Chapter 10

# Circles

## Chapter Overview and Pacing

*Year-long pacing: pages T20–T21.*

| LESSON OBJECTIVES | PACING (days) | | | |
|---|---|---|---|---|
| | Regular | | Block | |
| | Basic/ Average | Advanced | Basic/ Average | Advanced |
| **10-1 Circles and Circumference** *(pp. 522–528)*<br>• Identify and use parts of circles.<br>• Solve problems involving the circumference of a circle. | 1 | 1 | 0.5 | 0.5 |
| **10-2 Angles and Arcs** *(pp. 529–535)*<br>• Recognize major arcs, minor arcs, semicircles, and central angles and their measures.<br>• Find arc length. | 2 | 2 | 1 | 1 |
| **10-3 Arcs and Chords** *(pp. 536–543)*<br>• Recognize and use relationships between arcs and chords.<br>• Recognize and use relationships between chords and diameters. | 2 | 2 | 1 | 1 |
| **10-4 Inscribed Angles** *(pp. 544–551)*<br>• Find measures of inscribed angles.<br>• Find measures of angles of inscribed polygons. | 2 | 2 | 1 | 1 |
| **10-5 Tangents** *(pp. 552–560)*<br>• Use properties of tangents.<br>• Solve problems involving circumscribed polygons.<br>***Follow-Up:*** To construct inscribed and circumscribed triangles. | 2 (with 10-5 Follow-Up) | 2 (with 10-5 Follow-Up) | 1 (with 10-5 Follow-Up) | 1 (with 10-5 Follow-Up) |
| **10-6 Secants, Tangents, and Angle Measures** *(pp. 561–568)*<br>• Find measures of angles formed by lines intersecting on or inside a circle.<br>• Find measures of angles formed by lines intersecting outside the circle. | 2 | 2 | 1 | 1 |
| **10-7 Special Segments in a Circle** *(pp. 569–574)*<br>• Find measures of segments that intersect in the interior of a circle.<br>• Find measures of segments that intersect in the exterior of a circle. | 2 | 2 | 1 | 1 |
| **10-8 Equations of Circles** *(pp. 575–580)*<br>• Write the equation of a circle.<br>• Graph a circle on the coordinate plane. | 1 | 1 | 0.5 | 0.5 |
| **Study Guide and Practice Test** *(pp. 581–587)*<br>**Standardized Test Practice** *(pp. 588–589)* | 1 | 1 | 0.5 | 0.5 |
| **Chapter Assessment** | 1 | 1 | 0.5 | 0.5 |
| **TOTAL** | **16** | **16** | **8** | **8** |

*An electronic version of this chapter is available on **StudentWorks**™. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.*

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T5 and T21.

# Chapter Resource Manager

| Chapter 10 Resource Masters | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Study Guide and Intervention | Practice (Skills and Average) | Reading to Learn Mathematics | Enrichment | Assessment | Prerequisite Skills Workbook | Applications* | 5-Minute Check Transparencies | Interactive Chalkboard | GeomPASS: Tutorial Plus (lessons) | Materials |
| 541–542 | 543–544 | 545 | 546 | | 11–12, 23–24, 45–48 | SC 19 | 10-1 | 10-1 | | |
| 547–548 | 549–550 | 551 | 552 | 603 | 31–32, 61–64, 67–68, 71–72, 105–106, 109–110 | | 10-2 | 10-2 | | compass, protractor |
| 553–554 | 555–556 | 557 | 558 | | | | 10-3 | 10-3 | | compass, patty paper, centimeter ruler, scissors, protractor |
| 559–560 | 561–562 | 563 | 564 | 603, 605 | 41–42 | | 10-4 | 10-4 | | compass, protractor, straightedge |
| 565–566 | 567–568 | 569 | 570 | | 15–16 | GCC 35, 36 SC 20 | 10-5 | 10-5 | | compass, straightedge (*Follow-Up:* straightedge, compass, paper) |
| 571–572 | 573–574 | 575 | 576 | 604 | 17–18 | | 10-6 | 10-6 | | compass, straightedge |
| 577–578 | 579–580 | 581 | 582 | | 35–36, 51–52 | | 10-7 | 10-7 | | compass, straightedge |
| 583–584 | 585–586 | 587 | 588 | 604 | | | 10-8 | 10-8 | 19 | grid paper, compass, straightedge |
| | | | | 589–602, 606–608 | | | | | | |

**Key to Abbreviations:* GCC = Graphing Calculator and Computer Masters
SC = School-to-Career Masters

# Chapter 10 Mathematical Connections and Background

## Continuity of Instruction

### Prior Knowledge

Students solved equations for a variable and used the Quadratic Formula in previous courses. In Chapter 4, students found the measures of the angles in isosceles triangles. In Chapter 7, they found the missing side length in a right triangle and used the converse of the Pythagorean Theorem to determine whether figures were right triangles.

### This Chapter

This chapter focuses exclusively on circles and their special properties. A circle is a unique geometric shape in which the angles, arcs, and segments intersecting that circle have special relationships. In this chapter, students identify the parts of a circle and solve problems involving circumference. They find arc and angle measures and the measures of segments in a circle. In addition, students write the equation of a circle and graph circles in the coordinate plane.

### Future Connections

Students will use their knowledge of circles to find the area of a circle in Chapter 11. They will also need to understand a circle to understand a sphere, which is introduced in Chapter 12.

## 10-1 Circles and Circumference

A circle is the locus of all points in a plane equidistant from a given point, which is the center of the circle. A circle is usually named by its center point. Any segment with endpoints on the circle is a chord of the circle. A chord that contains the center of the circle is a diameter of the circle. Any segment with endpoints that are the center and a point on the circle is a radius. All radii of a circle are congruent and all diameters are congruent.

The circumference of a circle is the distance around the circle. The ratio of the circumference to the diameter of a circle is always equal to $\pi$. For a circumference of $C$ units and a diameter of $d$ units or a radius of $r$ units, $C = \pi d$ or $C = 2\pi r$.

## 10-2 Angles and Arcs

A central angle of a circle has the center of the circle as its vertex, and its sides are two radii of the circle. The sum of the measures of the central angles of a circle with no interior points in common is 360. A central angle separates the circle into two parts, each of which is an arc.

The measure of each arc is related to the measure of its central angle. A minor arc degree measure equals the measure of the central angle and is less than 180. A major arc degree measure equals 360 minus the measure of the minor arc and is greater than 180. A semicircle is also considered an arc and measures 180°. In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

In a circle graph, the central angles divide a circle into wedges, often expressed as percents. The size of the angle is proportional to the percent. By multiplying the percent by 360, you can determine the measure of the central angle. Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is part of the circumference. The ratio of the arc degree measure to 360 is equal to the ratio of the arc length to the circumference. You can use these ratios to solve for arc length.

## 10-3 Arcs and Chords

The endpoints of a chord are also endpoints of an arc. Arcs and chords have a special relationship. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent. In a circle or congruent circles, two chords are congruent if and only if they are equidistant from the center of the circle.

The chords of adjacent arcs can form a polygon. Such a polygon is said to be *inscribed* in the circle because all its vertices lie on the circle. The circle circumscribes the polygon.

Diameters that are perpendicular to chords create special segment and arc relationships. In a circle, if a diameter or radius is perpendicular to a chord, then it bisects the chord and its arc.

## 10-4 Inscribed Angles

An inscribed angle is an angle that has its vertex on the circle and its sides contained in chords of the circle. If an angle is inscribed in a circle, then the measure of the angle equals one-half of the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle). If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

Inscribed polygons also have special properties. An inscribed triangle with a side that is a diameter is a special type of triangle. If an inscribed angle intercepts a semicircle, the angle is a right angle. If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

## 10-5 Tangents

A tangent intersects a circle in exactly one point. This point is called the *point of tangency*. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency. The converse of that statement is also true: If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

More than one line can be tangent to the same circle. If two segments from the same exterior point are tangent to a circle, then they are congruent.

Circles can be inscribed in polygons, just as polygons can be inscribed in circles. If a circle is inscribed in a polygon, then every side of the polygon is tangent to the circle. You can use what you know about tangents to solve problems involving inscribed circles.

## 10-6 Secants, Tangents, and Angle Measures

A line that intersects a circle in exactly two points is called a *secant*. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept. If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

A secant can also intersect a tangent at the point of tangency. If this occurs, then the measure of each angle formed is one-half the measure of its intercepted arc.

Secants and tangents can intersect outside a circle as well. If two secants, a tangent and a secant, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

## 10-7 Special Segments in a Circle

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal. You can also use intersecting chords to measure arcs.

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment. This product can also be used if a tangent segment and a secant segment are drawn to a circle from an exterior point. In this case, the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

## 10-8 Equations of Circles

An equation for a circle with center at $(h, k)$ and radius of $r$ units is $(x - h)^2 + (y - k)^2 = r^2$. You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane. Once you know the coordinates of the center and the radius of a circle, you can graph the circle. In fact, if you know just three points on a circle, you can graph it and write its equation. By graphing the points as a triangle and constructing two perpendicular bisectors, you can locate the center of the circle. Then you can use the Distance Formula to calculate the radius. Finally, write an equation for the circle.

# Chapter 10

# DAILY INTERVENTION and Assessment

*Key to Abbreviations:*
TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

| | Type | Student Edition | Teacher Resources | Technology/Internet |
|---|---|---|---|---|
| INTERVENTION | Ongoing | Prerequisite Skills, pp. 521, 528, 535, 543, 551, 558, 568, 574<br>Practice Quiz 1, p. 543<br>Practice Quiz 2, p. 568 | 5-Minute Check Transparencies<br>*Prerequisite Skills Workbook*, pp. 11–12, 15–18, 23–24, 31–32, 35–36, 41–42, 45–48, 51–52, 61–64, 67–68, 71–72, 105–106, 109–110<br>Quizzes, *CRM* pp. 603–604<br>Mid-Chapter Test, *CRM* p. 605<br>Study Guide and Intervention, *CRM* pp. 541–542, 547–548, 553–554, 559–560, 565–566, 571–572, 577–578, 583–584 | GeomPASS: Tutorial Plus, Lesson 19<br>www.geometryonline.com/self_check_quiz<br>www.geometryonline.com/extra_examples |
| | Mixed Review | pp. 528, 535, 543, 551, 558, 568, 574, 580 | Cumulative Review, *CRM* p. 606 | |
| | Error Analysis | Find the Error, pp. 539, 571<br>Common Misconceptions, p. 555 | Find the Error, *TWE* pp. 539, 571<br>Unlocking Misconceptions, *TWE* p. 532<br>Tips for New Teachers, *TWE* pp. 524, 562 | |
| | Standardized Test Practice | pp. 525, 526, 528, 535, 543, 551, 558, 567, 574, 580, 587, 588, 589 | *TWE* pp. 588–589<br>Standardized Test Practice, *CRM* pp. 607–608 | Standardized Test Practice CD-ROM<br>www.geometryonline.com/standardized_test |
| ASSESSMENT | Open-Ended Assessment | Writing in Math, pp. 527, 534, 542, 551, 558, 567, 574, 579<br>Open Ended, pp. 525, 532, 539, 548, 555, 564, 572, 577<br>Standardized Test, p. 589 | Modeling: *TWE* pp. 551, 574<br>Speaking: *TWE* pp. 528, 568, 580<br>Writing: *TWE* pp. 535, 543, 558<br>Open-Ended Assessment, *CRM* p. 601 | |
| | Chapter Assessment | Study Guide, pp. 581–586<br>Practice Test, p. 587 | Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 589–594<br>Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 595–600<br>Vocabulary Test/Review, *CRM* p. 602 | ExamView® Pro (see below)<br>MindJogger Videoquizzes<br>www.geometryonline.com/vocabulary_review<br>www.geometryonline.com/chapter_test |

Yearly ProgressPro

*For more information on Yearly ProgressPro, see p. 400.*

| Geometry Lesson | Yearly ProgressPro Skill Lesson |
|---|---|
| 10-1 | Circles |
| 10-2 | Angles and Arcs |
| 10-3 | Arcs and Chords |
| 10-4 | Inscribed Angles |
| 10-5 | Tangents |
| 10-6 | Secants, Tangents, and Angle Measures |
| 10-7 | Special Segments in a Circle |
| 10-8 | Equations of Circles |

### ExamView® Pro

Use the networkable **ExamView® Pro** to:
- Create **multiple versions** of tests.
- Create **modified** tests for *Inclusion* students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your test from a program bank of artwork.

***For more information on Intervention and Assessment, see pp. T8–T11.***

# Reading and Writing in Mathematics

*Glencoe Geometry* provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

## Student Edition

- Foldables Study Organizer, p. 521
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 525, 532, 539, 548, 555, 564, 571, 577)
- Writing in Math questions in every lesson, pp. 527, 534, 542, 551, 558, 567, 574, 579
- Reading Study Tip, pp. 522, 536
- WebQuest, pp. 527, 580

## Teacher Wraparound Edition

- Foldables Study Organizer, pp. 521, 581
- Study Notebook suggestions, pp. 526, 533, 539, 548, 556, 560, 564, 571, 577
- Modeling activities, pp. 551, 574
- Speaking activities, pp. 528, 568, 580
- Writing activities, pp. 535, 543, 558
- Differentiated Instruction (Verbal/Linguistic), p. 525
- ELL Resources, pp. 520, 525, 527, 534, 541, 550, 557, 565, 573, 579, 581

## Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 10 Resource Masters,* pp. vii-viii)
- Proof Builder helps students learn and understand theorems and postulates from the chapter. (*Chapter 10 Resource Masters,* pp. ix–x)
- Reading to Learn Mathematics master for each lesson (*Chapter 10 Resource Masters*, pp. 545, 551, 557, 563, 569, 575, 581, 587)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading Strategies for the Mathematics Classroom*
- *WebQuest and Project Resources*

***For more information on Reading and Writing in Mathematics, see pp. T6–T7.***

## ELL ENGLISH LANGUAGE LEARNERS

**Lesson 10-1**

### Reading and Writing

Have students list what they already know about circles and what they want to learn. Lead a discussion with the class about what the students already know about circles. At the completion of the lesson, have students fill in what they have learned about circles. Have students review their lists after studying each lesson in this chapter.

**Lesson 10-3**

### Language Experience

Draw a circle on the board with an inscribed triangle and a circumscribed square. Have the class identify the circumscribed and inscribed figures. Discuss with the class the prefixes *circum* and *in.* Understanding the meaning of the terms will help students understand the concepts.

**Lesson 10-8**

### Alternative Assessment

Have your class compile their work on circles into a portfolio. Include drawings, definitions, and examples of vocabulary terms, as well as constructions.

# Chapter 10 Notes

## What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

## Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

| Lesson | NCTM Standards | Local Objectives |
|---|---|---|
| 10-1 | 3, 4, 6, 8, 9, 10 | |
| 10-2 | 3, 6, 8, 9, 10 | |
| 10-3 | 3, 4, 6, 8, 9, 10 | |
| 10-4 | 3, 6, 8, 9, 10 | |
| 10-5 | 3, 6, 8, 9, 10 | |
| 10-4 and 10-5 Follow-Up | 3, 6 | |
| 10-6 | 3, 6, 8, 9, 10 | |
| 10-7 | 3, 4, 6, 8, 9, 10 | |
| 10-8 | 3, 4, 6, 8, 9, 10 | |

**Key to NCTM Standards:**

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

# Chapter 10 Circles

## What You'll Learn

- **Lessons 10-1** Identify parts of a circle and solve problems involving circumference.
- **Lessons 10-2, 10-3, 10-4, and 10-6** Find arc and angle measures in a circle.
- **Lessons 10-5 and 10-7** Find measures of segments in a circle.
- **Lesson 10-8** Write the equation of a circle.

### Key Vocabulary

- chord (p. 522)
- circumference (p. 523)
- arc (p. 530)
- tangent (p. 552)
- secant (p. 561)

## Why It's Important

A circle is a unique geometric shape in which the angles, arcs, and segments intersecting that circle have special relationships. You can use a circle to describe a safety zone for fireworks, a location on Earth seen from space, and even a rainbow. *You will learn about angles of a circle when satellites send signals to Earth in Lesson 10-6.*

520 Chapter 10 Circles

Michael Dunning/Getty Images

## Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 10 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 10 test.

# Getting Started

**Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 10.

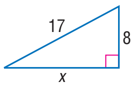
### For Lesson 10-1 — Solve Equations

Solve each equation for the given variable. *(For review, see pages 737 and 738.)*

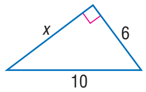
1. $\frac{4}{9}p = 72$ for $p$ **162**
2. $6.3p = 15.75$ **2.5**
3. $3x + 12 = 8x$ for $x$ **2.4**
4. $7(x + 2) = 3(x - 6)$ **−8**
5. $C = 2pr$ for $r$ **$r = \frac{C}{2p}$**
6. $r = \frac{C}{6.28}$ for C **$C = 6.28r$**

### For Lesson 10-5 — Pythagorean Theorem

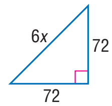
Find *x*. Round to the nearest tenth if necessary. *(For review, see Lesson 7-2.)*

7. 

**15**

8. 

**8**

9. 

**17.0**

### For Lesson 10-7 — Quadratic Formula

Solve each equation by using the Quadratic Formula. Round to the nearest tenth.

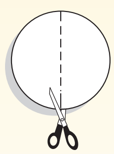
10. $x^2 - 4x = 10$ **5.7, −1.7**
11. $3x^2 - 2x - 4 = 0$ **1.5, −0.9**
12. $x^2 = x + 15$ **4.4, −3.4**
13. $2x^2 + x = 15$ **2.5, −3**

**FOLDABLES™ Study Organizer**

**Circles** Make this Foldable to help you organize your notes. Begin with five sheets of plain $8\frac{1}{2}$" by 11" paper, and cut out five large circles that are the same size.

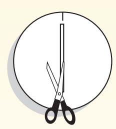
**Step 1 Fold and Cut**

Fold two of the circles in half and cut one-inch slits at each end of the folds.



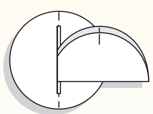
**Step 2 Fold and Cut**

Fold the remaining three circles in half and cut a slit in the middle of the fold.



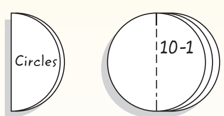
**Step 3 Slide**

Slide the two circles with slits on the ends through the large slit of the other circles.



**Step 4 Label**

Fold to make a booklet. Label the cover with the title of the chapter and each sheet with a lesson number.




**Reading and Writing** As you read and study each lesson, take notes and record concepts on the appropriate page of your Foldable.



# Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 10. Page references are included for additional student help.

Additional review is provided in the *Prerequisite Skills Workbook*, pages 11–12, 15–18, 23–24, 31–32, 35–36, 41–42, 45–48, 51–52, 61–64, 67–68, 71–72, 105–106, 109–110.

**Prerequisite Skills** in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

| For Lesson | Prerequisite Skill |
|---|---|
| 10-2 | Angle Addition, p. 528 |
| 10-3 | Isosceles Triangles, p. 535 |
| 10-4 | Solving Equations, p. 543 |
| 10-5 | Pythagorean Theorem, p. 551 |
| 10-6 | Solving Equations, p. 558 |
| 10-7 | Solving Equations by Factoring, p. 568 |
| 10-8 | Distance Formula, p. 574 |

**FOLDABLES™ Study Organizer**

For more information about Foldables, see *Teaching Mathematics with Foldables.*

**Organization of Data and Expository Writing** Use this Foldable for student writing about circles, angles, arcs, chords, tangents, secants, angle measurement, and equations. Students can use their Foldable to take notes, define terms, record concepts, use properties, and write and sketch examples. Ask students to write about circles in such a manner that someone who did not know what a circle was or understand how to solve problems using arcs and diameters will understand after reading what students have written.

# 10-1 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-1** Use as a quiz or review of Chapter 9.

**Mathematical Background** notes are available for this lesson on p. 520C.

**How** far does a carousel animal travel in one rotation?

*Ask students:*

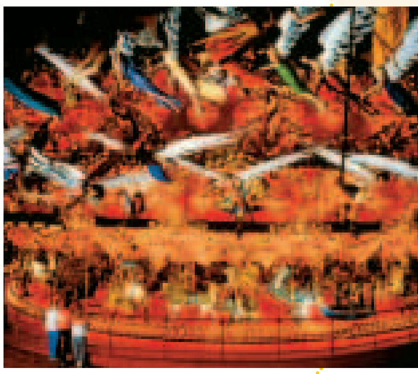
- Explain why an animal travels farther on the outside of the carousel than near the middle of the carousel. **The circumference of a circle with a large radius is greater than the circumference of a circle with a smaller radius.**
- Are there as many animals on the carousel in Wisconsin as there are degrees in a circle? Explain. **No; there are 100 degrees more in a circle than there are animals on the carousel.**

# 10-1 Circles and Circumference

**What You'll Learn**

- Identify and use parts of circles.
- Solve problems involving the circumference of a circle.

**How** far does a carousel animal travel in one rotation?



The largest carousel in the world still in operation is located in Spring Green, Wisconsin. It weighs 35 tons and contains 260 animals, none of which is a horse! The rim of the carousel base is a circle. The width, or diameter, of the circle is 80 feet. The distance that one of the animals on the outer edge travels can be determined by special segments in a circle.

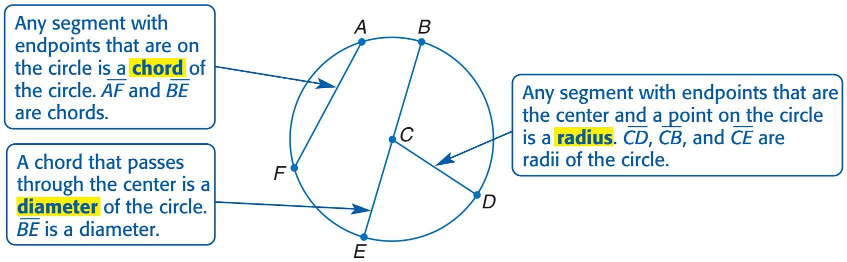
**Vocabulary**

- circle
- center
- chord
- radius
- diameter
- circumference
- pi ($\pi$)

**Study Tip**

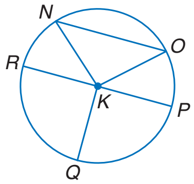
*Reading Mathematics*
The plural of radius is *radii*, pronounced RAY-dee-eye. The term *radius* can mean a segment or the measure of that segment. This is also true of the term *diameter*.

**PARTS OF CIRCLES** A **circle** is the locus of all points in a plane equidistant from a given point called the **center** of the circle. A circle is usually named by its center point. The figure below shows circle C, which can be written as ⊙C. Several special segments in circle C are also shown.




Note that diameter $\overline{BE}$ is made up of collinear radii $\overline{CB}$ and $\overline{CE}$.

### Example 1 Identify Parts of a Circle




**a. Name the circle.**

The circle has its center at *K*, so it is named circle *K*, or ⊙*K*.

In this textbook, the center of a circle will always be shown in the figure with a dot.

**b. Name a radius of the circle.**

Five radii are shown: $\overline{KN}$, $\overline{KO}$, $\overline{KP}$, $\overline{KQ}$, and $\overline{KR}$.

**c. Name a chord of the circle.**

Two chords are shown: $\overline{NO}$ and $\overline{RP}$.

**d. Name a diameter of the circle.**

$\overline{RP}$ is the only chord that goes through the center, so $\overline{RP}$ is a diameter.




## Resource Manager

**Workbook and Reproducible Masters**

***Chapter 10 Resource Masters***
- Study Guide and Intervention, pp. 541–542
- Skills Practice, p. 543
- Practice, p. 544
- Reading to Learn Mathematics, p. 545
- Enrichment, p. 546

***School-to-Career Masters,*** p. 19
***Prerequisite Skills Workbook,*** pp. 11–12, 23–24, 45–48
***Teaching Geometry With Manipulatives Masters,*** p. 161

**Transparencies**
5-Minute Check Transparency 10-1
Answer Key Transparencies

**Technology**
Interactive Chalkboard

**Study Tip**

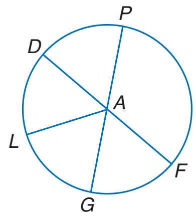
***Radii and Diameters***

There are an infinite number of radii in each circle. Likewise, there are an infinite number of diameters.

By the definition of a circle, the distance from the center to any point on the circle is always the same. Therefore, all radii are congruent. A diameter is composed of two radii, so all diameters are congruent. The letters $d$ and $r$ are usually used to represent diameter and radius in formulas. So, $d = 2r$ and $r = \frac{d}{2}$ or $\frac{1}{2}d$.

## Example 2 Find Radius and Diameter

**Circle $A$ has diameters $\overline{DF}$ and $\overline{PG}$.**




**a. If $DF = 10$, find $DA$.**

$r = \frac{1}{2}d$ Formula for radius

$r = \frac{1}{2}(10)$ or 5 Substitute and simplify.

**b. If $PA = 7$, find $PG$.**

$d = 2r$ Formula for diameter

$d = 2(7)$ or 14 Substitute and simplify.

**c. If $AG = 12$, find $LA$.**

Since all radii are congruent, $LA = AG$. So, $LA = 12$.

Circles can intersect. The segment connecting the centers of the two intersecting circles contains a radius of each circle.

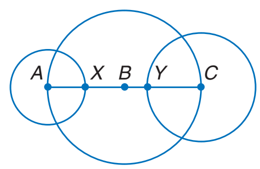
**Study Tip**

***Congruent Circles***

The circles shown in Example 3 have different radii. They are *not* congruent circles. For two circles to be *congruent circles*, they must have congruent radii or congruent diameters.

## Example 3 Find Measures in Intersecting Circles

**The diameters of $\odot A$, $\odot B$, and $\odot C$ are 10 inches, 20 inches, and 14 inches, respectively.**




**a. Find $XB$.**

Since the diameter of $\odot A$ is 10, $AX = 5$.

Since the diameter of $\odot B$ is 20, $AB = 10$ and $BC = 10$.

$\overline{XB}$ is part of radius $\overline{AB}$.

$AX + XB = AB$ Segment Addition Postulate

$5 + XB = 10$ Substitution

$XB = 5$ Subtract 5 from each side.

**b. Find $BY$.**

$\overline{BY}$ is part of $\overline{BC}$.

Since the diameter of $\odot C$ is 14, $YC = 7$.

$BY + YC = BC$ Segment Addition Postulate

$BY + 7 = 10$ Substitution

$BY = 3$ Subtract 7 from each side.

**CIRCUMFERENCE** The **circumference** of a circle is the distance around the circle. Circumference is most often represented by the letter $C$.

www.geometryonline.com/extra_examples



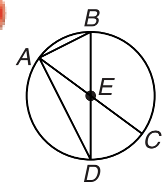
## 2 Teach

### PARTS OF CIRCLES

**In-Class Examples** Power Point®

***Teaching Tip*** Remind students that a diameter is a special chord of a circle because its endpoints are on the circle.

**1**



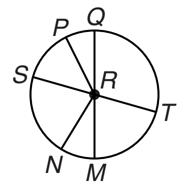

**a.** Name the circle. $\odot E$

**b.** Name a radius of the circle. $\overline{EB}$, $\overline{EA}$, $\overline{EC}$, or $\overline{ED}$

**c.** Name a chord of the circle. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, or $\overline{AD}$

**d.** Name a diameter of the circle. $\overline{AC}$ or $\overline{BD}$

**2** Circle $R$ has diameters $\overline{ST}$ and $\overline{QM}$.

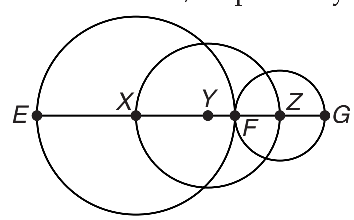



**a.** If $ST = 18$, find $RS$. **9**

**b.** If $RM = 24$, find $QM$. **48**

**c.** If $RN = 2$, find $RP$. **2**

**3** The diameters of $\odot X$, $\odot Y$, and $\odot Z$ are 22 millimeters, 16 millimeters, and 10 millimeters, respectively.




**a.** Find $EZ$. **27 mm**

**b.** Find $XF$. **11 mm**

**Interactive Chalkboard**
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Try These exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

## CIRCUMFERENCE

### In-Class Example

Power Point®

*Teaching Tip* Tell students that $\pi$ can also be approximated by $\frac{22}{7}$ if students are using a nonscientific calculator that does not include $\pi$ on its keyboard.

**4** a. Find $C$ if $r = 13$ inches. $26\pi$ or $\approx 81.68$ in.

b. Find $C$ if $d = 6$ millimeters. $6\pi$ or $\approx 18.85$ mm

c. Find $d$ and $r$ to the nearest hundredth if $C = 65.4$ feet. $d \approx 20.82$ ft; $r \approx 10.41$ ft

**Tips for New Teachers**

If students wonder why they are given two formulas for the circumference of a circle when they already know that the diameter is twice the radius, tell them that the two formulas lead them to look closely at a question to determine if the problem gives a radius or a diameter. Explain that a common mistake is to erroneously calculate the circumference of a circle as $\pi r$.

## Geometry Activity

### Circumference Ratio

A special relationship exists between the circumference of a circle and its diameter.

**Gather Data and Analyze**

Collect ten round objects.

1. Measure the circumference and diameter of each object using a millimeter measuring tape. Record the measures in a table like the one at the right. See students' work.
2. Compute the value of $\frac{C}{d}$ to the nearest hundredth for each object. Record the result in the fourth column of the table. Each ratio should be near 3.1.

| Object | $C$ | $d$ | $\frac{C}{d}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| ⋮ | | | |
| 10 | | | |

**Make a Conjecture**

3. What seems to be the relationship between the circumference and the diameter of the circle? $C \approx 3.14d$

**Study Tip**

*Value of $\pi$*
In this book, we will use a calculator to evaluate expressions involving $\pi$. If no calculator is available, 3.14 is a good estimate for $\pi$.

The Geometry Activity suggests that the circumference of any circle can be found by multiplying the diameter by a number slightly larger than 3. By definition, the ratio $\frac{C}{d}$ is an irrational number called **pi**, symbolized by the Greek letter $\pi$. Two formulas for the circumference can be derived using this definition.

$\frac{C}{d} = \pi$ Definition of pi

$C = \pi d$ Multiply each side by $d$.

$C = \pi d$

$C = \pi(2r)$ $d = 2r$

$C = 2\pi r$ Simplify.

**Key Concept** *Circumference*

For a circumference of $C$ units and a diameter of $d$ units or a radius of $r$ units, $C = \pi d$ or $C = 2\pi r$.

If you know the diameter or radius, you can find the circumference. Likewise, if you know the circumference, you can find the diameter or radius.

### Example 4 Find Circumference, Diameter, and Radius

**a. Find $C$ if $r = 7$ centimeters.**

$C = 2\pi r$ Circumference formula
$= 2\pi(7)$ Substitution
$= 14\pi$ or about 43.98 cm

**b. Find $C$ if $d = 12.5$ inches.**

$C = \pi d$ Circumference formula
$= \pi(12.5)$ Substitution
$= 12.5\pi$ or 39.27 in.

**c. Find $d$ and $r$ to the nearest hundredth if $C = 136.9$ meters.**

$C = \pi d$ Circumference formula
$136.9 = \pi d$ Substitution
$\frac{136.9}{\pi} = d$ Divide each side by $\pi$.
$43.58 \approx d$ Use a calculator.
$d \approx 43.58$ m

$r = \frac{1}{2}d$ Radius formula
$\approx \frac{1}{2}(43.58)$ $d \approx 43.58$
$\approx 21.79$ m Use a calculator.



Aaron Haupt

## Geometry Activity

- You can provide students a handout with a blank 11-row by 4-column table.
- Ask students why they think they are measuring the objects in millimeters. Students should note that millimeters provide very accurate values for comparison in this activity.
- Point out that the relationship between the circumference and diameter of a circle is an extremely interesting concept that has been analyzed for centuries, and there are many books written just on this subject.

You can also use other geometric figures to help you find the circumference of a circle.

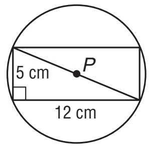
**Standardized Test Practice**

## Example 5 Use Other Figures to Find Circumference

**Multiple-Choice Test Item**

Find the exact circumference of ⊙$P$.

(A) 13 cm
(B) $12\pi$ cm
(C) 40.84 cm
(D) $13\pi$ cm




**Read the Test Item**

You are given a figure that involves a right triangle and a circle. You are asked to find the exact circumference of the circle.

**Solve the Test Item**

The diameter of the circle is the same as the hypotenuse of the right triangle.

| | |
|---|---|
| $a^2 + b^2 = c^2$ | Pythagorean Theorem |
| $5^2 + 12^2 = c^2$ | Substitution |
| $169 = c^2$ | Simplify. |
| $13 = c$ | Take the square root of each side. |

So the diameter of the circle is 13 centimeters.

| | |
|---|---|
| $C = \pi d$ | Circumference formula |
| $C = \pi(13)$ or $13\pi$ | Substitution |

Because we want the exact circumference, the answer is D.

### Test-Taking Tip

Notice that the problem asks for an exact answer. Since you know that an exact circumference contains $\pi$, you can eliminate choices A and C.

