger Videoquizzes www.geometryonline.com/vocabulary_review www.geometryonline.com/chapter_test

Yearly ProgressPro

For more information on Yearly ProgressPro, see p. 590.

Geometry Lesson	Yearly ProgressPro Skill Lesson
11-1	Areas of Parallelograms
11-2	Areas of Triangles, Trapezoids, and Rhombi
11-3	Areas of Regular Polygons and Circles
11-4	Areas of Irregular Figures
11-5	Geometric Probability

ExamView® Pro

Use the networkable **ExamView® Pro** to:
- Create **multiple versions** of tests.
- Create **modified** tests for *Inclusion* students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your test from a program bank of artwork.

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Geometry provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 593
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 598, 605, 613, 619, 625)
- Reading Mathematics, p. 594
- Writing in Math questions in every lesson, pp. 600, 608, 616, 620, 627
- Reading Study Tip, p. 617
- WebQuest, p. 618

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 592, 628
- Study Notebook suggestions, pp. 594, 598, 605, 613, 619, 624
- Modeling activities, pp. 616, 627
- Speaking activities, p. 600
- Writing activities, pp. 609, 621
- ELL Resources, pp. 592, 594, 599, 606, 614, 620, 626, 628

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 11 Resource Masters,* pp. vii-viii)
- Proof Builder helps students learn and understand theorems and postulates from the chapter. (*Chapter 11 Resource Masters,* pp. ix–x)
- Reading to Learn Mathematics master for each lesson (*Chapter 11 Resource Masters*, pp. 615, 621, 627, 633, 639)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading Strategies for the Mathematics Classroom*
- *WebQuest and Project Resources*

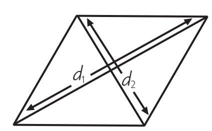
For more information on Reading and Writing in Mathematics, see pp. T6–T7.

PROJECT CRISS Study Skill

Study cards can be a helpful study aid for students learning definitions and formulas. The cards at the right show some common shapes and the formulas to find the areas of each. A name and sketch is on the front of the card and the area formula is on the back of the card.

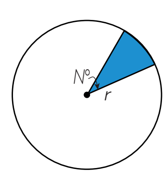
As students work through the chapter, have them make study cards for other concepts and topics. They can use the cards to quiz themselves or each other to help prepare for tests.

Rhombus



$$A = \frac{1}{2}d_1d_2$$

Sector



$$A = \frac{N}{360}\pi r^2$$

CReating Independence Through Student-Owned Strategies

Chapter 11 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
11-1	3, 6, 8, 9, 10	
11-2	3, 6, 8, 9, 10	
11-3	3, 6, 8, 9, 10	
11-4	3, 6, 8, 9, 10	
11-5	3, 5, 6, 8, 9, 10	

Key to NCTM Standards:
1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 11 Areas of Polygons and Circles

What You'll Learn

- **Lessons 11-1, 11-2, and 11-3** Find areas of parallelograms, triangles, rhombi, trapezoids, regular polygons, and circles.
- **Lesson 11-4** Find areas of irregular figures.
- **Lesson 11-5** Find geometric probability and areas of sectors and segments of circles.

Key Vocabulary

- **apothem** (p. 610)
- **irregular figure** (p. 617)
- **geometric probability** (p. 622)
- **sector** (p. 623)
- **segment** (p. 624)

Why It's Important

Skydivers use geometric probability when they attempt to land on a target marked on the ground. They can determine the chances of landing in the center of the target. *You will learn about skydiving in Lesson 11-5.*

Ken Fisher/Getty Images

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 11 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 11 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 11.

For Lesson 11-1 — Area of a Rectangle

The area and width of a rectangle are given. Find the length of the rectangle.
(For review, see pages 732–733.)

1. $A = 150, w = 15$ **10**
2. $A = 38, w = 19$ **2**
3. $A = 21.16, w = 4.6$ **4.6**
4. $A = 2000, w = 32$ **62.5**
5. $A = 450, w = 25$ **18**
6. $A = 256, w = 20$ **12.8**

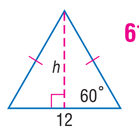
For Lessons 11-2 and 11-4 — Evaluate a Given Expression

Evaluate each expression if $a = 6$, $b = 8$, $c = 10$, and $d = 11$. *(For review, see page 736.)*

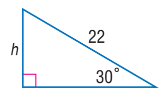
7. $\frac{1}{2}a(b + c)$ **54**
8. $\frac{1}{2}ab$ **24**
9. $\frac{1}{2}(2b + c)$ **13**
10. $\frac{1}{2}d(a + c)$ **88**
11. $\frac{1}{2}(b + c)$ **9**
12. $\frac{1}{2}cd$ **55**

For Lesson 11-3 — Height of a Triangle

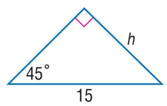
Find h in each triangle. *(For review, see Lesson 7-3.)*

13. 

$6\sqrt{3}$

14. 

11

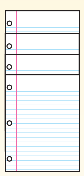
15. 

$\frac{15\sqrt{2}}{2}$

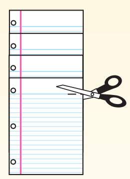
FOLDABLES™ Study Organizer

Areas of Polygons and Circles Make this Foldable to help you organize your notes. Begin with five sheets of notebook paper.

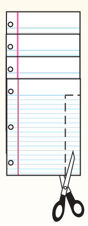
Step 1 Stack
Stack 4 of the 5 sheets of notebook paper as illustrated.



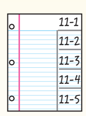
Step 2 Cut
Cut in about 1 inch along the heading line on the top sheet of paper.



Step 3 Cut
Cut the margins off along the right edge.



Step 4 Stack
Stack in order of cuts, placing the uncut fifth sheet at the back. Label the tabs as shown.



Reading and Writing As you read and study the chapter, take notes and record examples of areas of polygons and circles.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 11. Page references are included for additional student help.

Additional review is provided in the *Prerequisite Skills Workbook*, pages 27–28, 43–44, 107–108.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
11-2	Evaluating expressions, p. 600
11-3	Trigonometric ratios in right triangles, p. 609
11-4	Special right triangles, p. 616

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Summarizing Use this Foldable for student writing about polygons and area. After students make their Foldable, have them label the side tabs to correspond to the five lessons in this chapter. Students use their Foldable to take notes, define terms, record concepts, solve problems, and explain how to find areas. At the end of each lesson, ask students to write a summary of the lesson, or write in their own words what the lesson was about. Summaries are useful for condensing data.

Getting Started

Before referring to the Student Edition, you may want to give groups of students the prefixes listed in the table on p. 594 as cutouts in an envelope. Ask groups to make lists of everyday words and their meanings that start with that prefix. Then ask different groups to compare their lists.

Teach

Prefixes In this activity, students will learn that knowing the prefix or meaning of the root of a word can help them learn vocabulary. Encourage students to make their lists of everyday words as extensive as possible. Use the Internet or a dictionary to expand the lists.

Assess

Study Notebook

Ask students to summarize what they have learned about using prefixes or roots of words to learn vocabulary in their notebooks.

ELL English Language Learners may benefit from writing key concepts from this activity in their Study Notebooks in their native language and then in English.

Reading Mathematics

Prefixes

Many of the words used in mathematics use the same prefixes as other everyday words. Understanding the meaning of the prefixes can help you understand the terminology better.

Prefix	Meaning	Everyday Words	Meaning
bi-	2	bicycle	a 2-wheeled vehicle
		bipartisan	involving members of 2 political parties
tri-	3	triangle	closed figure with 3 sides
		tricycle	a 3-wheeled vehicle
		triplet	one of 3 children born at the same time
quad-	4	quadrilateral	closed figure with 4 sides
		quadriceps	muscles with 4 parts
		quadruple	four times as many
penta-	5	pentagon	closed figure with 5 sides
		pentathlon	athletic contest with 5 events
hexa-	6	hexagon	closed figure with 6 sides
hept-	7	heptagon	closed figure with 7 sides
oct-	8	octagon	closed figure with 8 sides
		octopus	animal with 8 legs
dec-	10	decagon	closed figure with 10 sides
		decade	a period of 10 years
		decathlon	athletic contest with 10 events

Several pairs of words in the chart have different prefixes, but the same root word. *Pentathlon* and *decathlon* are both athletic contests. *Heptagon* and *octagon* are both closed figures. Knowing the meaning of the root of the term as well as the prefix can help you learn vocabulary.

Reading to Learn

Use a dictionary to find the meanings of the prefix and root for each term. Then write a definition of the term. **1–6. See margin.**

1. bisector **2.** polygon **3.** equilateral

4. concentric **5.** circumscribe **6.** collinear

7. **RESEARCH** Use a dictionary to find the meanings of the prefix and root of *circumference*. ***circum-*** **around, about; *ferre-* to carry**

8. **RESEARCH** Use a dictionary or the Internet to find as many words as you can with the prefix *poly-* and the definition of each. **See margin.**

Answers

1. ***bi-*** **2, *sector-* a subdivision or region; divide into 2 regions**
2. ***poly-*** **many, *gon-* closed figure; closed figure with many sides**
3. ***equi-*** **equal, *lateral-* sides; having sides of equal length**
4. ***co-*** **together, *centr-* center; circles with a common center**
5. ***circum-*** **around, *scribe-* write; to write around (a geometrical figure)**
6. ***co-*** **together, *linear-* line; together on the same line**
8. **Sample answers: polychromatic—multicolored, polymer—a chemical compound composed of a repeating structural unit, polysyllabic—a word with more than three syllables**

11-1 Areas of Parallelograms

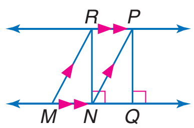
What You'll Learn

- Find perimeters and areas of parallelograms.
- Determine whether points on a coordinate plane define a parallelogram.

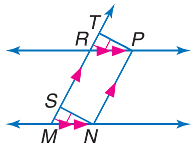
How is area related to garden design?

This composition of square-cut granite and moss was designed by Shigemori Mirei in Kyoto, Japan. How could you determine how much granite was used in this garden?

AREAS OF PARALLELOGRAMS Recall that a *parallelogram* is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a base. For each base, there is a corresponding altitude that is perpendicular to the base.



In ▱$MNPR$, if $\overline{MN}$ is the base, $\overline{RN}$ and $\overline{PQ}$ are altitudes. The length of an altitude is called the *height* of the parallelogram. If $\overline{MR}$ is the base, then the altitudes are $\overline{PT}$ and $\overline{NS}$.

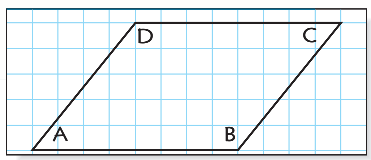


Geometry Activity

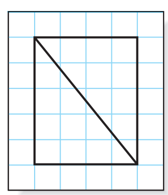
Area of a Parallelogram

Model

- Draw parallelogram *ABCD* on grid paper. Label the vertices on the interior of the angles with letters *A, B, C,* and *D*.
- Fold ▱*ABCD* so that *A* lies on *B* and *C* lies on *D*, forming a rectangle.



Analyze



1. What is the area of the rectangle? **20 units^2**
2. How many rectangles form the parallelogram? **2**
3. What is the area of the parallelogram? **40 units^2**
4. How do the base and altitude of the parallelogram relate to the length and width of the rectangle?
5. **Make a conjecture** Use what you observed to write a formula for the area of a parallelogram. **$A = bh$**

4. The base of the parallelogram is twice the length of the rectangle. The altitude of the parallelogram is the same length as the width of the rectangle.

11-1 Lesson Notes

1 Focus

5-Minute Check Transparency 11-1 Use as a quiz or review of Chapter 10.

Mathematical Background notes are available for this lesson on p. 592C.

How is area related to garden design?

Ask students:

- What kind of polygon is used in the garden design? **square**
- How could grid paper help model this garden design? **Sample answer: You could shade squares of the grid paper to represent the moss.**
- What other polygons are used in garden design? **Sample answer: triangles and rectangles**
- What is another real-world example of special garden designs? **Sample answer: a rose garden made in the form of a geometric design**

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters
- Study Guide and Intervention, pp. 611–612
- Skills Practice, p. 613
- Practice, p. 614
- Reading to Learn Mathematics, p. 615
- Enrichment, p. 616

Graphing Calculator and Computer Masters, pp. 37, 38
School-to-Career Masters, p. 21
Teaching Geometry With Manipulatives Masters, pp. 1, 180

Transparencies

5-Minute Check Transparency 11-1
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

AREAS OF PARALLELOGRAMS

Tips for New Teachers

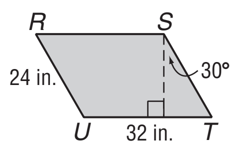
Intervention Point out that many different parallelograms can be drawn with the same altitude, with their bases congruent, and thus with the same area. Use a geoboard or similar modeling device to show different parallelograms with these same characteristics, but different slants.

In-Class Examples

Power Point®

Teaching Tip Point out that each parallelogram has two altitudes. Ask students to sketch the other altitude in this figure.

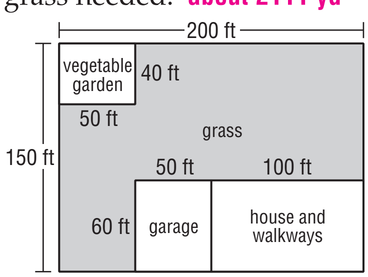
1 Find the area and perimeter of ▱*RSTU*.



area = $384\sqrt{3}$ or about 665.1 in^2; perimeter = 112 in.

Teaching Tip In Example 2, point out that another way to find the square yardage of the rooms they need to recarpet is to find the total area of the large rectangle and subtract the area of the noncarpeted section.

2 The Kanes are planning to sod some parts of their yard. Find the number of square yards of grass needed. **about 2111 yd^2**



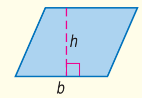
The Geometry Activity leads to the formula for the area of a parallelogram.

Study Tip

Units
Length is measured in linear units, and area is measured in square units.

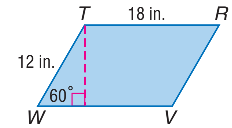
Key Concept — *Area of a Parallelogram*

If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



Example 1 Perimeter and Area of a Parallelogram

Find the perimeter and area of ▱*TRVW*.



Base and Side: Each pair of opposite sides of a parallelogram has the same measure. Each base is 18 inches long, and each side is 12 inches long.

Perimeter: The perimeter of a polygon is the sum of the measures of its sides. So, the perimeter of ▱*TRVW* is 2(18) + 2(12) or 60 inches.

Study Tip

Look Back
To review **perimeter of polygons**, see Lesson 1-6.

Height: Use a 30°-60°-90° triangle to find the height. Recall that if the measure of the leg opposite the 30° angle is x, then the length of the hypotenuse is $2x$, and the length of the leg opposite the 60° angle is $x\sqrt{3}$.

$12 = 2x$ Substitute 12 for the hypotenuse.

$6 = x$ Divide each side by 2.

So, the height of the parallelogram is $x\sqrt{3}$ or $6\sqrt{3}$ inches.

Area:

$A = bh$ Area of a parallelogram

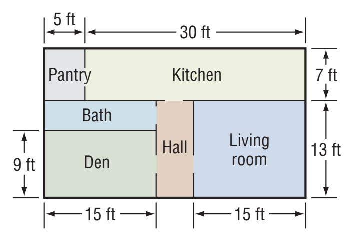
$= 18(6\sqrt{3})$ $b = 18$, $h = 6\sqrt{3}$

$= 108\sqrt{3}$ or about 187.1

The perimeter of ▱*TRVW* is 60 inches, and the area is about 187.1 square inches.

Example 2 Use Area to Solve a Real-World Problem

INTERIOR DESIGN **The Waroners are planning to recarpet part of the first floor of their house. Find the amount of carpeting needed to cover the living room, den, and hall.**



To estimate how much they can spend on carpeting, they need to find the square yardage of each room.

Living Room: $w = 13$ ft, $\ell = 15$ ft

Den: $w = 9$ ft, $\ell = 15$ ft

Hall: It is the same width as the living room, so $w = 13$. The total length of the house is 35 feet. So, $\ell = 35 - 15 - 15$ or 5 feet.

Area

Living Room	Den	Hall
$A = \ell w$	$A = \ell w$	$A = \ell w$
$= 13 \cdot 15$	$= 9 \cdot 15$	$= 5 \cdot 13$
$= 195$ ft^2	$= 135$ ft^2	$= 65$ ft^2

Geometry Activity

Materials: grid paper

To help students see the relationship between the area of a parallelogram and the area of a rectangle, have students cut out several different rectangles. Ask them to cut the rectangle on the diagonal to form two triangles. Then reposition the triangles to form a parallelogram. The area of the parallelogram is the same as the area of the rectangle.

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The total area is 195 + 135 + 65 or 395 square feet. There are 9 square feet in one square yard, so divide by 9 to convert from square feet to square yards.

$$395 \text{ ft}^2 \div \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 395 \text{ ft}^2 \times \frac{1 \text{ yd}^2}{9 \text{ ft}^2}$$
$$\approx 43.9 \text{ yd}^2$$

Therefore, 44 square yards of carpeting are needed to cover these areas.

PARALLELOGRAMS ON THE COORDINATE PLANE Recall the properties of quadrilaterals that you studied in Chapter 8. Using these properties as well as the formula for slope and the Distance Formula, you can find the areas of quadrilaterals on the coordinate plane.

Study Tip

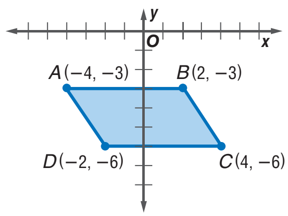
Look Back
To review **properties of parallelograms**, **rectangles**, and **squares**, see Lessons 8-3, 8-4, and 8-5.

Example 3 Area on the Coordinate Plane

COORDINATE GEOMETRY The vertices of a quadrilateral are $A(-4, -3)$, $B(2, -3)$, $C(4, -6)$, and $D(-2, -6)$.

a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*.

First graph each point and draw the quadrilateral. Then determine the slope of each side.



$$\text{slope of } \overline{AB} = \frac{-3-(-3)}{-4-2} = \frac{0}{-6} \text{ or } 0$$

$$\text{slope of } \overline{CD} = \frac{-6-(-6)}{4-(-2)} = \frac{0}{6} \text{ or } 0$$

$$\text{slope of } \overline{BC} = \frac{-3-(-6)}{2-4} = \frac{3}{-2}$$

$$\text{slope of } \overline{AD} = \frac{-3-(-6)}{-4-(-2)} = \frac{3}{-2}$$

Opposite sides have the same slope, so they are parallel. *ABCD* is a parallelogram. The slopes of the consecutive sides are *not* negative reciprocals of each other, so the sides are not perpendicular. Thus, the parallelogram is neither a square nor a rectangle.

b. Find the area of quadrilateral *ABCD*.

Base: $\overline{CD}$ is parallel to the *x*-axis, so subtract the *x*-coordinates of the endpoints to find the length: $CD = |4 - (-2)|$ or 6.

Height: Since $\overline{AB}$ and $\overline{CD}$ are horizontal segments, the distance between them, or the height, can be measured on any vertical segment. Reading from the graph, the height is 3.

$A = bh$ Area formula
$= 6(3)$ $b = 6, h = 3$
$= 18$ Simplify.

The area of ▱*ABCD* is 18 square units.

www.geometryonline.com/extra_examples

PARALLELOGRAMS ON THE COORDINATE PLANE

In-Class Example Power Point®

3 The vertices of a quadrilateral are at $A(-2, 3)$, $B(4, 1)$, $C(3, -2)$, and $D(-3, 0)$.

a. Determine whether the quadrilateral is a *square*, a *rectangle*, or a *parallelogram*. **rectangle**

b. Find the area of quadrilateral *ABCD*. **20 units²**

DAILY INTERVENTION

Differentiated Instruction

Logical For a parallelogram graphed on the coordinate plane, if two sides have slope 0, students should reason that an altitude of the parallelogram can be found easily by counting the number of grid units between the sides.

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Try These exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include the formula for the areas of a rectangle, square, and parallelogram with base b and height h.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Areas of Parallelograms:** 9–17, 27, 28, 31
- **Parallelograms on the Coordinate Plane:** 20–25

Odd/Even Assignments
Exercises 9–31 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 9–17 odd, 21–29 odd, 35–53

Average: 9–35 odd, 36–53

Advanced: 10–34 even, 35–49 (optional: 50–53)

Answer

1. The area of a rectangle is the product of the length and the width. The area of a parallelogram is the product of the base and the height. For both quadrilaterals, the measure of the length of one side is multiplied by the length of the altitude.

Check for Understanding

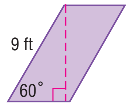
Concept Check

1. **Compare and contrast** finding the area of a rectangle and the area of a parallelogram. **See margin.**
2. **OPEN ENDED** Make and label a scale drawing of your bedroom. Then find its area in square yards. **See students' work.**

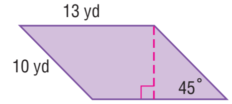
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
3–5	1
6–7	3
8	2

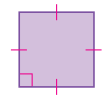
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

3. 

28 ft; 39.0 ft²

4. 

46 yd; 91.9 yd²

5. 

12.8 m; 10.2 m²

Given the coordinates of the vertices of quadrilateral *TVXY*, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of *TVXY*.

6. $T(0, 0)$, $V(2, 6)$, $X(6, 6)$, $Y(4, 0)$ **parallelogram, 24 units²**
7. $T(10, 16)$, $V(2, 18)$, $X(-3, -2)$, $Y(5, -4)$ **rectangle, 170 units²**

Application

8. **DESIGN** Mr. Kang is planning to stain his deck. To know how much stain to buy, he needs to find the area of the deck. What is the area? **1170 ft²**

★ indicates increased difficulty

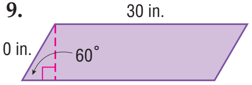
Practice and Apply

Homework Help

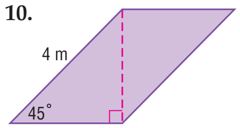
For Exercises	See Examples
9–14	1
15–17, 27, 28, 31	2
20–25	3

Extra Practice
See page 776.