## Check for Understanding

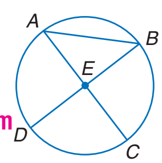
**Concept Check**

1. **Describe** how the value of $\pi$ can be calculated. **See margin.**
2. **Write** two equations that show how the diameter of a circle is related to the radius of a circle. $d = 2r$, $r = \frac{1}{2}d$
3. **OPEN ENDED** Explain why a diameter is the longest chord of a circle. **See margin.**

**Guided Practice**

| GUIDED PRACTICE KEY | |
|---|---|
| Exercises | Examples |
| 4–7 | 1 |
| 8, 9 | 2 |
| 10–12 | 3 |
| 13, 14 | 4 |
| 15 | 5 |

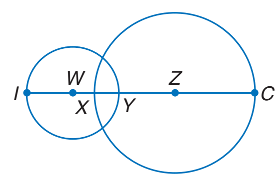
**For Exercises 4–9, refer to the circle at the right.** 6. $\overline{AB}$, $\overline{AC}$, or $\overline{BD}$




4. Name the circle. ⊙$E$
5. Name a radius. $\overline{EA}$, $\overline{EB}$, $\overline{EC}$, or $\overline{ED}$
6. Name a chord.
7. Name a diameter. $\overline{AC}$ or $\overline{BD}$
8. Suppose $BD = 12$ millimeters. Find the radius of the circle. **6 mm**
9. Suppose $CE = 5.2$ inches. Find the diameter of the circle. **10.4 in.**

**Circle $W$ has a radius of 4 units, ⊙$Z$ has a radius of 7 units, and $XY = 2$. Find each measure.**

10. $YZ$ **5**
11. $IX$ **6**
12. $IC$ **20**






## In-Class Example

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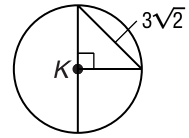
5 Find the exact circumference of ⊙$K$. **B**




**A** $3\sqrt{2}\pi$ **B** $6\pi$
**C** $6\sqrt{2}\pi$ **D** $12\pi$

## Answers

1. Sample answer: The value of $\pi$ is calculated by dividing the circumference of a circle by the diameter.
3. Except for a diameter, two radii and a chord of a circle can form a triangle. The Triangle Inequality Theorem states that the sum of two sides has to be greater than the third. So, $2r$ has to be greater than the measure of any chord that is not a diameter, but $2r$ is the measure of the diameter. So the diameter has to be longer than any other chord of the circle.

**DAILY INTERVENTION**

### Differentiated Instruction ELL

**Verbal/Linguistic** Have students write about the parts of a circle and its circumference in their own words. They can write a paragraph that explains each vocabulary term and the relationship of the terms to each other, or they can list the terms and write a brief explanation and/or provide an example for each. Students can use these explanations for their study notebooks.

# 3 Practice/Apply

## Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include one circle that is labeled to demonstrate each vocabulary term in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

## About the Exercises...

**Organization by Objective**
- **Parts of Circles:** 16–43
- **Circumference:** 48–52

**Odd/Even Assignments**
Exercises 16–55 are structured so that students practice the same concepts whether they are assigned odd or even problems.

## Assignment Guide

**Basic:** 17–35 odd, 39, 41, 45–49 odd, 53–63 odd, 64, 66–80 (optional: 65)

**Average:** 17–61 odd, 63, 64, 66–80 (optional: 65)

**Advanced:** 16–60 even, 61–74 (optional: 75–80)

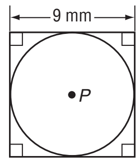
**The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.**

**13.** $r = 5$ m, $d =$ _?_, $C =$ _?_ **10 m, 31.42 m**

**14.** $C = 2368$ ft, $d =$ _?_, $r =$ _?_ **753.76 ft, 376.88 ft**

Standardized Test Practice

**15.** Find the exact circumference of the circle. **B**




(A) 4.5π mm
(B) 9π mm
(C) 18π mm
(D) 81π mm

★ indicates increased difficulty

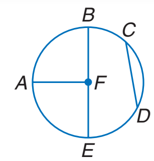
## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 16–25 | 1 |
| 26–31 | 2 |
| 32–43 | 3 |
| 48–51 | 4 |
| 52 | 5 |

**Extra Practice**
See page 773.

**For Exercises 16–20, refer to the circle at the right.**




**16.** Name the circle. **⊙F**

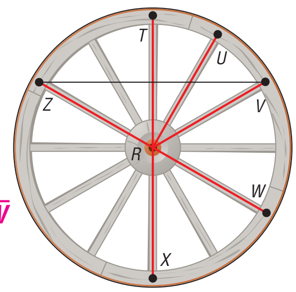
**17.** Name a radius. **$\overline{FA}$, $\overline{FB}$, or $\overline{FE}$**

**18.** Name a chord. **$\overline{BE}$ or $\overline{CD}$**

**19.** Name a diameter. **$\overline{BE}$**

**20.** Name a radius not contained in a diameter. **$\overline{FA}$**

**HISTORY** **For Exercises 21–31, refer to the model of a Conestoga wagon wheel.**




**21.** Name the circle. **⊙R**

**22.** Name a radius of the circle. **$\overline{RT}$, $\overline{RU}$, $\overline{RV}$, $\overline{RW}$, $\overline{RX}$, or $\overline{RZ}$**

**23.** Name a chord of the circle. **$\overline{ZV}$, $\overline{TX}$, or $\overline{WZ}$**

**24.** Name a diameter of the circle. **$\overline{TX}$ or $\overline{WZ}$**

**25.** Name a radius not contained in a diameter. **$\overline{RU}$, $\overline{RV}$**

**26.** Suppose the radius of the circle is 2 feet. Find the diameter. **4 ft**

**27.** The larger wheel of the wagon was often 5 or more feet tall. What is the radius of a 5-foot wheel? **2.5 ft**

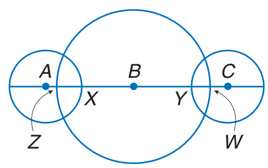
**28.** If $TX = 120$ centimeters, find $TR$. **60 cm**

**29.** If $RZ = 32$ inches, find $ZW$. **64 in. or 5 ft 4 in.**

**30.** If $UR = 18$ inches, find $RV$. **18 in.**

**31.** If $XT = 1.2$ meters, find $UR$. **0.6 m**

**The diameters of ⊙A, ⊙B, and ⊙C are 10, 30, and 10 units, respectively. Find each measure if $\overline{AZ} \cong \overline{CW}$ and $CW = 2$.**




**32.** $AZ$ **2**

**33.** $ZX$ **3**

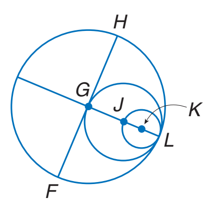
**34.** $BX$ **12**

**35.** $BY$ **12**

**36.** $YW$ **3**

★**37.** $AC$ **34**

**Circles G, J, and K all intersect at L. If $GH = 10$, find each measure.**




**38.** $FG$ **10**

**39.** $FH$ **20**

**40.** $GL$ **10**

**41.** $GJ$ **5**

**42.** $JL$ **5**

★**43.** $JK$ **2.5**



## Answer

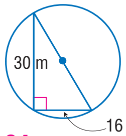
62. Sample answer: about 251.3 feet. Answers should include the following.
- The distance the animal travels is approximated by the circumference of the circle.
- The diameter for the circle on which the animal is located becomes 80 − 2 or 78. The circumference of this circle is 78π. Multiply by 22 to get a total distance of 22(78π) or 5391 feet. This is a little over a mile.

44. 14 mm, 43.98 mm
45. 13.4 cm, 84.19 cm
46. 26 mi, 13 mi
47. 24.32 m, 12.16 m
48. $6\frac{1}{4}$yd, 39.27 yd
49. $13\frac{1}{2}$ in., 42.41 in.

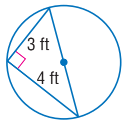
**The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.**

44. $r = 7$ mm, $d =$ __?__, $C =$ __?__
45. $d = 26.8$ cm, $r =$ __?__, $C =$ __?__
46. $C = 26\pi$ mi, $d =$ __?__, $r =$ __?__
47. $C = 76.4$ m, $d =$ __?__, $r =$ __?__
48. $d = 12\frac{1}{2}$yd, $r =$ __?__, $C =$ __?__
49. $r = 6\frac{3}{4}$in., $d =$ __?__, $C =$ __?__
50. $d = 2a$, $r =$ __?__, $C =$ __?__ **a, 6.28a**
★51. $r = \frac{a}{6}$, $d =$ __?__, $C =$ __?__ **0.33a, 1.05a**

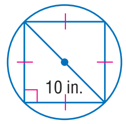
**Find the exact circumference of each circle.**

52. 

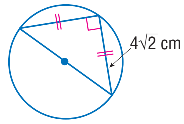
**34π m**

53. 

**5π ft**

54. 

**$10\pi\sqrt{2}$ in.**

55. 

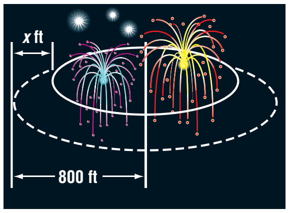
**8π cm**

56. 1; This description is the definition of a radius.

56. **PROBABILITY** Find the probability that a segment with endpoints that are the center of the circle and a point on the circle is a radius. Explain.

57. **PROBABILITY** Find the probability that a chord that does not contain the center of a circle is the longest chord of the circle.
**0; The longest chord of a circle is the diameter, which contains the center.**

**FIREWORKS** **For Exercises 58–60, use the following information.**
Every July 4th Boston puts on a gala with the Boston Pops Orchestra, followed by a huge fireworks display. The fireworks are shot from a barge in the river. There is an explosion circle inside which all of the fireworks will explode. Spectators sit outside a safety circle that is 800 feet from the center of the fireworks display.




58. Find the approximate circumference of the safety circle. **5026.5 ft**

59. If the safety circle is 200 to 300 feet farther from the center than the explosion circle, find the range of values for the radius of the explosion circle. **500–600 ft**

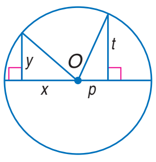
60. Find the least and maximum circumference of the explosion circle to the nearest foot. **3142 ft; 3770 ft**

**WebQuest**

Drawing a radius and circle on the map is the last clue to help you find the hidden treasure. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

***Online Research*** **Data Update** Find the largest firework ever made. How does its dimension compare to the Boston display? Visit www.geometryonline.com/data_update to learn more.

61. **CRITICAL THINKING** In the figure, $O$ is the center of the circle, and $x^2 + y^2 + p^2 + t^2 = 288$. What is the exact circumference of $\odot O$? **24π units**




62. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

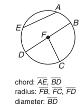
**How far does a carousel animal travel in one rotation?**

Include the following in your answer:
- a description of how the circumference of a circle relates to the distance traveled by the animal, and
- whether an animal located one foot from the outside edge of the carousel travels a mile when it makes 22 rotations for each ride.



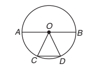
## Study Guide and Intervention, p. 541 (shown) and p. 542

**Parts of Circles** A **circle** consists of all points in a plane that are a given distance, called the **radius**, from a given point called the **center**.



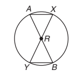

A segment or line can intersect a circle in several ways.
- A segment with endpoints that are the center of the circle and a point of the circle is a **radius**.
- A segment with endpoints that lie on the circle is a **chord**.
- A chord that contains the circle's center is a **diameter**.

*Example*




a. **Name the circle.**
The name of the circle is $\odot O$.
b. **Name radii of the circle.**
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$, and $\overline{DO}$ are radii.
c. **Name chords of the circle.**
$\overline{AB}$ and $\overline{CD}$ are chords.
d. **Name a diameter of the circle.**
$\overline{AB}$ is a diameter.

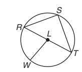
*Exercises*




1. Name the circle. **$\odot R$**
2. Name radii of the circle. **$\overline{RA}$, $\overline{RB}$, $\overline{RY}$, and $\overline{RX}$**
3. Name chords of the circle. **$\overline{BY}$, $\overline{AX}$, $\overline{AB}$, and $\overline{XY}$**
4. Name diameters of the circle. **$\overline{AB}$ and $\overline{XY}$**
5. Find $AR$ if $AB$ is 18 millimeters. **9 mm**
6. Find $AR$ and $AB$ if $RY$ is 10 inches. **$AR = 10$ in.; $AB = 20$ in.**
7. Is $\overline{AB} \cong \overline{XY}$? Explain. **Yes; all diameters of the same circle are congruent.**

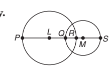
## Skills Practice, p. 543 and Practice, p. 544 (shown)

**For Exercises 1–5, refer to the circle.**




1. Name the circle. **$\odot L$**
2. Name a radius. **$\overline{LR}$, $\overline{LT}$, or $\overline{LW}$**
3. Name a chord. **$\overline{RT}$, $\overline{RS}$, or $\overline{ST}$**
4. Name a diameter. **$\overline{RT}$**
5. Name a radius not drawn as part of a diameter. **$\overline{LW}$**
6. Suppose the radius of the circle is 3.5 yards. Find the diameter. **7 yd**
7. If $RT = 19$ meters, find $LW$. **9.5 m**

**The diameters of $\odot L$ and $\odot M$ are 20 and 13 units, respectively. Find each measure if $QR = 4$.**

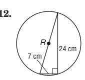



8. $LQ$ **6**
9. $RM$ **2.5**

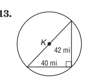
**The radius, diameter, or circumference of a circle is given. Find the missing measures to the nearest hundredth.**

10. $r = 7.5$ mm
$d =$ **15 mm**, $C \approx$ **47.12 mm**
11. $C = 227.6$ yd
$d \approx$ **72.45 yd**, $r \approx$ **36.22 yd**

**Find the exact circumference of each circle.**

12. 

**25π cm**

13. 

**58π mi**

**SUNDIALS** **For Exercises 14 and 15, use the following information.**
Herman purchased a sundial to use as the centerpiece for a garden. The diameter of the sundial is 9.5 inches.

14. Find the radius of the sundial. **4.75 in.**
15. Find the circumference of the sundial to the nearest hundredth. **29.85 in.**

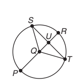
## Reading to Learn Mathematics, p. 545 ELL

**Pre-Activity** **How far does a carousel animal travel in one rotation?**

Read the introduction to Lesson 10-1 at the top of page 522 in your textbook.

How could you measure the approximate distance around the circular carousel using everyday measuring devices? **Sample answer: Place a piece of string along the rim of the carousel. Cut off a length of string that covers the perimeter of the circle. Straighten the string and measure it with a yardstick.**

**Reading the Lesson**




1. Refer to the figure.
   a. Name the circle. **$\odot Q$**
   b. Name four radii of the circle. **$\overline{QP}$, $\overline{QR}$, $\overline{QS}$, and $\overline{QT}$**
   c. Name a diameter of the circle. **$\overline{PR}$**
   d. Name two chords of the circle. **$\overline{PR}$ and $\overline{ST}$**

2. Match each description from the first column with the best term from the second column. (Some terms in the second column may be used more than once or not at all.)
   a. a segment whose endpoints are on a circle **iii**
   b. the set of all points in a plane that are the same distance from a given point **iv**
   c. the distance between the center of a circle and any point on the circle **i**
   d. a chord that passes through the center of a circle **ii**
   e. a segment whose endpoints are the center and any point on a circle **i**
   f. a chord made up of two collinear radii **ii**
   g. the distance around a circle **v**

   i. radius
   ii. diameter
   iii. chord
   iv. circle
   v. circumference

3. Which equations correctly express a relationship in a circle? **A, D, G**
   A. $d = 2r$ B. $C = \pi r$ C. $C = 2d$ D. $d = \frac{C}{\pi}$
   E. $r = \frac{d}{\pi}$ F. $C = r^2$ G. $C = 2\pi r$ H. $d = \frac{1}{2}r$

**Helping You Remember**

4. A good way to remember a new geometric term is to relate the word or its parts to geometric terms you already know. Look up the origins of the two parts of the word *diameter* in your dictionary. Explain the meaning of each part and give a term you already know that shares the origin of that part. **Sample answer: The first part comes from *dia*, which means *across* or *through*, as in *diagonal*. The second part comes from *metron*, which means *measure*, as in *geometry*.**

## Enrichment, p. 546

***The Four Color Problem***

Mapmakers have long believed that only four colors are necessary to distinguish among any number of different countries on a plane map. Countries that meet only at a point may have the same color provided they do not have an actual border. The conjecture that four colors are sufficient for every conceivable plane map eventually attracted the attention of mathematicians and became known as the "four-color problem." Despite extraordinary efforts over many years to solve the problem, no definite answer was obtained until the 1980s. Four colors are indeed sufficient, and the proof was accomplished by making ingenious use of computers.

The following problems will help you appreciate some of the complexities of the four-color problem. For these "maps," assume that each closed region is a different country.

1. What is the minimum number of colors necessary for each map?
   a. b. c.

## 4 Assess

### Open-Ended Assessment

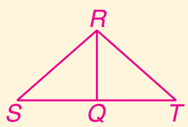
**Speaking** Students can practice the vocabulary terms in this lesson by describing selected circles and defining terms aloud. For example, find a circle in the lesson without values, and call on students to name its parts. Then ask students to state the values for the radius and circumference of the circle if the diameter is 10 units, 20 units, etc.

### Getting Ready for Lesson 10-2

**Prerequisite Skill** Students will learn about angles and arcs in Lesson 10-2. They will use angle addition to find angle measures in circles. Use Exercises 75–80 to determine your students' familiarity with angle addition.

### Answers

73. Given: $\overline{RQ}$ bisects $\angle SRT$.
Prove: $m\angle SQR > m\angle SRQ$



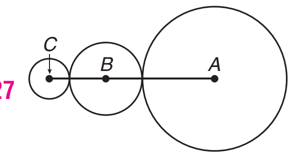

Proof:

Statements (Reasons)

1. $\overline{RQ}$ bisects $\angle SRT$. (Given)
2. $\angle SRQ \cong \angle QRT$ (Def. of $\angle$ bisector)
3. $m\angle SRQ = m\angle QRT$ (Def. of $\cong$ ∆s)
4. $m\angle SQR = m\angle T + m\angle QRT$ (Exterior Angle Theorem)
5. $m\angle SQR > m\angle QRT$ (Def. of Inequality)
6. $m\angle SQR > m\angle SRQ$ (Substitution)

### Standardized Test Practice

63. **GRID IN** In the figure, the radius of circle $A$ is twice the radius of circle $B$ and four times the radius of circle $C$. If the sum of the circumferences of the three circles is $42\pi$, find the measure of $\overline{AC}$. **27**

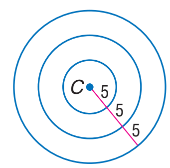



64. **ALGEBRA** There are $k$ gallons of gasoline available to fill a tank. After $d$ gallons have been pumped, what percent of gasoline, in terms of $k$ and $d$, has been pumped? **A**

(A) $\frac{100d}{k}\%$ (B) $\frac{k}{100d}\%$ (C) $\frac{100k}{d}\%$ (D) $\frac{100k-d}{k}\%$

### Extending the Lesson

65. **CONCENTRIC CIRCLES** Circles that have the same center, but different radii, are called *concentric circles*. Use the figure at the right to find the exact circumference of each circle. List the circumferences in order from least to greatest. **$10\pi$, $20\pi$, $30\pi$**




## Maintain Your Skills

### Mixed Review

**Find the magnitude to the nearest tenth and direction to the nearest degree of each vector.** *(Lesson 9-6)*

66. $\overrightarrow{AB} = \langle 1, 4\rangle$ **4.1; 76°**
67. $\vec{v} = \langle 4, 9\rangle$ **9.8; 66°**
68. $\overrightarrow{AB}$ if $A(4, 2)$ and $B(7, 22)$ **20.2; 81°**
69. $\overrightarrow{CD}$ if $C(0, -20)$ and $D(40, 0)$ **44.7; 27°**

**Find the measure of the dilation image of $\overline{AB}$ for each scale factor $k$.** *(Lesson 9-5)*

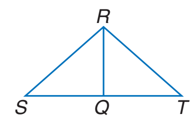
70. $AB = 5, k = 6$ **30**
71. $AB = 16, k = 1.5$ **24**
72. $AB = \frac{2}{3}, k = -\frac{1}{2}$ **$\frac{1}{3}$**

73. **PROOF** Write a two-column proof. *(Lesson 5-3)*

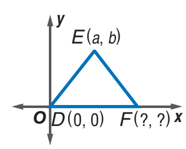
**Given:** $\overline{RQ}$ bisects $\angle SRT$.

**Prove:** $m\angle SQR > m\angle SRQ$

**See margin.**



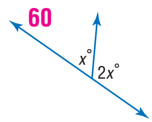

74. **COORDINATE GEOMETRY** Name the missing coordinates if $\triangle DEF$ is isosceles with vertex angle $E$. *(Lesson 4-3)* **$(2a, 0)$**



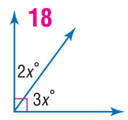

### Getting Ready for the Next Lesson

**PREREQUISITE SKILL** Find $x$. *(To review **angle addition**, see Lesson 1-4.)*

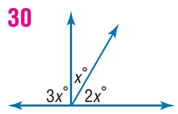
75. **60**



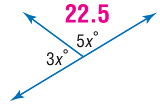

76. **18**



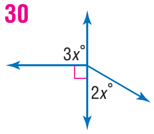

77. **30**



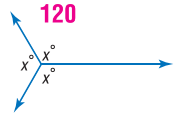

78. **22.5**




79. **30**




80. **120**






### Teacher to Teacher

Kim A. Halvorson, DeSoto County High School Arcadia, FL

My students are asked to decorate a T-shirt with a "pi" theme. Then they wear them on March 14 (3.14). The rest of the school (via morning announcements) is encouraged to ask the geometry students to discuss their shirts.

# 10-2 Angles and Arcs

## *What* You'll Learn

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

### *Vocabulary*

- central angle
- arc
- minor arc
- major arc
- semicircle

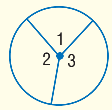
## *What* kinds of angles do the hands on a clock form?

Most clocks on electronic devices are digital, showing the time as numerals. Analog clocks are often used in decorative furnishings and wrist watches. An analog clock has moving hands that indicate the hour, minute, and sometimes the second. This clock face is a circle. The three hands form three central angles of the circle.

**ANGLES AND ARCS** In Chapter 1, you learned that a degree is $\frac{1}{360}$ of the circular rotation about a point. This means that the sum of the measures of the angles about the center of the clock above is 360. Each of the angles formed by the clock hands is called a central angle. A **central angle** has the center of the circle as its vertex, and its sides contain two radii of the circle.

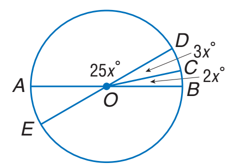
### Key Concept — *Sum of Central Angles*

- **Words** The sum of the measures of the central angles of a circle with no interior points in common is 360.
- **Example** $m\angle 1 + m\angle 2 + m\angle 3 = 360$




### *Example* 1 *Measures of Central Angles*

**ALGEBRA** **Refer to ⊙O.**




**a. Find** $m\angle AOD$.

$\angle AOD$ and $\angle DOB$ are a linear pair, and the angles of a linear pair are supplementary.

$$m\angle AOD + m\angle DOB = 180$$

$$m\angle AOD + m\angle DOC + m\angle COB = 180 \quad \text{Angle Sum Theorem}$$

$$25x + 3x + 2x = 180 \quad \text{Substitution}$$

$$30x = 180 \quad \text{Simplify.}$$

$$x = 6 \quad \text{Divide each side by 60.}$$

Use the value of $x$ to find $m\angle AOD$.

$$m\angle AOD = 25x \quad \text{Given}$$

$$= 25(6) \text{ or } 150 \quad \text{Substitution}$$





# 10-2 Lesson Notes

## 1 *Focus*

**5-Minute Check Transparency 10-2** Use as a quiz or review of Lesson 10-1.

**Mathematical Background** notes are available for this lesson on p. 520C.

**What kinds of angles do the hands on a clock form?**

*Ask students*:

- Do the three angles on the clock appear to be acute, obtuse, or right angles? **2 acute, 1 obtuse**
- Why do you think the angles formed by the three hands are called central angles? **Because the three angles share the center of the circle as a vertex.**

## Resource Manager

### Workbook and Reproducible Masters

***Chapter 10 Resource Masters***

- Study Guide and Intervention, pp. 547–548
- Skills Practice, p. 549
- Practice, p. 550
- Reading to Learn Mathematics, p. 551
- Enrichment, p. 552
- Assessment, p. 603

***Prerequisite Skills Workbook,*** pp. 31–32, 61–64, 67–68, 71–72, 105–106, 109–110

***Teaching Geometry With Manipulatives Masters,*** p. 16

### Transparencies

5-Minute Check Transparency 10-2
Answer Key Transparencies

### Technology

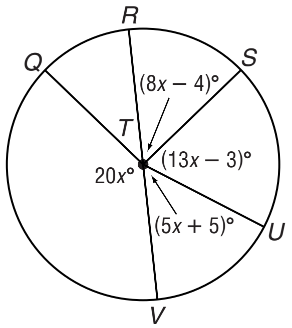
Interactive Chalkboard

# 2 Teach

## ANGLES AND ARCS

**In-Class Example** Power Point®

**1** **ALGEBRA** $\overline{RV}$ is a diameter of $\odot T$.




**a.** Find $m\angle RTS$. **52**

**b.** Find $m\angle QTR$. **40**

**Study Tip**

*Naming Arcs*
Do not assume that because an arc is named by three letters that it is a semicircle or major arc. You can also correctly name a minor arc using three letters.

**b. Find $m\angle AOE$.**

$\angle AOE$ and $\angle AOD$ form a linear pair.

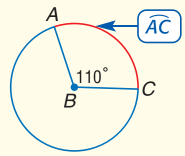
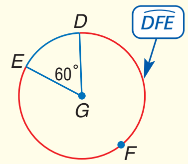
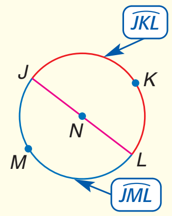
$m\angle AOE + m\angle AOD = 180$ Linear pairs are supplementary.

$m\angle AOE + 150 = 180$ Substitution

$m\angle AOE = 30$ Subtract 150 from each side.

A central angle separates the circle into two parts, each of which is an **arc**. The measure of each arc is related to the measure of its central angle.

### Key Concept — Arcs of a Circle

| Type of Arc: | minor arc | major arc | semicircle |
|---|---|---|---|
| Example: |   |   |   |
| Named: | usually by the letters of the two endpoints $\overset{\frown}{AC}$ | by the letters of the two endpoints and another point on the arc $\overset{\frown}{DFE}$ | by the letters of the two endpoints and another point on the arc $\overset{\frown}{JML}$ and $\overset{\frown}{JKL}$ |
| Arc Degree Measure Equals: | the measure of the central angle and is less than 180 $m\angle ABC = 110$, so $m\overset{\frown}{AC} = 110$ | 360 minus the measure of the minor arc and is greater than 180 $m\overset{\frown}{DFE} = 360 - m\overset{\frown}{DE}$ $m\overset{\frown}{DFE} = 360 - 60$ or 300 | 360 ÷ 2 or 180 $m\overset{\frown}{JML} = 180$ $m\overset{\frown}{JML} = 180$ |

Arcs with the same measure in the same circle or in congruent circles are congruent.

### Theorem 10.1

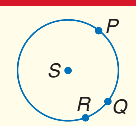
In the same or in congruent circles, two arcs are congruent if and only if their corresponding central angles are congruent.

You will prove Theorem 10.1 in Exercise 54.

Arcs of a circle that have exactly one point in common are *adjacent arcs*. Like adjacent angles, the measures of adjacent arcs can be added.

### Postulate 10.1

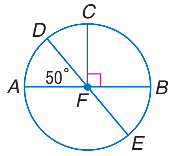
**Arc Addition Postulate** The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.




**Example:** In $\odot S$, $m\overset{\frown}{PQ} + m\overset{\frown}{QR} = m\overset{\frown}{PQR}$.

## Example 2 Measures of Arcs

**In $\odot F$, $m\angle DFA = 50$ and $\overline{CF} \perp \overline{FB}$. Find each measure.**




**a. $m\widehat{BE}$**

$\widehat{BE}$ is a minor arc, so $m\widehat{BE} = m\angle BFE$.

| | |
|---|---|
| $\angle BFE \cong \angle DFA$ | Vertical angles are congruent. |
| $m\angle BFE = m\angle DFA$ | Definition of congruent angles |
| $m\widehat{BE} = m\angle DFA$ | Transitive Property |
| $m\widehat{BE} = 50$ | Substitution |

**b. $m\widehat{CBE}$**

$\widehat{CBE}$ is composed of adjacent arcs, $\widehat{CB}$ and $\widehat{BE}$.

| | |
|---|---|
| $m\widehat{CB} = m\angle CFB$ | |
| $= 90$ | $\angle CFB$ is a right angle. |
| $m\widehat{CBE} = m\widehat{CB} + m\widehat{BE}$ | Arc Addition Postulate |
| $m\widehat{CBE} = 90 + 50$ or $140$ | Substitution |

**c. $m\widehat{ACE}$**

One way to find $m\widehat{ACE}$ is by using $\widehat{ACB}$ and $\widehat{BE}$. $\widehat{ACB}$ is a semicircle.

| | |
|---|---|
| $m\widehat{ACE} = m\widehat{ACB} + \widehat{BE}$ | Arc Addition Postulate |
| $m\widehat{ACE} = 180 + 50$ or $230$ | Substitution |

In a circle graph, the central angles divide a circle into wedges to represent data, often expressed as a percent. The size of the angle is proportional to the percent.

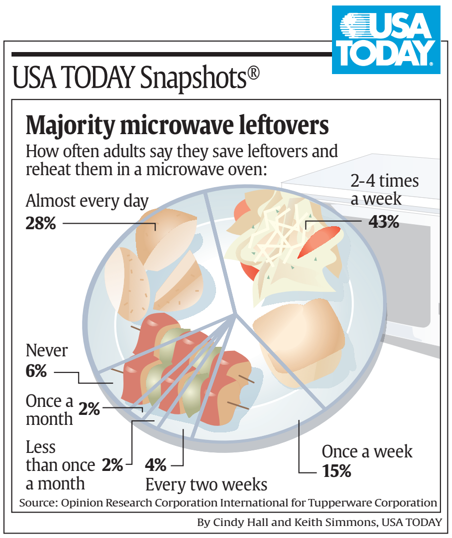
## Example 3 Circle Graphs

**FOOD** **Refer to the graphic.**

**a. Find the measurement of the central angle for each category.**

The sum of the percents is 100% and represents the whole. Use the percents to determine what part of the whole circle (360°) each central angle contains.

$2\%(360°) = 7.2°$
$6\%(360°) = 21.6°$
$28\%(360°) = 100.8°$
$43\%(360°) = 154.8°$
$15\%(360°) = 54°$
$4\%(360°) = 14.4°$




**b. Use the categories to identify any arcs that are congruent.**

The arcs for the wedges named *Once a month* and *Less than once a month* are congruent because they both represent 2% or 7.2° of the circle.

**Log on for:**
- Updated data
- More activities on circle graphs

www.geometryonline.com/usa_today

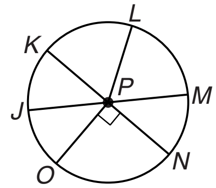
www.geometryonline.com/extra_examples



## In-Class Examples

Power Point®

**2** In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ bisects $\angle KPM$, and $\overline{OP} \perp \overline{KN}$. Find each measure.



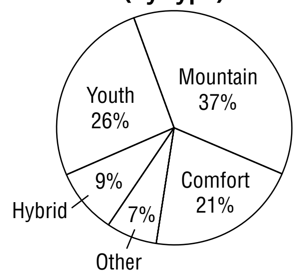

**a.** $m\widehat{OK}$ **90**

**b.** $m\widehat{LM}$ **67**

**c.** $m\widehat{JKO}$ **316**

**3** **BICYCLES** This graph shows the percent of each type of bicycle sold in the United States in 2001.

**Bicycles Bought Last Year (by type)**




**a.** Find the measurement of the central angle representing each category. List them from least to greatest. **25.2°, 32.4°, 75.6°, 93.6°, 133.2°**

**b.** Is the arc for the wedge named *Youth* congruent to the arc for the combined wedges named *Other* and *Comfort*? **no**

DAILY INTERVENTION

## Differentiated Instruction

**Interpersonal** Draw a circle segmented with different sizes of central angles. Shade each portion of the circle with a different color. Repeat for two other circles the same size, but with different central angles. Laminate the paper, cut out the circles, and separate each portion. Provide the cutouts to groups of students who can fit the pieces together to form the three circles, find the central angle measures, arc measures, circumferences and arc lengths. Groups can compare to check results and/or determine which group is the most efficient at finding all the correct information.

## ARC LENGTH

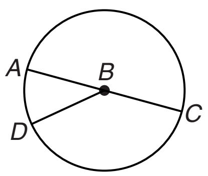
*Teaching Tip* Tell students that they can set up a proportion to find an arc length because they are finding a *portion* of the circumference. Explain that this process is very similar to finding a percent of a whole.

### In-Class Example

Power Point®

*Teaching Tip* If students want to see this problem another way, explain that 120 is $\frac{1}{3}$ of 360 as each arc length would be equal to $\frac{1}{3}$ of the total circumference. So, students can divide $30\pi$ by 3 and get the same answer.

**4** In $\odot B$, $AC = 9$ and $m\angle ABD = 40$. Find the length of $\widehat{AD}$.

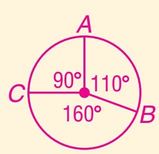



**$\pi$ units or about 3.14 units**

*Teaching Tip* Have students construct a circle like the one in Example 4 and measure its radius. Have them use string to trace the circumference of the circle. Mark on the string the points that are the endpoints of the arc. After calculating the circumference, have them use a ruler to verify the arc length.

### Answers

1. Sample answer: $\widehat{AB}$, $\widehat{BC}$, $\widehat{AC}$, $\widehat{ABC}$, $\widehat{BCA}$, $\widehat{CAB}$; $m\widehat{AB} = 110$, $m\widehat{BC} = 160$, $m\widehat{AC} = 90$, $m\widehat{ABC} = 270$, $m\widehat{BCA} = 250$, $m\widehat{CAB} = 200$




2. A diameter divides the circle into two congruent arcs. Without the third letter, it is impossible to know which semicircle is being referenced.
3. Sample answer: Concentric circles have the same center, but different radius measures; congruent circles usually have different centers but the same radius measure.

## ARC LENGTH

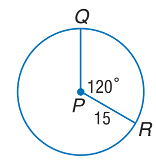
**ARC LENGTH** Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

**Study Tip**

*Look Back*
To review proportions, see Lesson 6-1.

### Example 4 Arc Length

**In $\odot P$, $PR = 15$ and $m\angle QPR = 120$. Find the length of $\widehat{QR}$.**




In $\odot P$, $r = 15$, so $C = 2\pi(15)$ or $30\pi$ and $m\widehat{QR} = m\angle QPR$ or 120. Write a proportion to compare each part to its whole.

$$\frac{\text{degree measure of arc} \rightarrow}{\text{degree measure of whole circle} \rightarrow}\ \frac{120}{360} = \frac{\ell}{30\pi}\ \frac{\leftarrow \text{arc length}}{\leftarrow \text{circumference}}$$

Now solve the proportion for $\ell$.

$$\frac{120}{360} = \frac{\ell}{30\pi}$$

$$\frac{120}{360}(30\pi) = \ell \qquad \text{Multiply each side by } 30\pi.$$

$$10\pi = \ell \qquad \text{Simplify.}$$

The length of $\widehat{QR}$ is $10\pi$ units or about 31.42 units.

The proportion used to find the arc length in Example 4 can be adapted to find the arc length in any circle.

### Key Concept — Arc Length

$$\frac{\text{degree measure of arc} \rightarrow}{\text{degree measure of whole circle} \rightarrow}\ \frac{A}{360} = \frac{\ell}{2\pi r}\ \frac{\leftarrow \text{arc length}}{\leftarrow \text{circumference}}$$

This can also be expressed as $\frac{A}{360} \cdot C = \ell$.

## Check for Understanding

*Concept Check*

1. **OPEN-ENDED** Draw a circle and locate three points on the circle. Name all of the arcs determined by the three points and use a protractor to find the measure of each arc. **1–3. See margin.**
2. **Explain** why it is necessary to use three letters to name a semicircle.
3. **Describe** the difference between *concentric* circles and *congruent* circles.

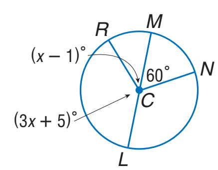
*Guided Practice*

GUIDED PRACTICE KEY

| Exercises | Examples |
|---|---|
| 4–7 | 1 |
| 8–11 | 2 |
| 12 | 3 |
| 13 | 4 |

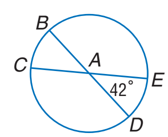
**ALGEBRA** **Find each measure.**

4. $m\angle NCL$ **120**
5. $m\angle RCL$ **137**
6. $m\angle RCM$ **43**
7. $m\angle RCN$ **103**




**In $\odot A$, $m\angle EAD = 42$. Find each measure.**

8. $m\widehat{BC}$ **42**
9. $m\widehat{CBE}$ **180**
10. $m\widehat{EDB}$ **222**
11. $m\widehat{CD}$ **138**




12. Points $T$ and $R$ lie on $\odot W$ so that $WR = 12$ and $m\angle TWR = 60$. Find the length of $\widehat{TR}$. **$4\pi \approx 12.57$ units**



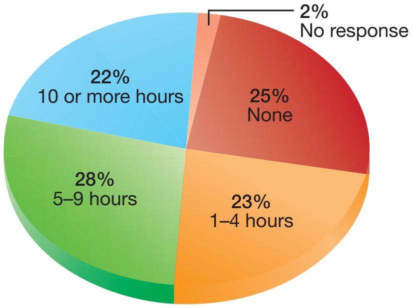
### DAILY INTERVENTION — Unlocking Misconceptions

**Arcs** Students may sometimes confuse the terms *arc measure* and *arc length*. Explain that they can remember that angles have degree measure, denoted $m\angle ABC$; similarly, arcs have degree measure, denoted $m\widehat{AC}$. Just as segment length is a distance along a line, arc length is a distance along a curve that you can actually follow or draw with a pencil. Point out that students should be careful to determine whether they need to find the *measure* or *length* of an arc.

*Application*

**13. SURVEYS** The graph shows the results of a survey of 1400 chief financial officers who were asked how many hours they spend working on the weekend. Determine the measurement of each angle of the graph. Round to the nearest degree. **Sample answer: 25% = 90°, 23% = 83°, 28% = 101°, 22% = 79°, 2% = 7°**

**Executives Working on the Weekend**




**Source:** Accountemps

★ indicates increased difficulty

## Practice and Apply

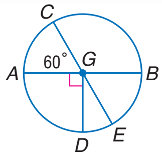
**Homework Help**

| For Exercises | See Examples |
|---|---|
| 14–23 | 1 |
| 24–39 | 2 |
| 40–43 | 3 |
| 44–45 | 4 |

**Extra Practice**
See page 774.

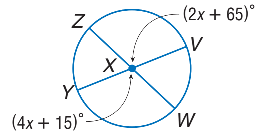
Find each measure.

14. $m\angle CGB$ **120**
15. $m\angle BGE$ **60**
16. $m\angle AGD$ **90**
17. $m\angle DGE$ **30**
18. $m\angle CGD$ **150**
19. $m\angle AGE$ **120**



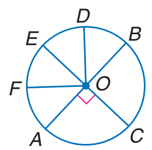

**ALGEBRA** Find each measure.

20. $m\angle ZXV$ **115**
21. $m\angle YXW$ **115**
22. $m\angle ZXY$ **65**
23. $m\angle VXW$ **65**



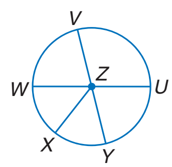

In $\odot O$, $\overline{EC}$ and $\overline{AB}$ are diameters, and $\angle BOD \cong \angle DOE \cong \angle EOF \cong \angle FOA$. Find each measure.

24. $m\overparen{BC}$ **90**
25. $m\overparen{AC}$ **90**
26. $m\overparen{AE}$ **90**
27. $m\overparen{EB}$ **90**
28. $m\overparen{ACB}$ **180**
29. $m\overparen{AD}$ **135**
30. $m\overparen{CBF}$ **225**
31. $m\overparen{ADC}$ **270**



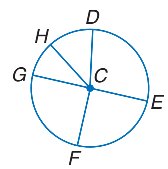

**ALGEBRA** In $\odot Z$, $\angle WZX \cong \angle XZY$, $m\angle VZU = 4x$, $m\angle UZY = 2x + 24$, and $\overline{VY}$ and $\overline{WU}$ are diameters. Find each measure.

32. $m\overparen{UY}$ **76**
33. $m\overparen{WV}$ **76**
34. $m\overparen{WX}$ **52**
35. $m\overparen{XY}$ **52**
36. $m\overparen{WUY}$ **256**
37. $m\overparen{YVW}$ **256**
★38. $m\overparen{XVY}$ **308**
★39. $m\overparen{WUX}$ **308**




The diameter of $\odot C$ is 32 units long. Find the length of each arc for the given angle measure.

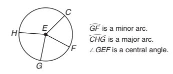
40. $\overparen{DE}$ if $m\angle DCE = 100$ **40.** $\frac{188\pi}{9} \approx 27.93$ **units**
41. $\overparen{DHE}$ if $m\angle DCE = 90$ **41.** $24\pi \approx 75.40$ **units**
42. $\overparen{HDF}$ if $m\angle HCF = 125$ $\frac{188\pi}{9} \approx 65.62$ **units**
43. $\overparen{HD}$ if $m\angle DCH = 45$ $4\pi \approx 12.57$ **units**






## Study Guide and Intervention, p. 547 (shown) and p. 548

**Angles and Arcs** A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.

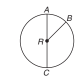



Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360. $m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$
- The measure of a minor arc equals the measure of its central angle. $m\overparen{CF} = m\angle CEF$
- The measure of a major arc is 360 minus the measure of the minor arc. $m\overparen{CGF} = 360 - m\overparen{CF}$
- Two arcs are congruent if and only if their corresponding central angles are congruent. $\overparen{CF} \cong \overparen{FG}$ if and only if $\angle CEF \cong \angle FEG$.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. **(Arc Addition Postulate)** $m\overparen{CF} + m\overparen{FG} = m\overparen{CG}$

**Example** **In $\odot R$, $m\angle ARB = 42$ and $\overline{AC}$ is a diameter. Find $m\overparen{AB}$ and $m\overparen{ACB}$.**

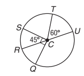
$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\overparen{AB} = 42$.
Thus $m\overparen{ACB} = 360 - 42$ or 318.




**Exercises**

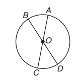
**Find each measure.**

1. $m\angle SCT$ **75**
2. $m\angle SCU$ **135**
3. $m\angle SCQ$ **90**
4. $m\angle QCT$ **165**




**If $m\angle BOA = 44$, find each measure.**

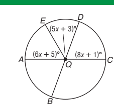
5. $m\overparen{BA}$ **44**
6. $m\overparen{BC}$ **136**
7. $m\overparen{CD}$ **44**
8. $m\overparen{ACB}$ **316**
9. $m\overparen{BCD}$ **180**
10. $m\overparen{AD}$ **136**




## Skills Practice, p. 549 and Practice, p. 550 (shown)

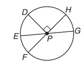
**ALGEBRA** In $\odot Q$, $\overline{AC}$ and $\overline{BD}$ are diameters. Find each measure.

1. $m\angle AQE$ **59**
2. $m\angle DQE$ **48**
3. $m\angle CQD$ **73**
4. $m\angle BQC$ **107**
5. $m\angle CQE$ **121**
6. $m\angle AQD$ **107**



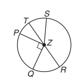

**In $\odot P$, $m\angle GPH = 38$. Find each measure.**

7. $m\overparen{EF}$ **38**
8. $m\overparen{DE}$ **52**
9. $m\overparen{FG}$ **142**
10. $m\overparen{DHG}$ **128**
11. $m\overparen{DFG}$ **232**
12. $m\overparen{DGE}$ **308**




**The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.**

13. $\overparen{QPT}$ if $m\angle QZT = 120$ **$9\pi \approx 28.27$ units**
14. $\overparen{QR}$ if $m\angle QZR = 60$ **$4.5\pi \approx 14.14$ units**
15. $\overparen{PQR}$ if $m\angle PZR = 150$ **$11.25\pi \approx 35.34$ units**
16. $\overparen{QPS}$ if $m\angle QZS = 160$ **$12\pi \approx 37.70$ units**




**HOMEWORK** For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

| Homework | |
|---|---|
| Less than 1 hour | 8% |
| 1–2 hours | 29% |
| 2–3 hours | 58% |
| 3–4 hours | 3% |
| Over 4 hours | 2% |

17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category? **28.8°, 104.4°, 208.8°, 10.8°, 7.2°**

18. Describe the arcs associated with each category. **The arc associated with 2–3 hours is a major arc; minor arcs are associated with the remaining categories.**

## Reading to Learn Mathematics, p. 551 ELL

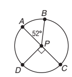
**Pre-Activity** **What kinds of angles do the hands on a clock form?**

Read the introduction to Lesson 10-2 at the top of page 529 in your textbook.

- What is the measure of the angle formed by the hour hand and the minute hand of the clock at 5:00? **150**
- What is the measure of the angle formed by the hour hand and the minute hand at 10:30? (Hint: How has each hand moved since 10:00?) **135**

### Reading the Lesson

1. Refer to $\odot P$. Indicate whether each statement is *true* or *false*.
   a. $\overparen{DAB}$ is a major arc. **false**
   b. $\overparen{ADC}$ is a semicircle. **true**
   c. $\overparen{AD} \cong \overparen{CD}$ **true**
   d. $\overparen{DA}$ and $\overparen{AB}$ are adjacent arcs. **true**
   e. $\angle BPC$ is an acute central angle. **false**
   f. $\angle DPA$ and $\angle BPA$ are supplementary central angles. **false**




2. Refer to the figure in Exercise 1. Give each of the following arc measures.
   a. $m\overparen{AB}$ **52**
   b. $m\overparen{CD}$ **90**
   c. $m\overparen{BC}$ **128**
   d. $m\overparen{ADC}$ **180**
   e. $m\overparen{DAB}$ **142**
   f. $m\overparen{DCB}$ **218**
   g. $m\overparen{DAC}$ **270**
   h. $m\overparen{BDA}$ **308**

3. Underline the correct word or number to form a true statement.
   a. The arc measure of a semicircle is (90/180/360).
   b. Arcs of a circle that have exactly one point in common are (congruent/opposite/adjacent) arcs.
   c. The measure of a major arc is greater than (0/90/180) and less than (90/180/360).
   d. Suppose a set of central angles of a circle have interiors that do not overlap. If the angles and their interiors contain all points of the circle, then the sum of the measures of the central angles is (90/270/360).
   e. The measure of an arc formed by two adjacent arcs is the (sum/difference/product) of the measures of the two arcs.
   f. The measure of a minor arc is greater than (0/90/180) and less than (90/180/360).

### Helping You Remember

4. A good way to remember something is to explain it to someone else. Suppose your classmate Luis does not like to work with proportions. What is a way that he can find the length of a minor arc of a circle without solving a proportion? **Sample answer: Divide the measure of the central angle of the arc by 360 to form a fraction. Multiply this fraction by the circumference of the circle to find the length of the arc.**

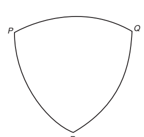
## Enrichment, p. 552

***Curves of Constant Width***

A circle is called a curve of constant width because no matter how you turn it, the greatest distance across it is always the same. However, the circle is not the only figure with this property.

The figure at the right is called a Reuleaux triangle.

1. Use a metric ruler to find the distance from $P$ to any point on the opposite side. **4.6 cm**
2. Find the distance from $Q$ to the opposite side. **4.6 cm**
3. What is the distance from $R$ to the opposite side? **4.6 cm**




The Reuleaux triangle is made of three arcs. In the example shown, $\overparen{PQ}$ has center $R$, $\overparen{QR}$ has center $P$, and $\overparen{PR}$ has center $Q$.

4. Trace the Reuleaux triangle above on a piece of paper and cut it out. Make a square with sides the length you found in Exercise 1. Show that you can turn the triangle inside the

## 3 Practice/Apply

### Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

### About the Exercises…

**Organization by Objective**

- **Angles and Arcs:** 14–43
- **Arc Length:** 44–45

**Odd/Even Assignments**

Exercises 14–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

***Alert!*** Exercise 46 requires a compass.

### Assignment Guide

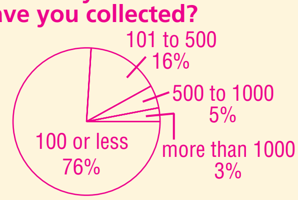
**Basic:** 15–37 odd, 41–55 odd, 57–76

**Average:** 15–55 odd, 57–76

**Advanced:** 14–50 even, 51, 52, 54, 55–70 (optional: 71–76)

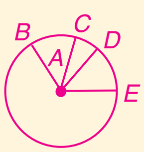
### Answers

46. How many free files have you collected?




54. Given: $\angle BAC \cong \angle DAE$

Prove: $\overset{\frown}{BC} \cong \overset{\frown}{DE}$

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**ONLINE MUSIC** **For Exercises 44–46, refer to the table and use the following information.**
A recent survey asked online users how many legally free music files they have collected. The results are shown in the table.

| Free Music Downloads | |
|---|---|
| How many free music files have you collected? | |
| 100 files or less | 76% |
| 101 to 500 files | 16% |
| 501 to 1000 files | 5% |
| More than 1000 files | 3% |

**Source:** QuickTake.com

44. If you were to construct a circle graph of this information, how many degrees would be needed for each category? **44. Sample answer: 76% = 273°, 16% = 58°, 5% = 18°, 3% = 11°**

45. Describe the kind of arc associated with each category. **45. The first category is a major arc, and the other three categories are minor arcs.**

★46. Construct a circle graph for these data. **See margin.**

**Determine whether each statement is *sometimes*, *always*, or *never* true.**

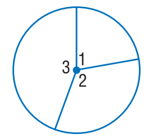
47. The measure of a major arc is greater than 180. **always**

48. The central angle of a minor arc is an acute angle. **sometimes**

49. The sum of the measures of the central angles of a circle depends on the measure of the radius. **never**

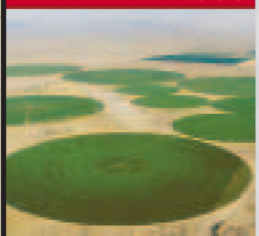
50. The semicircles of two congruent circles are congruent. **always**

51. **CRITICAL THINKING** Central angles 1, 2, and 3 have measures in the ratio 2 : 3 : 4. Find the measure of each angle. **$m\angle 1 = 80$, $m\angle 2 = 120$, $m\angle 3 = 160$**




52. **CLOCKS** The hands of a clock form the same angle at various times of the day. For example, the angle formed at 2:00 is congruent to the angle formed at 10:00. If a clock has a diameter of 1 foot, what is the distance along the edge of the clock from the minute hand to the hour hand at 2:00? **$2\pi$ in. ≈ 6.3 in.**

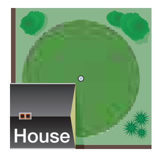
### More About. . .



***Irrigation*** In the Great Plains of the United States, farmers use center-pivot irrigation systems to water crops. New low-energy spray systems water circles of land that are thousands of feet in diameter with minimal water loss to evaporation from the spray.

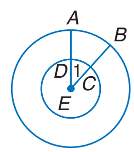
**Source:** U.S. Geological Survey

53. **IRRIGATION** Some irrigation systems spray water in a circular pattern. You can adjust the nozzle to spray in certain directions. The nozzle in the diagram is set so it does not spray on the house. If the spray has a radius of 12 feet, what is the approximate length of the arc that the spray creates? **56.5 ft**




54. **PROOF** Write a proof of Theorem 10.1. **See margin.**

55. **CRITICAL THINKING** The circles at the right are concentric circles that both have point $E$ as their center. If $m\angle 1 = 42$, determine whether $\overset{\frown}{AB} \cong \overset{\frown}{CD}$. Explain. **No; the radii are not equal, so the proportional part of the circumferences would not be the same. Thus, the arcs would not be congruent.**




56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**What kind of angles do the hands of a clock form?**

Include the following in your answer:

- the kind of angle formed by the hands of a clock, and
- several times of day when these angles are congruent.



Craig Aurness/CORBIS

---

**Proof:**

**Statements (Reasons)**

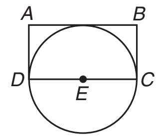
1. $\angle BAC \cong \angle DAE$ (Given)
2. $m\angle BAC = m\angle DAE$ (Def. of ≅ ∡)
3. $m\overset{\frown}{BC} = m\overset{\frown}{DE}$ (Def. of arc measure)
4. $\overset{\frown}{BC} \cong \overset{\frown}{DE}$ (Def. of ≅ arcs)

56. Sample answer: The hands of the clock form central angles. Answers should include the following.

- The hands form acute, right, and obtuse angles.
- Some times when the angles formed by the minute and hour hand are congruent are at 1:00 and 11:00, 2:00 and 10:00, 3:00 and 9:00, 4:00 and 8:00, and 5:00 and 7:00. They also form congruent angles at many other times of the day, such as 3:05 and 8:55.

**Standardized Test Practice**

57. Compare the circumference of circle $E$ with the perimeter of rectangle $ABCD$. Which statement is true? **B**




Ⓐ The perimeter of $ABCD$ is greater than the circumference of circle $E$.

Ⓑ The circumference of circle $E$ is greater than the perimeter of $ABCD$.

Ⓒ The perimeter of $ABCD$ equals the circumference of circle $E$.

Ⓓ There is not enough information to determine this comparison.

58. **SHORT RESPONSE** A circle is divided into three central angles that have measures in the ratio 3 : 5 : 10. Find the measure of each angle. **60, 100, 200**

## Maintain Your Skills

*Mixed Review*

**The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary.** *(Lesson 10-1)*

**59. 20; 62.83**
**60. 6.5; 40.84**

59. $r = 10$, $d =$ __?__, $C =$ __?__

60. $d = 13$, $r =$ __?__, $C =$ __?__

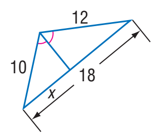
61. $C = 28\pi$, $d =$ __?__, $r =$ __?__ **28; 14**

62. $C = 75.4$, $d =$ __?__, $r =$ __?__ **24.00; 12.00**

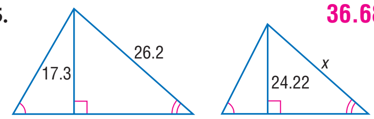
63. **SOCCER** Two soccer players kick the ball at the same time. One exerts a force of 72 newtons east. The other exerts a force of 45 newtons north. What are the magnitude to the nearest tenth and direction to the nearest degree of the resultant force on the soccer ball? *(Lesson 9-6)* **84.9 newtons, 32° north of due east**

**ALGEBRA** **Find $x$.** *(Lesson 6-5)*

64. **$8\frac{2}{11}$**




65. **36.68**




**Find the exact distance between each point and line or pair of lines.** *(Lesson 3-6)*

66. point $Q(6, -2)$ and the line with the equation $y - 7 = 0$ **9 units**

67. parallel lines with the equations $y = x + 3$ and $y = x - 4$ **$\sqrt{24.5}$**

68. Angle $A$ has a measure of 57.5. Find the measures of the complement and supplement of $\angle A$. *(Lesson 2-8)* **32.5, 122.5**

**Use the following statement for Exercises 69 and 70.**
*If ABC is a triangle, then ABC has three sides.* *(Lesson 2-3)*

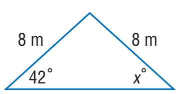
69. Write the converse of the statement. **If *ABC* has three sides, then *ABC* is a triangle.**

70. Determine the truth value of the statement and its converse. **Both are true.**

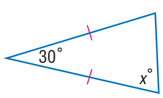
*Getting Ready for the Next Lesson*

**PREREQUISITE SKILL** **Find $x$.** *(To review **isosceles triangles**, see Lesson 4-6.)*

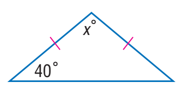
71. **42**



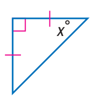

72. **75**



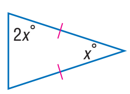

73. **100**



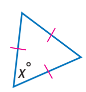

74. **45**




75. **36**




76. **60**




www.geometryonline.com/self_check_quiz



## 4 Assess

### Open-Ended Assessment

**Writing** Provide examples on the board of circles marked with central angles, and have students take turns coming to the board and writing angle measure(s), arc measure(s), and arc length(s) for each example.

### *Getting Ready for Lesson 10-3*

**Prerequisite Skill** Students will learn about arcs and chords in Lesson 10-3. They will use isosceles triangles to write proofs and to find arc measures and chord lengths. Use Exercises 71–76 to determine your students' familiarity with isosceles triangles.

### Assessment Options

**Quiz (Lessons 10-1 and 10-2)** is available on p. 603 of the *Chapter 10 Resource Masters*.

Lesson 10-2 Angles and Arcs 535

# 10-3 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-3** Use as a quiz or review of Lesson 10-2.

**Mathematical Background** notes are available for this lesson on p. 520C.

**How** do the grooves in a Belgian waffle iron model segments in a circle?

*Ask students:*

- Excluding the diameter, how many chords can you count in the lower semicircle of the top heated plate on the waffle iron? **4**
- If the radius of the top heated plate measures 11 cm, then what is the circumference of the plate? **$22\pi$ cm or about 69.12 cm**
- If one central angle of the waffle iron measures 90°, then what is the measure of its corresponding minor arc? **90**

# 10-3 Arcs and Chords

**What You'll Learn**

- Recognize and use relationships between arcs and chords.
- Recognize and use relationships between chords and diameters.

**How do the grooves in a Belgian waffle iron model segments in a circle?**

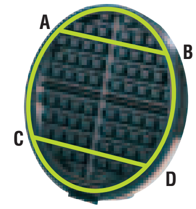
Waffle irons have grooves in each heated plate that result in the waffle pattern when the batter is cooked. One model of a Belgian waffle iron is round, and each groove is a chord of the circle.

**Vocabulary**

- inscribed
- circumscribed

## ARCS AND CHORDS

The endpoints of a chord are also endpoints of an arc. If you trace the waffle pattern on patty paper and fold along the diameter, $\overline{AB}$ and $\overline{CD}$ match exactly, as well as $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$. This suggests the following theorem.




**Study Tip**

*Reading Mathematics*
Remember that the phrase *if and only if* means that the conclusion and the hypothesis can be switched and the statement is still true.

### Theorem 10.2

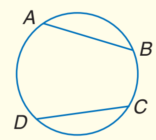
In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

**Abbreviations:**
*In ⊙, 2 minor arcs are ≅, corr. chords are ≅.*
*In ⊙, 2 chords are ≅, corr. minor arcs are ≅*

**Examples**

If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.



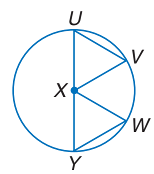

You will prove part 2 of Theorem 10.2 in Exercise 4.

### Example 1 Prove Theorems

**PROOF** Theorem 10.2 (part 1)

**Given:** $\odot X$, $\overset{\frown}{UV} \cong \overset{\frown}{YW}$

**Prove:** $\overline{UV} \cong \overline{YW}$




**Proof:**

| Statements | Reasons |
|---|---|
| 1. $\odot X$, $\overset{\frown}{UV} \cong \overset{\frown}{YW}$ | 1. Given |
| 2. $\angle UXV \cong \angle WXY$ | 2. If arcs are ≅, their corresponding central ∡ are ≅. |
| 3. $\overline{UX} \cong \overline{XV} \cong \overline{XW} \cong \overline{XY}$ | 3. All radii of a circle are congruent. |
| 4. $\triangle UXV \cong \triangle WXY$ | 4. SAS |
| 5. $\overline{UV} \cong \overline{YW}$ | 5. CPCTC |





## Resource Manager

**Workbook and Reproducible Masters**

***Chapter 10 Resource Masters***
- Study Guide and Intervention, pp. 553–554
- Skills Practice, p. 555
- Practice, p. 556
- Reading to Learn Mathematics, p. 557
- Enrichment, p. 558

***Teaching Geometry With Manipulatives Masters,*** pp. 16, 17, 162, 163, 164

**Transparencies**

5-Minute Check Transparency 10-3
Real-World Transparency 10
Answer Key Transparencies

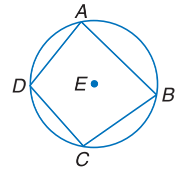
**Technology**

Interactive Chalkboard

**Study Tip**

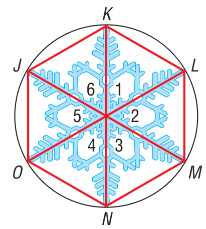
**Inscribed and Circumscribed**
A circle can also be inscribed in a polygon, so that the polygon is circumscribed about the circle. You will learn about this in Lesson 10-5.

The chords of adjacent arcs can form a polygon. Quadrilateral *ABCD* is an **inscribed** polygon because all of its vertices lie on the circle. Circle E is **circumscribed** about the polygon because it contains all the vertices of the polygon.




## Example 2 Inscribed Polygons

**SNOWFLAKES** **The main veins of a snowflake create six congruent central angles. Determine whether the hexagon containing the flake is regular.**




$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4 \cong \angle 5 \cong \angle 6$ — Given

$\widehat{KL} \cong \widehat{LM} \cong \widehat{MN} \cong \widehat{NO} \cong \widehat{OJ} \cong \widehat{JK}$ — If central $\angle$s are $\cong$, corresponding arcs are $\cong$.

$\overline{KL} \cong \overline{LM} \cong \overline{MN} \cong \overline{NO} \cong \overline{OJ} \cong \overline{JK}$ — In $\odot$, 2 minor arcs $\cong$, corr. chords are $\cong$.

Because all the central angles are congruent, the measure of each angle is 360 ÷ 6 or 60.

Let $x$ be the measure of each base angle in the triangle containing $\overline{KL}$.

$m\angle 1 + x + x = 180$ — Angle Sum Theorem

$60 + 2x = 180$ — Substitution

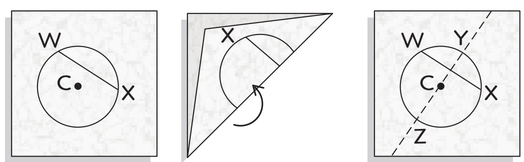
$2x = 120$ — Subtract 60 from each side.

$x = 60$ — Divide each side by 2.

This applies to each triangle in the figure, so each angle of the hexagon is 2(60) or 120. Thus the hexagon has all sides congruent and all vertex angles congruent.

## DIAMETERS AND CHORDS

Diameters that are perpendicular to chords create special segment and arc relationships. Suppose you draw circle *C* and one of its chords $\overline{WX}$ on a piece of patty paper and fold the paper to construct the perpendicular bisector. You will find that the bisector also cuts $\widehat{WX}$ in half and passes through the center of the circle, making it contain a diameter.

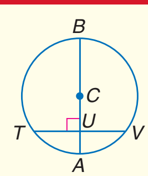



This is formally stated in the next theorem.

### Theorem 10.3

In a circle, if a diameter (or radius) is perpendicular to a chord, then it bisects the chord and its arc.

**Example:** If $\overline{BA} \perp \overline{TV}$, then $\overline{UT} \cong \overline{UV}$ and $\widehat{AT} \cong \widehat{AV}$.




You will prove Theorem 10.3 in Exercise 36.

www.geometryonline.com/extra_examples



# 2 Teach

## ARCS AND CHORDS

### In-Class Examples

Power Point®

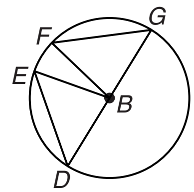
**1** **PROOF** Write a proof.

**Given:** $\overline{DE} \cong \overline{FG}$

$m\angle EBF = 24$

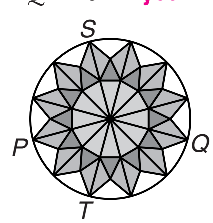
$\widehat{DFG}$ is a semicircle.

**Prove:** $m\angle FBG = 78$




1. $\overline{DE} \cong \overline{FG}$; $m\angle EBF = 24$; $\widehat{DFG}$ is a semicircle. (Given)
2. $m\widehat{DFG} = 180$ (Def. of semicircle)
3. $\widehat{DE} \cong \widehat{FG}$ (In $\odot$, 2 chords are $\cong$, corr. minor arcs are $\cong$.)
4. $m\widehat{DE} = m\widehat{FG}$ (Def. of $\cong$ arcs)
5. $m\widehat{EF} = 24$ (Def. of arc measure)
6. $m\widehat{ED} + m\widehat{EF} + m\widehat{FG} = m\widehat{DFG}$ (Arc Addition Post.)
7. $m\widehat{FG} + 24 + m\widehat{FG} = 180$ (Substitution)
8. $2(m\widehat{FG}) = 156$ (Subtr. Prop. and simplify)
9. $m\widehat{FG} = 78$ (Div. Prop.)
10. $m\widehat{FG} = m\angle FBG$ (Def. of arc measure)
11. $m\angle FBG = 78$ (Substitution)

**2** **TESSELLATIONS** The rotations of a tessellation can create twelve congruent central angles. Determine whether $\widehat{PQ} \cong \widehat{ST}$. **yes**




**DAILY INTERVENTION**

## Differentiated Instruction

**Auditory/Musical** Summarize the three major concepts of this lesson aloud for auditory students. Explain that they now know that congruent chords share endpoints with congruent arcs and vice versa; plus, the chords are only congruent if and only if they are equidistant from the center of the circle. They have also learned that a diameter perpendicularly bisects any chord of a circle.

## DIAMETERS AND CHORDS

### Building on Prior Knowledge

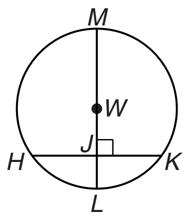
Students learned about the Pythagorean Theorem in Chapter 1. They determined how to prove triangle congruence in Chapter 4, and they learned about segment bisectors in Chapter 5. Students will apply all of these concepts in this lesson as they prove triangle congruence and find segment lengths in circles.