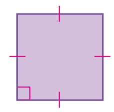
Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

9. 

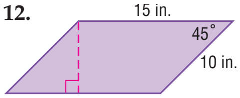
80 in.; 259.8 in²

10. 

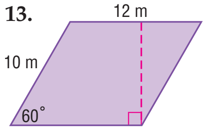
13.7 m; 8 m²

11. 

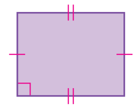
21.6 cm; 29.2 cm²

12. 

50 in.; 106.1 in²

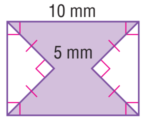
13. 

44 m; 103.9 m²

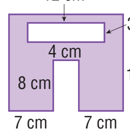
14. 

19.2 ft; 22.7 ft²

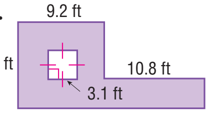
Find the area of each shaded region. Round to the nearest tenth if necessary.

15. 

45.7 mm²

16. 

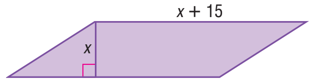
202 cm²

17. 

108.5 ft²

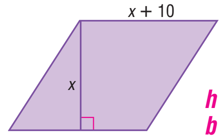
Find the height and base of each parallelogram given its area.

★18. 100 square units



$h = 5$ units, $b = 20$ units

★19. 2000 square units



$h = 40$ units, $b = 50$ units

598 Chapter 11 Areas of Polygons and Circles

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

20. parallelogram, 20 units^2
21. parallelogram, 56 units^2
22. parallelogram, 50 units^2
23. parallelogram, 64 units^2

20. $A(0, 0)$, $B(4, 0)$, $C(5, 5)$, $D(1, 5)$

21. $E(-5, -3)$, $F(3, -3)$, $G(5, 4)$, $H(-3, 4)$

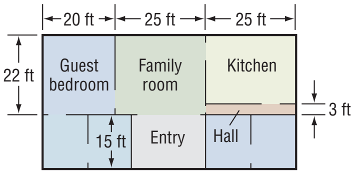
22. $J(-1, -4)$, $K(4, -4)$, $L(6, 6)$, $M(1, 6)$

23. $N(-6, 2)$, $O(2, 2)$, $P(4, -6)$, $Q(-4, -6)$

24. $R(-2, 4)$, $S(8, 4)$, $T(8, -3)$, $U(-2, -3)$ rectangle, 70 units^2

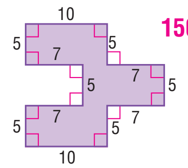
25. $V(1, 10)$, $W(4, 8)$, $X(2, 5)$, $Y(-1, 7)$ square, 13 units^2

26. INTERIOR DESIGN The Bessos are planning to have new carpet installed in their guest bedroom, family room, and hallway. Find the number of square yards of carpet they should order. 119 yd^2

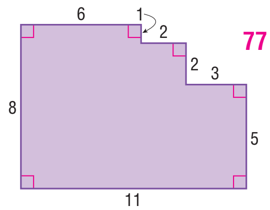


Find the area of each figure.

27. 150 units^2



28. 77 units^2



More About . . .

Art

A triptych originally referred to a Roman writing tablet with three panels that were hinged together.

Source: www.artlex.com

ART For Exercises 29 and 30, use the following information.

A *triptych* painting is a series of three pieces with a similar theme displayed together. Suppose the center panel is a 12-inch square and the panels on either side are 12 inches by 5 inches. The panels are 2 inches apart with a 3 inch wide border around the edges.

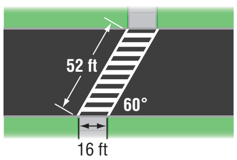
Study for a Triptych, by Albert Gleizes

29. Determine whether the triptych will fit a 45-inch by 20-inch frame. Explain.

30. Find the area of the artwork. 576 in^2

29. Yes; the dimensions are 32 in. by 18 in.

31. CROSSWALKS A crosswalk with two stripes each 52 feet long is at a 60° angle to the curb. The width of the crosswalk at the curb is 16 feet. Find the perpendicular distance between the stripes of the crosswalk. ≈ 13.9 ft



VARYING DIMENSIONS For Exercises 32–34, use the following information.

A parallelogram has a base of 8 meters, sides of 11 meters, and a height of 10 meters.

32. Find the perimeter and area of the parallelogram. 38 m, 80 m^2

★ **33.** Suppose the dimensions of the parallelogram were divided in half. Find the perimeter and the area.

★ **34.** Compare the perimeter and area of the parallelogram in Exercise 33 with the original.

33. The perimeter is 19 m, half of 38 m. The area is 20 m^2.

34. The new perimeter is half of the original. The new area is one half squared or one fourth the area of the original parallelogram.

35. CRITICAL THINKING A piece of twine 48 inches long is cut into two lengths. Each length is then used to form a square. The sum of the areas of the two squares is 74 square inches. Find the length of each side of the smaller square and the larger square. 5 in., 7 in.

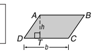
www.geometryonline.com/self_check_quiz

(l)State Hermitage Museum, St. Petersburg, Russia/CORBIS, (r)Bridgeman Art Library

Study Guide and Intervention, p. 611 (shown) and p. 612

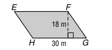
Areas of Parallelograms A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Any side of a parallelogram can be called a **base**. Each base has a corresponding **altitude**, and the length of the altitude is the **height** of the parallelogram. The area of a parallelogram is the product of the base and the height.

Area of a Parallelogram	If a parallelogram has an area of A square units, a base of b units, and a height of h units, then $A = bh$.



The area of parallelogram $ABCD$ is $CD \cdot AT$.

Example Find the area of parallelogram *EFGH*.



$A = bh$ Area of a parallelogram

$= 30(18)$ $b = 30$, $h = 18$

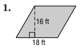
$= 540$ Multiply.

The area is 540 square meters.

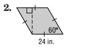
Exercises

Find the area of each parallelogram.

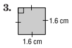
1. 288 ft^2



2. $288\sqrt{3}$ in^2

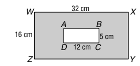


3. 2.56 cm^2

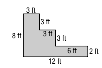


Find the area of each shaded region.

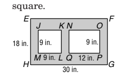
4. *WXYZ* and *ABCD* are rectangles. 452 cm^2



5. All angles are right angles. 51 ft^2



6. *EFGH* and *NOPQ* are rectangles; *JKLM* is a square. 351 in^2



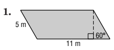
7. The area of a parallelogram is 3.36 square feet. The base is 2.8 feet. If the measures of the base and height are each doubled, find the area of the resulting parallelogram. 13.44 ft^2

8. A rectangle is 4 meters longer than it is wide. The area of the rectangle is 252 square meters. Find the length. 18 m

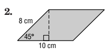
Skills Practice, p. 613 and Practice, p. 614 (shown)

Find the perimeter and area of each parallelogram. Round to the nearest tenth if necessary.

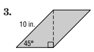
1. 32 m, 47.6 m^2



2. 36 cm, 56.6 cm^2

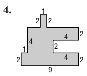


3. 34.1 in., 50 in^2

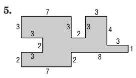


Find the area of each figure.

4. 44 units^2



5. 65 units^2



COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.

6. $C(-4, -1)$, $D(-4, 2)$, $F(1, 2)$, $G(1, -1)$ rectangle, 15 units^2

7. $W(2, 2)$, $X(1, -2)$, $Y(-2, -2)$, $Z(-1, 2)$ parallelogram, 12 units^2

8. $M(0, 4)$, $N(4, 6)$, $O(6, 2)$, $P(2, 0)$ square, 20 units^2

9. $P(-5, 2)$, $Q(4, 2)$, $R(5, 5)$, $S(-4, 5)$ parallelogram, 27 units^2

FRAMING For Exercises 10–12, use the following information.

A rectangular poster measures 42 inches by 26 inches. A frame shop fitted the poster with a half-inch mat border.

10. Find the area of the poster. 1092 in^2

11. Find the area of the mat border. 69 in^2

12. Suppose the wall is marked where the poster will hang. The marked area includes an additional 12-inch space around the poster and frame. Find the total wall area that has been marked for the poster. 3417 in^2

Reading to Learn Mathematics, p. 615 ELL

Pre-Activity How is area related to garden design?

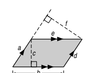
Read the introduction to Lesson 11-1 at the top of page 595 in your textbook.

How could you describe the pattern you see in the picture of the garden so that someone who doesn't have the picture will know what it looks like?

Sample answer: There are squares of granite and squares of moss of the same size placed in a checkerboard design.

Reading the Lesson

1. Which expression gives the area of the parallelogram? (Hint: There can be more than one correct response.) B, D, E, G

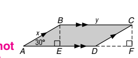


A. ab **B.** cb **C.** ed

D. af **E.** ce **F.** cd

G. df **H.** bf **I.** cf

2. Refer to the figure. Determine whether each statement is *true* or *false*. If the statement is false, explain why.



a. $\overline{AB}$ is an altitude of the parallelogram. False; $\overline{AB}$ is not perpendicular to any side of the parallelogram.

b. $\overline{CD}$ is a base of parallelogram $ABCD$. true

c. The perimeter of $ABCD$ is $(2x + 2y)$ units^2. False; perimeter is measured in linear units, not square units. The perimeter is $(2x + 2y)$ units.

d. $BE = CF$ true

e. $BE = \frac{\sqrt{3}}{2}x$ False; $\overline{BE}$ is opposite the 30° angle in a 30°-60°-90° triangle, so $BE = \frac{1}{2}x$ or $\frac{x}{2}$.

f. The area of $ABCD$ is $2xy$ units^2. False; since $BE = \frac{x}{2}$, the area of $ABCD$ is $\frac{xy}{2}$ units^2.

Helping You Remember

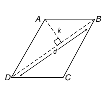
3. A good way to remember a new formula in geometry is to relate it to a formula you already know. How can you use the formula for the area of a rectangle to help you remember the formula for the area of a parallelogram?

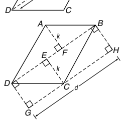
Sample answer: To find the area of a rectangle, you multiply the lengths of two segments that are perpendicular to each other. To find the area of a parallelogram, you do the same thing, but the height is not necessarily one of the sides.

Enrichment, p. 616

Area of a Parallelogram

You can prove some interesting results using the formula you have proved for the area of a parallelogram by drawing auxiliary lines to form congruent regions. Consider the top parallelogram shown at the right. In the figure, d is the length of the diagonal $\overline{BD}$, and k is the length of the perpendicular segment from A to $\overline{BD}$. Now consider the second figure, which shows the same parallelogram with a number of auxiliary perpendiculars added. Use what you know about perpendicular lines, parallel lines, and congruent triangles to answer the following.





1. What kind of figure is *DBHG*?

rectangle

4 Assess

Open-Ended Assessment

Speaking Have students describe how to find the area of a parallelogram in the coordinate plane.

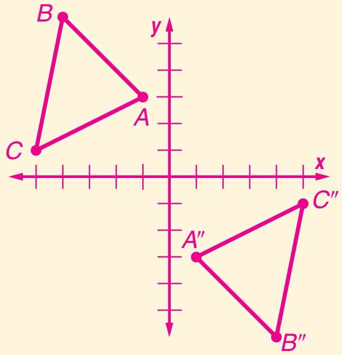
Getting Ready for Lesson 11-2

Prerequisite Skill Students will learn about the areas of triangles, rhombi, and trapezoids in Lesson 11-2. They will substitute values and evaluate expressions to find areas. Use Exercises 50–53 to determine your students' familiarity with evaluating expressions.

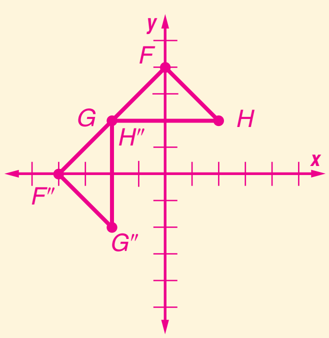
Answers

36. Sample answer: Area is used when designing a garden to find the total amount of materials needed. Answers should include the following.
    - Find the area of one square and multiply by the number of squares in the garden.
    - Knowing the area is useful when planning a stone walkway or fencing in flowers or vegetables.

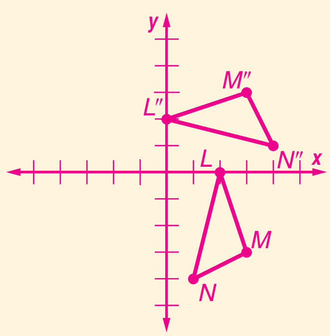
46.



47.



48.



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36. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

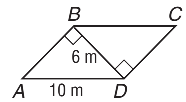
How is area related to garden design?

Include the following in your answer:
- how to determine the total area of granite squares, and
- other uses for area.

Standardized Test Practice

37. What is the area of ▱$ABCD$? **C**

Ⓐ 24 m^2 Ⓑ 30 m^2 Ⓒ 48 m^2 Ⓓ 60 m^2



38. ALGEBRA Which statement is correct? **D**

Ⓐ $x^2 > (x-1)^2$ Ⓒ $x^2 < (x-1)^2$

Ⓑ $x^2 = (x-1)^2$ Ⓓ The relationship cannot be determined.

Maintain Your Skills

Mixed Review **Determine the coordinates of the center and the measure of the radius for each circle with the given equation.** *(Lesson 10-8)*

39. $(x-5)^2 + (y-2)^2 = 49$ **(5, 2), $r = 7$**

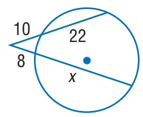
40. $(x+3)^2 + (y+9)^2 - 81 = 0$ **(−3, −9), $r = 9$**

41. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{9}\right)^2 - \frac{4}{9} = 0$ **$\left(-\frac{2}{3}, \frac{1}{9}\right)$, $r = \frac{2}{3}$**

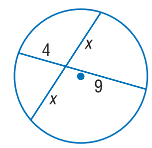
42. $(x-2.8)^2 + (y+7.6)^2 = 34.81$ **(2.8, −7.6), $r = 5.9$**

Find x. Assume that segments that appear to be tangent are tangent. *(Lesson 10-7)*

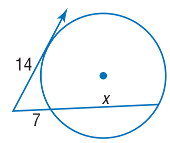
43. **32**



44. **6**



45. **21**



COORDINATE GEOMETRY **Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.** *(Lesson 9-3)* **46–48. See margin for figures.**

46. △ABC with vertices $A(-1, 3)$, $B(-4, 6)$, and $C(-5, 1)$, reflected in the y-axis and then the x-axis **$A''(1, -3)$, $B''(4, -6)$, $C''(5, -1)$; 180°**

47. △FGH with vertices $F(0, 4)$, $G(-2, 2)$, and $H(2, 2)$, reflected in $y = x$ and then the y-axis **$F''(-4, 0)$, $G''(-2, -2)$, $H''(-2, 2)$; 90° counterclockwise**

48. △LMN with vertices $L(2, 0)$, $M(3, -3)$, and $N(1, -4)$, reflected in the y-axis and then the line $y = -x$ **$L''(0, 2)$, $M''(3, 3)$, $N''(4, 1)$; 90° counterclockwise**

49. BIKES Nate is making a ramp for bike jumps. The ramp support forms a right angle. The base is 12 feet long, and the height is 5 feet. What length of plywood does Nate need for the ramp? *(Lesson 7-2)* **13 ft**

Getting Ready for the Next Lesson PREREQUISITE SKILL **Evaluate each expression if $w = 8$, $x = 4$, $y = 2$, and $z = 5$.**

*(To review **evaluating expressions**, see page 736.)*

50. $\frac{1}{2}(7y)$ **7**

51. $\frac{1}{2}wx$ **16**

52. $\frac{1}{2}z(x + y)$ **15**

53. $\frac{1}{2}x(y + w)$ **20**

600 Chapter 11 Areas of Polygons and Circles

11-2 Areas of Triangles, Trapezoids, and Rhombi

What You'll Learn

- Find areas of triangles.
- Find areas of trapezoids and rhombi.

How is the area of a triangle related to beach umbrellas?

Umbrellas can protect you from rain, wind, and sun. The umbrella shown at the right is made of triangular panels. To cover the umbrella frame with canvas panels, you need to know the area of each panel.

AREAS OF TRIANGLES You have learned how to find the areas of squares, rectangles, and parallelograms. The formula for the area of a triangle is related to these formulas.

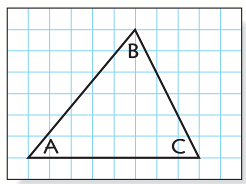
Geometry Activity

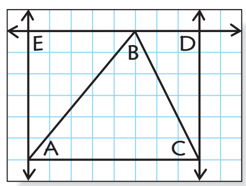
Area of a Triangle

Model

You can determine the area of a triangle by using the area of a rectangle.

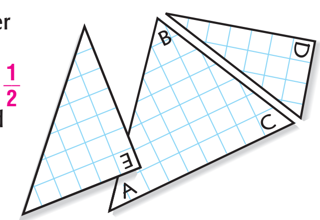
- Draw a triangle on grid paper so that one edge is along a horizontal line. Label the vertices on the interior of the angles of the triangle as A, B, and C.
- Draw a line perpendicular to $\overline{AC}$ through A.
- Draw a line perpendicular to $\overline{AC}$ through C.
- Draw a line parallel to $\overline{AC}$ through B.
- Label the points of intersection of the lines drawn as D and E as shown.
- Find the area of rectangle $ACDE$ in square units.
- Cut out rectangle $ACDE$. Then cut out $\triangle ABC$. Place the two smaller pieces over $\triangle ABC$ to completely cover the triangle.





Analyze 1. Together the two smaller triangles are the same size as $\triangle ABC$.

1. What do you observe about the two smaller triangles and $\triangle ABC$?
2. What fraction of rectangle $ACDE$ is $\triangle ABC$? $\frac{1}{2}$
3. Derive a formula that could be used to find the area of $\triangle ABC$. $A = \frac{1}{2}bh$



11-2 Lesson Notes

1 Focus

5-Minute Check Transparency 11-2 Use as a quiz or review of Lesson 11-1.

Mathematical Background notes are available for this lesson on p. 592C.

How is the area of a triangle related to beach umbrellas?

Ask students:

- What is the shape of the umbrella fabric when it is flat? **a many-sided polygon**
- Why are triangles the shape used to make umbrellas? **Sample answer: They fit together at one vertex.**
- Give another example of an item made from a triangular piece of fabric. **Sample answer: a sail**

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters

- Study Guide and Intervention, pp. 617–618
- Skills Practice, p. 619
- Practice, p. 620
- Reading to Learn Mathematics, p. 621
- Enrichment, p. 622
- Assessment, p. 655

Teaching Geometry With Manipulatives Masters, pp. 1, 181, 182

Transparencies

5-Minute Check Transparency 11-2
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

AREAS OF TRIANGLE

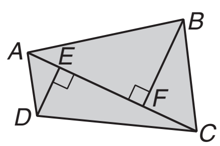
Tips for New Teachers

Intervention Help students understand the relationship between the area of a triangle and the area of a parallelogram or rectangle by showing them a model. Cut a piece of 8.5 × 11 paper in half along the diagonal to demonstrate that the area of a triangle is one-half the area of a rectangle. Then cut a right triangle from an end of another sheet of 8.5 × 11 paper so that it has the same height as the original paper. Form a parallelogram from the sheet by sliding it to the other side of the sheet. Then cut it in half along the diagonal. The area of a triangle is half the area of this corresponding parallelogram.

In-Class Example

Power Point®

1 Find the area of quadrilateral *ABCD* if $AC = 35$, $BF = 18$, and $DE = 10$.



490 units²

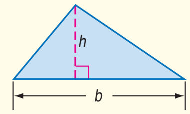
Study Tip

Look Back
To review **the height and altitude of a triangle**, see Lesson 5-1.

The Geometry Activity suggests the formula for finding the area of a triangle.

Key Concept — *Area of a Triangle*

If a triangle has an area of *A* square units, a base of *b* units, and a corresponding height of *h* units, then $A = \frac{1}{2}bh$.



Example 1 Areas of Triangles

Find the area of quadrilateral *XYZW* if $XZ = 39$, $HW = 20$, and $YG = 21$.

The area of the quadrilateral is equal to the sum of the areas of △*XWZ* and △*XYZ*.

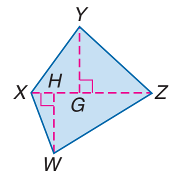
area of *XYZW* = area of △*XYZ* + area of △*XWZ*

$= \frac{1}{2}bh_1 + \frac{1}{2}bh_2$

$= \frac{1}{2}(39)(21) + \frac{1}{2}(39)(20)$ Substitution

$= 409.5 + 390$ Simplify.

$= 799.5$

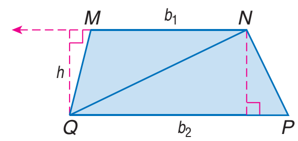


The area of quadrilateral *XYZW* is 799.5 square units.

AREAS OF TRAPEZOIDS AND RHOMBI

The formulas for the areas of trapezoids and rhombi are related to the formula for the area of a triangle.

Trapezoid *MNPQ* has diagonal $\overline{QN}$ with parallel bases $\overline{MN}$ and $\overline{PQ}$. Therefore, the altitude *h* from vertex *Q* to the extension of base $\overline{MN}$ is the same length as the altitude from vertex *N* to the base $\overline{QP}$. Since the area of the trapezoid is the area of two nonoverlapping parts, we can write the following equation.



area of trapezoid *MNPQ* = area of △*MNQ* + area of △*NPQ*

$A = \frac{1}{2}(b_1)h + \frac{1}{2}(b_2)h$ Let the area be *A*, *MN* be b_1, and *QP* be b_2.

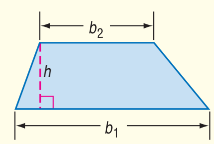
$A = \frac{1}{2}(b_1 + b_2)h$ Factor.

$A = \frac{1}{2}h(b_1 + b_2)$ Commutative Property

This is the formula for the area of any trapezoid.