### In-Class Example

Power Point®

*Teaching Tip* Remind students that they can add any known information to a figure to help them solve problems, as radius $\overline{OC}$ has been added to the figure in the example. Tell students to remember that angles, segment lengths, arcs, radii, and diameters all exist even if they are not drawn. Students need to be careful to follow geometric conditions/definitions when they add anything to a figure.

**3** Circle $W$ has a radius of 10 centimeters. Radius $\overline{WL}$ is perpendicular to chord $\overline{HK}$, which is 16 centimeters long.

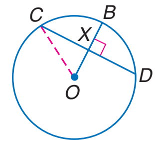



**a.** If $m\overset{\frown}{HL} \approx 53$, find $m\overset{\frown}{MK}$. **≈ 127**

**b.** Find $JL$. **4**

### Example 3 Radius Perpendicular to a Chord

**Circle $O$ has a radius of 13 inches. Radius $\overline{OB}$ is perpendicular to chord $\overline{CD}$, which is 24 inches long.**




**a. If $m\overset{\frown}{CD} = 134$, find $m\overset{\frown}{CB}$.**

$\overline{OB}$ bisects $\overset{\frown}{CD}$, so $m\overset{\frown}{CB} = \frac{1}{2}m\overset{\frown}{CD}$.

| | |
|---|---|
| $m\overset{\frown}{CB} = \frac{1}{2}m\overset{\frown}{CD}$ | Definition of arc bisector |
| $m\overset{\frown}{CB} = \frac{1}{2}(134)$ or 67 | $m\overset{\frown}{CD} = 134$ |

**b. Find $OX$.**

Draw radius $\overline{OC}$. $\triangle CXO$ is a right triangle.

| | |
|---|---|
| $CO = 13$ | $r = 13$ |
| $\overline{OB}$ bisects $\overline{CD}$. | A radius perpendicular to a chord bisects it. |
| $CX = \frac{1}{2}(CD)$ | Definition of segment bisector |
| $= \frac{1}{2}(24)$ or 12 | $CD = 24$ |

Use the Pythagorean Theorem to find $XO$.

| | |
|---|---|
| $(CX)^2 + (OX)^2 = (CO)^2$ | Pythagorean Theorem |
| $12^2 + (OX)^2 = 13^2$ | $CX = 12$, $CO = 13$ |
| $144 + (OX)^2 = 169$ | Simplify. |
| $(OX)^2 = 25$ | Subtract 144 from each side. |
| $OX = 5$ | Take the square root of each side. |

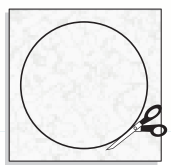
In the next activity, you will discover another property of congruent chords.

### Geometry Activity

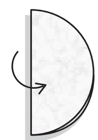
**Congruent Chords and Distance**

**Model**

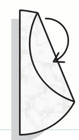
**Step 1** Use a compass to draw a large circle on patty paper. Cut out the circle.



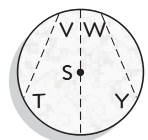
**Step 2** Fold the circle in half.



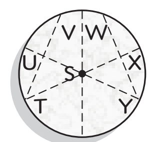
**Step 3** Without opening the circle, fold the edge of the circle so it does not intersect the first fold.



**Step 4** Unfold the circle and label as shown.




**Step 5** Fold the circle, laying point $V$ onto $T$ to bisect the chord. Open the circle and fold again to bisect $\overline{WY}$. Label as shown.




**Analyze** **1. $\overline{SU}$ and $\overline{SX}$ are perpendicular bisectors of $\overline{VT}$ and $\overline{WY}$, respectively.**

1. What is the relationship between $\overline{SU}$ and $\overline{VT}$? $\overline{SX}$ and $\overline{WY}$?
2. Use a centimeter ruler to measure $\overline{VT}$, $\overline{WY}$, $\overline{SU}$, and $\overline{SX}$. What do you find? **$VT = WY$, $SU = SX$**
3. **Make a conjecture** about the distance that two chords are from the center when they are congruent.

**Sample answer: When the chords are congruent, they are equidistant from the center of the circle.**



### Geometry Activity

**Materials:** compass, patty paper, centimeter ruler

- Students can use a ruler to draw $\overline{VT}$, $\overline{WY}$, $\overline{ST}$, and $\overline{SY}$.
- To reinforce concepts, have students measure the central angles and determine if $\overset{\frown}{TV} \cong \overset{\frown}{WY}$.

The Geometry Activity suggests the following theorem.

**Theorem 10.4**

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

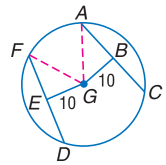
You will prove Theorem 10.4 in Exercises 37 and 38.

## Example 4 Chords Equidistant from Center

**Chords $\overline{AC}$ and $\overline{DF}$ are equidistant from the center. If the radius of $\odot G$ is 26, find $AC$ and $DE$.**

$\overline{AC}$ and $\overline{DF}$ are equidistant from $G$, so $\overline{AC} \cong \overline{DF}$.

Draw $\overline{AG}$ and $\overline{GF}$ to form two right triangles. Use the Pythagorean Theorem.




| | |
|---|---|
| $(AB)^2 + (BG)^2 = (AG)^2$ | Pythagorean Theorem |
| $(AB)^2 + 10^2 = 26^2$ | $BG = 10$, $AG = 26$ |
| $(AB)^2 + 100 = 676$ | Simplify. |
| $(AB)^2 = 576$ | Subtract 100 from each side. |
| $AB = 24$ | Take the square root of each side. |

$AB = \frac{1}{2}(AC)$, so $AC = 2(24)$ or 48.

$\overline{AC} \cong \overline{DF}$, so $DF$ also equals 48. $DE = \frac{1}{2}DF$, so $DE = \frac{1}{2}(48)$ or 24.

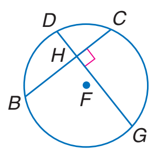
## Check for Understanding

### Concept Check

1–2. See margin.

3. Tokei; to bisect the chord, it must be a diameter and be perpendicular.

1. Explain the difference between an inscribed polygon and a circumscribed circle.
2. **OPEN ENDED** Construct a circle and inscribe any polygon. Draw the radii to the vertices of the polygon and use a protractor to determine whether any sides of the polygon are congruent.
3. **FIND THE ERROR** Lucinda and Tokei are writing conclusions about the chords in $\odot F$. Who is correct? Explain your reasoning.




| Lucinda | Tokei |
|---|---|
| Because $\overline{DG} \perp \overline{BC}$, $\angle DHB \cong \angle DHC \cong \angle CHG \cong \angle BHG$, and $\overline{DG}$ bisects $\overline{BC}$. | $\overline{DG} \perp \overline{BC}$, but $\overline{DG}$ does not bisect $\overline{BC}$ because it is not a diameter. |

### Guided Practice

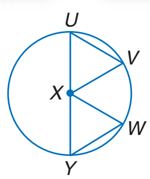
**GUIDED PRACTICE KEY**

| Exercises | Examples |
|---|---|
| 4 | 1 |
| 5–7 | 3 |
| 8–9 | 4 |
| 10 | 2 |

4. **PROOF** Prove part 2 of Theorem 10.2.

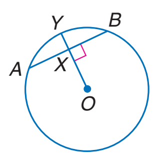
Given: $\odot X$, $\overline{UV} \cong \overline{WY}$

Prove: $\overparen{UV} \cong \overparen{WY}$ **See margin.**




**Circle $O$ has a radius of 10, $AB = 10$, and $m\overparen{AB} = 60$. Find each measure.**

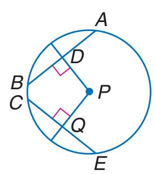
5. $m\overparen{AY}$ **30**
6. $AX$ **5**
7. $OX$ **$5\sqrt{3}$**




Exercises 5–7

**In $\odot P$, $PD = 10$, $PQ = 10$, and $QE = 20$. Find each measure.**

8. $AB$ **40**
9. $PE$ **$10\sqrt{5} \approx 22.36$**



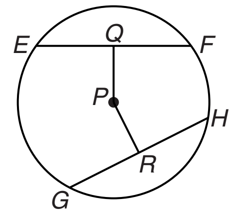

Exercises 8–9



## In-Class Example

Power Point®

4 Chords $\overline{EF}$ and $\overline{GH}$ are equidistant from the center. If the radius of $\odot P$ is 15 and $EF = 24$, find $PR$ and $RH$. **9; 12**




## 3 Practice/Apply

### Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

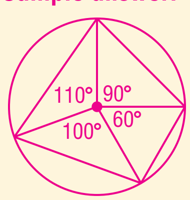
**DAILY INTERVENTION**

**FIND THE ERROR** Stress the importance of carefully analyzing figures to determine relationships. Point out that since a diameter or radius that is perpendicular to a chord does bisect the chord, no other segment perpendicular to the chord could bisect it.

## Answers

1. Sample answer: An inscribed polygon has all vertices on the circle. A circumscribed circle means the circle is drawn around so that the polygon lies in its interior and all vertices lie on the circle.

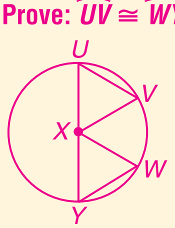
2. Sample answer:




None of the sides are congruent.

4. Given: $\odot X$, $\overline{UV} \cong \overline{WY}$

Prove: $\overparen{UV} \cong \overparen{WY}$




Proof: Because all radii are congruent, $\overline{XU} \cong \overline{XV} \cong \overline{XW} \cong \overline{XY}$. You are given that $\overline{UV} \cong \overline{WY}$, so $\triangle UVX \cong \triangle WYX$, by SSS. Thus, $\angle UXV \cong \angle WXY$ by CPCTC. Since the central angles have the same measure, their intercepted arcs have the same measure and are therefore, congruent. Thus, $\overparen{UV} \cong \overparen{WY}$.

## About the Exercises...

### Organization by Objective

- **Arcs and Chords:** 23–25, 36–38
- **Diameters and Chords:** 11–22, 26–33

### Odd/Even Assignments

Exercises 11–33 are structured so that students practice the same concepts whether they are assigned odd or even problems.

***Alert!*** Exercises 46–47 require a compass.

## Assignment Guide

**Basic:** 11–19 odd, 23–35 odd, 39, 45–49 odd, 50–65

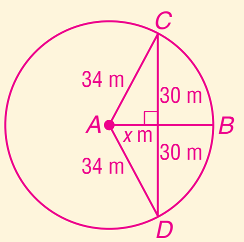
**Average:** 11–49 odd, 50–65

**Advanced:** 12–44 even, 45, 46, 48, 50, 52–59 (optional: 60–65)

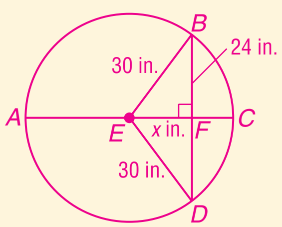
**All:** Quiz 1 (1–10)

### Answers (page 541)

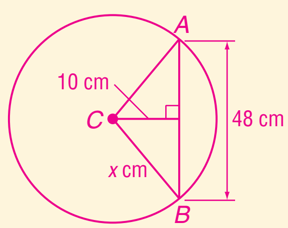
**40.**



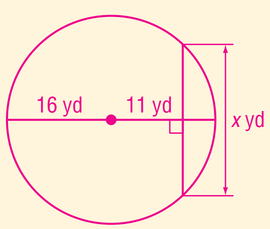

**41.**




**42.**



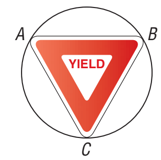

**43.**






*Application*

**10. TRAFFIC SIGNS** A yield sign is an equilateral triangle. Find the measure of each arc of the circle circumscribed about the yield sign. **Each arc measures 120°.**




★ indicates increased difficulty

## Practice and Apply

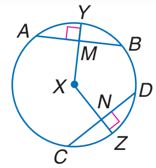
| For Exercises | See Examples |
|---|---|
| 11–22 | 3 |
| 23–25 | 2 |
| 26–33 | 4 |
| 36–38 | 1 |

**Homework Help**

**Extra Practice**
See page 774.

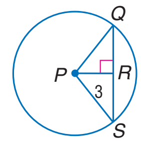
**In ⊙X, AB = 30, CD = 30, and $m\widehat{CZ} = 40$. Find each measure.**

**11.** $AM$ **15** **12.** $MB$ **15**
**13.** $CN$ **15** **14.** $ND$ **15**
**15.** $m\widehat{DZ}$ **40** **16.** $m\widehat{CD}$ **80**
**17.** $m\widehat{AB}$ **80** **18.** $m\widehat{YB}$ **40**




**The radius of ⊙P is 5 and PR = 3. Find each measure.**

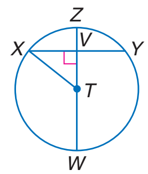
**19.** $QR$ **4** **20.** $QS$ **8**



Exercises 19–20

**In ⊙T, ZV = 1, and TW = 13. Find each measure.**

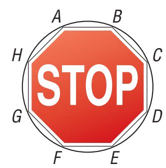
★ **21.** $XV$ **5** **22.** $XY$ **10**

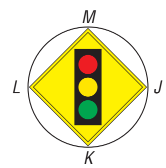


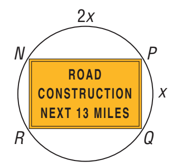
Exercises 21–22

**TRAFFIC SIGNS** **Determine the measure of each arc of the circle circumscribed about the traffic sign.**

**23.** regular octagon **24.** square **25.** rectangle



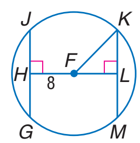

**23.** $m\widehat{AB} = m\widehat{BC} = m\widehat{CD} = m\widehat{DE} = m\widehat{EF} = m\widehat{FG} = m\widehat{GH} = m\widehat{HA} = 45$

**24.** $m\widehat{LM} = m\widehat{MJ} = m\widehat{JK} = m\widehat{KL} = 90$

**25.** $m\widehat{NP} = m\widehat{RQ} = 120$; $m\widehat{NR} = m\widehat{PQ} = 60$

**In ⊙F, $\overline{FH} \cong \overline{FL}$ and FK = 17. Find each measure.**

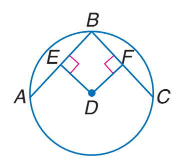
**26.** $LK$ **15** **27.** $KM$ **30**
**28.** $JG$ **30** **29.** $JH$ **15**



Exercises 26–29

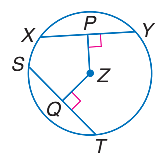
**In ⊙D, CF = 8, DE = FD, and DC = 10. Find each measure.**

**30.** $FB$ **8** **31.** $BC$ **16**
**32.** $AB$ **16** **33.** $ED$ **6**



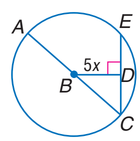
Exercises 30–33

**34. ALGEBRA** In ⊙Z, $PZ = ZQ$, $XY = 4a - 5$, and $ST = -5a + 13$. Find $SQ$. **1.5**



Exercise 34

**35. ALGEBRA** In ⊙B, the diameter is 20 units long, and $m\angle ACE = 45$. Find $x$. $\sqrt{2} \approx 1.41$ **units**



Exercise 35

## More About . . .

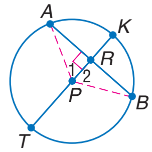
### Sayings

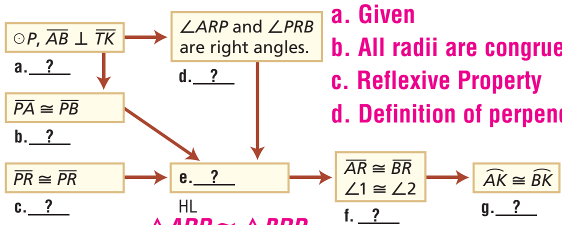
Many everyday sayings or expressions have a historical origin. The square peg comment is attributed to Sydney Smith (1771–1845), a witty British lecturer, who said "Trying to put those two together is like trying to put a square peg in a round hole."

36. **PROOF** Copy and complete the flow proof of Theorem 10.3.

**Given:** $\odot P$, $\overline{AB} \perp \overline{TK}$

**Prove:** $\widehat{AR} \cong \widehat{BR}$, $\widehat{AK} \cong \widehat{BK}$



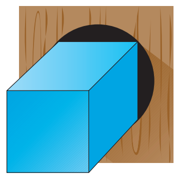

a. Given
b. All radii are congruent.
c. Reflexive Property
d. Definition of perpendicular lines
e. $\triangle ARP \cong \triangle BRP$
f. CPCTC
g. If central ∠s are ≅, intercepted arcs are ≅.

**PROOF** Write a proof for each part of Theorem 10.4. **37–38. See 589A.**

★ 37. In a circle, if two chords are equidistant from the center, then they are congruent.

★ 38. In a circle, if two chords are congruent, then they are equidistant from the center.

39. **SAYINGS** An old adage states that "You can't fit a square peg in a round hole." Actually, you can, it just won't fill the hole. If a hole is 4 inches in diameter, what is the approximate width of the largest square peg that fits in the round hole? **2.82 in.**



**40–43. See margin for sample figures.**

**For Exercises 40–43, draw and label a figure. Then solve.**

★ 40. The radius of a circle is 34 meters long, and a chord of the circle is 60 meters long. How far is the chord from the center of the circle? **16 m**

★ 41. The diameter of a circle is 60 inches, and a chord of the circle is 48 inches long. How far is the chord from the center of the circle? **18 in.**

★ 42. A chord of a circle is 48 centimeters long and is 10 centimeters from the center of the circle. Find the radius. **26 cm**

★ 43. A diameter of a circle is 32 yards long. A chord is 11 yards from the center. How long is the chord? $2\sqrt{135} \approx 23.24$ **yd**

**44. The line through the midpoint bisects the chord and is perpendicular to the chord, so the line is a diameter of the circle. Where two diameters meet would locate the center of the circle.**

### Study Tip

*Finding the Center of a Circle*

The process Mr. Ortega used can be done by construction and is often called *locating the center of a circle.*

44. **CARPENTRY** Mr. Ortega wants to drill a hole in the center of a round picnic table for an umbrella pole. To locate the center of the circle, he draws two chords of the circle and uses a ruler to find the midpoint for each chord. Then he uses a framing square to draw a line perpendicular to each chord at its midpoint. Explain how this process locates the center of the tabletop.

45. **CRITICAL THINKING** A diameter of $\odot P$ has endpoints $A$ and $B$. Radius $\overline{PQ}$ is perpendicular to $\overline{AB}$. Chord $\overline{DE}$ bisects $\overline{PQ}$ and is parallel to $\overline{AB}$. Does $DE = \frac{1}{2}(AB)$? Explain.

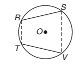
**45. Let $r$ be the radius of $\odot P$. Draw radii to points $D$ and $E$ to create triangles. The length $DE$ is $r\sqrt{3}$ and $AB = 2r$; $r\sqrt{3} \neq \frac{1}{2(2r)}$.**



## Study Guide and Intervention, p. 553 (shown) and p. 554

**Arcs and Chords** Points on a circle determine both chords and arcs. Several properties are related to points on a circle.

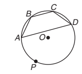
- In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.
- If all the vertices of a polygon lie on a circle, the polygon is said to be **inscribed** in the circle and the circle is **circumscribed** about the polygon.




$\widehat{RS} \cong \widehat{TV}$ if and only if $\overline{RS} \cong \overline{TV}$.
$RSVT$ is inscribed in $\odot O$.
$\odot O$ is circumscribed about $RSVT$.

**Example** **Trapezoid $ABCD$ is inscribed in $\odot O$. If $\overline{AB} \cong \overline{BC} \cong \overline{CD}$ and $m\widehat{BC} = 50$, what is $m\widehat{APD}$?**

Chords $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$ are congruent, so $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CD}$ are congruent. $m\widehat{BC} = 50$, so $m\widehat{AB} + m\widehat{BC} + m\widehat{CD} = 50 + 50 + 50 = 150$. Then $m\widehat{APD} = 360 - 150$ or 210.



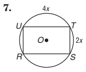

**Exercises**

**Each regular polygon is inscribed in a circle. Determine the measure of each arc that corresponds to a side of the polygon.**

1. hexagon **60**
2. pentagon **72**
3. triangle **120**
4. square **90**
5. octagon **45**
6. 36-gon **10**

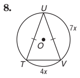
**Determine the measure of each arc of the circle circumscribed about the polygon.**

7.



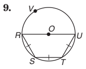

$m\widehat{UT} = m\widehat{RS} = 120$
$m\widehat{ST} = m\widehat{RU} = 60$

8.




$m\widehat{UT} = m\widehat{UV} = 140$
$m\widehat{TV} = 80$

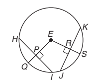
9.




$m\widehat{RS} = m\widehat{ST} = m\widehat{TU} = 60$
$m\widehat{RVU} = 180$

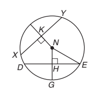
## Skills Practice, p. 555 and Practice, p. 556 (shown)

**In $\odot E$, $m\widehat{HQ} = 48$, $HI = JK$, and $JR = 7.5$. Find each measure.**



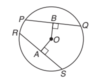

1. $m\widehat{HI}$ **96**
2. $m\widehat{QI}$ **48**
3. $m\widehat{JK}$ **96**
4. $HI$ **15**
5. $PI$ **7.5**
6. $JK$ **15**

**The radius of $\odot N$ is 18, $NK = 9$, and $m\widehat{DE} = 120$. Find each measure.**



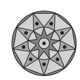

7. $m\widehat{GE}$ **60**
8. $m\angle HNE$ **60**
9. $m\angle HEN$ **30**
10. $HN$ **9**

**The radius of $\odot O = 32$, $\widehat{PQ} \cong \widehat{RS}$, and $PQ = 56$. Find each measure.**




11. $PB$ **28**
14. $BQ$ **28**
12. $OB$ $4\sqrt{15} \approx 15.49$
16. $RS$ **56**

13. **MANDALAS** The base figure in a mandala design is a nine-pointed star. Find the measure of each arc of the circle circumscribed about the star. **Each arc measures 40°.**



## Reading to Learn Mathematics, p. 557 ELL

**Pre-Activity** **How do the grooves in a Belgian waffle iron model segments in a circle?**

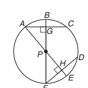
Read the introduction to Lesson 10-3 at the top of page 536 in your textbook.

What do you observe about any two of the grooves in the waffle iron shown in the picture in your textbook? **They are either parallel or perpendicular.**

**Reading the Lesson**

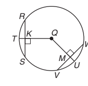
1. Supply the missing words or phrases to form true statements.
   a. In a circle, if a radius is **perpendicular** to a chord, then it bisects the chord and its **arc**.
   b. In a circle or in **congruent** circles, two **minor arcs** are congruent if and only if their corresponding chords are congruent.
   c. In a circle or in **congruent** circles, two chords are congruent if they are **equidistant** from the center.
   d. A polygon is inscribed in a circle if all of its **vertices** lie on the circle.
   e. All of the sides of an inscribed polygon are **chords** of the circle.

2. If $\odot P$ has a diameter 40 centimeters long, and $AC = FD = 24$ centimeters, find each measure.




   a. $PA$ **20 cm**
   b. $AG$ **12 cm**
   c. $PE$ **20 cm**
   d. $PH$ **16 cm**
   e. $HE$ **4 cm**
   f. $FG$ **36 cm**

3. In $\odot Q$, $RS = VW$ and $m\widehat{RS} = 70$. Find each measure.




   a. $m\widehat{RT}$ **35**
   b. $m\widehat{ST}$ **35**
   c. $m\widehat{VW}$ **70**
   d. $m\widehat{VU}$ **35**

4. Find the measure of each arc of a circle that is circumscribed about the polygon.
   a. an equilateral triangle **120**
   b. a regular pentagon **72**
   c. a regular hexagon **60**
   d. a regular decagon **36**
   e. a regular dodecagon **30**
   f. a regular $n$-gon $\frac{360}{n}$

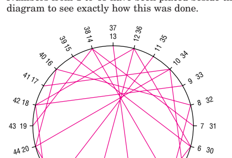
**Helping You Remember**

5. Some students have trouble distinguishing between *inscribed* and *circumscribed* figures. What is an easy way to remember which is which? **Sample answer: The *in*scribed figure is *in*side the circle.**

## Enrichment, p. 558

***Patterns from Chords***

Some beautiful and interesting patterns result if you draw chords to connect evenly spaced points on a circle. On the circle shown below, 24 points have been marked to divide the circle into 24 equal parts. Numbers from 1 to 48 have been placed beside the points. Study the diagram to see exactly how this was done.



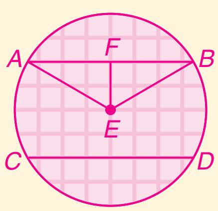
## Answers

46. The chords and the radii of the circle are congruent by construction. Thus, all triangles formed by these segments are equilateral triangles. That means each angle of the hexagon measures 120°, making all angles of the hexagon congruent and all sides congruent.

47. The six arcs making up the circle are congruent because the chords intercepting them were congruent by construction. Each of the three chords drawn intercept two of the congruent chords. Thus, the three larger arcs are congruent. So, the three chords are congruent, making this an equilateral triangle.

51. Sample answer: The grooves of a waffle iron are chords of the circle. The ones that pass horizontally and vertically through the center are diameters. Answers should include the following.

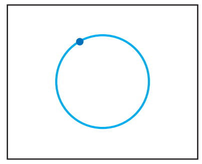
- If you know the measure of the radius and the distance the chord is from the center, you can use the Pythagorean Theorem to find the length of half of the chord and then multiply by 2.
- There are four grooves on either side of the diameter, so each groove is about 1 in. from the center. In the figure, $EF = 2$ and $EB = 4$ because the radius is half the diameter. Using the Pythagorean Theorem, you find that $FB \approx 3.464$ in. so $AB \approx 6.93$ in. Approximate lengths for other chords are 5.29 in. and 7.75 in., but exactly 8 in. for the diameter.



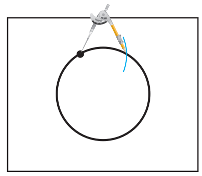



**CONSTRUCTION** **Use the following steps for each construction in Exercises 46 and 47.**

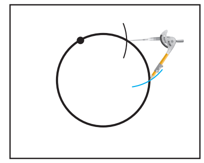
1. Construct a circle, and place a point on the circle.



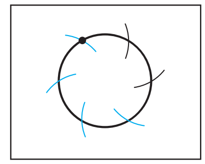
2. Using the same radius, place the compass on the point and draw a small arc to intercept the circle.



3. Using the same radius, place the compass on the intersection and draw another small arc to intercept the circle.



4. Continue the process in Step 3 until you return to the original point.

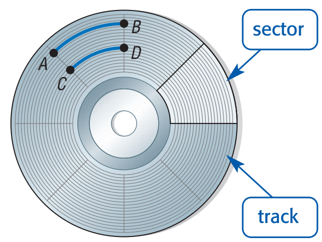


46–47. See margin for verifications.

**46.** Connect the intersections with chords of the circle. What type of figure is formed? Verify you conjecture. **inscribed regular hexagon**

**47.** Repeat the construction. Connect every other intersection with chords of the circle. What type of figure is formed? Verify your conjecture. **inscribed equilateral triangle**

**COMPUTERS** **For Exercises 48 and 49, use the following information.**
The hard drive of a computer contains platters divided into tracks, which are defined by concentric circles, and sectors, which are defined by radii of the circles.

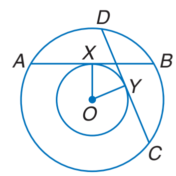



**48.** In the diagram of a hard drive platter at the right, what is the relationship between $m\widehat{AB}$ and $m\widehat{CD}$? $m\widehat{AB} = m\widehat{CD}$

**49.** Are $\widehat{AB}$ and $\widehat{CD}$ congruent? Explain. **No; congruent arcs must be in the same circle or congruent circles, but these are in concentric circles.**

50. $\overline{AB} \cong \overline{CD}$; in the smaller circle, $\overline{OX} \cong \overline{OY}$ because they are radii. This means that in the larger circle, $\overline{AB}$ and $\overline{CD}$ are equidistant from the center, making them congruent chords.

**50. CRITICAL THINKING** The figure shows two concentric circles with $\overline{OX} \perp \overline{AB}$ and $\overline{OY} \perp \overline{CD}$. Write a statement relating $\overline{AB}$ and $\overline{CD}$. Verify your reasoning.




**51. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**How do the grooves in a Belgian waffle iron model segments in a circle?**

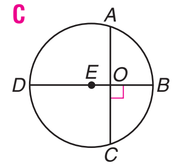
Include the following in your answer:
- a description of how you might find the length of a groove without directly measuring it, and
- a sketch with measurements for a waffle iron that is 8 inches wide.

## Standardized Test Practice

52. Refer to the figure. Which of the following statements is true? **C**

I. $\overline{DB}$ bisects $\overline{AC}$. II. $\overline{AC}$ bisects $\overline{DB}$. III. $OA = OC$

(A) I and II (B) II and III
(C) I and III (D) I, II, and III



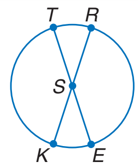

53. **SHORT RESPONSE** According to the 2000 census, the population of Bridgeworth was 204 thousand, and the population of Sutterly was 216 thousand. If the population of each city increased by exactly 20% ten years later, how many more people will live in Sutterly than in Bridgeworth in 2010? **14,400**

## Maintain Your Skills

### Mixed Review

**In ⊙S, $m\angle TSR = 42$. Find each measure.** *(Lesson 10-2)*

54. $m\widehat{KT}$ **138** 55. $m\widehat{ERT}$ **180** 56. $m\widehat{KRT}$ **222**



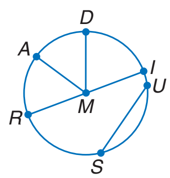

Exercises 54–56

**Refer to ⊙M.** *(Lesson 10-1)*

57. Name a chord that is not a diameter. **$\overline{SU}$**

58. If $MD = 7$, find $RI$. **14**

59. Name congruent segments in ⊙M.
**$\overline{RM}$, $\overline{AM}$, $\overline{DM}$, $\overline{IM}$**




Exercises 57–59

### Getting Ready for the Next Lesson

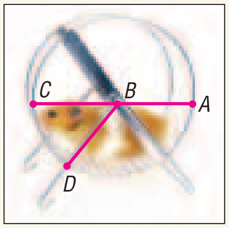
**PREREQUISITE SKILL** **Solve each equation.**
*(To review **solving equations**, see pages 742 and 743.)*

60. $\frac{1}{2}x = 120$ **240**
61. $\frac{1}{2}x = 25$ **50**
62. $2x = \frac{1}{2}(45 + 35)$ **20**
63. $3x = \frac{1}{2}(120 - 60)$ **10**
64. $45 = \frac{1}{2}(4x + 30)$ **15**
65. $90 = \frac{1}{2}(6x + 3x)$ **20**

## Practice Quiz 1

*Lessons 10-1 through 10-3*

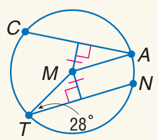
**PETS** **For Exercises 1–6, refer to the front circular edge of the hamster wheel shown at the right.** *(Lessons 10-1 and 10-2)*

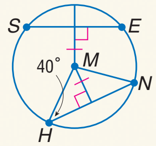


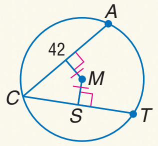

1. Name three radii of the wheel. **$\overline{BC}$, $\overline{BD}$, $\overline{BA}$**
2. If $BD = 3x$ and $CB = 7x - 3$, find $AC$. **4.5**
3. If $m\angle CBD = 85$, find $m\widehat{AD}$. **95**
4. If $r = 3$ inches, find the circumference of circle $B$ to the nearest tenth of an inch. **18.8 in.**
5. There are 40 equally-spaced rungs on the wheel. What is the degree measure of an arc connecting two consecutive rungs? **9**
6. What is the length of $\widehat{CAD}$ to the nearest tenth if $m\angle ABD = 150$ and $r = 3$? **17.3 units**

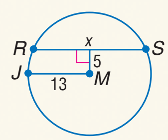
**Find each measure.** *(Lesson 10-3)*

7. $m\angle CAM$ **28**
8. $m\widehat{ES}$ **100**
9. $SC$ **21**
10. $x$ **24**

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## 4 Assess

### Open-Ended Assessment

**Writing** Have students draw three circles on a sheet of paper. Tell students to draw a chord anywhere on the first circle and then construct and label a perpendicular bisector for this chord. For the second circle, have students draw two segments extending from the center of the circle so that the chords perpendicular to these segments are congruent. Ask students to place two chords on the third circle so that their corresponding arcs are congruent. Tell students to write the rule they used from the lesson underneath each figure.

### Getting Ready for Lesson 10-4

**Prerequisite Skill** Students will learn about inscribed angles in Lesson 10-4. They will apply concepts of solving equations to find the measures of inscribed angles. Use Exercises 60–65 to determine your students' familiarity with solving equations.

### Assessment Options

**Practice Quiz 1** The quiz provides students with a brief review of the concepts and skills in Lessons 10-1 through 10-3. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

# 10-4 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-4** Use as a quiz or review of Lesson 10-3 .

**Mathematical Background** notes are available for this lesson on p. 520D.

**How** is a socket like an inscribed polygon?

*Ask students:*

- Why do you think socket wrenches and nuts have a hexagonal shape? **Because a hexagon offers good strength and leverage while still distributing the force applied by the user evenly.**
- What are the advantages of the wrench mechanism to which the hexagonal cylinder is attached? **This mechanism gives more leverage to the user than just the hexagon itself and allows more maneuverability and flexibility.**

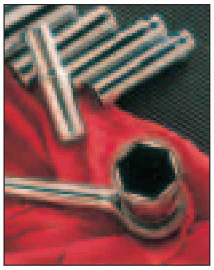
# 10-4 Inscribed Angles

## What You'll Learn

- Find measures of inscribed angles.
- Find measures of angles of inscribed polygons.

## How is a socket like an inscribed polygon?

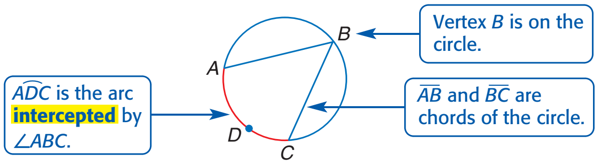
A socket is a tool that comes in varying diameters. It is used to tighten or unscrew nuts or bolts. The "hole" in the socket is a hexagon cast in a metal cylinder.



**Vocabulary**

- intercepted

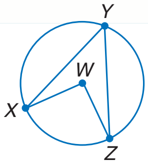
**INSCRIBED ANGLES** In Lesson 10-3, you learned that a polygon that has its vertices on a circle is called an inscribed polygon. Likewise, an *inscribed angle* is an angle that has its vertex on the circle and its sides contained in chords of the circle.




## Geometry Activity

### Measure of Inscribed Angles

**Model**




- Use a compass to draw a circle and label the center $W$.
- Draw an inscribed angle and label it $XYZ$.
- Draw $\overline{WX}$ and $\overline{WZ}$.

**Analyze**

1. Measure $\angle XYZ$ and $\angle XWZ$. **See students' work.**
2. Find $m\widehat{XZ}$ and compare it with $m\angle XYZ$. $m\widehat{XZ} = 2(m\angle XYZ)$
3. **Make a conjecture** about the relationship of the measure of an inscribed angle and the measure of its intercepted arc.

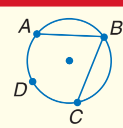
**3. The measure of an inscribed angle is one-half the measure of its intercepted arc.**

This activity suggests the following theorem.

## Theorem 10.5

**Inscribed Angle Theorem** If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

**Example:** $m\angle ABC = \frac{1}{2}(m\widehat{ADC})$ or $2(m\angle ABC) = m\widehat{ADC}$






## Resource Manager

### Workbook and Reproducible Masters

***Chapter 10 Resource Masters***

- Study Guide and Intervention, pp. 559–560
- Skills Practice, p. 561
- Practice, p. 562
- Reading to Learn Mathematics, p. 563
- Enrichment, p. 564
- Assessment, pp. 603, 605

***Prerequisite Skills Workbook,*** pp. 41–42

***Teaching Geometry With Manipulatives Masters,*** pp. 16, 17, 165, 166, 167

### Transparencies

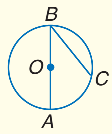
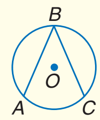
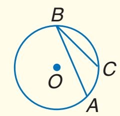
5-Minute Check Transparency 10-4

Answer Key Transparencies

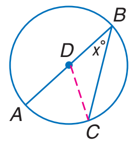
### Technology

Interactive Chalkboard

To prove Theorem 10.5, you must consider three cases.

| | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| Model of Angle Inscribed in $\odot O$ |   |   |   |
| Location of center of circle | on a side of the angle | in the interior of the angle | in the exterior of the angle |

**Proof** *Theorem 10.5 (Case 1)*




**Given:** $\angle ABC$ inscribed in $\odot D$ and $\overline{AB}$ is a diameter.

**Prove:** $m\angle ABC = \frac{1}{2}m\widehat{AC}$

Draw $\overline{DC}$ and let $m\angle B = x$.

**Proof:**

Since $\overline{DB}$ and $\overline{DC}$ are congruent radii, $\triangle BDC$ is isosceles and $\angle B \cong \angle C$. Thus, $m\angle B = m\angle C = x$. By the Exterior Angle Theorem, $m\angle ADC = m\angle B + m\angle C$. So $m\angle ADC = 2x$. From the definition of arc measure, we know that $m\widehat{AC} = m\angle ADC$ or $2x$. Comparing $m\widehat{AC}$ and $m\angle ABC$, we see that $m\widehat{AC} = 2(m\angle ABC)$ or that $m\angle ABC = \frac{1}{2}m\widehat{AC}$.

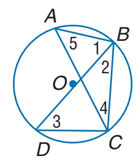
You will prove Cases 2 and 3 of Theorem 10.5 in Exercises 35 and 36.

**Study Tip**

*Using Variables*

You can also assign a variable to an unknown measure. So, if you let $m\widehat{AD} = x$, the second equation becomes $140 + 100 + x + x = 360$, or $240 + 2x = 360$. This last equation may seem simpler to solve.

**Example 1** *Measures of Inscribed Angles*




**In $\odot O$, $m\widehat{AB} = 140$, $m\widehat{BC} = 100$, and $m\widehat{AD} = m\widehat{DC}$. Find the measures of the numbered angles.**

First determine $m\widehat{DC}$ and $m\widehat{AD}$.

$m\widehat{AB} + m\widehat{BC} + m\widehat{DC} + m\widehat{AD} = 360$ Arc Addition Theorem

$140 + 100 + m\widehat{DC} + m\widehat{DC} = 360$ $m\widehat{AB} = 140$, $m\widehat{BC} = 100$, $m\widehat{DC} = m\widehat{AD}$

$240 + 2(m\widehat{DC}) = 360$ Simplify.

$2(m\widehat{DC}) = 120$ Subtract 240 from each side.

$m\widehat{DC} = 60$ Divide each side by 2.

So, $m\widehat{DC} = 60$ and $m\widehat{AD} = 60$.

$m\angle 1 = \frac{1}{2}m\widehat{AD}$
$= \frac{1}{2}(60)$ or 30

$m\angle 2 = \frac{1}{2}m\widehat{DC}$
$= \frac{1}{2}(60)$ or 30

$m\angle 3 = \frac{1}{2}m\widehat{BC}$
$= \frac{1}{2}(100)$ or 50

$m\angle 4 = \frac{1}{2}m\widehat{AB}$
$= \frac{1}{2}(140)$ or 70

$m\angle 5 = \frac{1}{2}m\widehat{BC}$
$= \frac{1}{2}(100)$ or 50

www.geometryonline.com/extra_examples

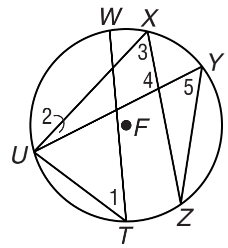


## 2 Teach

### INSCRIBED ANGLES

**In-Class Example** Power Point®

**1** In $\odot F$, $m\widehat{WX} = 20$, $m\widehat{XY} = 40$, $m\widehat{UZ} = 108$, and $m\widehat{UW} = m\widehat{YZ}$. Find the measures of the numbered angles.




**$m\angle 1 = 48$; $m\angle 2 = 20$; $m\angle 3 = 54$; $m\angle 4 = 106$; $m\angle 5 = 54$**

## Geometry Activity

**Materials:** compass, protractor, straightedge

Have some students start with an acute inscribed angle, some with a right inscribed angle, and some with an obtuse inscribed angle and compare results.

Lesson 10-4 Inscribed Angles 545

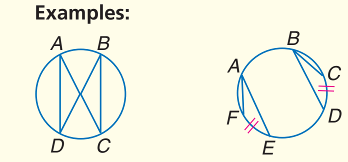
In Example 1, note that $\angle 3$ and $\angle 5$ intercept the same arc and are congruent.

## Theorem 10.6

If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

**Abbreviations:**
*Inscribed* $\measuredangle$s *of* $\cong$ *arcs are* $\cong$.
*Inscribed* $\measuredangle$s *of same arc are* $\cong$.

**Examples:**



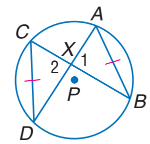

$\angle DAC \cong \angle DBC$    $\angle FAE \cong \angle CBD$

You will prove Theorem 10.6 in Exercise 37.

## Example 2 Proofs with Inscribed Angles

**Given:** $\odot P$ with $\overline{CD} \cong \overline{AB}$

**Prove:** $\triangle AXB \cong \triangle CXD$




**Proof:**

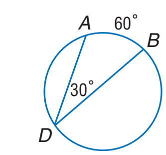
| **Statements** | **Reasons** |
|---|---|
| 1. $\angle DAB$ intercepts $\overset{\frown}{DB}$. $\angle DCB$ intercepts $\overset{\frown}{DB}$. | 1. Definition of intercepted arc |
| 2. $\angle DAB \cong \angle DCB$ | 2. Inscribed $\measuredangle$s of same arc are $\cong$. |
| 3. $\angle 1 \cong \angle 2$ | 3. Vertical $\measuredangle$s are $\cong$. |
| 4. $\overline{CD} \cong \overline{AB}$ | 4. Given |
| 5. $\triangle AXB \cong \triangle CXD$ | 5. AAS |

You can also use the measure of an inscribed angle to determine probability of a point lying on an arc.

## Example 3 Inscribed Arcs and Probability

**PROBABILITY Points $A$ and $B$ are on a circle so that $m\overset{\frown}{AB} = 60$. Suppose point $D$ is randomly located on the same circle so that it does not coincide with $A$ or $B$. What is the probability that $m\angle ADB = 30$?**

Since the angle measure is half the arc measure, inscribed $\angle ADB$ must intercept $\overset{\frown}{AB}$, so $D$ must lie on major arc $AB$. Draw a figure and label any information you know.




$$m\overset{\frown}{BDA} = 360 - m\overset{\frown}{AB}$$
$$= 360 - 60 \text{ or } 300$$

Since $\angle ADB$ must intercept $\overset{\frown}{AB}$, the probability that $m\angle ADB = 30$ is the same as the probability of $D$ being contained in $\overset{\frown}{BDA}$.

The probability that $D$ is located on $\overset{\frown}{ADB}$ is $\frac{5}{6}$. So, the probability that $m\angle ADB = 30$ is also $\frac{5}{6}$.

### Study Tip

**Eliminate the Possibilites**
Think about what would be true if $D$ was on minor arc $\overset{\frown}{AB}$. Then $\angle ADB$ would intercept the major arc. Thus, $m\angle ADB$ would be half of 300 or 150. This is not the desired angle measure in the problem, so you can eliminate the possibility that $D$ can lie on $\overset{\frown}{AB}$.



## In-Class Examples

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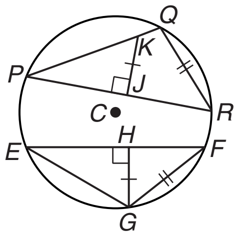
**2** **Given:** $\odot C$ with $\overline{QR} \cong \overline{GF}$ and $\overline{JK} \cong \overline{HG}$
**Prove:** $\triangle PJK \cong \triangle EHG$




**Statement (Reason)**

1. $\overline{QR} \cong \overline{GF}$ and $\overline{JK} \cong \overline{HG}$ (Given)
2. $\overset{\frown}{QR} \cong \overset{\frown}{GF}$ (If two chords are $\cong$, corr. minor arcs are $\cong$.)
3. $\angle GEF$ intercepts $\overset{\frown}{FG}$; $\angle QPR$ intercepts $\overset{\frown}{QR}$. (Def. of intercepted arc)
4. $\angle GEF \cong \angle QPR$ (Inscribed $\measuredangle$s of $\cong$ arcs are $\cong$.)
5. $\angle PJK \cong \angle EHG$ (Right $\measuredangle$s are congruent.)
6. $\triangle PJK \cong \triangle EHG$ (AAS)

**3** **PROBABILITY** Points $M$ and $N$ are on a circle so that $m\overset{\frown}{MN} = 72$. Suppose point $L$ is randomly located on the same circle so that it does not coincide with $M$ or $N$. What is the probability that $m\angle MLN = 144$? $\frac{1}{5}$

## DAILY INTERVENTION

### Differentiated Instruction

**Intrapersonal** Select or provide examples that cover each concept in the lesson so that students can sit quietly and work on them at their desks. Ask students to make a note if a particular type of problem gives them difficulty. Encourage students to reread and use the examples and theorems in the book to help them work and understand the problems.

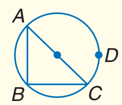
***Inscribed Polygons***
Remember that for a polygon to be an inscribed polygon, *all* of its vertices must lie on the circle.

**ANGLES OF INSCRIBED POLYGONS** An inscribed triangle with a side that is a diameter is a special type of triangle.

### Theorem 10.7

If an inscribed angle intercepts a semicircle, the angle is a right angle.

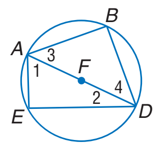
**Example:** $\overarc{ADC}$ is a semicircle, so $m\angle ABC = 90$.




You will prove Theorem 10.7 in Exercise 38.

## Example 4 Angles of an Inscribed Triangle

**ALGEBRA** **Triangles $ABD$ and $ADE$ are inscribed in $\odot F$ with $\overarc{AB} \cong \overarc{BD}$. Find the measure of each numbered angle if $m\angle 1 = 12x - 8$ and $m\angle 2 = 3x + 8$.**




$AED$ is a right angle because $\overarc{AED}$ is a semicircle.

$m\angle 1 + m\angle 2 + m\angle AED = 180$ Angle Sum Theorem

$(12x - 8) + (3x + 8) + 90 = 180$ $m\angle 1 = 12x - 8$, $m\angle 2 = 3x + 8$, $m\angle AED = 90$

$15x + 90 = 180$ Simplify.

$15x = 90$ Subtract 90 from each side.

$x = 6$ Divide each side by 15.

Use the value of $x$ to find the measures of $\angle 1$ and $\angle 2$.

| | | | |
|---|---|---|---|
| $m\angle 1 = 12x - 8$ | Given | $m\angle 2 = 3x + 8$ | Given |
| $= 12(6) - 8$ | $x = 6$ | $= 3(6) + 8$ | $x = 6$ |
| $= 64$ | Simplify. | $= 26$ | Simplify. |

Angle $ABD$ is a right angle because it intercepts a semicircle.

Because $\overarc{AB} \cong \overarc{BD}$, $\overline{AB} \cong \overline{BD}$, which leads to $\angle 3 \cong \angle 4$. Thus, $m\angle 3 = m\angle 4$.

$m\angle 3 + m\angle 4 + m\angle ABD = 180$ Angle Sum Theorem

$m\angle 3 + m\angle 3 + 90 = 180$ $m\angle 3 = m\angle 4$, $m\angle ABD = 90$

$2(m\angle 3) + 90 = 180$ Simplify.

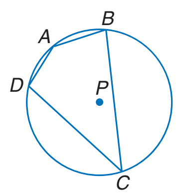
$2(m\angle 3) = 90$ Subtract 90 from each side.

$m\angle 3 = 45$ Divide each side by 2.

Since $m\angle 3 = m\angle 4$, $m\angle 4 = 45$.

## Example 5 Angles of an Inscribed Quadrilateral

**Quadrilateral $ABCD$ is inscribed in $\odot P$. If $m\angle B = 80$ and $m\angle C = 40$, find $m\angle A$ and $m\angle D$.**




Draw a sketch of this situation.

To find $m\angle A$, we need to know $m\overarc{BCD}$.

To find $m\overarc{BCD}$, first find $m\overarc{DAB}$.

$m\overarc{DAB} = 2(m\angle C)$ Inscribed Angle Theorem

$= 2(40)$ or $80$ $m\angle C = 40$

*(continued on the next page)*



## ANGLES OF INSCRIBED POLYGONS

### In-Class Examples

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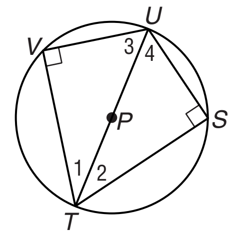
**4** **ALGEBRA** Triangles $TVU$ and $TSU$ are inscribed in $\odot P$ with $\overarc{VU} \cong \overarc{SU}$. Find the measure of each numbered angle if $m\angle 2 = x + 9$ and $m\angle 4 = 2x + 6$.




**$m\angle 1 = 34$; $m\angle 2 = 34$; $m\angle 3 = 56$; $m\angle 4 = 56$**

***Teaching Tip*** Students can also remember that the sum of the interior angles of a quadrilateral is 360 and subtract the first three angle measures from 360 to find the fourth angle measure.

**5** Quadrilateral $QRST$ is inscribed in $\odot M$. If $m\angle Q = 87$ and $m\angle R = 102$, find $m\angle S$ and $m\angle T$. **93; 78**

**Lesson 10-4** Inscribed Angles **547**

# 3 Practice/Apply

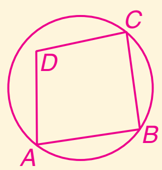
## Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include a definition/example for an inscribed angle.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

## Answers

**1. Sample answer:**

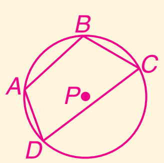



**2.** The measures of an inscribed angle and a central angle for the same intercepted arc can be calculated using the measure of the arc. However, the measure of the central angle equals the measure of the arc, while the measure of the inscribed angle is half the measure of the arc.

**4.** Given: Quadrilateral *ABCD* is inscribed in $\odot P$.

$m\angle C = \frac{1}{2} m\angle B$

Prove: $m\widehat{CDA} = 2(m\widehat{DAB})$




Proof: Given $m\angle C = \frac{1}{2}(m\angle B)$ means that $m\angle B = 2(m\angle C)$. Since $m\angle B = \frac{1}{2}(m\widehat{CDA})$ and $m\angle C = \frac{1}{2}(m\widehat{DAB})$, the equation becomes $\frac{1}{2}(m\widehat{CDA}) = 2[\frac{1}{2}(m\widehat{DAB})]$. Multiplying each side by 2 results in $m\widehat{CDA} = 2(m\widehat{DAB})$.



| | |
|---|---|
| $m\widehat{BCD} + m\widehat{DAB} = 360$ | Sum of angles in circle = 360 |
| $m\widehat{BCD} + 80 = 360$ | $m\widehat{DAB} = 80$ |
| $m\widehat{BCD} = 280$ | Subtract 80 from each side. |
| $m\widehat{BCD} = 2(m\angle A)$ | Inscribed Angle Theorem |
| $280 = 2(m\angle A)$ | Substitution |
| $140 = m\angle A$ | Divide each side by 2. |

To find $m\angle D$, we need to know $m\widehat{ABC}$, but first we must find $m\widehat{ADC}$.

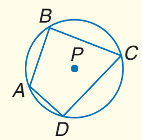
| | |
|---|---|
| $m\widehat{ADC} = 2(m\angle B)$ | Inscribed Angle Theorem |
| $m\widehat{ADC} = 2(80)$ or $160$ | $m\angle B = 80$ |
| $m\widehat{ABC} + m\widehat{ADC} = 360$ | Sum of angles in circle = 360 |
| $m\widehat{ABC} + 160 = 360$ | $m\widehat{ADC} = 160$ |
| $m\widehat{ABC} = 200$ | Subtract 160 from each side. |
| $m\widehat{ABC} = 2(m\angle D)$ | Inscribed Angle Theorem |
| $200 = 2(m\angle D)$ | Substitution |
| $100 = m\angle D$ | Divide each side by 2. |

In Example 5, note that the opposite angles of the quadrilateral are supplementary. This is stated in Theorem 10.8 and can be verified by considering that the arcs intercepted by opposite angles of an inscribed quadrilateral form a circle.

### Theorem 10.8

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

**Example:**
Quadrilateral *ABCD* is inscribed in $\odot P$.
$\angle A$ and $\angle C$ are supplementary.
$\angle B$ and $\angle D$ are supplementary.




You will prove this theorem in Exercise 39.

## Check for Understanding

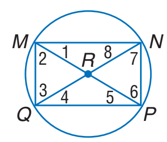
*Concept Check*

1. **OPEN ENDED** Draw a counterexample of an inscribed trapezoid. If possible, include at least one angle that is an inscribed angle. **1–2. See margin.**
2. **Compare and contrast** an inscribed angle and a central angle that intercepts the same arc.

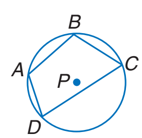
*Guided Practice*

| GUIDED PRACTICE KEY | |
|---|---|
| Exercises | Examples |
| 3 | 1 |
| 4 | 2 |
| 5 | 4 |
| 6 | 5 |
| 7 | 3 |

3. In $\odot R$, $m\widehat{MN} = 120$ and $m\widehat{MQ} = 60$. Find the measure of each numbered angle. **$m\angle 1 = 30$, $m\angle 2 = 60$, $m\angle 3 = 60$, $m\angle 4 = 30$, $m\angle 5 = 30$, $m\angle 6 = 60$, $m\angle 7 = 60$, $m\angle 8 = 30$**

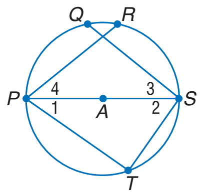



4. **PROOF** Write a paragraph proof. **See margin.**
**Given:** Quadrilateral *ABCD* is inscribed in $\odot P$.
$m\angle C = \frac{1}{2} m\angle B$
**Prove:** $m\widehat{CDA} = 2(m\widehat{DAB})$



**5. $m\angle 1 = 35$, $m\angle 2 = 55$, $m\angle 3 = 39$, $m\angle 4 = 39$**

5. **ALGEBRA** In ⊙$A$ at the right, $\widehat{PQ} \cong \widehat{RS}$. Find the measure of each numbered angle if $m\angle 1 = 6x + 11$, $m\angle 2 = 9x + 19$, $m\angle 3 = 4y - 25$, and $m\angle 4 = 3y - 9$.




6. Suppose quadrilateral $VWXY$ is inscribed in ⊙$C$. If $m\angle X = 28$ and $m\angle W = 110$, find $m\angle V$ and $m\angle Y$. **152, 70**

*Application*

7. **PROBABILITY** Points $X$ and $Y$ are endpoints of a diameter of ⊙$W$. Point $Z$ is another point on the circle. Find the probability that $\angle XZY$ is a right angle. **1**

★ indicates increased difficulty

## Practice and Apply

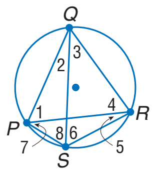
**Homework Help**

| For Exercises | See Examples |
|---|---|
| 8–10 | 1 |
| 11–12, 35–39 | 2 |
| 13–17 | 4 |
| 18–21, 26–29 | 5 |
| 31–34 | 3 |

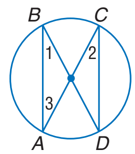
**Extra Practice**
See page 774.

Find the measure of each numbered angle for each figure.

8. $\overline{PQ} \cong \overline{RQ}$, $m\widehat{PS} = 45$, and $m\widehat{SR} = 75$

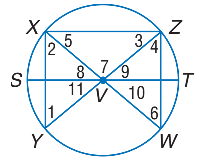



9. $m\angle BDC = 25$, $m\widehat{AB} = 120$, and $m\widehat{CD} = 130$




**$m\angle 1 = m\angle 2 = 30$, $m\angle 3 = 25$**

10. $m\widehat{XZ} = 100$, $\overline{XY} \perp \overline{ST}$, and $\overline{ZW} \perp \overline{ST}$

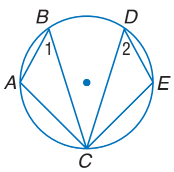



**8. $m\angle 1 = 60$, $m\angle 2 = 22.5$, $m\angle 3 = 37.5$, $m\angle 4 = 60$, $m\angle 5 = 22.5$, $m\angle 6 = 60$, $m\angle 7 = 37.5$, $m\angle 8 = 60$**

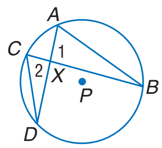
**10. $m\angle 1 = m\angle 2 = 50$, $m\angle 3 = 40$, $m\angle 4 = 50$, $m\angle 5 = 40$, $m\angle 6 = 50$, $m\angle 7 = 100$, $m\angle 8 = m\angle 9 = m\angle 10 = m\angle 11 = 40$**

**PROOF** Write a two-column proof. **11–12. See margin.**

11. **Given:** $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$
**Prove:** $\triangle ABC \cong \triangle EDC$

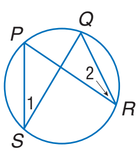



12. **Given:** ⊙$P$
**Prove:** $\triangle AXB \sim \triangle CXD$



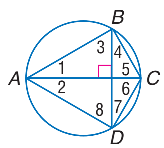

**ALGEBRA** Find the measure of each numbered angle for each figure.

13. $m\angle 1 = x$, $m\angle 2 = 2x - 13$



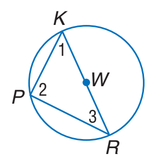

**$m\angle 1 = m\angle 2 = 13$**

14. $m\widehat{AB} = 120$




**$m\angle 1 = m\angle 2 = 30$, $m\angle 3 = 60$, $m\angle 4 = 30$, $m\angle 5 = m\angle 6 = 60$, $m\angle 7 = 30$, $m\angle 8 = 60$**

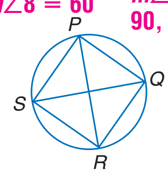
15. $m\angle R = \frac{1}{3}x + 5$, $m\angle K = \frac{1}{2}x$



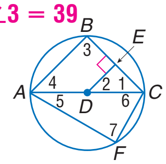

**$m\angle 1 = 51$, $m\angle 2 = 90$, $m\angle 3 = 39$**

16. $PQRS$ is a rhombus inscribed in a circle. Find $m\angle QRP$ and $m\widehat{SP}$. **45; 90**

★ 17. In ⊙$D$, $\overline{DE} \cong \overline{EC}$, $m\widehat{CF} = 60$, and $\overline{DE} \perp \overline{EC}$. Find $m\angle 4$, $m\angle 5$, and $m\widehat{AF}$. **45, 30, 120**




Exercise 16




Exercise 17



### About the Exercises...

**Organization by Objective**

- **Inscribed Angles:** 8–12, 31–39
- **Angles of Inscribed Polygons:** 13–21, 26–29

**Odd/Even Assignments**

Exercises 8–15, 22–29 are structured so that students practice the same concepts whether they are assigned odd or even problems.

### Assignment Guide

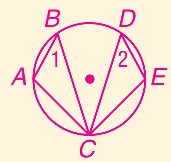
**Basic:** 9–15 odd, 19–27 odd, 31–43 odd, 44–58

**Average:** 9–43 odd, 44–58

**Advanced:** 8–42 even, 44–55 (optional: 56–58)

## Answers

**11. Given: $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$**
**Prove: $\triangle ABC \cong \triangle EDC$**

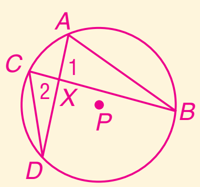



**Proof:**

**Statements (Reasons)**

1. $\widehat{AB} \cong \widehat{DE}$, $\widehat{AC} \cong \widehat{CE}$ (Given)
2. $m\widehat{AB} = m\widehat{DE}$, $m\widehat{AC} = m\widehat{CE}$ (Def. of ≅ arcs)
3. $\frac{1}{2}m\widehat{AB} = \frac{1}{2}m\widehat{DE}$, $\frac{1}{2}m\widehat{AC} = \frac{1}{2}m\widehat{CE}$ (Mult. Prop.)
4. $m\angle ACB = \frac{1}{2}m\widehat{AB}$, $m\angle ECD = \frac{1}{2}m\widehat{DE}$, $m\angle 1 = \frac{1}{2}m\widehat{AC}$, $m\angle 2 = \frac{1}{2}m\widehat{CE}$ (Inscribed ∠ Theorem)
5. $m\angle ACB = m\angle ECD$, $m\angle 1 = m\angle 2$ (Substitution)
6. $\angle ACB \cong \angle ECD$, $\angle 1 \cong \angle 2$ (Def. of ≅ ∠s)
7. $\overline{AB} \cong \overline{DE}$ (≅ arcs have ≅ chords.)
8. $\triangle ABC \cong \triangle EDC$ (AAS)

**12. Given: ⊙$P$**
**Prove: $\triangle AXB \sim \triangle CXD$**




**Proof:**

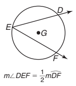
**Statements (Reasons)**

1. ⊙$P$ (Given)
2. $\angle A \cong \angle C$ (Inscribed ∠s intercepting same arc are ≅.)
3. $\angle 1 \cong \angle 2$ (Vertical ∠s are ≅.)
4. $\triangle AXB \sim \triangle CXD$ (AA Similarity)

## Study Guide and Intervention, p. 559 (shown) and p. 560

**Inscribed Angles** An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. In ⊙$G$, inscribed $\angle DEF$ **intercepts** $\overset{\frown}{DF}$.

| Inscribed Angle Theorem | If an angle is inscribed in a circle, then the measure of the angle equals one-half the measure of its intercepted arc. |
|---|---|



$m\angle DEF = \frac{1}{2}m\overset{\frown}{DF}$

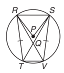
**Example** **In ⊙$G$ above, $m\overset{\frown}{DF} = 90$. Find $m\angle DEF$.**

$\angle DEF$ is an inscribed angle so its measure is half of the intercepted arc.

$$m\angle DEF = \frac{1}{2}m\overset{\frown}{DF} = \frac{1}{2}(90) \text{ or } 45$$

**Exercises**

**Use ⊙$P$ for Exercises 1–10. In ⊙$P$, $\overline{RS} \parallel \overline{TV}$ and $\overline{RT} \cong \overline{SV}$.**



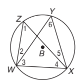

1. Name the intercepted arc for $\angle RTS$. $\overset{\frown}{RS}$
2. Name an inscribed angle that intercepts $\overset{\frown}{SV}$. $\angle SRV$ or $\angle STV$

**In ⊙$P$, $m\overset{\frown}{SV} = 120$ and $m\angle RPS = 76$. Find each measure.**

3. $m\angle PRS$ **52**
4. $m\overset{\frown}{RSV}$ **196**
5. $m\overset{\frown}{RT}$ **120**
6. $m\angle RVT$ **60**
7. $m\angle QRS$ **60**
8. $m\angle STV$ **60**
9. $m\overset{\frown}{TV}$ **44**
10. $m\angle SVT$ **98**

## Skills Practice, p. 561 and Practice, p. 562 (shown)

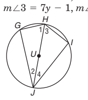
**In ⊙$B$, $m\overset{\frown}{WX} = 104$, $m\overset{\frown}{WZ} = 88$, and $m\angle ZWY = 26$. Find the measure of each angle.**




1. $m\angle 1$ **52**
2. $m\angle 2$ **26**
3. $m\angle 3$ **58**
4. $m\angle 4$ **44**
5. $m\angle 5$ **26**
6. $m\angle 6$ **52**

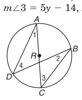
**ALGEBRA Find the measure of each numbered angle.**

7. $m\angle 1 = 5x + 2$, $m\angle 2 = 2x - 3$, $m\angle 3 = 7y - 1$, $m\angle 4 = 2y + 10$



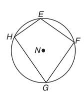

**$m\angle 1 = 67$, $m\angle 2 = 23$, $m\angle 3 = 62$, $m\angle 4 = 28$**

8. $m\angle 1 = 4x - 7$, $m\angle 2 = 2x + 11$, $m\angle 3 = 5y - 14$, $m\angle 4 = 3y + 8$



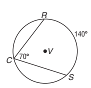

**$m\angle 1 = 29$, $m\angle 2 = 29$, $m\angle 3 = 41$, $m\angle 4 = 41$**

**Quadrilateral $EFGH$ is inscribed in ⊙$N$ such that $m\overset{\frown}{FG} = 97$, $m\overset{\frown}{GH} = 117$, and $m\overset{\frown}{EHG} = 164$. Find each measure.**




9. $m\angle E$ **107**
10. $m\angle F$ **82**
11. $m\angle G$ **73**
12. $m\angle H$ **98**

13. **PROBABILITY** In ⊙$V$, point $C$ is randomly located so that it does not coincide with points $R$ or $S$. If $m\overset{\frown}{RS} = 140$, what is the probability that $m\angle RCS = 70$? $\frac{11}{18}$




## Reading to Learn Mathematics, p. 563 ELL

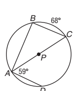
**Pre-Activity** **How is a socket like an inscribed polygon?**

Read the introduction to Lesson 10-4 at the top of page 544 in your textbook.