Key Concept — *Area of a Trapezoid*

If a trapezoid has an area of *A* square units, bases of b_1 units and b_2 units, and a height of *h* units, then $A = \frac{1}{2}h(b_1 + b_2)$.



Geometry Activity

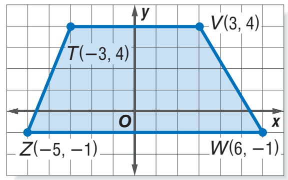
Materials: grid paper

- Ask students to recall what they know about perpendiculars and the altitudes of triangles before you do this activity.
- Some students may seem uninterested because they already know the formula for the area of a triangle. Ask them if they can demonstrate why the area formula works. In this activity, they will explore why the formula works and derive the formula for themselves.

602 Chapter 11 Areas of Polygons and Circles

Example 2 Area of a Trapezoid on the Coordinate Plane

COORDINATE GEOMETRY **Find the area of trapezoid *TVWZ* with vertices $T(-3, 4)$, $V(3, 4)$, $W(6, -1)$, and $Z(-5, -1)$.**



Bases: Since $\overline{TV}$ and $\overline{ZW}$ are horizontal, find their length by subtracting the x-coordinates of their endpoints.

$TV = |-3 - 3|$ $= |-6|$ or 6

$ZW = |-5 - 6|$ $= |-11|$ or 11

Height: Because the bases are horizontal segments, the distance between them can be measured on a vertical line. That is, subtract the y-coordinates.

$h = |4 - (-1)|$ or 5

Area:

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(5)(6 + 11)$ $h = 5, b_1 = 6, b_2 = 11$

$= 42.5$ Simplify.

The area of trapezoid *TVWZ* is 42.5 square units.

The formula for the area of a triangle can also be used to derive the formula for the area of a rhombus.

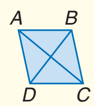
Study Tip

Area of a Rhombus
Because a rhombus is also a parallelogram, you can also use the formula $A = bh$ to determine the area.

Key Concept — Area of a Rhombus

If a rhombus has an area of A square units and diagonals of d_1 and d_2 units, then $A = \frac{1}{2}d_1d_2$.

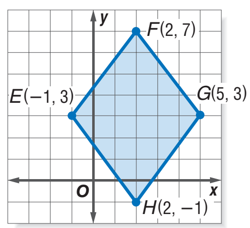
Example: $A = \frac{1}{2}(AC)(BD)$



You will derive this formula in Exercise 46.

Example 3 Area of a Rhombus on the Coordinate Plane

COORDINATE GEOMETRY **Find the area of rhombus *EFGH* with vertices at $E(-1, 3)$, $F(2, 7)$, $G(5, 3)$, and $H(2, -1)$.**



Explore To find the area of the rhombus, we need to know the lengths of each diagonal.

Plan Use coordinate geometry to find the length of each diagonal. Use the formula to find the area of rhombus *EFGH*.

Solve Let $\overline{EG}$ be d_1 and $\overline{FH}$ be d_2.

Subtract the x-coordinates of E and G to find that d_1 is 6.
Subtract the y-coordinates of F and H to find that d_2 is 8.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$= \frac{1}{2}(6)(8)$ or 24 $d_1 = 6, d_2 = 8$

Examine The area of rhombus *EFGH* is 24 square units.

If you know all but one measure in a quadrilateral, you can solve for the missing measure using the appropriate area formula.

www.geometryonline.com/extra_examples

AREAS OF TRAPEZOIDS AND RHOMBI

In-Class Examples

Power Point®

Teaching Tip Make sure students understand that the formula for the area of a trapezoid is derived from adding the areas of the two nonoverlapping triangles formed by one diagonal of the trapezoid. The height of the trapezoid is the same as the height of *each* triangle.

2 Find the area of trapezoid *RSTU* with vertices $R(4, 2)$, $S(6, -1)$, $T(-2, -1)$, and $U(-1, 2)$. **19.5 units^2**

3 Find the area of rhombus *MNPR* with vertices at $M(0, 1)$, $N(4, 2)$, $P(3, -2)$, and $R(-1, -3)$. **15 units^2**

Building on Prior Knowledge

In Chapter 8, students learned that if a parallelogram has all sides congruent, it is a rhombus. In this lesson, emphasize that the diagonals of a rhombus are used to find its area instead of the base and height.

DAILY INTERVENTION

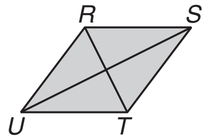
Differentiated Instruction

Visual/Spatial Stress that in some triangles, one side seems to be an altitude. This is not true unless the triangle is a right triangle, in which the legs are perpendicular. Students should not assume that angles are right angles unless they are clearly marked.

In-Class Examples

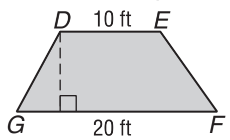
Power Point®

4 **a.** Rhombus *RSTU* has an area of 64 square inches. Find *US* if *RT* = 8 inches.

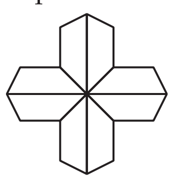


16 in.

b. Trapezoid *DEFG* has an area of 120 square feet. Find the height of *DEFG*. **8 ft**

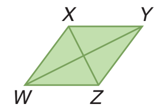


5 **STAINED GLASS** This stained glass window is composed of 8 congruent trapezoidal shapes. The total area of the design is 72 square feet. Each trapezoid has bases of 3 and 6 feet. Find the height of each trapezoid. **2 ft**



Example 4 Algebra: Find Missing Measures

a. Rhombus *WXYZ* has an area of 100 square meters. Find *WY* if *XZ* = 10 meters.



Use the formula for the area of a rhombus and solve for d_2.

$$A = \frac{1}{2}d_1d_2$$

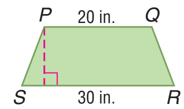
$$100 = \frac{1}{2}(10)(d_2)$$

$$100 = 5d_2$$

$$20 = d_2$$

WY is 20 meters long.

b. Trapezoid *PQRS* has an area of 250 square inches. Find the height of *PQRS*.



Use the formula for the area of a trapezoid and solve for *h*.

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$250 = \frac{1}{2}h(20 + 30)$$

$$250 = \frac{1}{2}(50)h$$

$$250 = 25h$$

$$10 = h$$

The height of trapezoid *PQRS* is 10 inches.

Since the dimensions of congruent figures are equal, the areas of congruent figures are also equal.

Postulate 11.1

Congruent figures have equal areas.

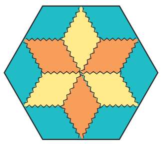
Study Tip

Look Back

To review **the properties of rhombi and trapezoids**, see Lessons 8-5 and 8-6.

Example 5 Area of Congruent Figures

QUILTING **This quilt block is composed of twelve congruent rhombi arranged in a regular hexagon. The height of the hexagon is 8 inches. If the total area of the rhombi is 48 square inches, find the lengths of each diagonal and the area of one rhombus.**



First, find the area of one rhombus. From Postulate 11.1, the area of each rhombus is the same. So, the area of each rhombus is 48 ÷ 12 or 4 square inches.

Next, find the length of one diagonal. The height of the hexagon is equal to the sum of the long diagonals of two rhombi. Since the rhombi are congruent, the long diagonals must be congruent. So, the long diagonal is equal to 8 ÷ 2, or 4 inches.

Use the area formula to find the length of the other diagonal.

$A = \frac{1}{2}d_1d_2$ — Area of a rhombus

$4 = \frac{1}{2}(4)\,d_2$ — $A = 4$, $d_1 = 4$

$2 = d_2$ — Solve for d_2.

Each rhombus in the pattern has an area of 4 square inches and diagonals 4 inches and 2 inches long.

604 Chapter 11 Areas of Polygons and Circles

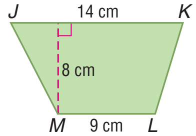
Check for Understanding

Concept Check

1. **OPEN ENDED** Draw an isosceles trapezoid that contains at least one isosceles triangle. **See margin.**

2. Kiku; she simplified the formula properly by adding the terms in the parentheses before multiplying.

2. **FIND THE ERROR** Robert and Kiku are finding the area of trapezoid *JKLM*.



Robert

$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(14) + 9$

$= 56 + 9$

$= 65\ cm^2$

Kiku

$A = \frac{1}{2}(8)(14 + 9)$

$= \frac{1}{2}(8)(23)$

$= 4(23)$

$= 92\ cm^2$

Who is correct? Explain your reasoning.

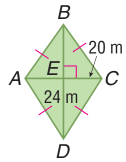
3. **Determine** whether it is *always, sometimes,* or *never* true that rhombi with the same area have the same diagonal lengths. Explain your reasoning. **Sometimes; two rhombi can have different corresponding diagonal lengths and have the same area.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
5, 7	1
6, 8	2
4, 9	3
10, 11	4
12	5

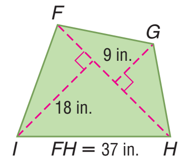
Find the area of each quadrilateral.

4.



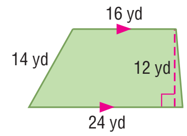
240 m²

5.



499.5 in²

6.



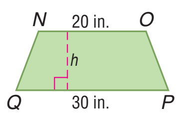
240 yd²

COORDINATE GEOMETRY **Find the area of each figure given the coordinates of the vertices.**

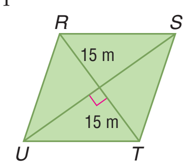
7. $\triangle ABC$ with $A(2, -3)$, $B(-5, -3)$, and $C(-1, 3)$ **21 units²**
8. trapezoid *FGHJ* with $F(-1, 8)$, $G(5, 8)$, $H(3, 4)$, and $J(1, 4)$ **16 units²**
9. rhombus *LMPQ* with $L(-4, 3)$, $M(-2, 4)$, $P(0, 3)$, and $Q(-2, 2)$ **4 units²**

ALGEBRA **Find the missing measure for each quadrilateral.**

10. Trapezoid *NOPQ* has an area of 250 square inches. Find the height of *NOPQ*. **10 in.**

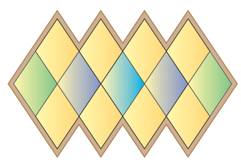


11. Rhombus *RSTU* has an area of 675 square meters. Find *SU*. **45 m**



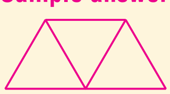
Application

12. **INTERIOR DESIGN** Jacques is designing a window hanging composed of 13 congruent rhombi. The total width of the window hanging is 15 inches, and the total area is $82\frac{7}{8}$ square inches. Find the length of each diagonal and the area of one rhombus.
$4\frac{1}{4}$ in., 3 in., $6\frac{3}{8}$ in²



Answer

1. Sample answer:



3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include the formulas and examples of the area of a triangle, rhombus, and trapezoid.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR In Exercise 2, caution students that the order of operations applies to all mathematical expressions, including area formulas. Kiku did the problem correctly by adding the terms in parentheses before multiplying by the other terms.

About the Exercises...

Organization by Objective
- **Areas of Triangles:** 13, 14, 19–21
- **Areas of Trapezoids and Rhombi:** 15–18, 22–44

Odd/Even Assignments
Exercises 13–44 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–43 odd, 47–57 odd, 58–60, 65–76 (optional: 61–64)

Average: 13–57 odd, 58–60, 65–76 (optional: 61–64)

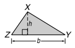
Advanced: 14–56 even, 57–73 (optional: 74–76)

All: Quiz 1 (1–5)

Study Guide and Intervention, p. 617 (shown) and p. 618

Areas of Triangles The area of a triangle is half the area of a rectangle with the same base and height as the triangle.

If a triangle has an area of A square units, a base of b units, and a corresponding height of h units, then $A = \frac{1}{2}bh$.



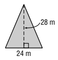
Example **Find the area of the triangle.**

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(24)(28)$ $b = 24, h = 28$

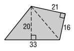
$= 336$ Multiply.

The area is 336 square meters.

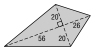


Exercises

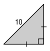
Find the area of each figure.

1. 

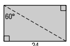
498 units²

2. 

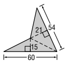
1640 units²

3. 

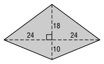
25 units²

4. 

332.6 units²

5. 

1017 units²

6. 

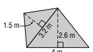
672 units²

7. The area of a triangle is 72 square inches. If the height is 8 inches, find the length of the base. **18 in.**

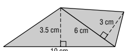
8. A right triangle has a perimeter of 36 meters, a hypotenuse of 15 meters, and a leg of 9 meters. Find the area of the triangle. **54 m²**

Skills Practice, p. 619 and Practice, p. 620 (shown)

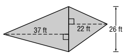
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

7.6 m²

2. 

26.5 cm²

3. 

767 ft²

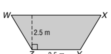
Find the area of each quadrilateral given the coordinates of the vertices.

4. trapezoid $ABCD$
$A(-7, 1), B(-4, 4), C(-4, -6), D(-7, -3)$
21 units²

5. rhombus $LMNO$
$L(6, 8), M(14, 4), N(6, 0), O(-2, 4)$
64 units²

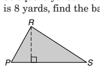
Find the missing measure for each figure.

6. Trapezoid $WXYZ$ has an area of 13.75 square meters. Find WX.



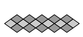
7.5 m

7. Triangle PRS has an area of 68 square yards. If the height of $\triangle PRS$ is 8 yards, find the base.



17 yd

DESIGN **For Exercises 8 and 9, use the following information.**
Mr. Hagarty used 16 congruent rhombi-shaped tiles to design the midsection of the backsplash area above a kitchen sink. The length of the design is 27 inches and the total area is 108 square inches.



8. Find the area of one rhombus.
$6\frac{3}{4}$ in²

9. Find the length of each diagonal.
$4\frac{1}{2}$ in., 3 in.

Reading to Learn Mathematics, p. 621 ELL

Pre-Activity **How is the area of a triangle related to beach umbrellas?**

Read the introduction to Lesson 11-2 at the top of page 601 in your textbook.

Classify the polygons in the panels of the beach umbrella.
Isosceles triangles and isosceles trapezoids

Reading the Lesson

1. Match each area formula from the first column with the corresponding polygon in the second column.

a. $A = \ell w$ **vi**	i. triangle
b. $A = \frac{1}{2}d_1d_2$ **iv**	ii. parallelogram
c. $A = s^2$ **v**	iii. trapezoid
d. $A = \frac{1}{2}h(b_1 + b_2)$ **iii**	iv. rhombus
e. $A = \frac{1}{2}bh$ **i**	v. square
f. $A = bh$ **ii**	vi. rectangle

2. Determine whether each statement is *always*, *sometimes*, or *never* true. In each case, explain your reasoning. **For explanations, sample answers are given.**
   a. The area of a square is half the product of its diagonals. **Always; a square is a rhombus, so you can use the rhombus formula.**
   b. The area of a triangle is half the product of two of its sides. **Sometimes; this is true only for a right triangle.**
   c. You can find the area of a rectangle by multiplying base times height. **Always; a rectangle is a parallelogram, so you can use the parallelogram formula. If the length of a rectangle is used as the base, then the width is the height.**
   d. You can find the area of a rectangle by multiplying the lengths of any two of its sides. **Sometimes; this is true only for a square. Otherwise, you must use two *consecutive* sides, not any two sides.**
   e. The area of a trapezoid is the product of its height and the sum of the bases. **Never; the area is one-half the product of its height and the sum of the bases.**
   f. The square of the length of a side of a square is equal to half the product of its diagonals. **Always; a square is a rhombus, so the formulas for a square and a rhombus must give the same answer whenever the rhombus is a square.**

Helping You Remember

3. A good way to remember a new geometric formula is to state it in words. Write a short sentence that tells how to find the area of a trapezoid in a way that is easy to remember. **Sample answer: Average the lengths of the bases and multiply by the height.**

★ indicates increased difficulty

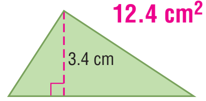
Practice and Apply

Homework Help

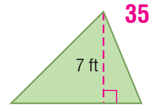
For Exercises	See Examples
13, 14, 19–21	1
15, 16, 22–25	2
17, 18, 26–29	3
30–35, 40–44	4
36–39	5

Extra Practice See page 776.

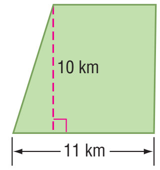
Find the area of each figure. Round to the nearest tenth if necessary.

13. 

12.4 cm²

14. 

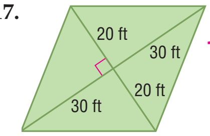
35.7 ft²

15. 

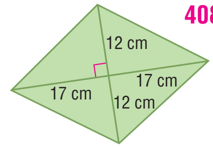
95 km²

16. 

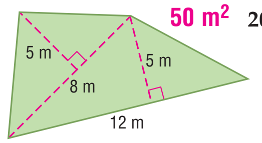
96.5 yd²

17. 

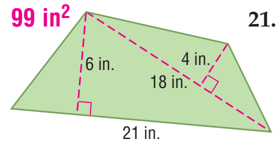
1200 ft²

18. 

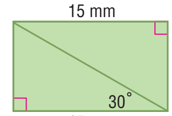
408 cm²

19. 

50 m²

20. 

99 in²

21. 

129.9 mm²

COORDINATE GEOMETRY **Find the area of trapezoid *PQRT* given the coordinates of the vertices.** **23. 55 units²**

22. $P(0, 3), Q(3, 7), R(5, 7), T(6, 3)$ **16 units²**

23. $P(-4, -5), Q(-2, -5), R(4, 6), T(-4, 6)$

24. $P(-3, 8), Q(6, 8), R(6, 2), T(1, 2)$ **42 units²**

25. $P(-6, 3), Q(1, 3), R(-2, -2), T(-4, -2)$ **22.5 units²**

COORDINATE GEOMETRY **Find the area of rhombus *JKLM* given the coordinates of the vertices.** **26. 30 units²** **27. 20 units²**

26. $J(2, 1), K(7, 4), L(12, 1), M(7, -2)$

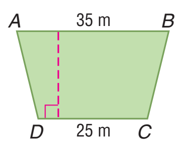
27. $J(-1, 1), K(1, 7), L(3, 2), M(1, -3)$

28. $J(-1, -4), K(2, 2), L(5, -4), M(2, -10)$ **36 units²**

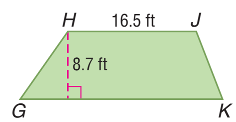
29. $J(2, 4), K(6, 6), L(10, 4), M(6, 2)$ **16 units²**

ALGEBRA **Find the missing measure for each figure.**

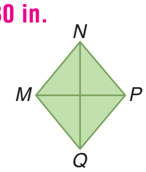
30. Trapezoid $ABCD$ has an area of 750 square meters. Find the height of $ABCD$. **25 m**



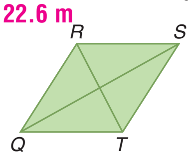
31. Trapezoid $GHJK$ has an area of 188.35 square feet. If HJ is 16.5 feet, find GK. **26.8 ft**



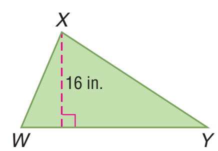
32. Rhombus $MNPQ$ has an area of 375 square inches. If MP is 25 inches, find NQ. **30 in.**



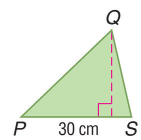
33. Rhombus $QRST$ has an area of 137.9 square meters. If RT is 12.2 meters, find QS. **22.6 m**



34. Triangle WXY has an area of 248 square inches. Find the length of the base. **31 in.**



35. Triangle PQS has an area of 300 square centimeters. Find the height. **20 cm**



Enrichment, p. 622

Areas of Similar Triangles

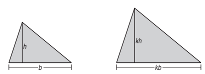
You have learned that if two triangles are similar, the ratio of the lengths of corresponding altitudes is equal to the ratio of the lengths of a pair of corresponding sides. However, there is a different relationship between the areas of the two triangles.

Theorem If two triangles are similar, the ratio of their areas is the square of the ratio of the lengths of a pair of corresponding sides.

Triangle II is k times larger than Triangle I. Thus, its base is k times as large as that of Triangle I and its height is k times as large as that of Triangle I.

$$\frac{\text{side of } \triangle II}{\text{side of } \triangle I} = \frac{kb}{b} \text{ or } \frac{k}{1}$$

$$\frac{\text{area of } \triangle II}{\text{area of } \triangle I} = \frac{\frac{1}{2}k^2bh}{\frac{1}{2}bh} \text{ or } \frac{k^2}{1}$$

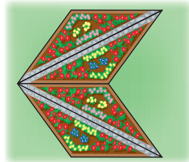


Triangle I
area $\triangle I = \frac{1}{2}bh$

Triangle II
area $\triangle II = \frac{1}{2}(kb)(kh) = \frac{1}{2}k^2bh$

GARDENS **For Exercises 36 and 37, use the following information.**

Keisha designed a garden that is shaped like two congruent rhombi. She wants the long diagonals lined with a stone walkway. The total area of the garden is 150 square feet, and the shorter diagonals are each 12 feet long.

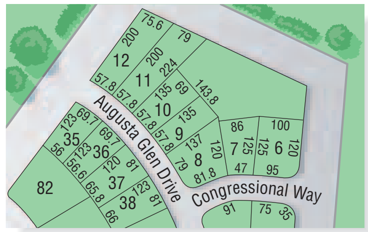


36. Find the length of each stone walkway. **12.5 ft**

37. Find the length of each side of the garden. **about 8.7 ft**

REAL ESTATE **For Exercises 38 and 39, use the following information.**

The map shows the layout and dimensions of several lot parcels in Linworth Village. Suppose Lots 35 and 12 are trapezoids.



38. If the height of Lot 35 is 122.81 feet, find the area of this lot. **7718.6 ft²**

39. If the height of Lot 12 is 199.8 feet, find the area of this lot. **13,326.7 ft²**

Career Choices

Real Estate Agent

Real estate agents must have a broad knowledge of the neighborhoods in their community. About two-thirds of real estate agents are self-employed. Success is dependent on selling properties.

Source: ww.bls.gov

Online Research **Data Update** Use the Internet or other resource to find the median price of homes in the United States. How does this compare to the median price of homes in your community? Visit www.geometryonline.com/data_update to learn more.