- Why do you think regular hexagons are used rather than squares for the "hole" in a socket? **Sample answer: If a square were used, the points might be too sharp for the tool to work smoothly.**
- Why do you think regular hexagons are used rather than regular polygons with more sides? **Sample answer: If there are too many sides, the polygon would be too close to a circle, so the wrench might slip.**

### Reading the Lesson

1. Underline the correct word or phrase to form a true statement.
   a. An angle whose vertex is on a circle and whose sides contain chords of the circle is called a(n) (central/inscribed/circumscribed) angle.
   b. Every inscribed angle that intercepts a semicircle is a(n) (acute/right/obtuse) angle.
   c. The opposite angles of an inscribed quadrilateral are (congruent/complementary/supplementary).
   d. An inscribed angle that intercepts a major arc is a(n) (acute/right/obtuse) angle.
   e. Two inscribed angles of a circle that intercept the same arc are (congruent/complementary/supplementary).
   f. If a triangle is inscribed in a circle and one of the sides of the triangle is a diameter of the circle, the diameter is (the longest side of an acute triangle/a leg of an isosceles triangle/the hypotenuse of a right triangle).
2. Refer to the figure. Find each measure.




   a. $m\angle ABC$ **90**
   b. $m\overset{\frown}{CD}$ **118**
   c. $m\overset{\frown}{AD}$ **62**
   d. $m\angle BAC$ **34**
   e. $m\angle BCA$ **56**
   f. $m\overset{\frown}{AB}$ **112**
   g. $m\overset{\frown}{BCD}$ **186**
   h. $m\overset{\frown}{BDA}$ **248**

### Helping You Remember

3. A good way to remember a geometric relationship is to visualize it. Describe how you could make a sketch that would help you remember the relationship between the measure of an inscribed angle and the measure of its intercepted arc. **Sample answer: Draw a diameter of the circle to divide it into two semicircles. Inscribe an angle in one of the semicircles; this angle will intercept the other semicircle. From your sketch, you can see that the inscribed angle is a right angle. The measure of the semicircle arc is 180, so the measure of the inscribed angle is half the measure of its intercepted arc.**

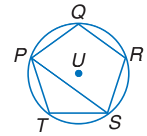
18. Quadrilateral $WRTZ$ is inscribed in a circle. If $m\angle W = 45$ and $m\angle R = 100$, find $m\angle T$ and $m\angle Z$. **135, 80**

19. Trapezoid $ABCD$ is inscribed in a circle. If $m\angle A = 60$, find $m\angle B$, $m\angle C$, and $m\angle D$. **$m\angle B = 120$, $m\angle C = 120$, $m\angle D = 60$**

20. Rectangle $PDQT$ is inscribed in a circle. What can you conclude about $\overline{PQ}$? **Sample answer: $\overline{PQ}$ is a diagonal of $PDQT$ and a diameter of the circle.**

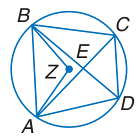
21. Square $EDFG$ is inscribed in a circle. What can you conclude about $\overline{EF}$? **Sample answer: $\overline{EF}$ is a diameter of the circle and a diagonal and angle bisector of $EDFG$.**

**Equilateral pentagon $PQRST$ is inscribed in ⊙$U$. Find each measure.**




22. $m\overset{\frown}{QR}$ **72**
23. $m\angle PSR$ **72**
24. $m\angle PQR$ **108**
25. $m\overset{\frown}{PTS}$ **144**

**Quadrilateral $ABCD$ is inscribed in ⊙$Z$ such that $m\angle BZA = 104$, $m\overset{\frown}{CB} = 94$, and $\overline{AB} \parallel \overline{DC}$. Find each measure.**

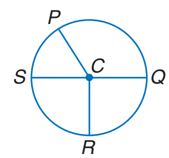



26. $m\overset{\frown}{BA}$ **104**
27. $m\overset{\frown}{ADC}$ **162**
★ 28. $m\angle BDA$ **52**
★ 29. $m\angle ZAC$ **9**

30. **SCHOOL RINGS** Some designs of class rings involve adding gold or silver to the surface of the round stone. The design at the right includes two inscribed angles. If $m\angle ABC = 50$ and $m\overset{\frown}{DBF} = 128$, find $m\overset{\frown}{AC}$ and $m\angle DEF$. **100, 64**


**PROBABILITY** **For Exercises 31–34, use the following information.**

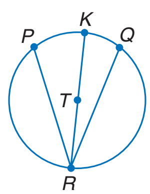
Point $T$ is randomly selected on ⊙$C$ so that it does not coincide with points $P$, $Q$, $R$, or $S$. $\overline{SQ}$ is a diameter of ⊙$C$.



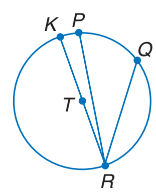

31. Find the probability that $m\angle PTS = 20$ if $m\overset{\frown}{PS} = 40$. $\frac{8}{9}$
32. Find the probability that $m\angle PTR = 55$ if $m\overset{\frown}{PSR} = 110$. $\frac{25}{36}$
33. Find the probability that $m\angle STQ = 90$. **1**
34. Find the probability that $m\angle PTQ = 180$. **0**

**PROOF** **Write the indicated proof for each theorem.** **35–39. See pp. 589A–589B.**

35. two-column proof:
Case 2 of Theorem 10.5
**Given:** $T$ lies inside $\angle PRQ$.
$\overline{RK}$ is a diameter of ⊙$T$.
**Prove:** $m\angle PRQ = \frac{1}{2}m\overset{\frown}{PKQ}$

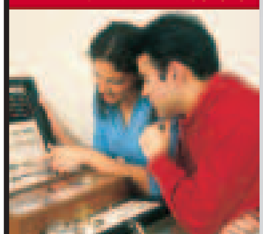



36. two-column proof:
Case 3 of Theorem 10.5
**Given:** $T$ lies outside $\angle PRQ$.
$\overline{RK}$ is a diameter of ⊙$T$.
**Prove:** $m\angle PRQ = \frac{1}{2}m\overset{\frown}{PQ}$




37. two-column proof: Theorem 10.6
38. paragraph proof: Theorem 10.7
39. paragraph proof: Theorem 10.8

**More About. . .**



**School Rings**

Many companies that sell school rings also offer schools and individuals the option to design their own ring.

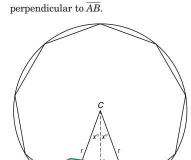


Aaron Haupt

## Enrichment, p. 564

***Formulas for Regular Polygons***

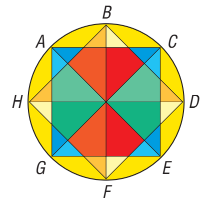
Suppose a regular polygon of $n$ sides is inscribed in a circle of radius $r$. The figure shows one of the isosceles triangles formed by joining the endpoints of one side of the polygon to the center $C$ of the circle. In the figure, $s$ is the length of each side of the regular polygon, and $a$ is the length of the segment from $C$ perpendicular to $\overline{AB}$.



41–43. See p. 589B for explanations.

41. isosceles right triangle

**STAINED GLASS** In the stained glass window design, all of the small arcs around the circle are congruent. Suppose the center of the circle is point *O*.




40. What is the measure of each of the small arcs? **45**

41. What kind of figure is $\triangle AOC$? Explain.

42. What kind of figure is quadrilateral *BDFH*? Explain. **square**

43. What kind of figure is quadrilateral *ACEG*? Explain. **square**

44. **CRITICAL THINKING** A trapezoid *ABCD* is inscribed in $\odot O$. Explain how you can verify that *ABCD* must be an isosceles trapezoid. **See margin.**

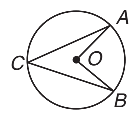
45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**How is a socket like an inscribed polygon?**

Include the following in your answer:
- a definition of an inscribed polygon, and
- the side length of a regular hexagon inscribed in a circle $\frac{3}{4}$ inch wide.

*Standardized Test Practice*

46. What is the ratio of the measure of $\angle ACB$ to the measure of $\angle AOB$? **C**




Ⓐ 1 : 1 Ⓑ 2 : 1

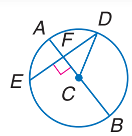
Ⓒ 1 : 2 Ⓓ not enough information

47. **GRID IN** The daily newspaper always follows a particular format. Each even-numbered page contains six articles, and each odd-numbered page contains seven articles. If today's paper has 36 pages, how many articles does it contain? **234**

## Maintain Your Skills

*Mixed Review*

**Find each measure.** *(Lesson 10-3)*




48. If $AB = 60$ and $DE = 48$, find *CF*. **18**

49. If $AB = 32$ and $FC = 11$, find *FE*.

50. If $DE = 60$ and $FC = 16$, find *AB*. **68**

49. $\sqrt{135} \approx 11.62$

**Points *Q* and *R* lie on $\odot P$. Find the length of $\widehat{QR}$ for the given radius and angle measure.** *(Lesson 10-2)*

51. $PR = 12$, and $m\angle QPR = 60$ **$4\pi$ units**

52. $m\angle QPR = 90$, $PR = 16$ **$8\pi$ units**

**Complete each sentence with *sometimes*, *always*, or *never*.** *(Lesson 4-1)*

53. Equilateral triangles are __?__ isosceles. **always**

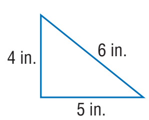
54. Acute triangles are __?__ equilateral. **sometimes**

55. Obtuse triangles are __?__ scalene. **sometimes**

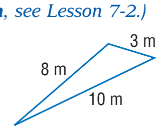
*Getting Ready for the Next Lesson*

**PREREQUISITE SKILL** **Determine whether each figure is a right triangle.** *(To review the **Pythagorean Theorem**, see Lesson 7-2.)*

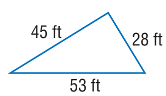
56. **no**




57. **no**




58. **yes**




www.geometryonline.com/self_check_quiz



## 4 Assess

### Open-Ended Assessment

**Modeling** Use a cork board, pushpins, a cutout circle, and a flexible rubber band to model inscribed angles. Place the circle on the cork board and put two pushpins on the circle to represent the points of an intercepted arc. Wrap the rubber band around the pins and use a pencil to drag the rubber band to the opposite end of the circle to represent the vertex of the inscribed angle. Students can move the pencil along the circle and use a protractor to note that the measure of the angle stays the same.

### *Getting Ready for Lesson 10-5*

**Prerequisite Skill** Students will learn about tangents in Lesson 10-5. They will apply the Pythagorean Theorem to determine if segments are tangents and find lengths. Use Exercises 56–58 to determine your students' familiarity with the Pythagorean Theorem.

### Assessment Options

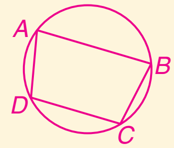
**Quiz (Lessons 10-3 and 10-4)** is available on p. 603 of the *Chapter 10 Resource Masters*.

**Mid-Chapter Test (Lessons 10-1 through 10-4)** is available on p. 605 of the *Chapter 11 Resource Masters*.

45. Sample answer: The socket is similar to an inscribed polygon because the vertices of the hexagon can be placed on a circle that is concentric with the outer circle of the socket. Answers should include the following.
- An inscribed polygon is one in which all of its vertices are points on a circle.
- The side of the regular hexagon inscribed in a circle $\frac{3}{4}$ inch wide is $\frac{3}{8}$ inch.

### Answers

44. Use the properties of trapezoids and inscribed quadrilaterals to verify that *ABCD* is isosceles.




$m\angle A + m\angle D = 180$ (same side interior angles = 180)
$m\angle A + m\angle C = 180$ (opposite angles of inscribed quadrilaterals = 180)
$m\angle A + m\angle D = m\angle A + m\angle C$ (Substitution)
$m\angle D = m\angle C$ (Subtraction Property)
$\angle D \cong \angle C$ (Def. of $\cong$ ∠s)
Trapezoid *ABCD* is isosceles because the base angles are congruent.

# 10-5 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-5** Use as a quiz or review of Lesson 10-4.

**Mathematical Background** notes are available for this lesson on p. 520D.

**How** are tangents related to track and field events?

*Ask students:*

- What does *tangent* mean? Where have you heard the word used? **Accept all reasonable answers that suggest tangent means touching.**
- What other situations can be modeled by a circle and a tangent? **Sample answers: fishing line unrolling from a spool, the string of a yo-yo, a line of paint being applied to a wall by a roller**

# 10-5 Tangents

**What You'll Learn**

- Use properties of tangents.
- Solve problems involving circumscribed polygons.

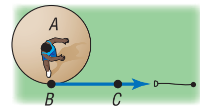
**Vocabulary**

- tangent
- point of tangency

**How are tangents related to track and field events?**

In July 2001, Yipsi Moreno of Cuba won her first major title in the hammer throw at the World Athletic Championships in Edmonton, Alberta, Canada, with a throw of 70.65 meters. The hammer is a metal ball, usually weighing 16 pounds, attached to a steel wire at the end of which is a grip. The ball is spun around by the thrower and then released, with the greatest distance thrown winning the event.

**TANGENTS** The figure models the hammer throw event. Circle $A$ represents the circular area containing the spinning thrower. Ray $BC$ represents the path the hammer takes when released. $\overrightarrow{BC}$ is **tangent** to $\odot A$, because the line containing $\overrightarrow{BC}$ intersects the circle in exactly one point. This point is called the **point of tangency**.




**Study Tip**

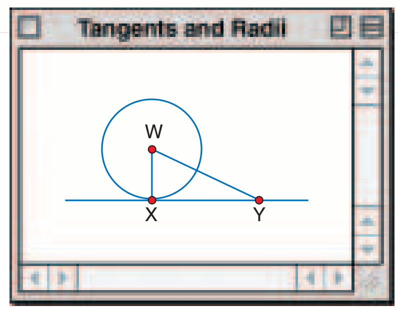
*Tangent Lines*
All of the theorems applying to tangent lines also apply to parts of the line that are tangent to the circle.

## Geometry Software Investigation

### Tangents and Radii

**Model**

- Use The Geometer's Sketchpad to draw a circle with center $W$. Then draw a segment tangent to $\odot W$. Label the point of tangency as $X$.
- Choose another point on the tangent and name it $Y$. Draw $\overline{WY}$.




**Think and Discuss**

1. What is $\overline{WX}$ in relation to the circle? **radius**
2. Measure $\overline{WY}$ and $\overline{WX}$. Write a statement to relate $WX$ and $WY$. **$WX < WY$**
3. Move point $Y$ along the tangent. How does the location of $Y$ affect the statement you wrote in Exercise 2? **It doesn't, unless $Y$ and $X$ coincide.**
4. Measure $\angle WXY$. What conclusion can you make? **$\overline{WX} \perp \overline{XY}$**
5. **Make a conjecture** about the shortest distance from the center of the circle to a tangent of the circle.

**5. Sample answer: The shortest distance from the center of a circle to the tangent is the radius of the circle, which is perpendicular to the tangent.**

This investigation suggests an indirect proof of Theorem 10.9.




## Resource Manager

### Workbook and Reproducible Masters

***Chapter 10 Resource Masters***
- Study Guide and Intervention, pp. 565–566
- Skills Practice, p. 567
- Practice, p. 568
- Reading to Learn Mathematics, p. 569
- Enrichment, p. 570

***Graphing Calculator and Computer Masters,*** pp. 35, 36
***School-to-Career Masters,*** p. 20
***Prerequisite Skills Workbook,*** pp. 15–16
***Teaching Geometry With Manipulatives Masters,*** pp. 17, 170, 171, 173

### Transparencies

5-Minute Check Transparency 10-5
Answer Key Transparencies

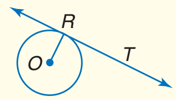
### Technology

Interactive Chalkboard

## Theorem 10.9

If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

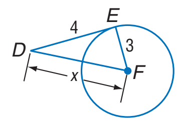
**Example:** If $\overleftrightarrow{RT}$ is a tangent, $\overline{OR} \perp \overleftrightarrow{RT}$.




## Example 1 *Find Lengths*

**ALGEBRA** $\overline{ED}$ **is tangent to $\odot F$ at point $E$. Find $x$.**

Because the radius is perpendicular to the tangent at the point of tangency, $\overline{EF} \perp \overline{DE}$. This makes $\angle DEF$ a right angle and $\triangle DEF$ a right triangle. Use the Pythagorean Theorem to find $x$.




| | |
|---|---|
| $(EF)^2 + (DE)^2 = (DF)^2$ | Pythagorean Theorem |
| $3^2 + 4^2 = x^2$ | $EF = 3, DE = 4, DF = x$ |
| $25 = x^2$ | Simplify. |
| $\pm 5 = x$ | Take the square root of each side. |

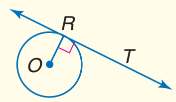
Because $x$ is the length of $\overline{DF}$, ignore the negative result. Thus, $x = 5$.

The converse of Theorem 10.9 is also true.

## Theorem 10.10

If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

**Example:** If $\overline{OR} \perp \overleftrightarrow{RT}$, $\overleftrightarrow{RT}$ is a tangent.




You will prove this theorem in Exercise 22.

### Study Tip

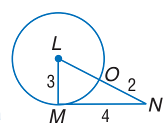
*Identifying Tangents*

Never assume that a segment is tangent to a circle by appearance unless told otherwise. The figure must either have a right angle symbol or include the measurements that confirm a right angle.

## Example 2 *Identify Tangents*

**a. Determine whether $\overline{MN}$ is tangent to $\odot L$.**

First determine whether $\triangle LMN$ is a right triangle by using the converse of the Pythagorean Theorem.

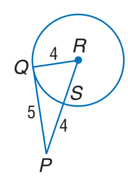



| | |
|---|---|
| $(LM)^2 + (MN)^2 \stackrel{?}{=} (LN)^2$ | Converse of Pythagorean Theorem |
| $3^2 + 4^2 \stackrel{?}{=} 5^2$ | $LM = 3, MN = 4, LN = 3 + 2$ or $5$ |
| $25 = 25$ ✓ | Simplify. |

Because the converse of the Pythagorean Theorem is true, $\triangle LMN$ is a right triangle and $\angle LMN$ is a right angle. Thus, $\overline{LM} \perp \overline{MN}$, making $\overline{MN}$ a tangent to $\odot L$.

**b. Determine whether $\overline{PQ}$ is tangent to $\odot R$.**

Since $RQ = RS$, $RP = 4 + 4$ or $8$ units.




| | |
|---|---|
| $(RQ)^2 + (PQ)^2 \stackrel{?}{=} (RP)^2$ | Converse of Pythagorean Theorem |
| $4^2 + 5^2 \stackrel{?}{=} 8^2$ | $RQ = 4, PQ = 5, RP = 8$ |
| $41 \neq 64$ | Simplify. |

Because the converse of the Pythagorean Theorem did not prove true in this case, $\triangle RQP$ is not a right triangle.

So, $\overline{PQ}$ is not tangent to $\odot R$.

www.geometryonline.com/extra_examples



# 2 Teach

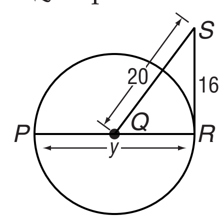
## TANGENTS

### In-Class Examples

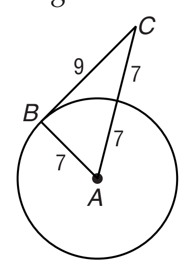
Power Point®

**Teaching Tip** Explain that even though a tangent intersects a circle, there is never any part of a tangent contained inside a circle. The only point that the tangent and the circle have in common is the point of intersection.

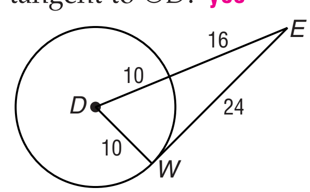
**1** **ALGEBRA** $\overline{RS}$ is tangent to $\odot Q$ at point $R$. Find $y$. **24**




**2** **a.** Determine whether $\overline{BC}$ is tangent to $\odot A$. **no**




**b.** Determine whether $\overline{WE}$ is tangent to $\odot D$. **yes**



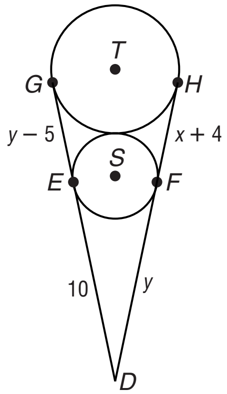

## Geometry Software Investigation

**Tangents and Radii** Tell students that since a radius is perpendicular to a tangent at the point of tangency, the diameter containing that radius is also perpendicular to the tangent at the same point. Students can also repeat the activity for other points of tangency. They can start with a new circle, or you can ask students where they could place another tangent on $\odot W$ that is perpendicular to $\overline{WY}$.

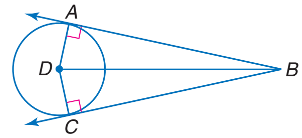
## In-Class Example

Power Point®

**3** **ALGEBRA** Find $x$. Assume that segments that appear tangent to circles are tangent. **1**




More than one line can be tangent to the same circle. In the figure, $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot D$. So, $(AB)^2 + (AD)^2 = (DB)^2$ and $(BC)^2 + (CD)^2 = (DB)^2$.



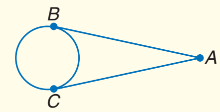

| | |
|---|---|
| $(AB)^2 + (AD)^2 = (BC)^2 + (CD)^2$ | Substitution |
| $(AB)^2 + (AD)^2 = (BC)^2 + (AD)^2$ | $AD = CD$ |
| $(AB)^2 = (BC)^2$ | Subtract $(AD)^2$ from each side. |
| $AB = BC$ | Take the square root of each side. |

The last statement implies that $\overline{AB} \cong \overline{BC}$. This is a proof of Theorem 10.10.

### Theorem 10.11

If two segments from the same exterior point are tangent to a circle, then they are congruent.

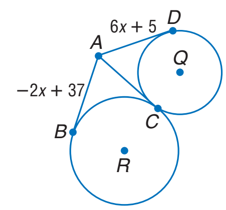
**Example:** $\overline{AB} \cong \overline{AC}$




You will prove this theorem in Exercise 27.

### Example 3 Solve a Problem Involving Tangents

**ALGEBRA** **Find $x$. Assume that segments that appear tangent to circles are tangent.**



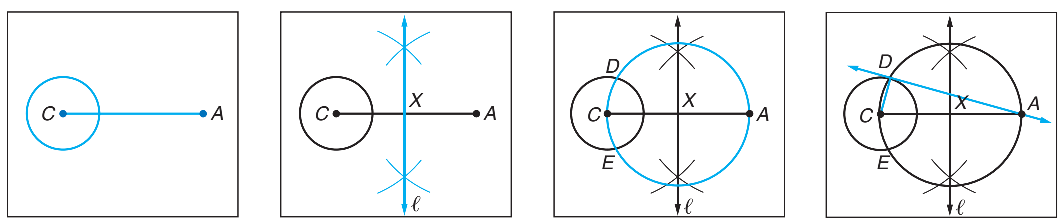

$\overline{AD}$ and $\overline{AC}$ are drawn from the same exterior point and are tangent to $\odot Q$, so $\overline{AD} \cong \overline{AC}$. $\overline{AC}$ and $\overline{AB}$ are drawn from the same exterior point and are tangent to $\odot R$, so $\overline{AC} \cong \overline{AB}$. By the Transitive Property, $\overline{AD} \cong \overline{AB}$.

| | |
|---|---|
| $AD = AB$ | Definition of congruent segments |
| $6x + 5 = -2x + 37$ | Substitution |
| $8x + 5 = 37$ | Add $2x$ to each side. |
| $8x = 32$ | Subtract 5 from each side. |
| $x = 4$ | Divide each side by 8. |

## Construction

### Line Tangent to a Circle Through a Point Exterior to the Circle

1. Construct a circle. Label the center $C$. Draw a point outside $\odot C$. Then draw $\overline{CA}$.
2. Construct the perpendicular bisector of $\overline{CA}$ and label it line $\ell$. Label the intersection of $\ell$ and $\overline{CA}$ as point $X$.
3. Construct circle $X$ with radius $\overline{XC}$. Label the points where the circles intersect as $D$ and $E$.
4. Draw $\overleftrightarrow{AD}$. $\triangle ADC$ is inscribed in a semicircle. So $\angle ADC$ is a right angle, and $\overleftrightarrow{AD}$ is a tangent.




You will construct a line tangent to a circle through a point on the circle in Exercise 21.



DAILY INTERVENTION

## Differentiated Instruction

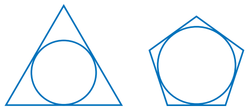
**Kinesthetic** Have students place a compact disc on a sheet of blank paper and trace around it. Then have students place two metric rulers beside the disc to model two tangents. Have students arrange the rulers so that they form two tangents that intersect on the sheet. Then have students lightly draw the two tangents from the circle to the point of intersection and measure the distance from the circle to the point of intersection to confirm that the measures are equal.

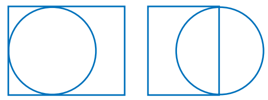
## Study Tip

**Common Misconceptions**

Just because the circle is tangent to one or more of the sides of a polygon does not mean that the polygon is circumscribed about the circle, as shown in the second pair of figures.

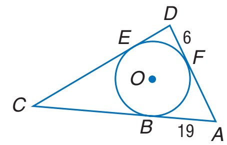
**CIRCUMSCRIBED POLYGONS** In Lesson 10-3, you learned that circles can be circumscribed about a polygon. Likewise, polygons can be circumscribed about a circle, or the circle is inscribed in the polygon. Notice that the vertices of the polygon *do not* lie on the circle, but every side of the polygon is tangent to the circle.


Polygons are circumscribed.


Polygons are *not* circumscribed.

### Example 4 Triangles Circumscribed About a Circle

**Triangle $ADC$ is circumscribed about $\odot O$. Find the perimeter of $\triangle ADC$ if $EC = DE + AF$.**




Use Theorem 10.10 to determine the equal measures.
$AB = AF = 19$, $FD = DE = 6$, and $EC = CB$.
We are given that $EC = DE + AF$, so $EC = 6 + 19$ or 25.

$P = AB + BC + EC + DE + FD + AF$ Definition of perimeter

$= 19 + 25 + 25 + 6 + 6 + 19$ or 100 Substitution

The perimeter of $\triangle ADC$ is 100 units.

## Check for Understanding

**Concept Check**

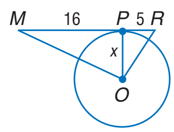
1. **Determine** the number of tangents that can be drawn to a circle for each point. Explain your reasoning. **See margin for reasoning.**
   a. containing a point outside the circle **two**
   b. containing a point inside the circle **none**
   c. containing a point on the circle **one**
2. Write an argument to support or provide a counterexample to the statement *If two lines are tangent to the same circle, they intersect.*
3. **OPEN ENDED** Draw an example of a circumscribed polygon and an example of an inscribed polygon. **See margin.**

**2. If the lines are tangent at the endpoints of a diameter, they are parallel and thus, not intersecting.**

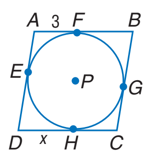
**Guided Practice**

| GUIDED PRACTICE KEY | |
|---|---|
| Exercises | Examples |
| 4 | 1 |
| 5 | 2 |
| 6 | 4 |
| 7 | 3 |

**For Exercises 4 and 5, use the figure at the right.**

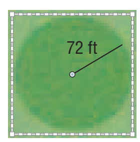



4. Tangent $\overline{MP}$ is drawn to $\odot O$. Find $x$ if $MO = 20$. **12**
5. If $RO = 13$, determine whether $\overline{PR}$ is tangent to $\odot O$. **Yes; $5^2 + 12^2 = 13^2$.**
6. Rhombus $ABCD$ is circumscribed about $\odot P$ and has a perimeter of 32. Find $x$. **5**




**Application**

7. **AGRICULTURE** A pivot-circle irrigation system waters part of a fenced square field. If the spray extends to a distance of 72 feet, what is the total length of the fence around the field? **576 ft**

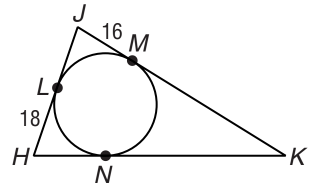





## CIRCUMSCRIBED POLYGONS

**In-Class Example** Power Point®

4 Triangle $HJK$ is circumscribed about $\odot G$. Find the perimeter of $\triangle HJK$ if $NK = JL + 29$.




**158 units**

## Answers

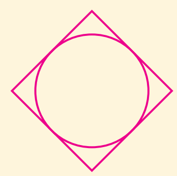
1a. From any point outside the circle, you can draw only two tangents.

1b. A line containing a point inside the circle would intersect the circle in two points. A tangent can only intersect a circle in one point.

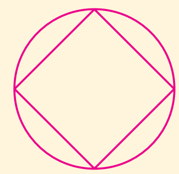
1c. Since a tangent intersects a circle in exactly one point, there is one tangent containing a point on the circle.

3. Sample answer:

polygon circumscribed about a circle:



polygon inscribed in a circle:



# 3 Practice/Apply

## Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include an example of a circumscribed polygon and an example that demonstrates how to use tangents to find segment lengths.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

## About the Exercises…

**Organization by Objective**

- **Tangents:** 8–20
- **Circumscribed Polygons:** 17–18

**Odd/Even Assignments**

Exercises 8–20, 23–26 are structured so that students practice the same concepts whether they are assigned odd or even problems.

*Alert!* Exercise 21 requires a compass and straightedge.

## Assignment Guide

**Basic:** 9–17 odd, 21–31 odd, 32–34 (optional: 35–36), 37–45

**Average:** 9–31 odd, 32–34 (optional: 35–36), 37–45

**Advanced:** 8–30 even, 31–41 (optional: 42–45)

★ indicates increased difficulty

## Practice and Apply

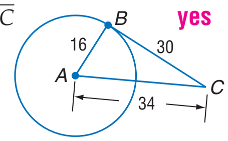
**Homework Help**

| For Exercises | See Examples |
|---|---|
| 8–11 | 2 |
| 12–20 | 1, 3 |
| 17, 18, 23–26 | 4 |

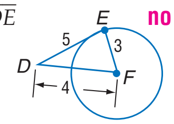
**Extra Practice**
See page 775.

**Determine whether each segment is tangent to the given circle.**

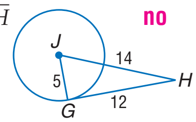
**8.** $\overline{BC}$ **yes**



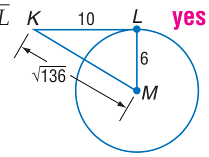

**9.** $\overline{DE}$ **no**




**10.** $\overline{GH}$ **no**

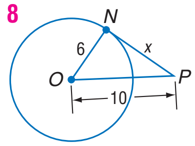



**11.** $\overline{KL}$ **yes**

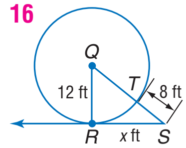



**Find x. Assume that segments that appear to be tangent are tangent.**

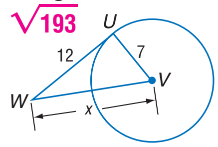
**12.** **8**



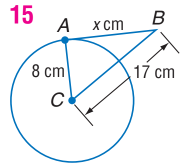

**13.** **16**



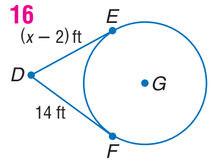

**14.** $\sqrt{193}$



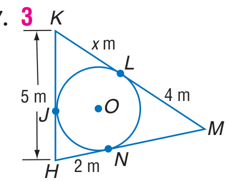

**15.** **15**



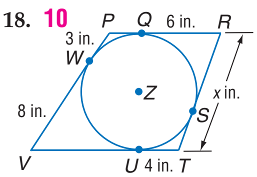

**16.** **16**



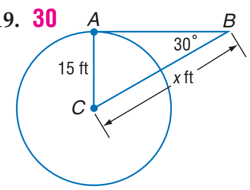

**17.** **3**



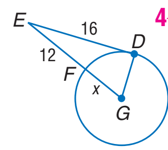

**18.** **10**




★ **19.** **30**




★ **20.** $4\frac{2}{3}$




**Study Tip**

*Look Back*
To review constructing perpendiculars to a line, see Lesson 3-6.

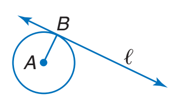
**21.** **CONSTRUCTION** Construct a line tangent to a circle through a point on the circle following these steps. **See students' work.**

- Construct a circle with center $T$.
- Locate a point $P$ on $\odot T$ and draw $\overrightarrow{TP}$.
- Construct a perpendicular to $\overrightarrow{TP}$ through point $P$.

**22.** **PROOF** Write an indirect proof of Theorem 10.10 by assuming that $\ell$ is not tangent to $\odot A$.

**Given:** $\ell \perp \overline{AB}$, $\overline{AB}$ is a radius of $\odot A$.

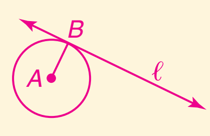
**Prove:** Line $\ell$ is tangent to $\odot A$. **See margin.**






## Answer

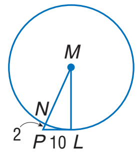
22. Given: $\ell \perp \overline{AB}$
$\overline{AB}$ is a radius of $\odot A$.
Prove: $\ell$ is tangent to $\odot A$.
Proof: Assume $\ell$ is not tangent to $\odot A$. Since $\ell$ intersects $\odot A$ at $B$, it must intersect the circle in another place. Call this point $C$. Then $AB = AC$. But if $\overline{AB} \perp \ell$, then $\overline{AB}$ must be the shortest segment from $A$ to $\ell$. If $AB = AC$, then $\overline{AC}$ is the shortest segment from $A$ to $\ell$. Since $B$ and $C$ are two different points on $\ell$, this is a contradiction. Therefore, $\ell$ is tangent to $\odot A$.



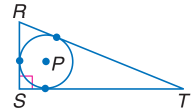

556 Chapter 10 Circles

Find the perimeter of each polygon for the given information.

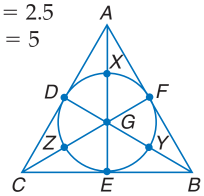
23. **60 units**



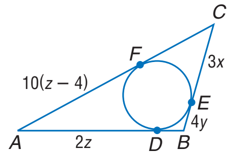

24. $ST = 18$, radius of $\odot P = 5$ **58.5 units**




25. $BY = CZ = AX = 2.5$ diameter of $\odot G = 5$ **$15\sqrt{3}$ units**

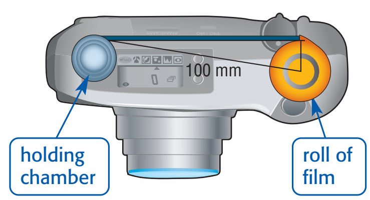



★26. $CF = 6(3 - x)$, $DB = 12y - 4$ **36 units**

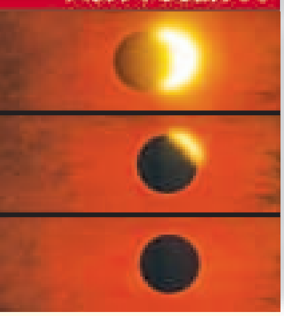



27. **PROOF** Write a two-column proof to show that if two segments from the same exterior point are tangent to a circle, then they are congruent. (Theorem 10.11) **See p. 589B.**

28. **PHOTOGRAPHY** The film in a 35-mm camera unrolls from a cylinder, travels across an opening for exposure, and then is forwarded into another circular chamber as each photograph is taken. The roll of film has a diameter of 25 millimeters, and the distance from the center of the roll to the intake of the chamber is 100 millimeters. To the nearest millimeter, how much of the film would be exposed if the camera were opened before the roll had been totally used? **99 mm**




**More About . . .**



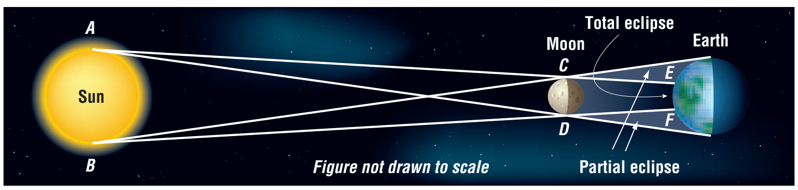
***Astronomy***

During the 20th century, there were 78 total solar eclipses, but only 15 of these affected parts of the United States. The next total solar eclipse visible in the U.S. will be in 2017.

**Source:** *World Almanac*

**ASTRONOMY** **For Exercises 29 and 30, use the following information.**

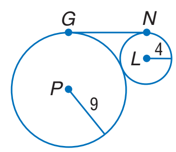
A solar eclipse occurs when the moon blocks the sun's rays from hitting Earth. Some areas of the world will experience a total eclipse, others a partial eclipse, and some no eclipse at all, as shown in the diagram below.




29. The blue section denotes a total eclipse on that part of Earth. Which tangents define the blue area? **$\overline{AE}$ and $\overline{BF}$**

30. The pink areas denote the portion of Earth that will have a partial eclipse. Which tangents define the northern and southern boundaries of the partial eclipse? **$\overline{AD}$ and $\overline{BC}$**

31. **CRITICAL THINKING** Find the measure of tangent $\overline{GN}$. Explain your reasoning. **See p. 589B.**




www.geometryonline.com/self_check_quiz

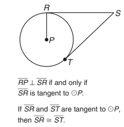


Ray Massey/Getty Images

## Study Guide and Intervention, p. 565 (shown) and p. 566

**Tangents** A tangent to a circle intersects the circle in exactly one point, called the **point of tangency**. There are three important relationships involving tangents.

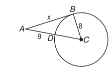
- If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.
- If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent to the circle.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.




$\overline{RP} \perp \overline{SR}$ if and only if $\overline{SR}$ is tangent to $\odot P$.

If $\overline{SR}$ and $\overline{ST}$ are tangent to $\odot P$, then $\overline{SR} \cong \overline{ST}$.

*Example* **$\overline{AB}$ is tangent to $\odot C$. Find $x$.**



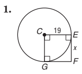

$\overline{AB}$ is tangent to $\odot C$, so $\overline{AB}$ is perpendicular to radius $\overline{BC}$. $\overline{CD}$ is a radius, so $CD = 8$ and $AC = 9 + 8$ or 17. Use the Pythagorean Theorem with right $\triangle ABC$.

| | |
|---|---|
| $(AB)^2 + (BC)^2 = (AC)^2$ | Pythagorean Theorem |
| $x^2 + 8^2 = 17^2$ | Substitution |
| $x^2 + 64 = 289$ | Multiply. |
| $x^2 = 225$ | Subtract 64 from each side. |
| $x = 15$ | Take the square root of each side. |

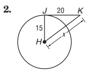
*Exercises*

**Find $x$. Assume that segments that appear to be tangent are tangent.**

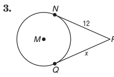
1. **19**



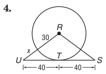
2. **25**



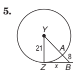
3. **12**



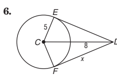
4. **20**



5. **20**



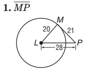
6. **12**



## Skills Practice, p. 567 and Practice, p. 568 (shown)

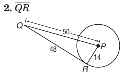
**Determine whether each segment is tangent to the given circle.**

1. $\overline{MP}$



**no**

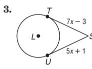
2. $\overline{QR}$



**yes**

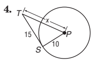
**Find $x$. Assume that segments that appear to be tangent are tangent.**

3.



**2**

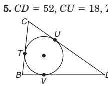
4.



**$5\sqrt{13}$**

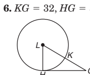
**Find the perimeter of each polygon for the given information. Assume that segments that appear to be tangent are tangent.**

5. $CD = 52$, $CU = 18$, $TB = 12$



**128 units**

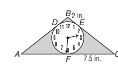
6. $KG = 32$, $HG = 56$



**154 units**

**CLOCKS For Exercises 7 and 8, use the following information.**

The design shown in the figure is that of a circular clock face inscribed in a triangular base. $AF$ and $FC$ are equal.




7. Find $AB$. **9.5 in.**

8. Find the perimeter of the clock. **34 in.**

## Reading to Learn Mathematics, p. 569 ELL

**Pre-Activity** **How are tangents related to track and field events?**

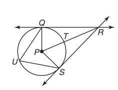
Read the introduction to Lesson 10-5 at the top of page 552 in your textbook.

How is the hammer throw event related to the mathematical concept of a tangent line?

**Sample answer: When the hammer is released, its initial path is a good approximation of a tangent line to the circular path around which it was traveling just before it was released.**

**Reading the Lesson**

1. Refer to the figure. Name each of the following in the figure.




   a. two lines that are tangent to $\odot P$ **$\overleftrightarrow{RQ}$ and $\overleftrightarrow{RS}$**
   b. two points of tangency **Q, S**
   c. two chords of the circle **$\overline{UQ}$ and $\overline{US}$**
   d. three radii of the circle **$\overline{PQ}$, $\overline{PS}$, and $\overline{PT}$**
   e. two right angles **$\angle PQR$ and $\angle PSR$**
   f. two congruent right triangles **$\triangle PQR$ and $\triangle PSR$**
   g. the hypotenuse or hypotenuses in the two congruent right triangles **$\overline{PR}$**
   h. two congruent central angles **$\angle QPT$ and $\angle SPT$**
   i. two congruent minor arcs **$\overset{\frown}{QT}$ and $\overset{\frown}{ST}$**
   j. an inscribed angle **$\angle QUS$**

2. Explain the difference between an *inscribed polygon* and a *circumscribed polygon*. Use the words *vertex* and *tangent* in your explanation.
**Sample answer: If a polygon is *inscribed* in a circle, every vertex of the polygon lies on the circle. If a polygon is *circumscribed* about a circle, every side of the polygon is tangent to the circle.**

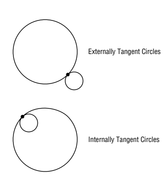
**Helping You Remember**

3. A good way to remember a mathematical term is to relate it to a word or expression that is used in a nonmathematical way. Sometimes a word or expression used in English is derived from a mathematical term. What does it mean to "go off on a tangent," and how is this meaning related to the geometric idea of a *tangent* line?
**Sample answer: To "go off on a tangent" means to suddenly change the subject when you are talking or writing. You can visualize this as being like a tangent line "going off" from a circle as you go farther from the point of tangency.**

## Enrichment, p. 570

***Tangent Circles***

Two circles in the same plane are **tangent circles** if they have exactly one point in common. Tangent circles with no common interior points are **externally tangent**. If tangent circles have common interior points, then they are **internally tangent**. Three or more circles are **mutually tangent** if each pair of them are tangent.




1. Make sketches to show all possible positions of three mutually tangent circles.

## 4 Assess

### Open-Ended Assessment

**Writing** Provide an example on the board with a triangle formed by a tangent, a radius, and the line from the center of the circle to a point on the tangent. Assign lengths to the figure and ask students to write the equation necessary to solve the problem. Have a volunteer write his or her equation on the board, and allow students to check their work. Repeat for the other concepts presented in this lesson.

### *Getting Ready for Lesson 10-6*

**Prerequisite Skill** Students will learn about secants, tangents, and angle measures in Lesson 10-6. They will apply concepts of solving equations to writing proofs and finding values. Use Exercises 42–45 to determine your students' familiarity with solving equations.

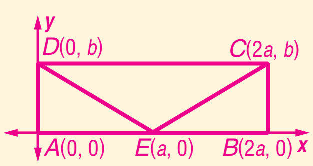
### Answers

**32. Sample answer: Many of the field events have the athlete moving in a circular motion and releasing an object (discus, hammer, shot). The movement of the athlete models a circle and the path of the released object models a tangent. Answers should include the following.**

- **The arm of the thrower, the handle, the wire, and hammer form the radius defining the circle when the hammer is spun around. The tangent is the path of the hammer when it is released.**
- **The distance the hammer was from the athlete was about 70.68 meters.**

**41. Sample answer:**

**Given: *ABCD* is a rectangle.**
**$E$ is the midpoint of $\overline{AB}$.**
**Prove: $\triangle CED$ is isosceles.**




**Proof: Let the coordinates of $E$ be $(a, 0)$. Since $E$ is the midpoint and is halfway between $A$ and $B$, the coordinates of $B$ will be $(2a, 0)$. Let the coordinates of $D$ be $(0, b)$. The coordinates of $C$ will be $(2a, b)$, because it is on the same horizontal as $D$ and the same vertical as $B$.**

$ED = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$EC = \sqrt{(a-2a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

**Since $ED = EC$, $\overline{ED} \cong \overline{EC}$. $\triangle DEC$ has two congruent sides, so it is isosceles.**



---

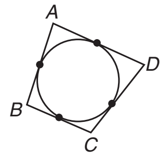
32. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**How are tangents related to track and field events?**

Include the following in your answer:
- how the hammer throw models a tangent, and
- the distance the hammer landed from the athlete if the wire and handle are 1.2 meters long and the athlete's arm is 0.8 meter long.

### *Standardized Test Practice*

33. **GRID IN** $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$ are tangent to a circle. If $AB = 19$, $BC = 6$, and $CD = 14$, find $AD$. **27**

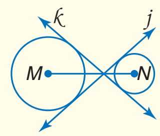
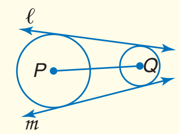



34. **ALGEBRA** Find the mean of all of the numbers from 1 to 1000 that end in 2. **B**

(A) 496 (B) 497
(C) 498 (D) 500

### *Extending the Lesson*

**A line that is tangent to two circles in the same plane is called a *common tangent*.**

| ***Common internal tangents*** intersect the segment connecting the centers. | ***Common external tangents*** do not intersect the segment connecting the centers. |
|---|---|
|  Lines $k$ and $j$ are common internal tangents. |  Lines $\ell$ and $m$ are common external tangents. |

**Refer to the diagram of the eclipse on page 557.**

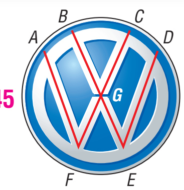
35. Name two common internal tangents. **$\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$**

36. Name two common external tangents. **$\overleftrightarrow{AE}$ and $\overleftrightarrow{BF}$**

## Maintain Your Skills

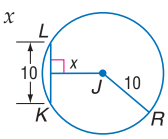
### *Mixed Review*

37. **LOGOS** Circles are often used in logos for commercial products. The logo at the right shows two inscribed angles and two central angles. If $\widehat{AC} \cong \widehat{BD}$, $m\widehat{AF} = 90$, $m\widehat{FE} = 45$, and $m\widehat{ED} = 90$, find $m\angle AFC$ and $m\angle BED$. *(Lesson 10-4)* **45, 45**



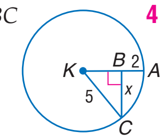

**Find each measure.** *(Lesson 10-3)*

38. $x$

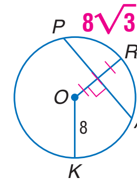



**$5\sqrt{3} \approx 8.7$**

39. $BC$ **4**




40. $AP$ **$8\sqrt{3} \approx 13.9$**




41. **PROOF** Write a coordinate proof to show that if $E$ is the midpoint of $\overline{AB}$ in rectangle $ABCD$, then $\triangle CED$ is isosceles. *(Lesson 8-7)* **See margin.**

### *Getting Ready for the Next Lesson*

**PREREQUISITE SKILL Solve each equation.**
*(To review **solving equations**, see pages 737 and 738.)*

42. $x + 3 = \frac{1}{2}[(4x + 6) - 10]$ **5**

43. $2x - 5 = \frac{1}{2}[(3x + 16) - 20]$ **6**

44. $2x + 4 = \frac{1}{2}[(x + 20) - 10]$ **$\frac{2}{3}$**

45. $x + 3 = \frac{1}{2}[(4x + 10) - 45]$ **20.5**

# Geometry Activity

A Follow-Up of Lessons 10-4 and 10-5

# Inscribed and Circumscribed Triangles

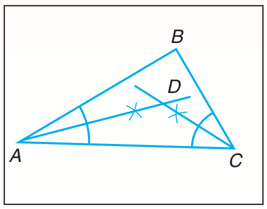
In Lesson 5-1, you learned that there are special points of concurrency in a triangle. Two of these will be used in these activities.

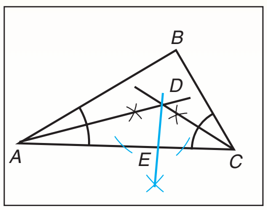
- The *incenter* is the point at which the angle bisectors meet. It is equidistant from the sides of the triangle.
- The *circumcenter* is the point at which the perpendicular bisectors of the sides intersect. It is equidistant from the vertices of the triangle.

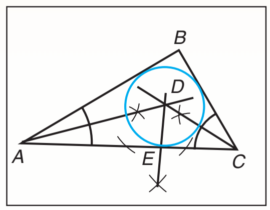
## Activity 1

**Construct a circle inscribed in a triangle.** *The triangle is circumscribed about the circle.*

1. Draw a triangle and label its vertices *A*, *B*, and *C*. Construct two angle bisectors of the triangle to locate the incenter. Label it *D*.
2. Construct a segment perpendicular to a side of $\triangle ABC$ through the incenter. Label the intersection *E*.
3. Use the compass to measure *DE*. Then put the point of the compass on *D*, and draw a circle with that radius.



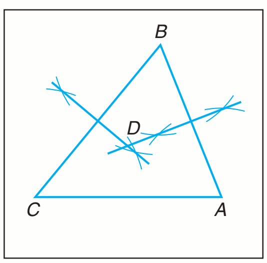



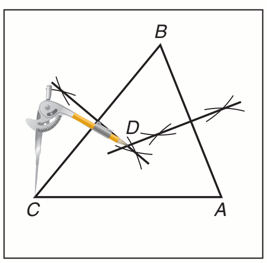



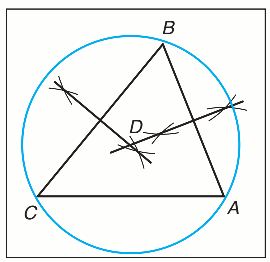

## Activity 2

**Construct a circle through any three noncollinear points.**
*This construction may be referred to as circumscribing a circle about a triangle.*

1. Draw a triangle and label its vertices *A*, *B*, and *C*. Construct perpendicular bisectors of two sides of the triangle to locate the circumcenter. Label it *D*.
2. Use the compass to measure the distance from the circumcenter *D* to any of the three vertices.
3. Using that setting, place the compass point at *D*, and draw a circle about the triangle.




*(continued on the next page)*



# Geometry Activity

**A Follow-Up of Lessons 10-4 and 10-5**

## Getting Started

Explain that students will use the incenter of a triangle to construct a circle so that the triangle is circumscribed about the circle, and they will use the circumcenter of a triangle to construct a circle in which the triangle is inscribed. They will also learn how to construct an equilateral triangle circumscribed about a circle.

**Objective** To construct inscribed and circumscribed triangles.

**Materials**
straightedge, compass, pencil, paper

## Teach

- For Activity 1, have students draw an acute triangle. Have them use an acute triangle or obtuse triangle for Activity 2.
- In Activity 1, remind students that they only need two angle bisectors to locate the incenter because by definition, students know that the third angle bisector would pass through the same point.
- Explain that students must construct the angle bisectors and perpendicular bisectors for these activities because they need very accurate positions for the incenter and circumcenter in order to complete the activities successfully.
- In Activity 3, tell students that the radius is the setting needed to construct six congruent arcs in a circle.

## Resource Manager

***Teaching Geometry with Manipulatives***
- p. 174 (student recording sheet)

***Glencoe Mathematics Classroom Manipulative Kit***
- compasses
- rulers

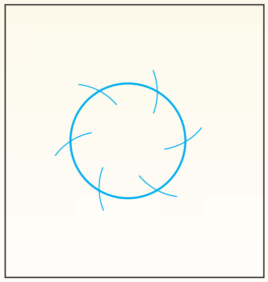
# Geometry Activity

For the next activity, refer to the construction of an inscribed regular hexagon on page 542.

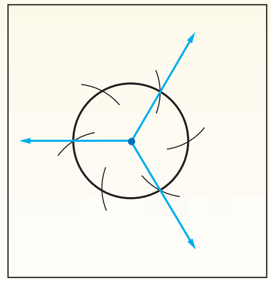
## Activity 3

**Construct an equilateral triangle circumscribed about a circle.**

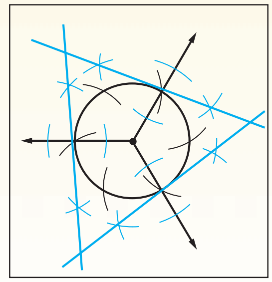
1. Construct a circle and divide it into six congruent arcs.



2. Place a point at every other arc. Draw rays from the center through these points.



3. Construct a line perpendicular to each of the rays through the points.



### Model

1. Draw an obtuse triangle and inscribe a circle in it. **1–3. See students' work.**
2. Draw a right triangle and circumscribe a circle about it.
3. Draw a circle of any size and circumscribe an equilateral triangle about it.

### Analyze

**Refer to Activity 1.** **4–5. See margin.**

4. Why do you only have to construct the perpendicular to one side of the triangle?
5. How can you use the Incenter Theorem to explain why this construction is valid?

**Refer to Activity 2.** **6–7. See margin.**

6. Why do you only have to measure the distance from the circumcenter to any one vertex?
7. How can you use the Circumcenter Theorem to explain why this construction is valid?

**Refer to Activity 3.**

8. What is the measure of each of the six congruent arcs? **60**
9. Write a convincing argument as to why the lines constructed in Step 3 form an equilateral triangle. **See margin.**
10. Why do you think the terms *incenter* and *circumcenter* are good choices for the points they define? **See margin.**



## Assess

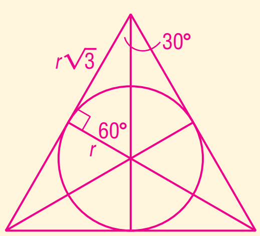
**Exercises 1–3** lead students to repeat the activities for different types of triangles and practice the constructions. Students draw upon their knowledge of circles and use **Exercises 4–10** to analyze their constructions and form a conjecture about the terms *incenter* and *circumcenter*.

### Study Notebook

Ask students to summarize what they have learned about inscribed and circumscribed triangles and the terms incenter and circumcenter.

### Answers

4. The incenter is equidistant from each side. The perpendicular to one side should be the same length as it is to the other two sides.
5. The incenter is equidistant from all the sides. The radius of the circle is perpendicular to the tangent sides and all radii are congruent, matching the distance from the incenter to the sides.
6. The circumcenter is equidistant from all three vertices, so the distance from the circumcenter to one vertex is the same as the distance to each of the others.
7. The circumcenter is equidistant from the vertices and all of the vertices must lie on the circle. So, this distance is the radius of the circle containing the vertices.
9. Suppose all six radii are drawn. Each central angle measures 60°. Thus, six 30°-60°-90° triangles are formed. Each triangle has a side which is a radius *r* units long. Using 30°-60°-90° side ratios, the segment tangent to the circle has length $r\sqrt{3}$, making each side of the circumscribed triangle $2r\sqrt{3}$. If all three sides have the same measure, then the triangle is equilateral.




10. The incenter is the point from which you can construct a circle "in" the triangle. Circum means *around*. So the circumcenter is the point from which you can construct a circle "around" the triangle.

# 10-6 Secants, Tangents, and Angle Measures

## What You'll Learn

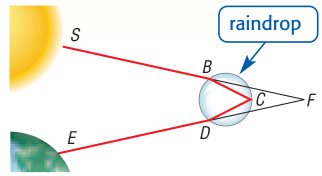
- Find measures of angles formed by lines intersecting on or inside a circle.
- Find measures of angles formed by lines intersecting outside the circle.

**Vocabulary**

- secant

## How is a rainbow formed by segments of a circle?

Droplets of water in the air refract or bend sunlight as it passes through them, creating a rainbow. The various angles of refraction result in an arch of colors. In the figure, the sunlight from point $S$ enters the raindrop at $B$ and is bent. The light proceeds to the back of the raindrop, and is reflected at $C$ to leave the raindrop at point $D$ heading to Earth. Angle $F$ represents the measure of how the resulting ray of light deviates from its original path.



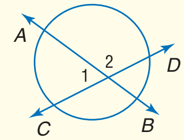

**INTERSECTIONS ON OR INSIDE A CIRCLE** A line that intersects a circle in exactly two points is called a **secant**. In the figure above, $\overline{SF}$ and $\overline{EF}$ are secants of the circle. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.

### Theorem 10.12

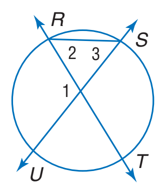
If two secants intersect in the interior of a circle, then the measure of an angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.

**Examples:** $m\angle 1 = \frac{1}{2}(m\widehat{AC} + m\widehat{BD})$

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$




### Proof — Theorem 10.12




**Given:** secants $\overleftrightarrow{RT}$ and $\overleftrightarrow{SU}$

**Prove:** $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$

Draw $\overline{RS}$. Label $\angle TRS$ as $\angle 2$ and $\angle USR$ as $\angle 3$.

**Proof:**

| Statements | Reasons |
|---|---|
| 1. $m\angle 1 = m\angle 2 + m\angle 3$ | 1. Exterior Angle Theorem |
| 2. $m\angle 2 = \frac{1}{2}m\widehat{ST}$, $m\angle 3 = \frac{1}{2}m\widehat{RU}$ | 2. The measure of inscribed $\angle$ = half the measure of the intercepted arc. |
| 3. $m\angle 1 = \frac{1}{2}m\widehat{ST} + \frac{1}{2}m\widehat{RU}$ | 3. Substitution |
| 4. $m\angle 1 = \frac{1}{2}(m\widehat{ST} + m\widehat{RU})$ | 4. Distributive Property |



# 10-6 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-6** Use as a quiz or review of Lesson 10-5.

**Mathematical Background** notes are available for this lesson on p. 520D.

**How is a rainbow formed by segments of a circle?**

*Ask students:*

- If you were to connect $B$ and $D$ in the figure, what would you have? **a triangle inscribed in the circle**
- Name some situations that allow you to see a rainbow formed by segments of a circle. **Sample answers: a rainy, misty day when the sun is low in the sky; a bright sunny day when you are spraying a mist of water from a hose**

## Resource Manager

### Workbook and Reproducible Masters

***Chapter 10 Resource Masters***

- Study Guide and Intervention, pp. 571–572
- Skills Practice, p. 573
- Practice, p. 574
- Reading to Learn Mathematics, p. 575
- Enrichment, p. 576
- Assessment, p. 604

***Prerequisite Skills Workbook,*** pp. 17–18

***Teaching Geometry With Manipulatives Masters,*** p. 17

### Transparencies

5-Minute Check Transparency 10-6

Answer Key Transparencies

### Technology

Interactive Chalkboard

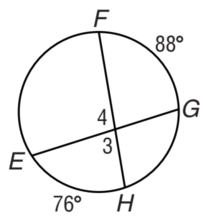
# 2 Teach

## INTERSECTIONS ON OR INSIDE A CIRCLE

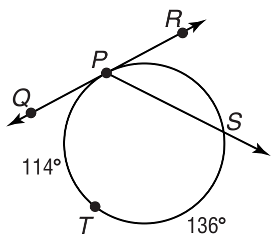
### In-Class Examples

Power Point®

**1** Find $m\angle 4$ if $m\overset{\frown}{FG} = 88$ and $m\overset{\frown}{EH} = 76$. **98**




**2** Find $m\angle RPS$ if $m\overset{\frown}{PT} = 114$ and $m\overset{\frown}{TS} = 136$. **55**

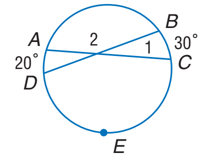



### Tips for New Teachers

Some students may ask you what the difference is between chords and secants and why there are two names for something that intersects a circle at two points. You may want to review how segments are parts of lines and explain that chords are segments of secants, which are lines that intersect circles. Tell students that every chord lies on a secant and every secant contains a chord.

## Example 1 Secant-Secant Angle

**Find $m\angle 2$ if $m\overset{\frown}{BC} = 30$ and $m\overset{\frown}{AD} = 20$.**




**Method 1**

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$

$= \frac{1}{2}(30 + 20)$ or $25$ Substitution

$m\angle 2 = 180 - m\angle 1$

$= 180 - 25$ or $155$

**Method 2**

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DEC})$

Find $m\overset{\frown}{AB} + m\overset{\frown}{DEC}$.

$m\overset{\frown}{AB} + m\overset{\frown}{DEC} = 360 - (m\overset{\frown}{BC} + m\overset{\frown}{AD})$

$= 360 - (30 + 20)$

$= 360 - 50$ or $310$

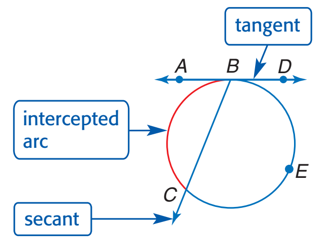
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DEC})$

$= \frac{1}{2}(310)$ or $155$

A secant can also intersect a tangent at the point of tangency. Angle $ABC$ intercepts $\overset{\frown}{BC}$, and $\angle DBC$ intercepts $\overset{\frown}{BEC}$. Each angle formed has a measure half that of the arc it intercepts.

$m\angle ABC = \frac{1}{2}m\overset{\frown}{BC}$ $\quad m\angle DBC = \frac{1}{2}m\overset{\frown}{BEC}$

This is stated formally in Theorem 10.13.



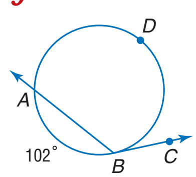

### Theorem 10.13

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

You will prove this theorem in Exercise 43.

## Example 2 Secant-Tangent Angle

**Find $m\angle ABC$ if $m\overset{\frown}{AB} = 102$.**




$m\overset{\frown}{ADB} = 360 - m\overset{\frown}{AB}$

$= 360 - 102$ or $258$

$m\angle ABC = \frac{1}{2}m\overset{\frown}{ADC}$

$= \frac{1}{2}(258)$ or $129$



### DAILY INTERVENTION — Differentiated Instruction

**Naturalist** Explain that the relationships presented in this chapter are naturally occurring relationships that have been mathematically defined and explained. Tell students that scientists from all fields can use these relationships to examine everything from raindrops and soap bubbles to cells and microorganisms.

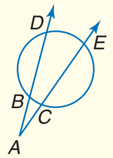
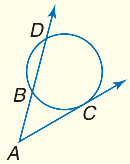
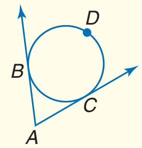
**INTERSECTIONS OUTSIDE A CIRCLE** Secants and tangents can also meet outside a circle. The measure of the angle formed also involves half of the measures of the arcs they intercept.

**Study Tip**

***Absolute Value***
The measure of each $\angle A$ can also be expressed as one-half the absolute value of the difference of the arc measures. In this way, the order of the arc measures does not affect the outcome of the calculation.

## Theorem 10.14

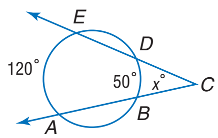
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one-half the positive difference of the measures of the intercepted arcs.

| Two Secants | Secant-Tangent | Two Tangents |
|---|---|---|
|  |  |  |
| $m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$ | $m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$ | $m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$ |

You will prove this theorem in Exercise 40.

### Example 3 Secant-Secant Angle

**Find *x*.**



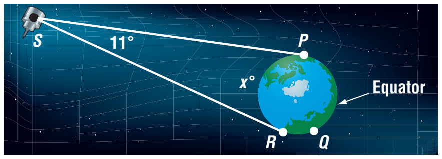

$m\angle C = \frac{1}{2}(m\widehat{EA} - m\widehat{DB})$

$x = \frac{1}{2}(120 - 50)$ Substitution

$x = \frac{1}{2}(70)$ or 35 Simplify.

### Example 4 Tangent-Tangent Angle

**SATELLITES** **Suppose a geostationary satellite *S* orbits about 35,000 kilometers above Earth rotating so that it appears to hover directly over the equator. Use the figure to determine the arc measure on the equator visible to this geostationary satellite.**




$\widehat{PR}$ represents the arc along the equator visible to the satellite *S*. If $x = m\widehat{PR}$, then $m\widehat{PQR} = 360 - x$. Use the measure of the given angle to find $m\widehat{PR}$.

$m\angle S = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$

$11 = \frac{1}{2}[(360 - x) - x]$ Substitution

$22 = 360 - 2x$ Multiply each side by 2 and simplify.

$-338 = -2x$ Subtract 360 from each side.

$169 = x$ Divide each side by $-2$.

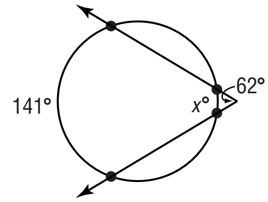
The measure of the arc on Earth visible to the satellite is 169.



## INTERSECTIONS OUTSIDE A CIRCLE

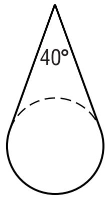
**In-Class Examples** Power Point®

**3** Find *x*. **17**




***Teaching Tip*** Remind students that a semicircle is formed when a chord passes through the center of the circle to make a diameter. Explain that students should always examine a figure for known information that might not be labeled.

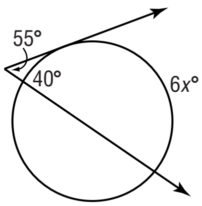
**4** **JEWELRY** A jeweler wants to craft a pendant with the shape shown. Use the figure to determine the measure of the arc at the bottom of the pendant. **220**



## In-Class Example

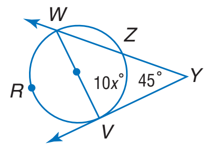
Power Point®

**5** Find $x$. **25**




## Example 5 Secant-Tangent Angle

**Find $x$.**




$\overset{\frown}{WRV}$ is a semicircle because $\overline{WV}$ is a diameter.

So, $m\overset{\frown}{WRV} = 180$.

$m\angle Y = \frac{1}{2}(m\overset{\frown}{WV} - m\overset{\frown}{ZV})$

$45 = \frac{1}{2}(180 - 10x)$ Substitution

$90 = 180 - 10x$ Multiply each side by 2.

$-90 = -10x$ Subtract 180 from each side.

$9 = x$ Divide each side by $-10$.

## 3 Practice/Apply

### Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include examples to demonstrate the relationships described in the theorems introduced in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

### About the Exercises...

**Organization by Objective**

- **Intersections on or Inside a Circle:** 12–19
- **Intersections Outside a Circle:** 21–32

**Odd/Even Assignments**

Exercises 12–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

### Assignment Guide

**Basic:** 13, 17–29 odd, 33–45 odd, 46–59

**Average:** 13–45 odd, 46–59

**Advanced:** 12–40 even, 41, 42, 44, 45–56 (optional: 57–59)

**All:** Quiz 2 (1–5)

## Check for Understanding

*Concept Check*

1. **Describe** the difference between a secant and a tangent. **1–2. See margin.**
2. **OPEN ENDED** Draw a circle and one of its diameters. Call the diameter $\overline{AC}$. Draw a line tangent to the circle at $A$. What type of angle is formed by the tangent and the diameter? Explain.