Find the area of each figure.

40. rhombus with a perimeter of 20 meters and a diagonal of 8 meters **24 m²**

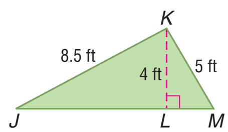
41. rhombus with a perimeter of 52 inches and a diagonal of 24 inches **120 in²**

42. isosceles trapezoid with a perimeter of 52 yards; the measure of one base is 10 yards greater than the other base, the measure of each leg is 3 yards less than twice the length of the shorter base **156 yd²**

43. equilateral triangle with a perimeter of 15 inches **≈ 10.8 in²**

★ **44.** scalene triangle with sides that measure 34.0 meters, 81.6 meters, and 88.4 meters. **1387.2 m²**

★ **45.** Find the area of $\triangle JKM$. **21 ft²**



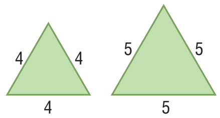
46. Derive the formula for the area of a rhombus using the formula for the area of a triangle. **See margin.**

47. Determine whether the statement *Two triangles that have the same area also have the same perimeter* is true or false. Give an example or counterexample. **See margin.**

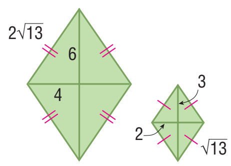
48–49. See margin.

Each pair of figures is similar. Find the area and perimeter of each figure. Describe how changing the dimensions affects the perimeter and area.

48.



49.

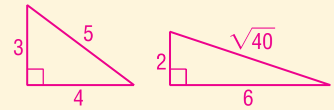


50. **RECREATION** Becky wants to cover a kite frame with decorative paper. If the length of one diagonal is 20 inches and the other diagonal measures 25 inches, find the area of the surface of the kite. **250 in²**

Answers

46. A rhombus is made up of two congruent triangles. Using d_1 and d_2 instead of b and h, its area in reference to $A = \frac{1}{2}bh$ is $2\left[\frac{1}{2}(d_1)\left(\frac{1}{2}d_2\right)\right]$ or $\frac{1}{2}d_1d_2$.

47. False; Sample answer:



The area for each of these right triangles is 6 square units. The perimeter of one triangle is 12 and the perimeter of the other is $8 + \sqrt{40}$ or about 14.3.

48. area ≈ 6.9, area ≈ 10.8; perimeter = 12, perimeter = 15; scale factor and ratio of perimeters is $\frac{5}{4}$, ratio of areas is $\left(\frac{5}{4}\right)^2$

49. area = 12, area = 3, perimeter = $8\sqrt{13}$, perimeter = $4\sqrt{13}$; scale factor and ratio of perimeters = $\frac{1}{2}$, ratio of areas = $\left(\frac{1}{2}\right)^2$

Answers

58. Sample answer: Umbrellas have triangular panels of fabric or nylon. In order to make the panels to fit the umbrella frame, the area of the triangles is needed. Answers should include the following.

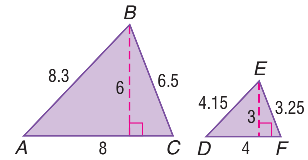
- Find the area of a triangle by multiplying the base and the height and dividing by two.
- Rhombi are composed of two congruent isosceles triangles, and trapezoids are composed of two triangles and a rectangle.

53. The ratio is the same.

55. 4:1; The ratio of the areas is the square of the scale factor.

SIMILAR FIGURES **For Exercises 51–56, use the following information.**

Triangle *ABC* is similar to triangle *DEF*.



51. Find the scale factor. $\frac{2}{1}$

52. Find the perimeter of each triangle. **22.8; 11.4**

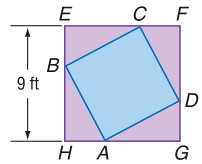
53. Compare the ratio of the perimeters of the triangles to the scale factor.

54. Find the area of each triangle. **24, 6**

55. Compare the ratio of the areas of the triangles to the scale factor.

56. Compare the ratio of the areas of the triangles to the ratio of the perimeters of the triangles. **The ratio of the areas is the square of the ratio of the perimeters.**

57. **CRITICAL THINKING** In the figure, the vertices of quadrilateral *ABCD* intersect square *EFGH* and divide its sides into segments with measures that have a ratio of 1:2. Find the area of *ABCD*. Describe the relationship between the areas of *ABCD* and *EFGH*. **45 ft^2; The ratio of the areas is 5:9.**



58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

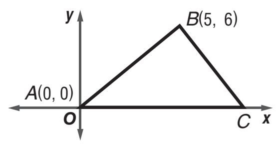
How is the area of a triangle related to beach umbrellas?

Include the following in your answer:
- how to find the area of a triangle, and
- how the area of a triangle can help you find the areas of rhombi and trapezoids.

Standardized Test Practice

59. In the figure, if point *B* lies on the perpendicular bisector of $\overline{AC}$, what is the area of $\triangle ABC$? **B**

Ⓐ 15 units^2 Ⓑ 30 units^2
Ⓒ 50 units^2 Ⓓ 1602 units^2



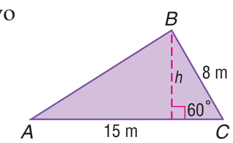
60. **ALGEBRA** What are the solutions of the equation $(2x - 7)(x + 10) = 0$? **D**

Ⓐ −3.5 and 10 Ⓑ 7 and −10 Ⓒ $\frac{2}{7}$ and −10 Ⓓ 3.5 and −10

Extending the Lesson

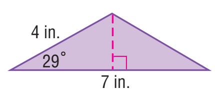
Trigonometric Ratios and the Areas of Triangles

The area of any triangle can be found given the measures of two sides of the triangle and the measure of the included angle. Suppose we are given $AC = 15$, $BC = 8$, and $m\angle C = 60$. To find the height of the triangle, use the sine ratio, $\sin C = \frac{h}{BC}$. Then use the value of h in the formula for the area of a triangle. So, the area is $\frac{1}{2}(15)(8 \sin 60°)$ or 52.0 square meters.

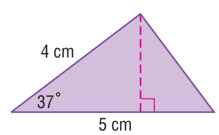


61. Derive a formula to find the area of any triangle, given the measures of two sides of the triangle and their included angle. **area = $\frac{1}{2}ab \sin C$**

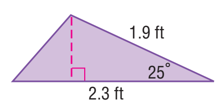
Find the area of each triangle. Round to the nearest hundredth.

62.


6.79 in^2

63.


6.02 cm^2

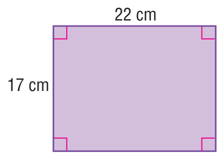
64.


0.92 ft^2

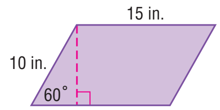
608 Chapter 11 Areas of Polygons and Circles

Maintain Your Skills

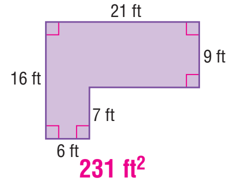
Mixed Review Find the area of each figure. Round to the nearest tenth. *(Lesson 11-1)*

65.


374 cm²

66.


129.9 in²

67.


231 ft²

Write an equation of circle R based on the given information. *(Lesson 10-8)*

68. center: $R(1, 2)$ radius: 7

69. center: $R\left(-4, \frac{1}{2}\right)$ radius: $\frac{11}{2}$

70. center: $R(-1.3, 5.6)$ radius: 3.5

68. $(x-1)^2 + (y-2)^2 = 49$
69. $(x+4)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{121}{4}$
70. $(x+1.3)^2 + (y-5.6)^2 = 12.25$

71. **CRAFTS** Andria created a pattern to appliqué flowers onto a quilt by first drawing a regular pentagon that was 3.5 inches long on each side. Then she added a semicircle onto each side of the pentagon to create the appearance of five petals. How many inches of gold trim does she need to edge 10 flowers? *(Lesson 10-1)* **275 in.**

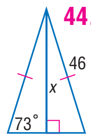
Given the magnitude and direction of a vector, find the component form with values rounded to the nearest tenth. *(Lesson 9-6)* **72. ⟨123.3, 57.5⟩**

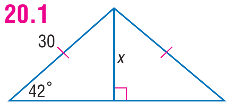
72. magnitude of 136 at a direction of 25 degrees with the positive x-axis

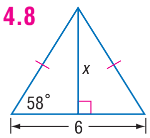
73. magnitude of 280 at a direction of 52 degrees with the positive x-axis **⟨172.4, 220.6⟩**

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find x. Round to the nearest tenth.
*(To review **trigonometric ratios in right triangles**, see Lesson 7-4.)*

74. **44.0**


75. **20.1**


76. **4.8**


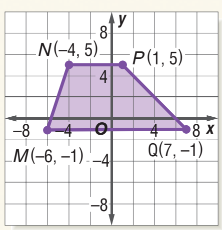
Practice Quiz 1

Lessons 11-1 and 11-2

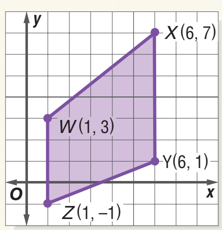
The coordinates of the vertices of quadrilateral $JKLM$ are $J(-8, 4)$, $K(-4, 0)$, $L(0, 4)$, and $M(-4, 8)$. *(Lesson 11-1)*

1. Determine whether $JKLM$ is a *square*, a *rectangle*, or a *parallelogram*. **square**
2. Find the area of $JKLM$. **32 units²**

Find the area of each trapezoid. *(Lesson 11-2)*

3.


54 units²

4.


25 units²

5. The area of a rhombus is 546 square yards. If d_1 is 26 yards long, find the length of d_2. *(Lesson 11-2)* **42 yd**

www.geometryonline.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Ask students to list all the formulas for area they have learned so far in this chapter. Ask them to include a labeled diagram corresponding to each formula.

Getting Ready for Lesson 11-3

Prerequisite Skill Students will use right triangle trigonometry in Lesson 11-3 to find the altitude of a regular polygon so that they can calculate the area of the polygon. Use Exercises 74–76 to determine your students' familiarity with trigonometric ratios in right triangles.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 11-1 and 11-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 11-1 and 11-2) is available on p. 655 of the *Chapter 11 Resource Masters*.

Teacher to Teacher

Sarah L. Waldrop, Forestview High School — Gastonia, NC

We play MATHO as a review. I give students (on the overhead) about 30–36 answers to be placed on their MATHO sheet. This sheet is arranged like bingo—with a free space. They fill in any 24 answers. After their card is filled in, I ask questions with the given answer. When they have MATHO they get a prize (candy, points on a quiz, etc.).

11-3 Lesson Notes

1 Focus

5-Minute Check Transparency 11-3 Use as a quiz or review of Lesson 11-2.

Mathematical Background notes are available for this lesson on p. 592C.

How can you find the area of a polygon?

Ask students:

- Which geometric shape is used most often in the construction of a gazebo? **a regular hexagon**
- Why is this shape used for gazebos? **Sample answer: nice shape; large area**
- What is another structure that uses this shape in its construction? **Sample answer: barns**
- How is this shape used in nature? **Sample answer: honeycombs**

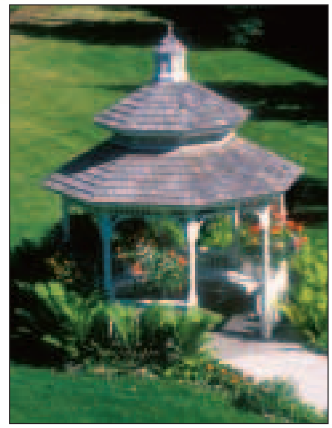
11-3 Areas of Regular Polygons and Circles

What You'll Learn

- Find areas of regular polygons.
- Find areas of circles.

How can you find the area of a polygon?

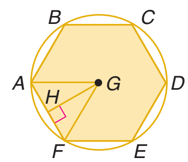
The foundations of most gazebos are shaped like regular hexagons. Suppose the owners of this gazebo would like to install tile on the floor. If tiles are sold in square feet, how can they find out the actual area of tiles needed to cover the floor?



Vocabulary

- apothem

AREAS OF REGULAR POLYGONS In regular hexagon *ABCDEF* inscribed in circle *G*, $\overline{GA}$ and $\overline{GF}$ are radii from the center of the circle *G* to two vertices of the hexagon. $\overline{GH}$ is drawn from the center of the regular polygon perpendicular to a side of the polygon. This segment is called an **apothem**.

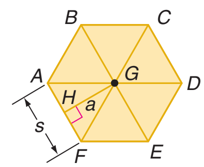


Study Tip

Look Back
To review **apothem**, see Lesson 1-4.

Triangle *GFA* is an isosceles triangle, since the radii are congruent. If all of the radii were drawn, they would separate the hexagon into 6 nonoverlapping congruent isosceles triangles.

The area of the hexagon can be determined by adding the areas of the triangles. Since $\overline{GH}$ is perpendicular to $\overline{AF}$, it is an altitude of $\triangle AGF$. Let *a* represent the length of $\overline{GH}$ and let *s* represent the length of a side of the hexagon.



$$\text{Area of } \triangle AGF = \frac{1}{2}bh$$
$$= \frac{1}{2}sa$$

The area of one triangle is $\frac{1}{2}sa$ square units. So the area of the hexagon is $6\left(\frac{1}{2}sa\right)$ square units. Notice that the perimeter *P* of the hexagon is 6*s* units. We can substitute *P* for 6*s* in the area formula. So, $A = 6\left(\frac{1}{2}sa\right)$ becomes $A = \frac{1}{2}Pa$. This formula can be used for the area of any regular polygon.

Key Concept *Area of a Regular Polygon*

If a regular polygon has an area of *A* square units, a perimeter of *P* units, and an apothem of *a* units, then $A = \frac{1}{2}Pa$.

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters

- Study Guide and Intervention, pp. 623–624
- Skills Practice, p. 625
- Practice, p. 626
- Reading to Learn Mathematics, p. 627
- Enrichment, p. 628
- Assessment, pp. 655, 657

Prerequisite Skills Workbook, pp. 43–44

Teaching Geometry With Manipulatives Masters, pp. 1, 18, 184, 185, 186, 189

Transparencies

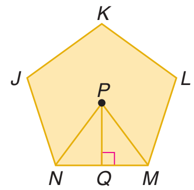
5-Minute Check Transparency 11-3
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 1 Area of a Regular Polygon

Find the area of a regular pentagon with a perimeter of 40 centimeters.



Apothem: The central angles of a regular pentagon are all congruent. Therefore, the measure of each angle is $\frac{360}{5}$ or 72. $\overline{PQ}$ is an apothem of pentagon $JKLMN$. It bisects $\angle NPM$ and is a perpendicular bisector of $\overline{NM}$. So, $m\angle MPQ = \frac{1}{2}(72)$ or 36. Since the perimeter is 40 centimeters, each side is 8 centimeters and $QM = 4$ centimeters. Write a trigonometric ratio to find the length of $\overline{PQ}$.

$\tan \angle MPQ = \frac{QM}{PQ}$ — $\tan \theta = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$

$\tan 36° = \frac{4}{PQ}$ — $m\angle MPQ = 36$, $QM = 4$

$(PQ) \tan 36° = 4$ — Multiply each side by PQ.

$PQ = \frac{4}{\tan 36°}$ — Divide each side by tan 36°.

$PQ \approx 5.5$ — Use a calculator.

Area: $A = \frac{1}{2}Pa$ — Area of a regular polygon

$\approx \frac{1}{2}(40)(5.5)$ — $P = 40$, $a \approx 5.5$

≈ 110 — Simplify.

So, the area of the pentagon is about 110 square centimeters.

Study Tip

Problem Solving
There is another method for finding the apothem of a regular polygon. You can use the Interior Angle Sum Theorem to find $m\angle PMQ$ and then write a trigonometric ratio to find PQ.

AREAS OF CIRCLES

You can use a calculator to help derive the formula for the area of a circle from the areas of regular polygons.

Geometry Activity

Area of a Circle

Collect Data

Suppose each regular polygon is inscribed in a circle of radius r.

1. Copy and complete the following table. Round to the nearest hundredth.

Inscribed Polygon	(triangle)	(pentagon)	(octagon)	(decagon)	(20-gon)	(50-gon)
Number of Sides	3	5	8	10	20	50
Measure of a Side	$1.73r$	$1.18r$	$0.77r$	$0.62r$	$0.31r$	$0.126r$
Measure of Apothem	$0.5r$	$0.81r$	$0.92r$	$0.95r$	$0.99r$	$0.998r$
Area	$1.30r^2$	$2.39r^2$	$2.83r^2$	$2.95r^2$	$3.07r^2$	$3.14r^2$

Analyze the Data

2. What happens to the appearance of the polygon as the number of sides increases? **The polygon appears to be a circle.**
3. What happens to the areas as the number of sides increases?
4. **Make a conjecture** about the formula for the area of a circle.

3. The areas of the polygons approach the area of the circle.

4. The formula for the area of a circle is πr^2 or about $3.14r^2$.

www.geometryonline.com/extra_examples

2 Teach

Building on Prior Knowledge

In Chapter 7, students learned how to use trigonometry to solve a triangle. In this lesson, they will use trigonometry to find the apothem of a regular polygon so that they can apply the area formula for a regular polygon. The area is one-half the product of the apothem and the perimeter.

Tips for New Teachers

Rounding Rounding during computation may result in a final answer that differs from the ones given in the Teacher Wraparound Edition.

AREAS OF REGULAR POLYGONS

In-Class Example Power Point®

Teaching Tip Students may have difficulty finding the apothem of a regular hexagon. Emphasize the construction of a 30°-60°-90° triangle from the center to a side of the hexagon. Then use the side relationships to find the apothem.

1 Find the area of a regular pentagon with a perimeter of 90 meters. **about 557 m²**

Geometry Activity

Materials: calculator

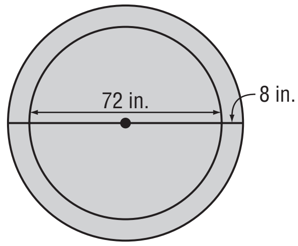
- Ask why the radius is left as a variable in this data. (The table can be developed for a circle with any radius r.
- Also point out that the measure of a side and measure of the apothem of an equilateral triangle is found using 30°-60°-90° triangle relationships. Trigonometric ratios are used to determine the measures of a side and an apothem for each polygon.

AREAS OF CIRCLES

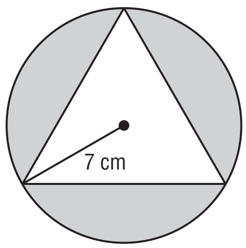
In-Class Examples

Power Point®

2 An outdoor accessories company manufactures circular covers for outdoor umbrellas. If the cover is 8 inches longer than the umbrella on each side, find the area of the cover in square yards. **4.7 yd²**



3 Find the area of the shaded region. Assume that the triangle is equilateral. Round to the nearest tenth.

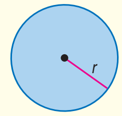


90.3 cm²

You can see from the Geometry Activity that the more sides a regular polygon has, the more closely it resembles a circle.

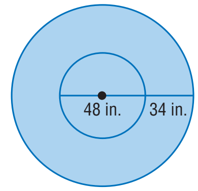
Key Concept — *Area of a Circle*

If a circle has an area of A square units and a radius of r units, then $A = \pi r^2$.



Example 2 Use Area of a Circle to Solve a Real-World Problem

SEWING **A caterer has a 48-inch diameter table that is 34 inches tall. She wants a tablecloth that will touch the floor. Find the area of the tablecloth in square yards.**



The diameter of the table is 48 inches, and the tablecloth must extend 34 inches in each direction. So the diameter of the tablecloth is 34 + 48 + 34 or 116 inches. Divide by 2 to find that the radius is 58 inches.

$A = \pi r^2$ — Area of a circle

$= \pi(58)^2$ — Substitution

$\approx 10{,}568.3$ — Use a calculator.

The area of the tablecloth is 10,568.3 square inches. To convert to square yards, divide by 1296. The area of the tablecloth is 8.2 square yards to the nearest tenth.

Study Tip

Square Yards

A square yard measures 36 inches by 36 inches or 1296 square inches.

You can use the properties of circles and regular polygons to find the areas of inscribed and circumscribed polygons.

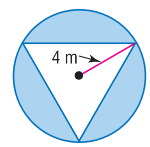
Study Tip

Look Back

To review **inscribed and circumscribed polygons,** see Lesson 10-4.

Example 3 Area of an Inscribed Polygon

Find the area of the shaded region. Assume that the triangle is equilateral.

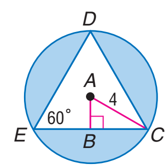


The area of the shaded region is the difference between the area of the circle and the area of the triangle. First, find the area of the circle.

$A = \pi r^2$ — Area of a circle

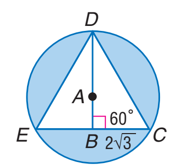
$= \pi(4)^2$ — Substitution

≈ 50.3 — Use a calculator.



To find the area of the triangle, use properties of 30°-60°-90° triangles. First, find the length of the base. The hypotenuse of $\triangle ABC$ is 4, so BC is $2\sqrt{3}$. Since $EC = 2(BC)$, $EC = 4\sqrt{3}$.

Next, find the height of the triangle, DB. Since $m\angle DCB$ is 60, $DB = 2\sqrt{3}(\sqrt{3})$ or 6.



Use the formula to find the area of the triangle.

$A = \frac{1}{2}bh$ — Area of a triangle

$= \frac{1}{2}(4\sqrt{3})(6)$ — $b = 4\sqrt{3}$, $h = 6$

≈ 20.8 — Use a calculator.

The area of the shaded region is 50.3 − 20.8 or 29.5 square meters to the nearest tenth.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students choose a partner. Ask one student to draw a polygon and give the radius of the circumscribed circle. Then ask the partner to find the area by first finding the apothem. Next switch roles and do the activity again. If students have difficulty finding the apothem of the polygon, ask them to discuss how right triangle trigonometry may be used.

Check for Understanding

Concept Check

1. **Explain** how to derive the formula for the area of a regular polygon. **See margin.**
2. **OPEN ENDED** Describe a method for finding the base or height of a right triangle given one acute angle and the length of one side. **See margin.**

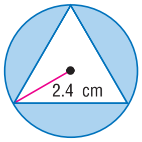
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
3–4	1
5–6	3
7	2

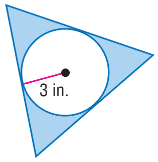
Find the area of each polygon. Round to the nearest tenth.

3. a regular hexagon with a perimeter of 42 yards **127.3 yd^2**
4. a regular nonagon with a perimeter of 108 meters **890.2 m^2**

Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

5. 

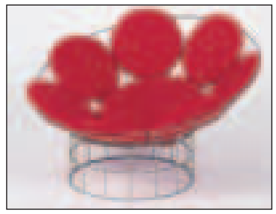
10.6 cm^2

6. 

18.5 in^2

Application

7. **UPHOLSTERY** Tyra wants to cover the cushions of her papasan chair with a different fabric. If there are seven circular cushions that are the same size with a diameter of 12 inches, around a center cushion with a diameter of 20 inches, find the area of fabric in square yards that she will need to cover both sides of the cushions. Allow an extra 3 inches of fabric around each cushion. **about 3.6 yd^2**



★ indicates increased difficulty

Practice and Apply

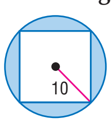
Homework Help	
For Exercises	See Examples
8–13, 26, 27	1
14–23, 37–42	3
24, 25, 28–31	2

Extra Practice
See page 777.

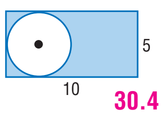
Find the area of each polygon. Round to the nearest tenth.

8. a regular octagon with a perimeter of 72 inches **391.1 in^2**
9. a square with a perimeter of $84\sqrt{2}$ meters **882 m^2**
10. a square with apothem length of 12 centimeters **576 cm^2**
11. a regular hexagon with apothem length of 24 inches **1995.3 in^2**
12. a regular triangle with side length of 15.5 inches **104.0 in^2**
13. a regular octagon with side length of 10 kilometers **482.8 km^2**

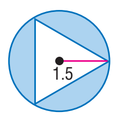
Find the area of each shaded region. Assume that all polygons that appear to be regular are regular. Round to the nearest tenth.

14. 

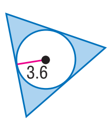
114.2 units^2

15. 

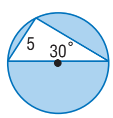
30.4 units^2

16. 

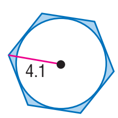
4.1 units^2

17. 

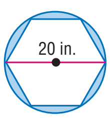
26.6 units^2

18. 

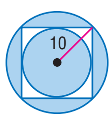
56.9 units^2

19. 

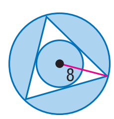
4.1 units^2

20. 

54.4 in^2

★21. 

271.2 units^2

★22. 

168.2 units^2

Concept Check

Ask students to describe how to find the area of an inscribed polygon. **First find the apothem, or the distance from the center of the polygon perpendicular to the midpoint of the opposite side. Then find half of the product of the apothem and the perimeter.**

3 Practice/Apply

Study Notebook

Have students—

- *add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.*
- *write the formula for the area of a regular polygon and the area of a circle. Ask them to include an example of each.*
- *include any other item(s) that they find helpful in mastering the skills in this lesson.*

About the Exercises...

Organization by Objective

- **Areas of Regular Polygons:** 8–13, 26, 27
- **Areas of Circles:** 14–25, 28–31, 37–42

Odd/Even Assignments

Exercises 8–42 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 9–19 odd, 23–37 odd, 38, 45–55 odd, 56–72

Average: 9–37 odd, 38, 39–55 odd, 56–72

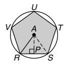
Advanced: 8–54 even, 55–68 (optional: 69–72)

Answers

1. Sample answer: Separate a hexagon inscribed in a circle into six congruent nonoverlapping isosceles triangles. The area of one triangle is one-half the product of one side of the hexagon and the apothem of the hexagon. The area of the hexagon is $6\left(\frac{1}{2}sa\right)$. The perimeter of the hexagon is $6s$, so the formula is $\frac{1}{2}Pa$.
2. Sample answer: Use the given angle measure, the given side length, and trigonometric ratios to find the missing lengths.

Study Guide and Intervention, p. 623 (shown) and p. 624

Areas of Regular Polygons In a regular polygon, the segment drawn from the center of the polygon perpendicular to the opposite side is called the **apothem**. In the figure at the right, $\overline{AP}$ is the apothem and $\overline{AR}$ is the radius of the circumscribed circle.



Area of a Regular Polygon	If a regular polygon has an area of A square units, a perimeter of P units, and an apothem of a units, then $A = \frac{1}{2}Pa$.

Example 1 **Verify the formula $A = \frac{1}{2}Pa$ for the regular pentagon above.**

For $\triangle RAS$, the area is $A = \frac{1}{2}bh = \frac{1}{2}(RS)(AP)$. So the area of the pentagon is $A = 5\left(\frac{1}{2}\right)(RS)(AP)$. Substituting P for $5RS$ and substituting a for AP, then $A = \frac{1}{2}Pa$.

Example 2 **Find the area of regular pentagon $RSTUV$ above if its perimeter is 60 centimeters.**

First find the apothem.

The measure of central angle RAS is $\frac{360}{5}$ or 72. Therefore $m\angle RAP = 36$. The perimeter is 60, so $RS = 12$ and $RP = 6$.

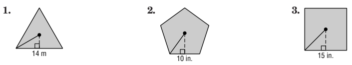
$$\tan \angle RAP = \frac{RP}{AP}$$
$$\tan 36° = \frac{6}{AP}$$
$$AP = \frac{6}{\tan 36°} \approx 8.26$$

So $A = \frac{1}{2}Pa = \frac{1}{2}60(8.26)$ or 247.7.

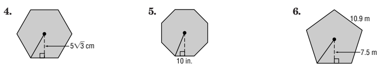
The area is about 248 square centimeters.

Exercises

Find the area of each regular polygon. Round to the nearest tenth.



1. 84.9 m²
2. 172.0 in²
3. 225 in²



4. 259.8 cm²
5. 482.8 in²
6. 204.4 m²

Skills Practice, p. 625 and Practice, p. 626 (shown)

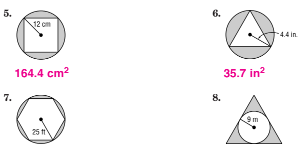
Find the area of each regular polygon. Round to the nearest tenth.

1. a nonagon with a perimeter of 117 millimeters
   1044.7 mm²
2. an octagon with a perimeter of 96 yards
   695.3 yd²

Find the area of each circle. Round to the nearest tenth.

3. a circle with a diameter of 26 feet
   530.9 ft²
4. a circle with a circumference of 88 kilometers
   616.2 km²

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth.



5. 164.4 cm²
6. 35.7 in²
7. 339.7 ft²
8. 166.4 m²

DISPLAYS For Exercises 9 and 10, use the following information.
A display case in a jewelry store has a base in the shape of a regular octagon. The length of each side of the base is 10 inches. The owners of the store plan to cover the base in black velvet.

9. Find the area of the base of the display case.
   about 482.8 in²
10. Find the number of square yards of fabric needed to cover the base.
   about 0.37 yd²

Reading to Learn Mathematics, p. 627 ELL

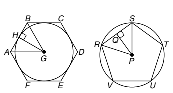
Pre-Activity **How can you find the area of a polygon?**

Read the introduction to Lesson 11-3 at the top of page 610 in your textbook.

How can you find the area of a regular hexagon without a new area formula? **Sample answer: Divide the hexagon into six congruent equilateral triangles. Use properties of the 30°-60°-90° triangle to find the area of one of these triangles and multiply the result by 6.**

Reading the Lesson

1. $ABCDEF$ and $RSTUV$ are regular polygons. Name each of the following in one of the figures.
   a. a circumscribed polygon **hexagon $ABCDEF$**
   b. an inscribed polygon **pentagon $RSTUV$**
   c. an apothem of a regular hexagon **$\overline{GH}$**
   d. an isosceles triangle **$\triangle PRS$ or $\triangle GAB$**
   e. a 30°-60°-90° triangle **$\triangle GAH$ or $\triangle GBH$**
   f. a central angle with a measure of 72 **$\angle RPS$**



2. Refer to the figures in Exercise 1. Match each item in the first column with an expression in the second column.

a. perimeter of $ABCDEF$ **v**	i. $\pi(PS)^2$
b. circumference of circle G **viii**	ii. $2\pi(PR)$
c. perimeter of $RSTUV$ **vii**	iii. $\frac{5}{2}(RS)(PQ)$
d. area of circle G **vi**	iv. $3(AB)(HG)$
e. area of $RSTUV$ **iii**	v. $6(CD)$
f. area of $ABCDEF$ **iv**	vi. $\pi(GH)^2$
g. area of circle P **i**	vii. $5(UV)$
h. circumference of circle P **ii**	viii. $2\pi(GH)$

3. Explain in your own words how to find the area of a circle if you know the circumference. **Sample answer: Divide the circumference by 2π to find the radius. Then square the radius and multiply by π to find the area.**

Helping You Remember

4. A good way to remember something is to explain it to someone else. Suppose your classmate Joelle is having trouble remembering which formula is for circumference and which is for area. How can you help her? **Sample answer: Circumference is measured in linear units, while area is measured in square units, so the formula containing r^2 must be the one for area.**

23. **ALGEBRA** A circle is inscribed in a square, which is circumscribed by another circle. If the diagonal of the square is $2x$, find the ratio of the area of the large circle to the area of the small circle. **2:1**

24. **CAKE** A bakery sells single-layer mini-cakes that are 3 inches in diameter for \$4 each. They also have a 9-inch cake for \$15. If both cakes are the same thickness, which option gives you more cake for the money, nine mini-cakes or one 9-inch cake? Explain.

24. The total area is equal. Nine mini-cakes are the same size as one 9-inch cake, but nine mini-cakes cost $9 \cdot \$4$ or \$36 while the 9-inch cake is only \$15.

25. **PIZZA** A pizza shop sells 8-inch pizzas for \$5 and 16-inch pizzas for \$10. Which would give you more pizza, two 8-inch pizzas or one 16-inch pizza? Explain.

25. One 16-inch pizza; the area of the 16-inch pizza is greater than the area of two 8-inch pizzas, so you get more pizza for the same price.

COORDINATE GEOMETRY The coordinates of the vertices of a regular polygon are given. Find the area of each polygon to the nearest tenth.

26. $T(0, 0)$, $U(-7, -7)$, $V(0, -14)$, $W(7, -7)$ **98 units²**

27. $G(-12, 0)$, $H(0, 4\sqrt{3})$, $J(0, -4\sqrt{3})$ **83.1 units²**

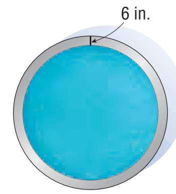
28. $J(5, 0)$, $K(2.5\sqrt{2}, -2.5\sqrt{2})$, $L(0, -5)$, $M(-2.5\sqrt{2}, -2.5\sqrt{2})$, $N(-5, 0)$, $P(-2.5\sqrt{2}, 2.5\sqrt{2})$, $Q(0, 5)$, $R(2.5\sqrt{2}, 2.5\sqrt{2})$ **70.7 units²**

29. $A(-2\sqrt{2}, 2\sqrt{2})$, $B(0, 4)$, $C(2\sqrt{2}, 2\sqrt{2})$, $D(4, 0)$, $E(2\sqrt{2}, -2\sqrt{2})$, $F(0, -4)$, $G(-2\sqrt{2}, -2\sqrt{2})$, $H(-4, 0)$ **45.3 units²**

Find the area of each circle. Round to the nearest tenth.

30. $C = 34\pi$ **907.9 units²**
31. $C = 17\pi$ **227.0 units²**
32. $C = 54.8$ **239.0 units²**
33. $C = 91.4$ **664.8 units²**

SWIMMING POOL For Exercises 34 and 35, use the following information.
The area of a circular pool is approximately 7850 square feet. The owner wants to replace the tiling at the edge of the pool.



34. The edging is 6 inches wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase? **629 tiles**

35. Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase? **triangles; 629 tiles**

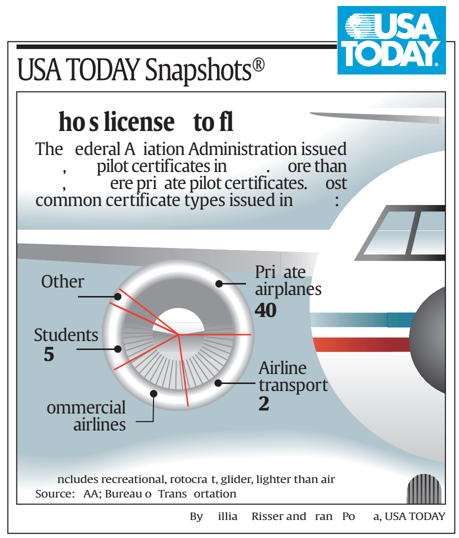
AVIATION For Exercises 36–38, refer to the circle graph.

36. Suppose the radius of the circle on the graph is 1.3 centimeters. Find the area of the circle on the graph. **5.3 cm²**

37. Francesca wants to use this circle graph for a presentation. She wants the circle to use as much space on a 22″ by 28″ sheet of poster board as possible. Find the area of the circle. **≈380.1 in²**

38. **CRITICAL THINKING** Make a conjecture about how you could determine the area of the region representing the pilots who are certified to fly private airplanes.

38. Sample answer: Multiply the total area by 40%.



Study Tip

Look Back
To review **circle graphs,** see Lesson 10-2.

Enrichment, p. 628

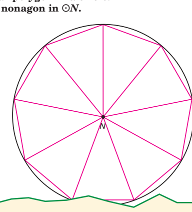
Areas of Inscribed Polygons

A protractor can be used to inscribe a regular polygon in a circle. Follow the steps below to inscribe a regular nonagon in ⊙N.

Step 1 Find the degree measure of each of the nine congruent arcs. **40**

Step 2 Draw 9 radii to form 9 angles with the measure you found in Step 1. The radii will intersect the circle in 9 points.

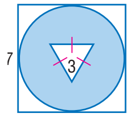
Step 3 Connect the nine points to form the nonagon.



1. Find the length of one side of the nonagon to the nearest tenth of a centimeter. What is the perimeter of the nonagon? **2.5 cm, P = 22.5 cm**

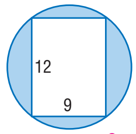
Find the area of each shaded region. Round to the nearest tenth.

★ **39.**



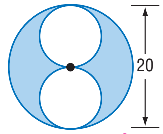
34.6 units^2

★ **40.**



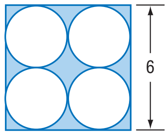
68.7 units^2

★ **41.**



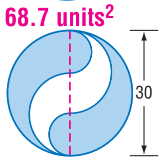
157.1 units^2

★ **42.**



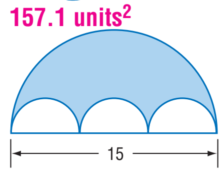
7.7 units^2

★ **43.**



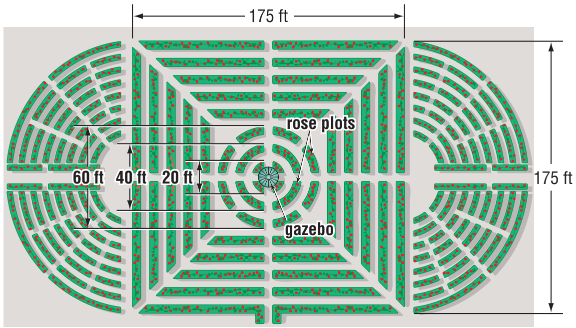
471.2 units^2

★ **44.**



58.9 units^2

GARDENS **For Exercises 45–47, use the following information.**
The Elizabeth Park Rose Garden in Hartford, Connecticut, was designed with a gazebo surrounded by two concentric rose garden plots. Wide paths emanate from the center, dividing the garden into square and circular sections.



45. Find the area and perimeter of the entire Rose Garden. Round to the nearest tenth. **54,677.8 ft^2; 899.8 ft**

46. What is the total of the circumferences of the three concentric circles formed by the gazebo and the two circular rose garden plots? (Ignore the width of the rose plots and the width of the paths.) **$120\pi \approx 377.0$ ft**

47. Each rose plot has a width of 5 feet. What is the area of the path between the outer two complete circles of rose garden plots? **$225\pi \approx 706.9$ ft^2**

More About...

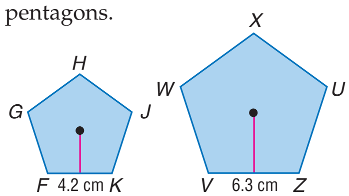
Architecture

This structure is a *pagoda*. Pagodas are characterized by having several hexagonal or octagonal stories each topped with a curved roof. In this temple, the sizes of the floors are in the ratio 1:3:5:7.

Source: www.infoplease.com

48. **ARCHITECTURE** The Anraku-ji Temple in Japan is composed of four octagonal floors of different sizes that are separated by four octagonal roofs of different sizes. Refer to the information at the left. Determine whether the areas of each of the four floors are in the same ratio as their sizes. Explain. **See margin.**

SIMILAR FIGURES **For Exercises 49–54, use the following information.**
Polygons *FGHJK* and *VWXUZ* are similar regular pentagons.



49. Find the scale factor. **2 : 3**

50. Find the perimeter of each pentagon.

51. Compare the ratio of the perimeters of the pentagons to the scale factor.

52. Find the area of each pentagon.

53. Compare the ratio of the areas of the pentagons to the scale factor.

54. Compare the ratio of the areas of the pentagons to the ratio of the perimeters of the pentagons. **The ratio of the areas is the square of the ratio of the perimeters.**

50. 21 cm, 31.5 cm
51. The ratio is the same.
52. ≈30.35 cm^2, ≈68.29 cm^2
53. The ratio of the areas is the square of the scale factor.

www.geometryonline.com/self_check_quiz

Answer

48. No; the areas of the floors will increase by the squares of 1, 3, 5, and 7, or 1, 9, 25, and 49. The ratio of area is the square of the scale factor.

4 Assess

Open-Ended Assessment

Modeling Ask students to demonstrate how to construct a circle with radius 6 feet circumscribed about an equilateral triangle on the floor of the classroom. Ask them to use the tiles in the floor to demonstrate how to estimate the area of the region outside the triangle but inside the circle. Then use the area formula to calculate it exactly.

Getting Ready for Lesson 11-4

Prerequisite Skill Students will learn about areas of irregular figures in Lesson 11-4. They will use the properties of special right triangles to break the region into known shapes so that they can find the area. Use Exercises 67–70 to determine your students' familiarity with special right triangles.

Quiz (Lesson 11-3) is available on p. 655 of the *Chapter 11 Resource Masters*.

Mid-Chapter Test (Lessons 11-1 through 11-3) is available on p. 567 of the *Chapter 11 Resource Masters*.

Answers

56. Sample answer: You can find the areas of regular polygons by finding the product of the perimeter and the apothem and then multiplying by one half. Answers should include the following.

- **We need to know the length of each side and the length of the apothem.**
- **One method is to divide the area of the floor by the area of each tile. Since the floor is hexagonal and not rectangular, tiles of different shapes will need to be ordered to cover the floor.**

55. **CRITICAL THINKING** A circle inscribes one regular hexagon and circumscribes another. If the radius of the circle is 10 units long, find the ratio of the area of the smaller hexagon to the area of the larger hexagon. **3 to 4**

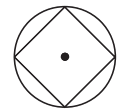
56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you find the area of a polygon?

Include the following in your answer:
- information needed about the gazebo floor to find the area, and
- how to find the number of tiles needed to cover the floor.

Standardized Test Practice

57. A square is inscribed in a circle of area 18π square units. Find the length of a side of the square. **B**



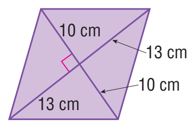
(A) 3 units (B) 6 units
(C) $3\sqrt{2}$ units (D) $6\sqrt{2}$ units

58. **ALGEBRA** The average of x numbers is 15. If the sum of the x numbers is 90, what is the value of x? **B**

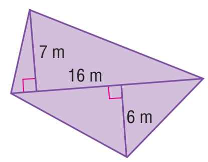
(A) 5 (B) 6 (C) 8 (D) 15

Maintain Your Skills

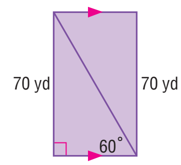
Mixed Review **Find the area of each quadrilateral.** *(Lesson 11-2)*

59.


260 cm²

60.


104 m²

61.


2829.0 yd²

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.** *(Lesson 11-1)*

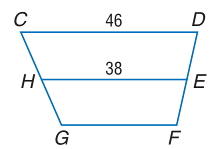
62. $A(-3, 2), B(4, 2), C(2, -1), D(-5, -1)$ **parallelogram; 21 units²**

63. $F(4, 1), G(4, -5), H(-2, -5), J(-2, 1)$ **square; 36 units²**

64. $K(-1, -3), L(-2, 5), M(1, 5), N(2, -3)$ **parallelogram; 24 units²**

65. $P(5, -7), Q(-1, -7), R(-1, -2), S(5, -2)$ **rectangle; 30 units²**

Refer to trapezoid *CDFG* with median $\overline{HE}$. *(Lesson 8-6)*



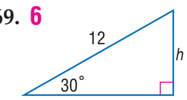
66. Find GF. **30**

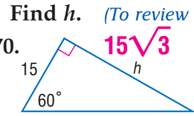
67. Let $\overline{WX}$ be the median of $CDEH$. Find WX. **42**

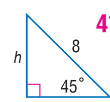
68. Let $\overline{YZ}$ be the median of $HEFG$. Find YZ. **34**

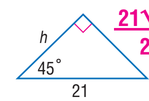
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find h.** *(To review **special right triangles**, see Lesson 7-3.)*

69. **6**


70. **$15\sqrt{3}$**


71. **$4\sqrt{2}$**


72. **$\frac{21\sqrt{2}}{2}$**


616 Chapter 11 Areas of Polygons and Circles

11-4 Areas of Irregular Figures

What You'll Learn
- Find areas of irregular figures.
- Find areas of irregular figures on the coordinate plane.

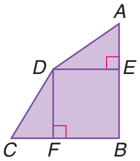
How do windsurfers use area?

The sail for a windsurf board cannot be classified as a triangle or a parallelogram. However, it can be separated into figures that can be identified, such as trapezoids and a triangle.

Vocabulary
- irregular figure
- irregular polygon

IRREGULAR FIGURES An **irregular figure** is a figure that cannot be classified into the specific shapes that we have studied. To find areas of irregular figures, separate the figure into shapes of which we can find the area. The sum of the areas of each is the area of the figure.

Auxiliary lines are drawn in quadrilateral $ABCD$. $\overline{DE}$ and $\overline{DF}$ separate the figure into $\triangle ADE$, $\triangle CDF$, and rectangle $BEDF$.

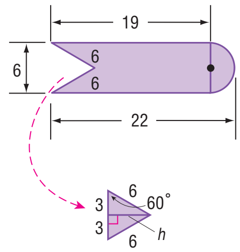


Postulate 11.2

The area of a region is the sum of all of its nonoverlapping parts.

Example 1 *Area of an Irregular Figure*

Find the area of the figure.



The figure can be separated into a rectangle with dimensions 6 units by 19 units, an equilateral triangle with sides each measuring 6 units, and a semicircle with a radius of 3 units.

Use 30°-60°-90° relationships to find that the height of the triangle is $3\sqrt{3}$.

area of irregular figure = area of rectangle − area of triangle + area of semicircle

$= \ell w - \frac{1}{2}bh + \frac{1}{2}\pi r^2$ Area formulas

$= 19 \cdot 6 - \frac{1}{2}(6)(3\sqrt{3}) + \frac{1}{2}\pi(3^2)$ Substitution

$= 114 - 9\sqrt{3} + \frac{9}{2}\pi$ Simplify.

≈ 112.5 Use a calculator.

The area of the irregular figure is 112.5 square units to the nearest tenth.

Study Tip

Reading Math

Irregular figures are also called *composite figures* because the regions can be separated into smaller regions.

www.geometryonline.com/extra_examples

11-4 Lesson Notes

1 Focus

5-Minute Check Transparency 11-4 Use as a quiz or review of Lesson 11-3.

Mathematical Background notes are available for this lesson on p. 592D.

How do windsurfers use area?

Ask students:
- What kind of shapes make up the shape of a windsurf sail? **triangles and trapezoids**
- Where are these same shapes also used? **Sample answer: building design**

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters
- Study Guide and Intervention, pp. 629–630
- Skills Practice, p. 631
- Practice, p. 632
- Reading to Learn Mathematics, p. 633
- Enrichment, p. 634
- Assessment, p. 656

School-to-Career Masters, p. 22

Teaching Geometry With Manipulatives Masters, p. 1

Transparencies

5-Minute Check Transparency 11-4
Answer Key Transparencies

Technology

GeomPASS: Tutorial Plus, Lesson 20
Interactive Chalkboard
Multimedia Applications: Virtual Activities

2 Teach

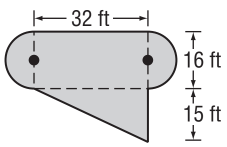
IRREGULAR FIGURES

In-Class Examples

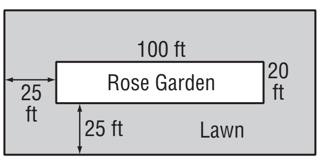
Power Point®

Teaching Tip Remind students that they may break up an irregular figure in different ways if they want to find the area. Suggest that they plan ahead to find the combination of shapes that are easiest to work with.

1 Find the area of the figure in square feet. Round to the nearest tenth if necessary. **953.1 ft²**



2 A rectangular rose garden is centered in a border of lawn. Find the area of the lawn around the garden in square feet. **8500 ft²**



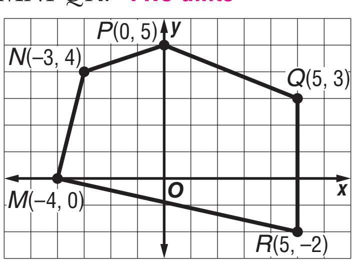
IRREGULAR FIGURES ON THE COORDINATE PLANE

In-Class Example

Power Point®

Teaching Tip Watch for students who think that the area formula for a regular polygon applies to any polygon. Stress that to find the area of a polygon that is not regular, you may need to divide the polygon into shapes with known formulas. Show students both types of polygons.

3 Find the area of polygon *MNPQR*. **44.5 units²**

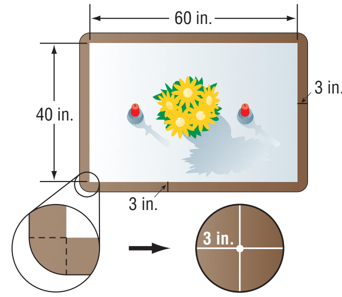


WebQuest

Identifying the polygons forming a region such as a tessellation will help you determine the type of tessellation. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

Example 2 Find the Area of an Irregular Figure to Solve a Problem

FURNITURE **Melissa's dining room table has hardwood around the outside. Find the area of wood around the edge of the table.**



First, draw auxiliary lines to separate the figure into regions. The table can be separated into four rectangles and four corners.

The four corners of the table form a circle with radius 3 inches.

area of wood edge = area of rectangles + area of circle

$= 2\ell w + 2\ell w + \pi r^2$ Area formulas

$= 2(3)(60) + 2(3)(40) + \pi(3^2)$ Substitution

$= 360 + 240 + 9\pi$ Simplify.

≈ 628.3 Use a calculator.

The area of the wood edge of the table is 628.3 square inches to the nearest tenth.

IRREGULAR FIGURES ON THE COORDINATE PLANE The formula for the area of a regular polygon does not apply to an **irregular polygon**, a polygon that is not regular. To find the area of an irregular polygon on the coordinate plane, separate the polygon into known figures.

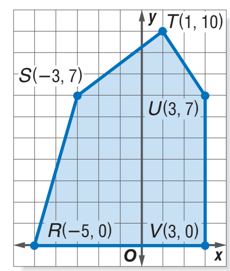
Study Tip

Estimation

Estimate the area of the simple closed curves by counting the unit squares. Use the estimate to determine if your answer is reasonable.

Example 3 Coordinate Plane

COORDINATE GEOMETRY **Find the area of polygon *RSTUV*.**



First, separate the figure into regions. Draw an auxiliary line from *S* to *U*. This divides the figure into triangle *STU* and trapezoid *RSUV*.

Find the difference between *x*-coordinates to find the length of the base of the triangle and the lengths of the bases of the trapezoid. Find the difference between *y*-coordinates to find the heights of the triangle and trapezoid.

area of *RSTUV* = area of $\triangle STU$ + area of trapezoid *RSUV*

$= \frac{1}{2}bh + \frac{1}{2}h(b_1 + b_2)$ Area formulas

$= \frac{1}{2}(6)(3) + \frac{1}{2}(7)(8 + 6)$ Substitution

$= 58$ Simplify.

The area of *RSTUV* is 58 square units.

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Have your students use string, masking tape, and a tiled floor to mark off irregular shapes on the floor. Ask them to estimate the area by counting squares and then verify the estimate by calculating the sum of the individual parts.

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Check for Understanding

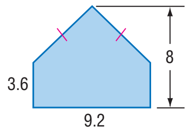
Concept Check

1. **OPEN ENDED** Sketch an irregular figure on a coordinate plane and find its area.
2. **Describe** the difference between an irregular figure and an irregular polygon. **1–2. See margin.**

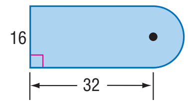
Guided Practice

Find the area of each figure. Round to the nearest tenth if necessary.

GUIDED PRACTICE KEY	
Exercises	Examples
3–4	1
5–6	3
7	2

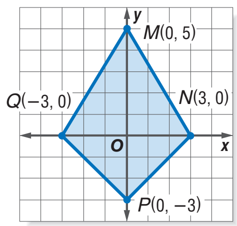
3. 

53.4 units²

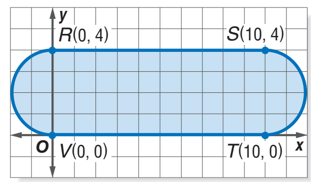
4. 

612.5 units²

COORDINATE GEOMETRY **Find the area of each figure.**

5. 

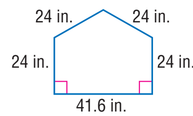
24 units²

6. 

52.6 units²

Application

7. **GATES** The Roths have a series of interlocking gates to form a play area for their baby. Find the area enclosed by the wall and gates. **1247.4 in²**



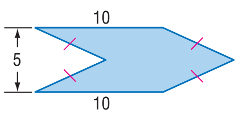
Practice and Apply

For Exercises	See Examples
8–13	1
14, 15, 23–27	2
16–22	3

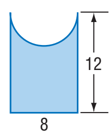
Homework Help

Extra Practice
See page 777.

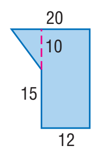
Find the area of each figure. Round to the nearest tenth if necessary.

8. 

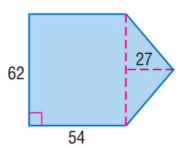
50 units²

9. 

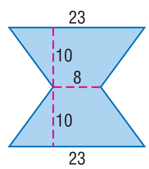
70.9 units²

10. 

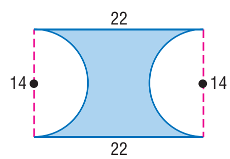
340 units²

11. 

4185 units²

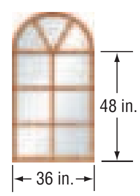
12. 

310 units²

13. 

154.1 units²

WINDOWS **For Exercises 14 and 15, use the following information.**
Mr. Cortez needs to replace this window in his house. The window panes are rectangles and sectors.



14. Find the perimeter of the window. **188.5 in.**
15. Find the area of the window. **2236.9 in²**

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include an example of finding the area of an irregular shape that is divided into known figures.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Irregular Figures:** 8–15, 23–27
- **Irregular Figures on the Coordinate Plane:** 16–22

Odd/Even Assignments

Exercises 8–22 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 24 requires the Internet or other research materials.

Assignment Guide

Basic: 9–27 odd, 28–42

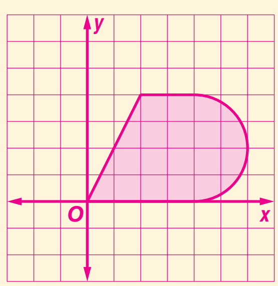
Average: 9–27 odd, 28–42

Advanced: 8–28 even, 29–38 (optional: 39–42)

All: Quiz 2 (1–5)

Answers

1. Sample answer: ≈18.3 units²

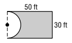


2. An irregular polygon is a polygon in which all sides are not congruent. If a shape can be separated into semicircles or smaller circular regions, it is an irregular figure.

Study Guide and Intervention, p. 629 (shown) and p. 630

Irregular Figures An **irregular** figure is one that cannot be classified as one of the previously-studied shapes. To find the area of an irregular figure, break it into familiar shapes. Find the area of each shape and add the areas.

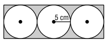
Example 1 **Find the area of the irregular figure.**



The figure is a rectangle minus one half of a circle. The radius of the circle is one half of 30 or 15.

$$A = lw - \frac{1}{2}\pi r^2$$
$$\approx 50(30) - 0.5(3.14)(15)^2$$
$$= 1146.6 \text{ or about } 1147 \text{ ft}^2$$

Example 2 **Find the area of the shaded region.**

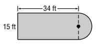


The dimensions of the rectangle are 10 centimeters and 30 centimeters. The area of the shaded region is

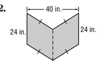
$$(10)(30) - 3\pi(5^2) = 300 - 75\pi \approx 64.4 \text{ cm}^2$$

Exercises

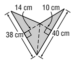
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

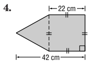
598.4 ft²

2. 

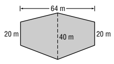
960 in²

3. 

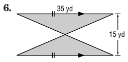
466 cm²

4. 

704 cm²

5. 

1920 m²

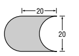
6. 

262.5 yd²

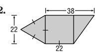
7. Refer to Example 2 above. Draw the largest possible square inside each of the three circles. What is the total area of the three squares? 150 cm²

Skills Practice, p. 631 and Practice, p. 632 (shown)

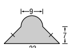
Find the area of each figure. Round to the nearest tenth if necessary.

1. 

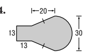
400 units²

2. 

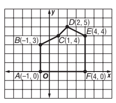
869.6 units²

3. 

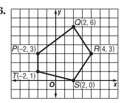
143.8 units²

4. 

952.4 units²

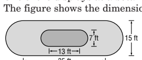
5. 

20.5 units²

6. 

22 units²

LANDSCAPING For Exercises 7 and 8, use the following information.
One of the displays at a botanical garden is a koi pond with a walkway around it. The figure shows the dimensions of the pond and the walkway.



7. Find the area of the pond to the nearest tenth.
129.5 ft²

8. Find the area of the walkway to the nearest tenth.
572.2 ft²

Reading to Learn Mathematics, p. 633

ELL

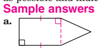
Pre-Activity **How do windsurfers use area?**

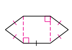
Read the introduction to Lesson 11-4 at the top of page 617 in your textbook.

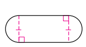
How do you think the areas of the figures outlined in the picture of the sail are related? **Sample answer: The areas get smaller as you move further up the sail. The area of the triangle is smaller than the area of any of the trapezoids.**

Reading the Lesson

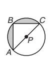
1. Use dashed segments to show how each figure can be subdivided into figures for which you have learned area formulas. Name the smaller figures that you have formed as specifically as possible and indicate whether any of them are congruent to each other.
Sample answers are given.

a. 
rectangle and isosceles triangle

b. 
square and two congruent isosceles triangles

c. 
rectangle and two congruent semicircles

2. In the figure, B is the midpoint of $\widehat{ABC}$. Complete the following steps to derive a formula for the area of the shaded region in terms of the radius r of the circle.



The area of circle P is $\underline{\pi r^2}$.

$m\angle ABC = \underline{90}$ because **Sample answer: It is an inscribed angle that intercepts a semicircle**.

$m\widehat{AB} = m\widehat{BC}$ because **Sample answer: B is the midpoint of $\widehat{ABC}$ (definition of midpoint)**.

$\overline{AB} \cong \overline{BC}$ because **Sample answer: If two minor arcs of a circle are congruent, their corresponding chords are congruent**.

Therefore, $\triangle ABC$ is a(n) **isosceles right or 45°-45°-90°** triangle.

$AC = \underline{2r}$, so $AB = \underline{\frac{2r}{\sqrt{2}} \text{ or } r\sqrt{2}}$ and $BC = \underline{\frac{2r}{\sqrt{2}} \text{ or } r\sqrt{2}}$.

The area of $\triangle ABC$ is $\frac{1}{2} \cdot \underline{r\sqrt{2}} \cdot \underline{r\sqrt{2}} = \underline{r^2}$.

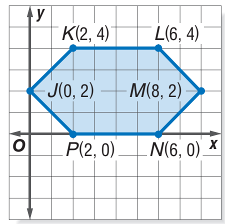
Therefore, the area of the shaded region is given by

$A = \underline{\frac{1}{2}\pi r^2} - \underline{r^2} = \underline{\left(\frac{\pi}{2} - 1\right)r^2}$.

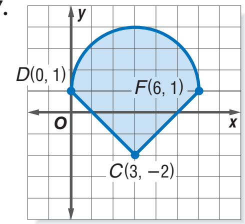
Helping You Remember

3. Rolando is having trouble remembering when to subtract an area when finding the area of an irregular figure. How can you help him remember? **Sample answer: Subtract when there is an indentation, or a hole in the figure.**

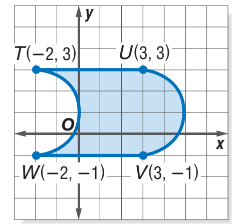
COORDINATE GEOMETRY **Find the area of each figure. Round to the nearest tenth if necessary.**

16. 

24 units²

17. 

23.1 units²

18. 

20 units²

COORDINATE GEOMETRY **The vertices of an irregular figure are given. Find the area of each figure.**

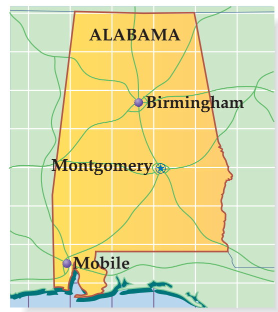
19. $M(-4, 0)$, $N(0, 3)$, $P(5, 3)$, $Q(5, 0)$ **21 units²**

20. $T(-4, -2)$, $U(-2, 2)$, $V(3, 4)$, $W(3, -2)$ **29 units²**

21. $G(-3, -1)$, $H(-3, 1)$, $I(2, 4)$, $J(5, -1)$, $K(1, -3)$ **33 units²**

22. $P(-8, 7)$, $Q(3, 7)$, $R(3, -2)$, $S(-1, 3)$, $T(-11, 1)$ **67 units²**

23. **GEOGRAPHY** Estimate the area of the state of Alabama. Each square on the grid represents 2500 square miles. **Sample answer: 57,500 mi²**



24. **RESEARCH** Find a map of your state or a state of your choice. Estimate the area. Then use the Internet or other source to check the accuracy of your estimate. **See students' work.**

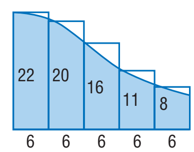
More About. . .

Calculus

Twenty percent of students in Japan take a calculus class in high school. In Korea and Switzerland, nearly all high school students take a course in calculus. In the U.S., only 1% of students take a calculus course in high school.

Source: NCTM

CALCULUS **For Exercises 25–27, use the following information.** **26. See margin.**
The irregular region under the curve has been approximated by rectangles of equal width.

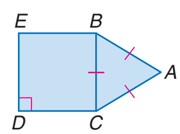


25. Use the rectangles to approximate the area of the region. **462**

26. Analyze the estimate. Do you think the actual area is larger or smaller than your estimate? Explain.

27. How could the irregular region be separated to give an estimate of the area that is more accurate? **Sample answer: Reduce the width of each rectangle.**

28. **CRITICAL THINKING** Find the ratio of the area of $\triangle ABC$ to the area of square $BCDE$. $\frac{\sqrt{3}}{4}$**:1**



29. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

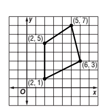
How do windsurfers use area?

Include the following in your answer:
- describe how to find the area of the sail, and
- another example of an irregular figure.

Enrichment, p. 634

Aerial Surveyors and Area

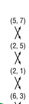
Many land regions have irregular shapes. Aerial surveyors often use coordinates when finding areas of such regions. The coordinate method described in the steps below can be used to find the area of *any* polygonal region. Study how this method is used to find the area of the region at the right.



Step 1 List the ordered pairs for the vertices in counter-clockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).

$$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$$
$$= 25 + 2 + 6 + 42 \text{ or } 75$$



Step 3 Find U, the sum of the upward diagonal products (from left to right).

U ... 7) + (2 · 5) + (6 · 1) ... (5 · 3) ... 6 + 1 ...

Answer

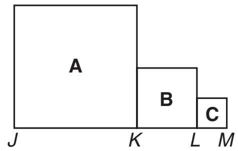
26. The actual area of the irregular region should be smaller than the estimate. The rectangles drawn are larger than the region.

Standardized Test Practice

30. In the figure consisting of squares A, B, and C, $JK = 2KL$ and $KL = 2LM$. If the perimeter of the figure is 66 units, what is the area? **B**

(A) 117 units^2 (B) 189 units^2

(C) 224 units^2 (D) 258 units^2



31. **ALGEBRA** For all integers n, $\boxed{n} = n^2$ if n is odd and $\boxed{n} = \sqrt{n}$ if n is even. What is the value of $\boxed{16} + \boxed{9}$? **C**

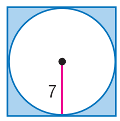
(A) 7 (B) 25 (C) 85 (D) 97

Maintain Your Skills

Mixed Review

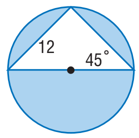
Find the area of each shaded region. Assume that all polygons are regular unless otherwise stated. Round to the nearest tenth. *(Lesson 11-3)*

32.



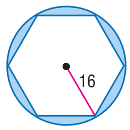
42.1 units^2

33.



154.2 units^2

34.



139.1 units^2

Find the area of each figure. Round to the nearest tenth if necessary. *(Lesson 11-2)*

35. equilateral triangle with perimeter of 57 feet **156.3 ft^2**

36. rhombus with a perimeter of 40 yards and a diagonal of 12 yards **96 yd^2**

37. isosceles trapezoid with a perimeter of 90 meters if the longer base is 5 meters less than twice as long as the other base and each leg is 3 meters less than the shorter base **≈ 384.0 m^2**

38. **COORDINATE GEOMETRY** The point (6, 0) is rotated 45° clockwise about the origin. Find the exact coordinates of its image. *(Lesson 9-3)* **$(3\sqrt{2}, -3\sqrt{2})$**

Getting Ready for the Next Lesson

BASIC SKILL **Express each fraction as a decimal to the nearest hundredth.**

39. $\frac{5}{8}$ **0.63** 40. $\frac{13}{16}$ **0.81** 41. $\frac{9}{47}$ **0.19** 42. $\frac{10}{21}$ **0.48**

Practice Quiz 2 — *Lessons 11-3 and 11-4*

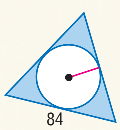
Find the area of each polygon. Round to the nearest tenth. *(Lesson 11-3)*

1. regular hexagon with apothem length of 14 millimeters **679.0 mm^2**

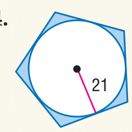
2. regular octagon with a perimeter of 72 inches **391.1 in^2**

Find the area of each shaded region. Assume that all polygons are regular. Round to the nearest tenth. *(Lesson 11-3)*

3. **1208.1 units^2**



4. **216.6 units^2**



5. **COORDINATE GEOMETRY** Find the area of $CDGHJ$ with vertices $C(-3, -2)$, $D(1, 3)$, $G(5, 5)$, $H(8, 3)$, and $J(5, -2)$. *(Lesson 11-4)* **44.5 units^2**

www.geometryonline.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Have students explain how to find the area of an irregular figure.

Getting Ready for Lesson 11-5

Prerequisite Skill Students will learn about geometric probability in Lesson 11-5. They will write fractions as decimals and percents to express probabilities involving geometric figures. Use Exercises 39–42 to determine your students' familiarity with writing fractions as decimals and percents.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 11-3 and 11-4. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lesson 11-4) is available on p. 656 of the *Chapter 11 Resource Masters*.

Answer

29. Sample answer: Windsurfers use the area of the sail to catch the wind and stay afloat on the water. Answers should include the following.

- To find the area of the sail, separate it into shapes. Then find the area of each shape. The sum of areas is the area of the sail.
- Sample answer: Surfboards and sailboards are also irregular figures.

11-5 Lesson Notes

1 Focus

5-Minute Check Transparency 11-5 Use as a quiz or review of Lesson 11-4.

Mathematical Background notes are available for this lesson on p. 592D.

How can geometric probability help you win a game of darts?

Ask students:

- What shapes are used in a dart board? **circles and trapezoids**
- How can you get the most points playing darts? **Throw the darts into the smaller circular regions.**
- How are concentric circles used in the real-world? **Sample answer: a roof landing pad for a helicopter**

11-5 Geometric Probability

What You'll Learn

- Solve problems involving geometric probability.
- Solve problems involving sectors and segments of circles.

How can geometric probability help you win a game of darts?

To win at darts, you have to throw a dart at either the center or the part of the dartboard that earns the most points. In games, probability can sometimes be used to determine chances of winning. Probability that involves a geometric measure such as length or area is called **geometric probability**.

Vocabulary

- geometric probability
- sector
- segment

GEOMETRIC PROBABILITY In Chapter 1, you learned that the probability that a point lies on a part of a segment can be found by comparing the length of the part to the length of the whole segment. Similarly, you can find the probability that a point lies in a part of a two-dimensional figure by comparing the area of the part to the area of the whole figure.

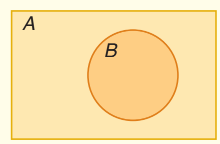
Study Tip

Look Back
To review **probability with line segments,** see page 20.

Key Concept — *Probability and Area*

If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is

$$P(B) = \frac{\text{area of region } B}{\text{area of region } A}.$$



When determining geometric probability with targets, we assume

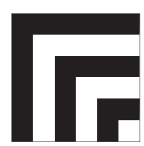
- that the object lands within the target area, and
- it is equally likely that the object will land anywhere in the region.

Standardized Test Practice

Example 1 Probability with Area

Grid-In Test Item

A square game board has black and white stripes of equal width as shown. What is the chance that a dart thrown at the board will land on a white stripe?



Read the Test Item

You want to find the probability of landing on a white stripe, not a black stripe.

Resource Manager

Workbook and Reproducible Masters

Chapter 11 Resource Masters

- Study Guide and Intervention, pp. 635–636
- Skills Practice, p. 637
- Practice, p. 638
- Reading to Learn Mathematics, p. 639
- Enrichment, p. 640
- Assessment, p. 656

Prerequisite Skills Workbook, pp. 27–28, 107–108

Teaching Geometry With Manipulatives Masters, pp. 1, 18

Transparencies

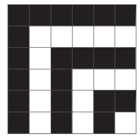
5-Minute Check Transparency 11-5
Real-World Transparency 11
Answer Key Transparencies

Technology

Interactive Chalkboard

Solve the Test Item

We need to divide the area of the white stripes by the total area of the game board. Extend the sides of each stripe. This separates the square into 36 small unit squares.



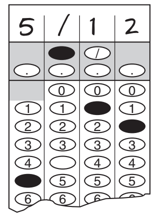
The white stripes have an area of 15 square units. The total area is 36 square units.

The probability of tossing a chip onto the white stripes is $\frac{15}{36}$ or $\frac{5}{12}$.

Fill in the Grid

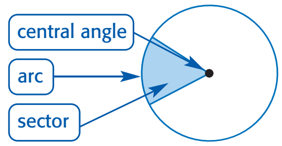
Write $\frac{5}{12}$ as 5/12 in the top row of the grid-in.

Then shade in the appropriate bubble under each entry.



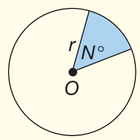
SECTORS AND SEGMENTS OF CIRCLES

Sometimes you need to know the area of a sector of a circle in order to find a geometric probability. A **sector** of a circle is a region of a circle bounded by a central angle and its intercepted arc.



Key Concept — *Area of a Sector*

If a sector of a circle has an area of A square units, a central angle measuring $N°$, and a radius of r units, then $A = \frac{N}{360}\pi r^2$.



Study Tip

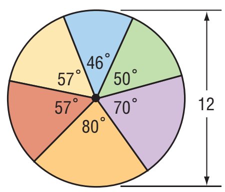
Common Misconceptions

The probability of an event can be expressed as a decimal or a fraction. These numbers are also sometimes represented by a percent.

Example 2 *Probability with Sectors*

a. Find the area of the blue sector.

Use the formula to find the area of the sector.



$A = \frac{N}{360}\pi r^2$ — Area of a sector

$= \frac{46}{360}\pi(6^2)$ — $N = 46, r = 6$

$= 4.6\pi$ — Simplify.

b. Find the probability that a point chosen at random lies in the blue region.

To find the probability, divide the area of the sector by the area of the circle. The area of the circle is πr^2 with a radius of 6.

$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ — Geometric probability formula

$= \frac{4.6\pi}{\pi \cdot 6^2}$ — Area of sector $= 4.6\pi$, area of circle $= \pi \cdot 6^2$

≈ 0.13 — Use a calculator.

The probability that a random point is in the blue sector is about 0.13 or 13%.

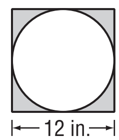
2 Teach

GEOMETRIC PROBABILITY

In-Class Example — PowerPoint®

Teaching Tip Some students may not believe that you can discuss probability in relation to geometric figures. Point out that many games already use the notion of probability to determine possible outcomes for the game. Ask volunteers for examples of games that use geometric probability.

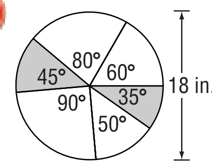
1 **GRID IN** A game board consists of a circle inscribed in a square. What is the chance that a dart thrown at the board will land in the shaded area? ≈ 0.215



SECTORS AND SEGMENTS OF CIRCLES

In-Class Examples — PowerPoint®

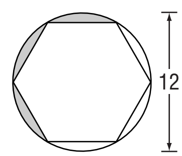
2



a. Find the total area of the shaded sectors. $\approx 56.5 \text{ in}^2$

b. Find the probability that a point chosen at random lies in the shaded region. $\frac{2}{9} \approx 0.22$

3 A regular hexagon is inscribed in a circle with a diameter of 12.



a. Find the area of the shaded regions. $\approx 9.78 \text{ units}^2$

b. Find the probability that a point chosen at random lies in the shaded regions. 0.087 or 8.7%

3 Practice/Apply

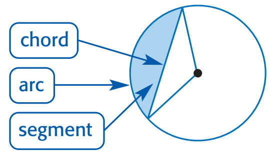
Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 11.
- include the formula for finding geometric probability.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR In Exercise 3, caution students that this equation can be set up in two ways. Rachel added the degree measures of each green sector first. Another method is to find the probability for each green sector and then add. If Taimi had multiplied the second term by $\pi(5^2)$, her method would be correct.

The region of a circle bounded by an arc and a chord is called a **segment** of a circle. To find the area of a segment, subtract the area of the triangle formed by the radii and the chord from the area of the sector containing the segment.



Example 3 Probability with Segments

A regular hexagon is inscribed in a circle with a diameter of 14.

a. Find the area of the red segment.

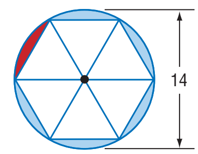
Area of the sector:

$A = \frac{N}{360}\pi r^2$ Area of a sector

$= \frac{60}{360}\pi(7^2)$ $N = 60, r = 7$

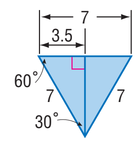
$= \frac{49}{6}\pi$ Simplify.

≈ 25.66 Use a calculator.



Area of the triangle:

Since the hexagon was inscribed in the circle, the triangle is equilateral, with each side 7 units long. Use properties of 30°-60°-90° triangles to find the apothem. The value of x is 3.5, the apothem is $x\sqrt{3}$ or $3.5\sqrt{3}$ which is approximately 6.06.



Next, use the formula for the area of a triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$= \frac{1}{2}(7)(6.06)$ $b = 7, h = 6.06$

≈ 21.22 Simplify.

Area of the segment:

area of segment = area of sector − area of triangle

$\approx 25.66 - 21.22$ Substitution

≈ 4.44 Simplify.

b. Find the probability that a point chosen at random lies in the red region.

Divide the area of the sector by the area of the circle to find the probability. First, find the area of the circle. The radius is 7, so the area is $\pi(7^2)$ or about 153.94 square units.

$P(\text{blue}) = \frac{\text{area of segment}}{\text{area of circle}}$

$\approx \frac{4.44}{153.94}$

≈ 0.03

The probability that a random point is on the red segment is about 0.03 or 3%.

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Ask students to name the similarities between probability and geometric probability. **The similarity is that the probability is still found by dividing the favorable outcomes by the total number of outcomes. In geometric probability, the favorable outcome is an area or a length, and the total number of outcomes is an area or a length.**

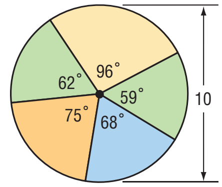
Check for Understanding

Concept Check

1. Multiply the measure of the central angle of the sector by the area of the circle and then divide the product by 360°.
2. Sample answer: darts, archery, shuffleboard

1. **Explain** how to find the area of a sector of a circle.
2. **OPEN ENDED** List three games that involve geometric probability.
3. **FIND THE ERROR** Rachel and Taimi are finding the probability that a point chosen at random lies in the green region.

Rachel	Taimi
$A = \frac{N}{360}\pi r^2$	$A = \frac{N}{360}\pi r^2$
$= \frac{59+62}{360}\pi(5^2)$	$= \frac{59}{360}\pi(5^2) + \frac{62}{360}$
≈ 26.4	≈ 13.0
$P(\text{green}) \approx \frac{26.4}{25\pi} \approx 0.34$	$P(\text{green}) \approx \frac{13.0}{25\pi} \approx 0.17$

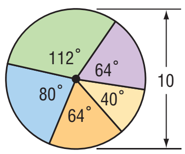


Who is correct? Explain your answer. Rachel; Taimi did not multiply $\frac{62}{360}$ by the area of the circle.

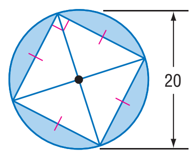
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4	2
5	3
6	1

Find the area of the blue region. Then find the probability that a point chosen at random will be in the blue region.

4. 

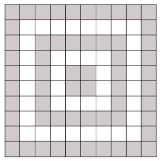
≈ 17.5 units^2, ≈ 0.22

5. 

≈ 114.2 units^2, ≈ 0.36

Standardized Test Practice

6. What is the chance that a point chosen at random lies in the shaded region? $\frac{3}{5}$ or 0.6

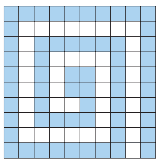


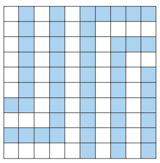
Practice and Apply

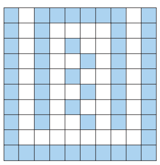
Homework Help	
For Exercises	See Examples
7–9, 16, 24–30	1
10–15, 20–23	2
17–19	3

Extra Practice
See page 777.

Find the probability that a point chosen at random lies in the shaded region.

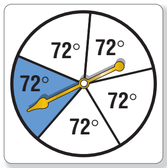
7. 
0.60

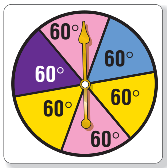
8. 
0.50

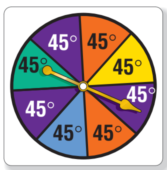
9. 
0.54

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of each spinner is 15 centimeters.

10. blue ≈ 35.3 units^2, 0.20
11. pink
12. purple







11. ≈ 58.9 units^2, $0.\overline{3}$
12. ≈ 66.3 units^2, 0.375

About the Exercises...

Organization by Objective

- **Geometric Probability:** 7–9, 16, 24–30
- **Sectors and Segments of Circles:** 10–15, 17–23

Odd/Even Assignments

Exercises 7–30 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 7–31 odd, 32–45

Average: 7–31 odd, 32–45

Advanced: 8–30 even, 31–45

DAILY INTERVENTION

Unlocking Misconceptions

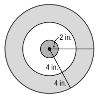
A common error is to believe that you can estimate a geometric probability by looking at the figure. Remind students to be careful that they calculate all areas by using formulas, and then compare the calculated probability and the estimate.

Study Guide and Intervention, p. 635 (shown) and p. 636

Geometric Probability The probability that a point in a figure will lie in a particular part of the figure can be calculated by dividing the area of the part of the figure by the area of the entire figure. The quotient is called the **geometric probability** for the part of the figure.

If a point in region A is chosen at random, then the probability $P(B)$ that the point is in region B, which is in the interior of region A, is
$P(B) = \frac{\text{area of region } B}{\text{area of region } A}$.

Example **Darts are thrown at a circular dartboard. If a dart hits the board, what is the probability that the dart lands in the bull's-eye?**



Area of bull's-eye: $A = \pi(2)^2$ or 4π

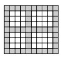
Area of entire dartboard: $A = \pi(10)^2$ or 100π

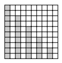
The probability of landing in the bull's-eye is

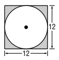
$\frac{\text{area of bull's-eye}}{\text{area of dartboard}} = \frac{4\pi}{100\pi} = \frac{1}{25}$ or 0.04.

Exercises

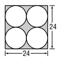
Find the probability that a point chosen at random lies in the shaded region. Round to the nearest hundredth if necessary.

1. 
0.53

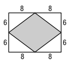
2. 
0.3

3. 

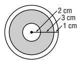
0.21

4. 

0.21

5. 

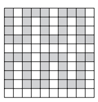
0.5

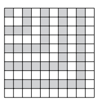
6. 

0.58

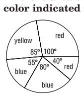
Skills Practice, p. 637 and Practice, p. 638 (shown)

Find the probability that a point chosen at random lies in the shaded region.

1. 
0.50

2. 
0.45

Find the area of the indicated sector. Then find the probability of spinning the color indicated if the diameter of the spinner is 9 meters.

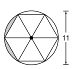


3. red 24.7 m^2, 0.39

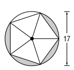
4. blue 23.9 m^2, 0.38

5. yellow 15.0 m^2, 0.24

Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume that all inscribed polygons are regular.

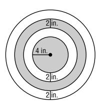
6. 

5.5 $units^2$, 0.058

7. 

44.2 $units^2$, 0.195

8. **ARCHERY** A target consists of four concentric rings. The radius of the center circle is 4 inches, and the circles are spaced 2 inches apart. Find the probability that an arrow shot at random by an inexperienced archer will land in a shaded region. 0.44



Reading to Learn Mathematics, p. 639

ELL

Pre-Activity **How can geometric probability help you win a game of darts?**

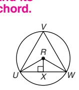
Read the introduction to Lesson 11-5 at the top of page 622 in your textbook.

To find the probability of winning at darts, would you use geometric probability to compare areas or lengths? areas

Reading the Lesson

1. Explain the difference between a sector of a circle and a segment of a circle **Sample answer: A sector of a circle is bounded by a central angle and its intercept arc, while a segment is bounded by an arc and a chord.**

2. Suppose you are playing a game of darts with a target like the one shown at the right. If your dart lands inside equilateral $\triangle UVW$, you get a point. Assume that every dart will land on the target. The radius of the circle is 1. Complete the following steps to figure out the probability of getting a point.



The area of circle R is __π__.

$\triangle URW$ is a(n) __isosceles__ triangle because $\overline{RU}$ and $\overline{RW}$ are __radii__ of the same __circle__.

$\angle URW$ is a(n) __central__ angle of the circle, and $m\angle URW$ = __120__.

$m\angle RUX$ = __30__ and $m\angle RWX$ = __30__.

The angle measures in $\triangle RUX$ are __30__, __60__, and __90__.

$\overline{RU}$ is a __radius__ of the circle, so RU = __1__.

$\overline{RX}$ is the leg of $\triangle RUX$ opposite the __30°__ angle, so RX = __$\frac{1}{2}$__.

Also, $\overline{UX}$ is the leg of $\triangle RUX$ opposite the __60°__ angle, so UX = __$\frac{\sqrt{3}}{2}$__.

UW = __$\sqrt{3}$__, so the area of $\triangle URW$ is $\frac{1}{2} \cdot$ __$\sqrt{3}$__ $\cdot$ __$\frac{1}{2}$__ = __$\frac{\sqrt{3}}{4}$__.

Then, the area of $\triangle UVW = 3 \cdot$ __$\frac{\sqrt{3}}{4}$__ = __$\frac{3\sqrt{3}}{4}$__.

Therefore, the probability that the dart will fall inside the triangle is the ratio of __$\frac{3\sqrt{3}}{4}$__ to __π__, which is approximately __0.413__ (to the nearest thousandth).

Helping You Remember

3. Many students find it difficult to remember a large number of geometric formulas. How can you use the formula for the area of a circle to find the area of a sector of a circle without having to learn a new formula? **Sample answer: First use $A = \pi r^2$ to find the area of the circle. Then use the measure of the central angle to find out what fraction of the circle the sector is. Multiply the area of the circle by this fraction and you will have the area of the sector.**

---

14. $\approx$ 84.9 $units^2$, $\approx$ 0.48

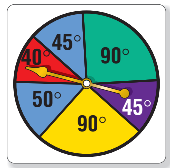
15. $\approx$ 74.6 $units^2$, $\approx$ 0.42

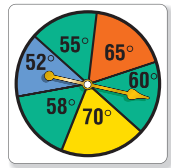
Find the area of the indicated sector. Then find the probability of choosing the color indicated if the diameter of each spinner is 15 centimeters.

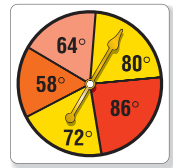
13. red $\approx$ 19.6 $units^2$, $\approx 0.\overline{1}$

14. green

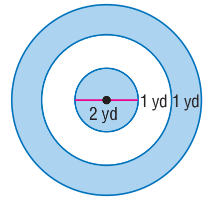
15. yellow



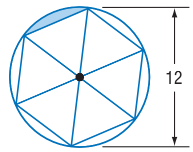




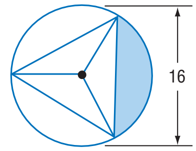
16. **PARACHUTES** A skydiver must land on a target of three concentric circles. The diameter of the center circle is 2 yards, and the circles are spaced 1 yard apart. Find the probability that she will land on the shaded area. $\frac{2}{3}$



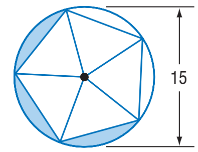
Find the area of the shaded region. Then find the probability that a point chosen at random is in the shaded region. Assume all inscribed polygons are regular.

17. 

$\approx$ 3.3 $units^2$, $\approx$ 0.03

18. 

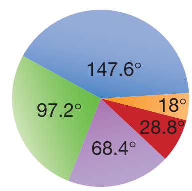
$\approx$ 39.3 $units^2$, $\approx$ 0.20

19. 

$\approx$ 25.8 $units^2$, $\approx$ 0.15

SURVEYS **For Exercises 20–23, use the following information.**
A survey was taken at a high school, and the results were put in a circle graph. The students were asked to list their favorite colors. The measurement of each central angle is shown. If a person is chosen at random from the school, find the probability of each response.

What's Your Favorite Color?



20. Favorite color is red. 0.08

21. Favorite color is blue or green. 0.68

22. Favorite color is *not* red or blue. 0.51

23. Favorite color is *not* orange or green. 0.68

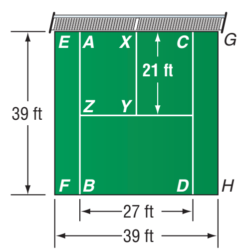
More About. . .

Tennis

In tennis, the linesman determines whether the hit ball is in or out. The umpire may only overrule the linesman if he or she immediately thinks the call was wrong without a doubt and never as a result of a player's protest.

Source: www.usta.com

TENNIS **For Exercises 24 and 25, use the following information.**
A tennis court has stripes dividing it into rectangular regions. For singles play, the inbound region is defined by segments $\overline{AB}$ and $\overline{CD}$. The doubles court is bound by the segments $\overline{EF}$ and $\overline{GH}$.



24. Find the probability that a ball in a singles game will land inside the court, but out of bounds. 0.31

25. When serving, the ball must land within $AXYZ$, the service box. Find the probability that a ball will land in the service box, relative to the court. 0.19

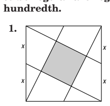
Stu Forster/Getty Images

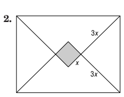
Enrichment, p. 640

Polygon Probability

Each problem on this page involves one or more regular polygons. To find the probability of a point chosen at random being in the shaded region, you need to find the ratio of the shaded area to the total area. If you wish, you may substitute numbers for the variables.

Find the probability that a point chosen at random in each figure is in the shaded region. Assume polygons that appear to be regular are regular. Round your answer to the nearest hundredth.

1. 

2. 

DARTS **For Exercises 26–30, use the following information.**
Each sector of the dartboard has congruent central angles. Find the probability that the dart will land on the indicated color. The diameter of the center circle is 2 units.

26. black **≈ 0.29** **27.** white **≈ 0.29** **28.** red **≈ 0.43**

29. Point values are assigned to each color. Should any of the colors have the same point value? Explain. **See margin.**

30. Which color should have the lowest point value? Explain. **See margin.**

31a–b. See margin.

31. CRITICAL THINKING Study each spinner in Exercises 13–15.
a. Are the chances of landing on each color equal? Explain.
b. Would this be considered a fair spinner to use in a game? Explain.

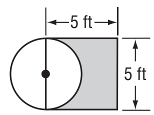
32. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How can geometric probability help you win a game of darts?

Include the following in your answer:
- an explanation of how to find the geometric probability of landing on a red sector, and
- an explanation of how to find the geometric probability of landing in the center circle.

Standardized Test Practice

33. One side of a square is a diameter of a circle. The length of one side of the square is 5 feet. To the nearest hundredth, what is the probability that a point chosen at random is in the shaded region? **C**



Ⓐ 0.08 Ⓑ 0.22 Ⓒ 0.44 Ⓓ 0.77

34. ALGEBRA If $4y = 16$, then $12 \div y =$ **C**

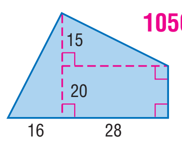
Ⓐ 1. Ⓑ 2. Ⓒ 3. Ⓓ 4.

Maintain Your Skills

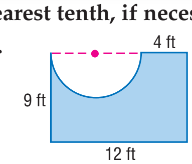
Mixed Review

Find the area of each figure. Round to the nearest tenth, if necessary. *(Lesson 11-4)*

35. **1050 units^2**



36. **82.9 ft^2**



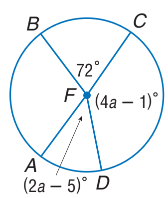
Find the area of each polygon. Round to the nearest tenth, if necessary. *(Lesson 11-3)*

37. a regular triangle with a perimeter of 48 feet **110.9 ft^2**

38. a square with a side length of 21 centimeters **441 cm^2**

39. a regular hexagon with an apothem length of 8 inches **221.7 in^2**

ALGEBRA **Find the measure of each angle on ⊙F with diameter $\overline{AC}$.** *(Lesson 10-2)*



40. $\angle AFB$ **41.** $\angle CFD$ **42.** $\angle AFD$ **43.** $\angle DFB$

40. 108
41. 123
42. 57
43. 165

Find the length of the third side of a triangle given the measures of two sides and the included angle of the triangle. Round to the nearest tenth. *(Lesson 7-7)*

44. $m = 6.8$, $n = 11.1$, $m\angle P = 57$ **$p = 9.3$**

45. $f = 32$, $h = 29$, $m\angle G = 41$ **$g = 21.5$**

www.geometryonline.com/self_check_quiz

4 Assess

Open-Ended Assessment

Modeling Have students design a gameboard. Ask them to draw a shaded region in polygons or squares whose area can be found. Then calculate the area of the shaded region and the corresponding probability.

Assessment Options

Quiz (Lesson 11-5) is available on p. 656 of the *Chapter 11 Resource Masters*.

Answers

29. The chances of landing on a black or white sector are the same, so they should have the same point value.

30. Of the three colors, there is the highest probability of landing on red, so red should have a lower point value than white or black.

31a. No; each colored sector has a different central angle.

31b. No; there is not an equal chance of landing on each color.

32. Sample answer: Geometric probability can help you determine the chance of a dart landing on the bullseye or high scoring sector. Answers should include the following.
- Find the area of the circles containing the red sector. Divide the difference by the area of the larger circle.
- Find the area of the center circle and divide by the area of the largest circle on the board.

Lesson 11-5 Geometric Probability 627

Chapter 11 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 11 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 11 is available on p. 654 of the *Chapter 11 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,
- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

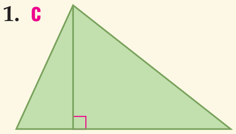
Chapter 11 Study Guide and Review

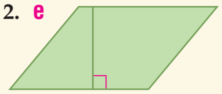
Vocabulary and Concept Check

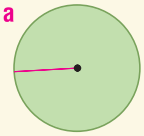
apothem (p. 610)	irregular figure (p. 617)	sector (p. 623)
geometric probability (p. 622)	irregular polygon (p. 618)	segment (p. 624)

A complete list of postulates and theorems can be found on pages R1–R8.

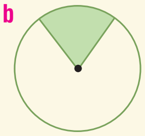
Exercises Choose the formula to find the area of each shaded figure.

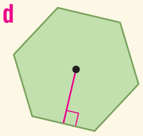
1. c


2. e


3. a


4. f


5. b


6. d


a. $A = \pi r^2$
b. $A = \frac{N}{360}\pi r^2$
c. $A = \frac{1}{2}bh$
d. $A = \frac{1}{2}Pa$
e. $A = bh$
f. $A = \frac{1}{2}h(b_1 + b_2)$

Lesson-by-Lesson Review

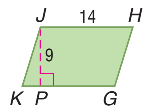
11-1 Area of Parallelograms

See pages 595–600.

Concept Summary
- The area of a parallelogram is the product of the base and the height.

Example Find the area of ▱*GHJK*.

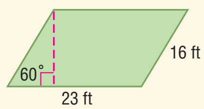
The area of a parallelogram is given by the formula $A = bh$.



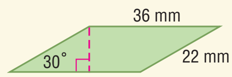
$A = bh$ Area of a parallelogram
$= 14(9)$ or 126 $b = 14, h = 9$

The area of the parallelogram is 126 square units.

Exercises Find the perimeter and area of each parallelogram. *See Example 1 on page 596.*

7. 

78 ft, 318.7 ft^2

8. 

116 mm, 396 mm^2

COORDINATE GEOMETRY Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral. *See Example 3 on page 597.*

9. $A(-6, 1), B(1, 1), C(1, -6), D(-6, -6)$ square; 49 units^2
10. $E(7, -2), F(1, -2), G(2, 2), H(8, 2)$ parallelogram; 24 units^2
11. $J(-1, -4), K(-5, 0), L(-5, 5), M(-1, 1)$ parallelogram; 20 units^2
12. $P(-7, -1), Q(-3, 3), R(-1, 1), S(-5, -3)$ rectangle; 16 units^2

www.geometryonline.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Have students look through the chapter to make sure they have included notes and examples in their Foldables for each lesson of Chapter 11.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Study Guide and Review

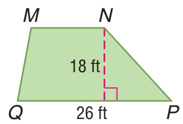
11-2 Areas of Triangles, Rhombi, and Trapezoids

See pages 601–609.

Concept Summary

- The formula for the area of a triangle can be used to find the areas of many different figures.
- Congruent figures have equal areas.

Example **Trapezoid *MNPQ* has an area of 360 square feet. Find the length of $\overline{MN}$.**



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$360 = \frac{1}{2}(18)(b_1 + 26)$ $A = 360, h = 18, b_2 = 26$

$360 = 9b_1 + 234$ Multiply.

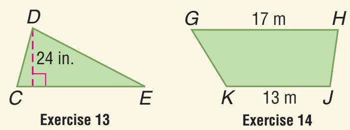
$14 = b_1$ Solve for b_1.

The length of $\overline{MN}$ is 14 feet.

Exercises **Find the missing measure for each quadrilateral.** *See Example 4 on page 604.*

13. Triangle *CDE* has an area of 336 square inches. Find *CE*. **28 in.**

14. Trapezoid *GHJK* has an area of 75 square meters. Find the height. **5 m**



11-3 Areas of Regular Polygons and Circles

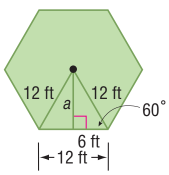
See pages 610–616.

Concept Summary

- A regular n-gon is made up of n congruent isosceles triangles.
- The area of a circle of radius r units is πr^2 square units.

Example **Find the area of a regular hexagon with a perimeter of 72 feet.**

Since the perimeter is 72 feet, the measure of each side is 12 feet. The central angle of a hexagon is 60°. Use the properties of 30°-60°-90° triangles to find that the apothem is $6\sqrt{3}$ feet.



$A = \frac{1}{2}Pa$ Area of a regular polygon

$= \frac{1}{2}(72)(6\sqrt{3})$ $P = 72, a = 6\sqrt{3}$

$= 216\sqrt{3}$ Simplify.

≈ 374.1

The area of the regular hexagon is 374.1 square feet to the nearest tenth.

Exercises **Find the area of each polygon. Round to the nearest tenth.**
See Example 1 on page 611.

15. a regular pentagon with perimeter of 100 inches **688.2 in^2**

16. a regular decagon with side length of 12 millimeters **1108.0 mm^2**

Chapter 11 Study Guide and Review 629

Chapter 11 For More ...
• Extra Practice, see pages 776–777.
• Mixed Problem Solving, see page 792.

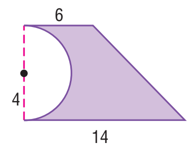
11-4 Areas of Irregular Figures

See pages 617–621.

Concept Summary

- The area of an irregular figure is the sum of the areas of its nonoverlapping parts.

Example **Find the area of the figure.**



Separate the figure into a rectangle and a triangle.

$$\text{area of irregular figure} = \text{area of rectangle} - \text{area of semicircle} + \text{area of triangle}$$

$= \ell w - \frac{1}{2}\pi r^2 + \frac{1}{2}bh$ — Area formulas

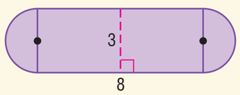
$= (6)(8) - \frac{1}{2}\pi(4^2) + \frac{1}{2}(8)(8)$ — Substitution

$= 48 - 8\pi + 32$ or about 54.9 — Simplify.

The area of the irregular figure is 54.9 square units to the nearest tenth.

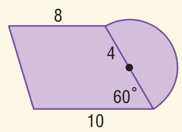
Exercises **Find the area of each figure to the nearest tenth.** *See Example 1 on page 617.*

17.



31.1 units^2

18.



87.5 units^2

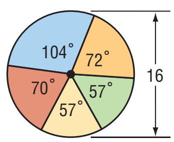
11-5 Geometric Probability

See pages 622–627.

Concept Summary

- To find a geometric probability, divide the area of a part of a figure by the total area.

Example **Find the probability that a point chosen at random will be in the blue sector.**



First find the area of the blue sector.

$A = \frac{N}{360}\pi r^2$ — Area of a sector

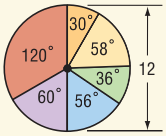
$= \frac{104}{360}\pi(8^2)$ or about 58.08 — Substitute and simplify.

To find the probability, divide the area of the sector by the area of the circle.

$P(\text{blue}) = \frac{\text{area of sector}}{\text{area of circle}}$ — Geometric probability formula

$= \frac{58.08}{\pi 8^2}$ or about 0.29 — The probability is about 0.29 or 29%.

Exercises **Find the probability that a point chosen at random will be in the sector of the given color.** *See Example 2 on page 623.*



19. red $0.\overline{3}$

20. purple or green ≈ 0.27

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Chapter 11 Practice Test

Vocabulary and Concepts

Choose the letter of the correct area formula for each figure.

1. regular polygon **a**
2. trapezoid **c**
3. triangle **b**

a. $A = \frac{1}{2}Pa$
b. $A = \frac{1}{2}bh$
c. $A = \frac{1}{2}h(b_1 + b_2)$

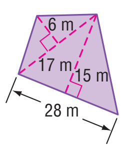
Skills and Applications

COORDINATE GEOMETRY **Given the coordinates of the vertices of a quadrilateral, determine whether it is a *square*, a *rectangle*, or a *parallelogram*. Then find the area of the quadrilateral.**

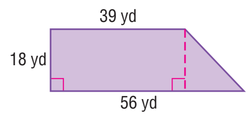
4. $R(-6, 8)$, $S(-1, 5)$, $T(-1, 1)$, $U(-6, 4)$ **parallelogram, 20 units²**
5. $R(7, -1)$, $S(9, 3)$, $T(5, 5)$, $U(3, 1)$ **square, 20 units²**
6. $R(2, 0)$, $S(4, 5)$, $T(7, 5)$, $U(5, 0)$ **parallelogram, 15 units²**
7. $R(3, -6)$, $S(9, 3)$, $T(12, 1)$, $U(6, -8)$ **rectangle, 39 units²**

Find the area of each figure. Round to the nearest tenth if necessary.

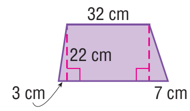
8. **261 m²**



9. **855 yd²**



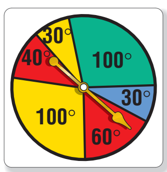
10. **814 cm²**



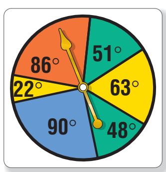
11. a regular octagon with apothem length of 3 ft **29.8 ft²**
12. a regular pentagon with a perimeter of 115 cm **910.1 cm²**

Each spinner has a diameter of 12 inches. Find the probability of spinning the indicated color.

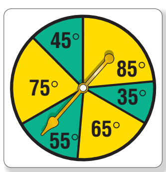
13. red **0.28**



14. orange **0.24**

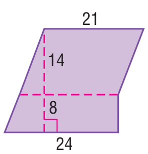


15. green **0.38**

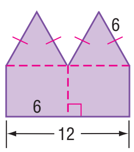


Find the area of each figure. Round to the nearest tenth.

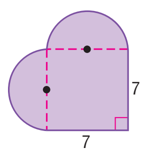
16. **474 units²**



17. **91.2 units²**



18. **87.5 units²**



19. **SOCCER BALLS** The surface of a soccer ball is made of a pattern of regular pentagons and hexagons. If each hexagon on a soccer ball has a perimeter of 9 inches, what is the area of a hexagon? **5.8 in²**

20. **STANDARDIZED TEST PRACTICE** What is the area of a quadrilateral with vertices at $(-3, -1)$, $(-1, 4)$, $(7, 4)$, and $(5, -1)$? **D**

(A) 50 units² (B) 45 units² (C) $8\sqrt{29}$ units² (D) 40 units²

www.geometryonline.com/chapter_test

Chapter 11 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 11 can be found on p. 654 of the *Chapter 11 Resource Masters*.

Chapter Tests There are six Chapter 11 Tests and an Open-Ended Assessment task available in the *Chapter 11 Resource Masters*.

Chapter 11 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	641–642
2A	MC	average	643–644
2B	MC	average	645–646
2C	FR	average	647–648
2D	FR	average	649–650
3	FR	advanced	651–652

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 11 can be found on p. 653 of the *Chapter 11 Resource Masters*. A sample scoring rubric for these tasks appears on p. A22.

ExamView® Pro

Use the networkable **ExamView® Pro** to:

- Create **multiple versions** of tests.
- Create **modified** tests for Inclusion students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your tests from a program bank of artwork.

Portfolio Suggestion

Introduction Areas of irregular figures are used in architecture.

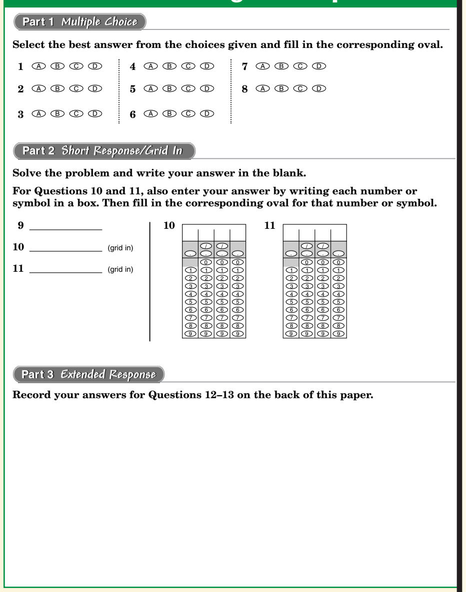
Ask Students Ask students to design the floor plan of an elaborate house or garden. Challenge them to include rooms of many different shapes. Have them find the area of each room, showing their calculations. Have students add their designs and area calculations to their portfolios.

Chapter 11 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 11 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ

2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ

3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 10 and 11, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

9 ________

10 ________ (grid in)

11 ________ (grid in)

Part 3 Extended Response

Record your answers for Questions 12–13 on the back of this paper.

Additional Practice

See pp. 659–660 in the *Chapter 11 Resource Masters* for additional standardized test practice.

Chapter 11 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Solve $3\left(\frac{2x-4}{-6}\right) = 18$. (Prerequisite Skill) **B**

Ⓐ −19 Ⓑ −16 Ⓒ 4 Ⓓ 12

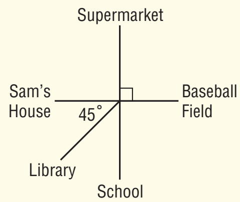
2. Sam rode his bike along the path from the library to baseball practice. What type of angle did he form during the ride? (Lesson 1-5) **B**

Ⓐ straight

Ⓑ obtuse

Ⓒ acute

Ⓓ right



3. What is the logical conclusion of these statements?

If you exercise, you will maintain better health.
If you maintain better health, you will live longer.
(Lesson 2-4) **A**

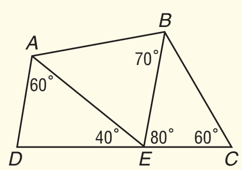
Ⓐ If you exercise, you will live longer.

Ⓑ If you do not exercise, you will not live longer.

Ⓒ If you do not exercise, you will not maintain better health.

Ⓓ If you maintain better health, you will not live longer.

4. Which segments are parallel? (Lesson 3-5) **C**



Ⓐ $\overline{AB}$ and $\overline{CD}$ Ⓑ $\overline{AD}$ and $\overline{BC}$

Ⓒ $\overline{AD}$ and $\overline{BE}$ Ⓓ $\overline{AE}$ and $\overline{BC}$

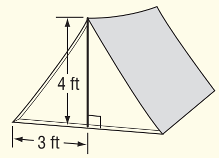
5. The front view of a pup tent resembles an isosceles triangle. The entrance to the tent is an angle bisector. The tent is secured by stakes. What is the distance between the two stakes? (Lesson 5-1) **D**

Ⓐ 3 ft

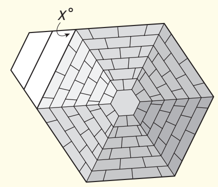
Ⓑ 4 ft

Ⓒ 5 ft

Ⓓ 6 ft



6. A carpenter is building steps leading to a hexagonal gazebo. The outside edges of the steps need to be cut at an angle. Find x. (Lesson 8-1) **D**



Ⓐ 180 Ⓑ 120 Ⓒ 72 Ⓓ 60

7. Which statement is *always* true? (Lesson 10-4) **A**

Ⓐ When an angle is inscribed in a circle, the angle's measure equals one-half of the measure of the intercepted arc.

Ⓑ In a circle, an inscribed quadrilateral will have consecutive angles that are supplementary.

Ⓒ In a circle, an inscribed angle that intercepts a semicircle is obtuse.

Ⓓ If two inscribed angles of a circle intercept congruent arcs, then the angles are complementary.

8. The apothem of a regular hexagon is 7.8 centimeters. If the length of each side is 9 centimeters, what is the area of the hexagon? (Lesson 11-3) **C**

Ⓐ 35.1 cm^2 Ⓑ 70.2 cm^2

Ⓒ 210.6 cm^2 Ⓓ 421.2 cm^2

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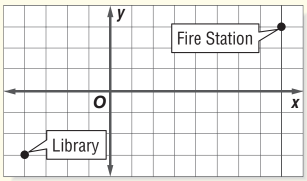
Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and state proficiency tests can be found on this CD-ROM.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 795–810.

Part 2 Short Response/Grid In

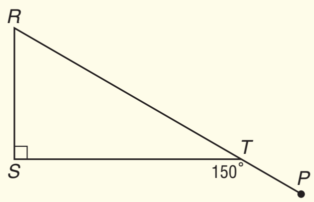
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The post office is located halfway between the fire station and the library. What are the coordinates of the post office? (Lesson 1-3)
(2, 0)

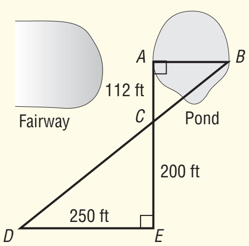


10. What is the slope of a line perpendicular to the line represented by the equation $3x - 6y = 12$? (Lesson 3-3) **−2**

11. $\triangle RST$ is a right triangle. Find $m\angle R$. (Lesson 4-2) **60**



12. If $\angle A$ and $\angle E$ are congruent, find AB, the distance in feet across the pond. (Lesson 6-3) **140**



13. If point $J(6, -3)$ is translated 5 units up and then reflected over the y-axis, what will the new coordinates of J' be? (Lesson 9-2) **(−6, 2)**

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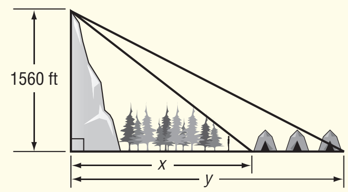
Test-Taking Tip

Question 4
To find the pair of parallel lines, first you need to find the missing angle measures. Use the Angle Sum Theorem to find the measures of the angles in each triangle.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

14. Lori and her family are camping near a mountain. Their campground is in a clearing next to a stretch of forest.



a. The angle of elevation from Lori's line of sight at the edge of the forest to the top of the mountain, is 38°. Find the distance x from the base of the mountain to the edge of the forest. Round to the nearest foot. (Lesson 7-5) **1997**

b. The angle of elevation from the far edge of the campground to the top of the mountain is 35°. Find the distance y from the base of the mountain to the far edge of the campground. Round to the nearest foot. (Lesson 7-5) **2228**

c. What is the width of the campground? Round to the nearest foot. (Lesson 7-5) **231**

15. Parallelogram $ABCD$ has vertices $A(0, 0)$, $B(3, 4)$, and $C(8, 4)$.

a. Find the possible coordinates for D. (Lesson 8-2) **(5, 0)**

b. Find the area of $ABCD$. (Lesson 11-1) **20 units^2**

Evaluating Extended Response Questions

Extended Response questions are graded by using a multilevel rubric that guides you in assessing a student's knowledge of a particular concept.

Goal: In Exercise 14, students use angles of elevation to find distances. In Exercise 15, students find the area of a parallelogram on a coordinate grid.

Sample Scoring Rubric: The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.

Score	Criteria
4	A correct solution that is supported by well-developed, accurate explanations
3	A generally correct solution, but may contain minor flaws in reasoning or computation
2	A partially correct interpretation and/or solution to the problem
1	A correct solution with no supporting evidence or explanation
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given