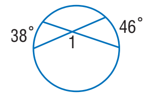
*Guided Practice*

**GUIDED PRACTICE KEY**

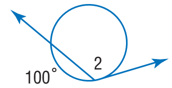
| Exercises | Examples |
|---|---|
| 3 | 1 |
| 4, 8 | 2 |
| 5 | 3 |
| 6 | 4 |
| 7 | 5 |

**Find each measure.**

3. $m\angle 1$ **138**

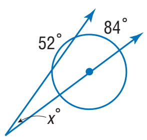



4. $m\angle 2$ **130**

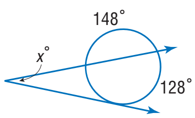



**Find $x$.**

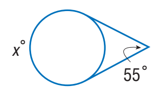
5. **20**




6. **22**



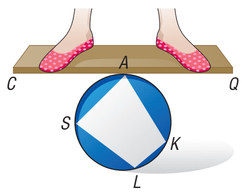

7. **235**




*Application*

**CIRCUS** **For Exercises 8–11, refer to the figure and the information below.**

One of the acrobatic acts in the circus requires the artist to balance on a board that is placed on a round drum as shown at the right. Find each measure if $\overline{SA} \parallel \overline{LK}$, $m\angle SLK = 78$, and $m\overset{\frown}{SA} = 46$.



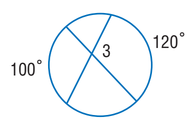

8. $m\angle CAS$ **23**
9. $m\angle QAK$ **55**
10. $m\overset{\frown}{KL}$ **94**
11. $m\overset{\frown}{SL}$ **110**

★ indicates increased difficulty

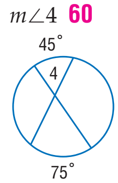
## Practice and Apply

**Find each measure.**

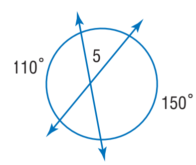
12. $m\angle 3$ **110**




13. $m\angle 4$ **60**




14. $m\angle 5$ **50**

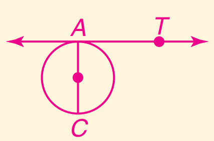





## Answers

1. Sample answer: A tangent intersects the circle in only one point and no part of the tangent is in the interior of the circle. A secant intersects the circle in two points and some of its points do lie in the interior of the circle.

2. Sample answer: $\angle TAC$ is a right angle; There are two reasons: (1) If the point of tangency is the endpoint of a diameter, then the tangent is perpendicular to the diameter at that point. (2) The arc intercepted by the secant (diameter) and the tangent is a semicircle. Thus the measure of the angle is half of 180 or 90.

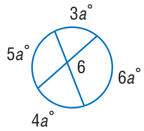


## Homework Help

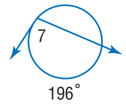
| For Exercises | See Examples |
|---|---|
| 12–15 | 1 |
| 16–19 | 2 |
| 21–24, 31 | 3 |
| 25–28, 32 | 5 |
| 29–30 | 4 |

**Extra Practice**
See page 775.

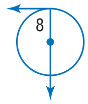
★ **15.** $m\angle 6$ **110**



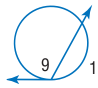

**16.** $m\angle 7$ **98**



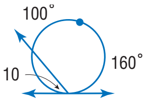

**17.** $m\angle 8$ **90**



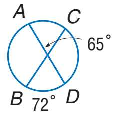

**18.** $m\angle 9$ **120**




**19.** $m\angle 10$ **50**

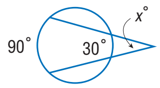



**20.** $m\widehat{AC}$ **58**

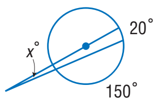



**Find *x*. Assume that any segment that appears to be tangent is tangent.**

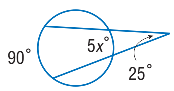
**21.** **30**



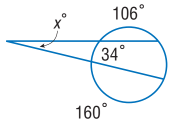

**22.** **5**



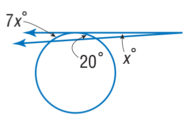

**23.** **8**



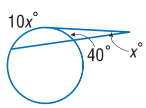

**24.** **13**



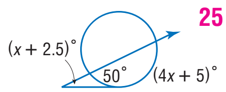

**25.** **4**



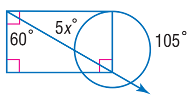

**26.** **5**



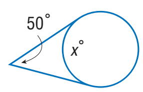

**27.** **25**



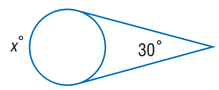

★ **28.** **9**



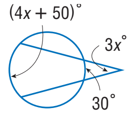

**29.** **130**



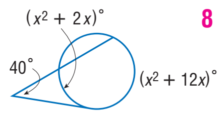

**30.** **210**



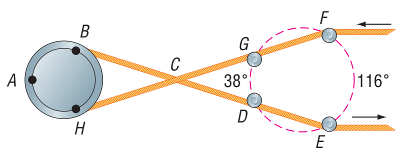

★ **31.** **10**




★ **32.** **8**




**33. WEAVING** Once yarn is woven from wool fibers, it is often dyed and then threaded along a path of pulleys to dry. One set of pulleys is shown below. Note that the yarn appears to intersect itself at *C*, but in reality it does not. Use the information from the diagram to find $m\widehat{BH}$. **141**

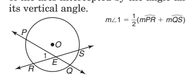





## Study Guide and Intervention, p. 571 (shown) and p. 572

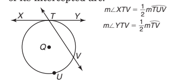
**Intersections On or Inside a Circle** A line that intersects a circle in exactly two points is called a **secant**. The measures of angles formed by secants and tangents are related to intercepted arcs.

- If two secants intersect in the interior of a circle, then the measure of the angle formed is one-half the sum of the measure of the arcs intercepted by the angle and its vertical angle.




$m\angle 1 = \frac{1}{2}(m\widehat{PR} + m\widehat{QS})$

- If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.

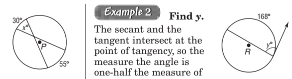



$m\angle XTV = \frac{1}{2}m\widehat{TUV}$

$m\angle YTV = \frac{1}{2}m\widehat{TV}$

**Example 1** **Find *x*.**

The two secants intersect inside the circle, so *x* is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.




$x = \frac{1}{2}(30 + 55)$
$= \frac{1}{2}(85)$
$= 42.5$

**Example 2** **Find *y*.**

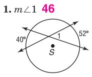
The secant and the tangent intersect at the point of tangency, so the measure the angle is one-half the measure of its intercepted arc.

$y = \frac{1}{2}(168)$
$= 84$

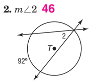
**Exercises**

**Find each measure.**

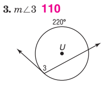
**1.** $m\angle 1$ **46**



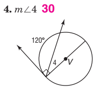
**2.** $m\angle 2$ **46**



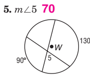
**3.** $m\angle 3$ **110**



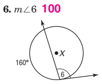
**4.** $m\angle 4$ **30**



**5.** $m\angle 5$ **70**



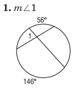
**6.** $m\angle 6$ **100**



## Skills Practice, p. 573 and Practice, p. 574 (shown)

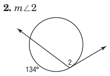
**Find each measure.**

**1.** $m\angle 1$



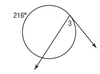
**79**

**2.** $m\angle 2$



**113**

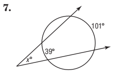
**3.** $m\angle 3$



**72**

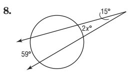
**Find *x*. Assume that any segment that appears to be tangent is tangent.**

**7.**



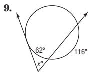
**31**

**8.**



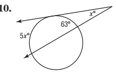
**14.5**

**9.**



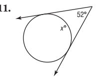
**60**

**10.**



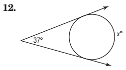

**21**

**11.**



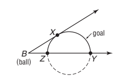

**128**

**12.**




**217**

**9. RECREATION** In a game of kickball, Rickie has to kick the ball through a semicircular goal to score. If $m\widehat{XZ} = 58$ and the $m\widehat{XY} = 122$, at what angle must Rickie kick the ball to score? Explain.




**Rickie must kick the ball at an angle less than 32° since the measure of the angle from the ground that a tangent would make with the goal post is 32°.**

## Reading to Learn Mathematics, p. 575 ELL

**Pre-Activity** **How is a rainbow formed by segments of a circle?**

Read the introduction to Lesson 10-6 at the top of page 561 in your textbook.

- How would you describe $\angle C$ in the figure in your textbook? **Sample answer: $\angle C$ is an inscribed angle in the circle that represents the raindrop.**
- When you see a rainbow, where is the sun in relation to the circle of which the rainbow is an arc? **Sample answer: behind you and opposite the center of the circle**

### Reading the Lesson

**1.** Underline the correct word to form a true statement.

**a.** A line can intersect a circle in at most (one/two/three) points.

**b.** A line that intersects a circle in exactly two points is called a (tangent/secant/radius).

**c.** A line that intersects a circle in exactly one point is called a (tangent/secant/radius).

**d.** Every secant of a circle contains a (radius/tangent/chord).

**2.** Determine whether each statement is *always*, *sometimes*, or *never* true.

**a.** A secant of a circle passes through the center of the circle. **sometimes**

**b.** A tangent to a circle passes through the center of the circle. **never**

**c.** A secant-secant angle is a central angle of the circle. **sometimes**

**d.** A vertex of a secant-tangent angle is a point on the circle. **sometimes**

**e.** A secant-tangent angle passes through the center of the circle. **sometimes**

**f.** The vertex of a tangent-tangent angle is a point on the circle. **never**

**g.** If one side of a secant-tangent angle passes through the center of the circle, the angle is a right angle. **always**

**h.** The measure of a secant-secant angle is one-half the positive difference of the measures of its intercepted arcs. **sometimes**

**i.** The sum of the measures of the arcs intercepted by a tangent-tangent angle is 360. **always**

**j.** The two arcs intercepted by a tangent-tangent angle are congruent. **never**

### Helping You Remember

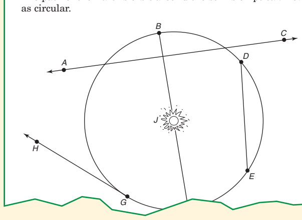
**4.** Some students have trouble remembering the difference between a *secant* and a *tangent*. What is an easy way to remember which is which?

**Sample answer: A secant cuts a circle, while a tangent just touches it at one point. You can associate *tangent* with *touches* because they both start with *t*. Then associate *secant* with *cuts*.**

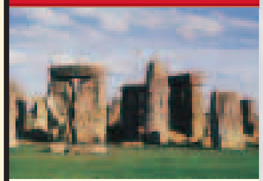
## Enrichment, p. 576

***Orbiting Bodies***

The path of the Earth's orbit around the sun is elliptical. However, it is often viewed as circular.



**More About . . .**

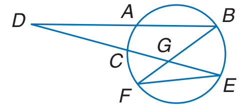


**Landmarks**

Stonehenge is located in southern England near Salisbury. In its final form, Stonehenge included 30 upright stones about 18 feet tall by 7 feet thick.

**Source:** *World Book Encyclopedia*

**Find each measure if $m\widehat{FE} = 118$, $m\widehat{AB} = 108$, $m\angle EGB = 52$, and $m\angle EFB = 30$.**

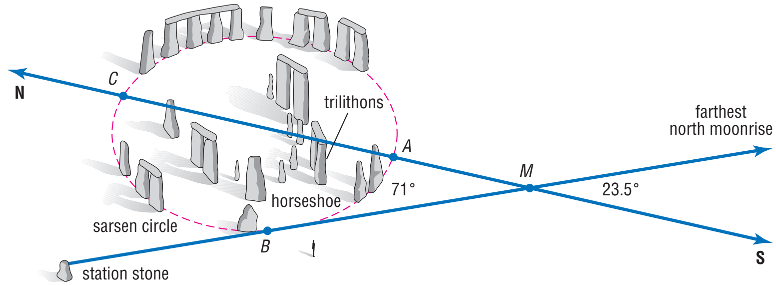



**34.** $m\widehat{AC}$ **30**

**35.** $m\widehat{CF}$ **44**

**36.** $m\angle EDB$ **15**

**LANDMARKS** **For Exercises 37–39, use the following information.**
Stonehenge is a British landmark made of huge stones arranged in a circular pattern that reflects the movements of Earth and the moon. The diagram shows that the angle formed by the north/south axis and the line aligned from the station stone to the northmost moonrise position measures 23.5°.



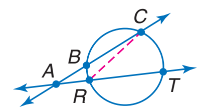

**37.** Find $m\widehat{BC}$. **118**

**38.** Is $\widehat{ABC}$ a semicircle? Explain. **No, its measure is 189.**

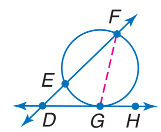
**39.** If the circle measures about 100 feet across, approximately how far would you walk around the circle from point $B$ to point $C$? **about 103 ft**

**40.** **PROOF** Write a two-column proof of Theorem 10.14. Consider each case.

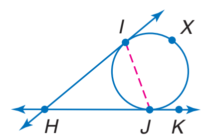
**a.** Case 1: Two Secants **See margin.**
**Given:** $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle.
**Prove:** $m\angle CAT = \frac{1}{2}(m\widehat{CT} - m\widehat{BR})$



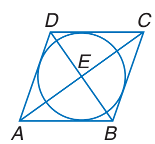

**b.** Case 2: Secant and a Tangent **See margin.**
**Given:** $\overleftrightarrow{DG}$ is a tangent to the circle. $\overleftrightarrow{DF}$ is a secant to the circle.
**Prove:** $m\angle FDG = \frac{1}{2}(m\widehat{FG} - m\widehat{GE})$




**c.** Case 3: Two Tangents **See margin.**
**Given:** $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle.
**Prove:** $m\angle IHJ = \frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ})$



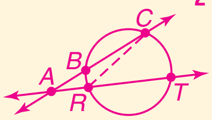

**41.** **CRITICAL THINKING** Circle $E$ is inscribed in rhombus $ABCD$. The diagonals of the rhombus are 10 centimeters and 24 centimeters long. To the nearest tenth centimeter, how long is the radius of circle $E$? (*Hint:* Draw an altitude from $E$.) **4.6 cm**






## Answers

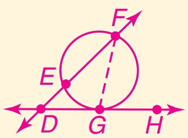
40a. Given: $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle.
Prove: $m\angle CAT = \frac{1}{2}(m\widehat{CT} - m\widehat{BR})$




**Statements (Reasons)**

1. $\overleftrightarrow{AC}$ and $\overleftrightarrow{AT}$ are secants to the circle. (Given)
2. $m\angle CRT = \frac{1}{2}m\widehat{CT}$, $m\angle ACR = \frac{1}{2}m\widehat{BR}$ (The meas. of an inscribed $\angle$ = $\frac{1}{2}$ the meas. of its intercepted arc.)
3. $m\angle CRT = m\angle ACR + m\angle CAT$ (Exterior $\angle$ Theorem)
4. $\frac{1}{2}m\widehat{CT} = \frac{1}{2}m\widehat{BR} + m\angle CAT$ (Substitution)
5. $\frac{1}{2}m\widehat{CT} - \frac{1}{2}m\widehat{BR} = m\angle CAT$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{CT} - m\widehat{BR}) = m\angle CAT$ (Distributive Prop.)

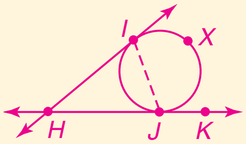
40b. Given: $\overleftrightarrow{DG}$ is a tangent to the circle. $\overleftrightarrow{DF}$ is a secant to the circle.
Prove: $m\angle FDG = \frac{1}{2}(m\widehat{FG} - m\widehat{GE})$




**Statements (Reasons)**

1. $\overleftrightarrow{DG}$ is a tangent to the circle. $\overleftrightarrow{DF}$ is a secant to the circle. (Given)
2. $m\angle DFG = \frac{1}{2}m\widehat{GE}$, $m\angle FGH = \frac{1}{2}m\widehat{FG}$ (The meas. of an inscribed $\angle$ = $\frac{1}{2}$ the meas. of its intercepted arc.)
3. $m\angle FGH = m\angle DFG + m\angle FDG$ (Exterior $\angle$ Theorem)
4. $\frac{1}{2}m\widehat{FG} = \frac{1}{2}m\widehat{GE} + m\angle FDG$ (Substitution)
5. $\frac{1}{2}m\widehat{FG} - \frac{1}{2}m\widehat{GE} = m\angle FDG$ (Subtraction Prop.)
6. $\frac{1}{2}(m\widehat{FG} - m\widehat{GE}) = m\angle FDG$ (Distributive Prop.)

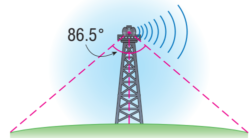
40c. Given: $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle.
Prove: $m\angle IHJ = \frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ})$




**Statements (Reasons)**

1. $\overleftrightarrow{HI}$ and $\overleftrightarrow{HJ}$ are tangents to the circle. (Given)
2. $m\angle IJK = \frac{1}{2}m\widehat{IXJ}$, $m\angle HIJ = \frac{1}{2}m\widehat{IJ}$ (The measure of a secant-tangent $\angle$ = $\frac{1}{2}$ the measure of its intercepted arc.)
3. $m\angle IJK = m\angle HIJ + m\angle IHJ$ (Ext. $\angle$ Th.)
4. $\frac{1}{2}m\widehat{IXJ} = \frac{1}{2}m\widehat{IJ} + m\angle IHJ$ (Substitution)
5. $\frac{1}{2}m\widehat{IXJ} - \frac{1}{2}m\widehat{IJ} = m\angle IHJ$ (Subtr. Prop.)
6. $\frac{1}{2}(m\widehat{IXJ} - m\widehat{IJ}) = m\angle IHJ$ (Distrib. Prop.)

42. **TELECOMMUNICATION** The signal from a telecommunication tower follows a ray that has its endpoint on the tower and is tangent to Earth. Suppose a tower is located at sea level as shown in the figure. Determine the measure of the arc intercepted by the two tangents. **93.5**

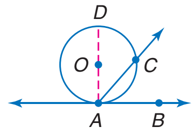


*Note:* Art not drawn to scale

43. **PROOF** Write a paragraph proof of Theorem 10.13

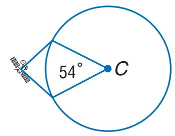
a. **Given:** $\overleftrightarrow{AB}$ is a tangent of $\odot O$.
$\overrightarrow{AC}$ is a secant of $\odot O$.
$\angle CAB$ is acute.

**Prove:** $m\angle CAB = \frac{1}{2}m\widehat{CA}$

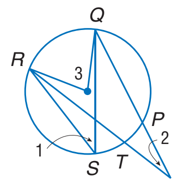



b. Prove Theorem 10.13 if the angle in part **a** is obtuse. **See margin.**

44. **SATELLITES** A satellite is orbiting so that it maintains a constant altitude above the equator. The camera on the satellite can detect an arc of 6000 kilometers on Earth's surface. This arc measures 54°. What is the measure of the angle of view of the camera located on the satellite? **126**




45. **CRITICAL THINKING** In the figure, ∠3 is a central angle. List the numbered angles in order from greatest measure to least measure. Explain your reasoning.




46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**How is a rainbow formed by segments of a circle?**

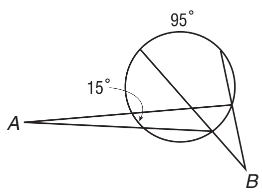
Include the following in your answer:

- the types of segments represented in the figure on page 561, and
- how you would calculate the angle representing how the light deviates from its original path.

**Standardized Test Practice**

47. What is the measure of ∠B if $m\angle A = 10$? **A**

(A) 30 (B) 35
(C) 47.5 (D) 90




48. **ALGEBRA** Which of the following sets of data can be represented by a linear equation? **C**

(A)

| x | y |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

(B)

| x | y |
|---|---|
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 4 |

(C)

| x | y |
|---|---|
| 2 | 2 |
| 4 | 3 |
| 6 | 4 |
| 8 | 5 |

(D)

| x | y |
|---|---|
| 1 | 1 |
| 3 | 9 |
| 5 | 25 |
| 7 | 49 |



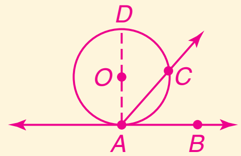
**45.** ∠3, ∠1, ∠2; $m\angle 3 = m\widehat{RQ}$, $m\angle 1 = \frac{1}{2}m\widehat{RQ}$ so $m\angle 3 > m\angle 1$, $m\angle 2 = \frac{1}{2}(m\widehat{RQ} - m\widehat{TP}) = \frac{1}{2}m\widehat{RQ} - \frac{1}{2}m\widehat{TP}$, which is less than $\frac{1}{2}m\widehat{RQ}$, so $m\angle 2 < m\angle 1$.

**46.** Sample answer: Each raindrop refracts light from the sun and sends the beam to Earth. The raindrop is actually spherical, but the angle of the light is an inscribed angle from the bent rays. Answers should include the following.

- ∠C is an inscribed angle and ∠F is a secant-secant angle.
- The measure of ∠F can be calculated by finding the positive difference between $m\widehat{BD}$ and the measure of the small intercepted arc containing point C.

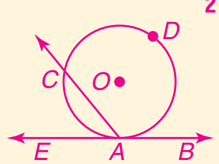
## Answers

**43a.** Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is acute.
Prove: $m\angle CAB = \frac{1}{2}m\widehat{CA}$




Proof: ∠DAB is a right ∠ with measure 90, and $\widehat{DCA}$ is a semicircle with measure 180, since if a line is tangent to a ⊙, it is ⊥ to the radius at the point of tangency. Since ∠CAB is acute, C is in the interior of ∠DAB, so by the Angle and Arc Addition Postulates, $m\angle DAB = m\angle DAC + m\angle CAB$ and $m\widehat{DCA} = m\widehat{DC} + m\widehat{CA}$. By substitution, $90 = m\angle DAC + m\angle CAB$ and $180 = m\widehat{DC} + m\widehat{CA}$. So, $90 = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by Division Prop., and $m\angle DAC + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$ by substitution. $m\angle DAC = \frac{1}{2}m\widehat{DC}$ since ∠DAC is inscribed, so substitution yields $\frac{1}{2}m\widehat{DC} + m\angle CAB = \frac{1}{2}m\widehat{DC} + \frac{1}{2}m\widehat{CA}$. By Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CA}$.

**43b.** Given: $\overleftrightarrow{AB}$ is a tangent to $\odot O$. $\overrightarrow{AC}$ is a secant to $\odot O$. $\angle CAB$ is obtuse.
Prove: $m\angle CAB = \frac{1}{2}m\widehat{CDA}$




Proof: ∠CAB and ∠CAE form a linear pair, so $m\angle CAB + m\angle CAE = 180$. Since ∠CAB is obtuse, ∠CAE is acute and Case 1 applies, so $m\angle CAE = \frac{1}{2}m\widehat{CA}$. $m\widehat{CA} + m\widehat{CDA} = 360$, so $\frac{1}{2}m\widehat{CA} + \frac{1}{2}m\widehat{CDA} = 180$ by Division Prop., and $m\angle CAE + \frac{1}{2}m\widehat{CDA} = 180$ by substitution. By the Transitive Prop., $m\angle CAB + m\angle CAE = m\angle CAE + \frac{1}{2}m\widehat{CDA}$, so by Subtraction Prop., $m\angle CAB = \frac{1}{2}m\widehat{CDA}$.

# 4 Assess

## Open-Ended Assessment

**Speaking** Select examples and ask students to call out the names of the segments in the figure. Then call on volunteers to explain how they would find missing angle measures or arc lengths.

## *Getting Ready for Lesson 10-7*

**Prerequisite Skill** Students will learn about special segments in a circle in Lesson 10-7. They will apply solving quadratic equations to find values for segments that intersect in the interior and exterior of a circle. Use Exercises 57–59 to determine your students' familiarity with solving quadratic equations by factoring.

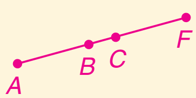
## Assessment Options

**Practice Quiz 2** The quiz provides students with a brief review of the concepts and skills in Lessons 10-4 through 10-6. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

**Quiz (Lessons 10-5 and 10-6)** is available on p. 604 of the *Chapter 10 Resource Masters*.

### Answer

**56. Given: $\overline{AC} \cong \overline{BF}$**
**Prove: $AB = CF$**




**Proof: By definition of congruent segments, $AC = BF$. Using the Segment Addition Postulate, we know that $AC = AB + BC$ and $BF = BC + CF$. Since $AC = BF$, this means that $AB + BC = BC + CF$. If $BC$ is subtracted from each side of this equation, the result is $AB = CF$.**

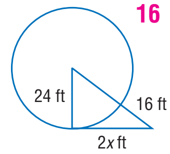


## Maintain Your Skills

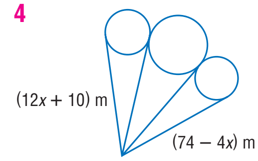
### *Mixed Review*

**Find $x$. Assume that segments that appear to be tangent are tangent.** *(Lesson 10-5)*

**49.** **16**




**50.** **4**

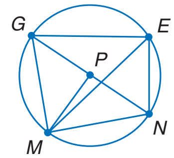



**In $\odot P$, $m\widehat{EN} = 66$ and $m\angle GPM = 89$. Find each measure.** *(Lesson 10-4)*

**51.** $m\angle EGN$ **33**

**52.** $m\angle GME$ **57**

**53.** $m\angle GNM$ **44.5**

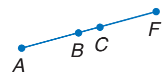



**RAMPS** **Use the following information for Exercises 54 and 55.**
The *Americans with Disabilities Act* (ADA), which went into effect in 1990, requires that wheelchair ramps have at least a 12-inch run for each rise of 1 inch. *(Lesson 3-3)*

**54.** Determine the slope represented by this requirement. $\frac{1}{12}$

**55.** The maximum length the law allows for a ramp is 30 feet. How many inches tall is the highest point of this ramp? **30 in.**

**56.** **PROOF** Write a paragraph proof to show that $AB = CF$ if $\overline{AC} \cong \overline{BF}$. *(Lesson 2-5)* **See margin.**




### *Getting Ready for the Next Lesson*

**PREREQUISITE SKILL** **Solve each equation by factoring.**
*(To review **solving equations by factoring**, see pages 750 and 751.)*

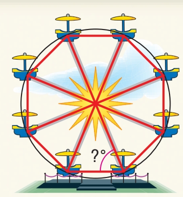
**57.** $x^2 + 6x - 40 = 0$ **4, −10**

**58.** $2x^2 + 7x - 30 = 0$ $-6, 2\frac{1}{2}$

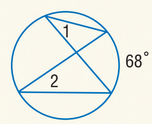
**59.** $3x^2 - 24x + 45 = 0$ **3, 5**

## Practice Quiz 2 — *Lessons 10-4 through 10-6*

**1.** **AMUSEMENT RIDES** A Ferris wheel is shown at the right. If the distances between the seat axles are the same, what is the measure of an angle formed by the braces attaching consecutive seats? *(Lesson 10-4)* **67.5**

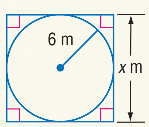



**2.** Find the measure of each numbered angle. *(Lesson 10-4)* $m\angle 1 = m\angle 2 = 34$

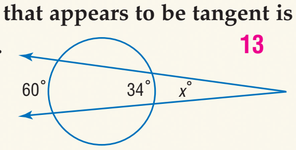



**Find $x$. Assume that any segment that appears to be tangent is tangent.** *(Lessons 10-5 and 10-6)*

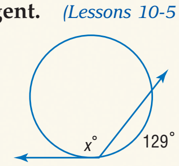
**3.** **12**




**4.** **13**




**5.** **115.5**



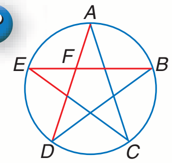
# 10-7 Special Segments in a Circle

## What You'll Learn

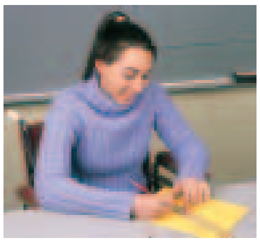
- Find measures of segments that intersect in the interior of a circle.
- Find measures of segments that intersect in the exterior of a circle.

## How are lengths of intersecting chords related?

The star is inscribed in a circle. It was formed by intersecting chords. Segments $AD$ and $EB$ are two of those chords. When two chords intersect, four smaller segments are defined.




**SEGMENTS INTERSECTING INSIDE A CIRCLE** In Lesson 10-2, you learned how to find lengths of parts of a chord that is intersected by the perpendicular diameter. But how do you find lengths for other intersecting chords?



1. $\angle PTS \cong \angle RTQ$ (Vertical ∠s are ≅.); $\angle P \cong \angle R$ (∠s intercepting same arc are ≅.); $\angle S \cong \angle Q$ (∠s intercepting same arc are ≅.)

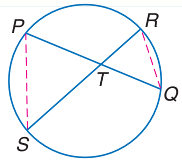
3. $\frac{PT}{RT} = \frac{ST}{TQ}$ or $PT \cdot TQ = RT \cdot ST$

### Geometry Activity

**Intersecting Chords**

**Make A Model**

- Draw a circle and two intersecting chords.
- Name the chords $\overline{PQ}$ and $\overline{RS}$ intersecting at $T$.
- Draw $\overline{PS}$ and $\overline{RQ}$.




**Analyze**

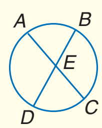
1. Name pairs of congruent angles. Explain your reasoning.
2. How are $\triangle PTS$ and $\triangle RTQ$ related? Why? **similar by AA Similarity**
3. **Make a conjecture** about the relationship of $\overline{PT}$, $\overline{TQ}$, $\overline{RT}$, and $\overline{ST}$.

The results of the activity suggest a proof for Theorem 10.15.

### Theorem 10.15

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.

**Example:** $AE \cdot EC = BE \cdot ED$



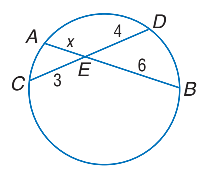

You will prove Theorem 10.15 in Exercise 21.

### Example 1 Intersection of Two Chords

**Find x.**

$AE \cdot EB = CE \cdot ED$

| | |
|---|---|
| $x \cdot 6 = 3 \cdot 4$ | Substitution |
| $6x = 12$ | Multiply. |
| $x = 2$ | Divide each side by 6. |






# 10-7 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-7** Use as a quiz or review of Lesson 10-6.

**Mathematical Background** notes are available for this lesson on p. 520D.

**How are lengths of intersecting chords related?**

*Ask students*:

- Name the segments that are defined by the intersection of $\overline{AD}$ and $\overline{EB}$ in the figure. **$\overline{AF}$, $\overline{FD}$, $\overline{EF}$, and $\overline{FB}$**
- Name the inscribed angles in the figure. **∠A, ∠B, ∠C, ∠D, and ∠E**
- Is $\triangle DFB$ inscribed in the circle? Why or why not? **No; because F is not a point on the circle.**

## Resource Manager

**Workbook and Reproducible Masters**

***Chapter 10 Resource Masters***

- Study Guide and Intervention, pp. 577–578
- Skills Practice, p. 579
- Practice, p. 580
- Reading to Learn Mathematics, p. 581
- Enrichment, p. 582

***Prerequisite Skills Workbook,*** pp. 35–36, 51–52

***Teaching Geometry With Manipulatives Masters,*** p. 17

**Transparencies**

5-Minute Check Transparency 10-7
Answer Key Transparencies

**Technology**

Interactive Chalkboard
Multimedia Applications: Virtual Activities

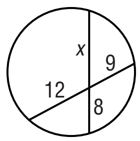
# 2 Teach

## SEGMENTS INTERSECTING INSIDE A CIRCLE

### In-Class Examples

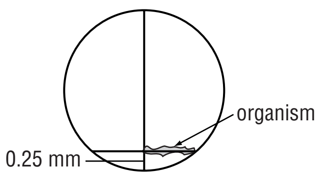
Power Point®

**1** Find $x$. **13.5**




*Teaching Tip* Remind students that a diameter can be drawn to bisect any chord of a circle.

**2** **BIOLOGY** Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located 0.25 mm from the bottom of the field of view. Round to the nearest hundredth. **0.66 mm**




## SEGMENTS INTERSECTING OUTSIDE A CIRCLE

### In-Class Example

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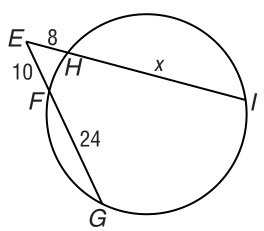
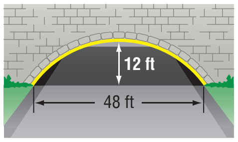
**3** Find $x$ if $EF = 10$, $EH = 8$, and $FG = 24$. **34.5**




Intersecting chords can also be used to measure arcs.

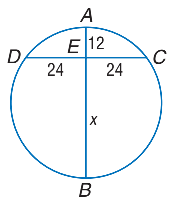
### Example 2 *Solve Problems*

**TUNNELS** **Tunnels are constructed to allow roadways to pass through mountains. What is the radius of the circle containing the arc if the opening is not a semicircle?**




Draw a model using a circle. Let $x$ represent the unknown measure of the segment of diameter $\overline{AB}$. Use the products of the lengths of the intersecting chords to find the length of the diameter.

$$
\begin{aligned}
AE \cdot EB &= DE \cdot EC && \text{Segment products} \\
12x &= 24 \cdot 24 && \text{Substitution} \\
x &= 48 && \text{Divide each side by 12.} \\
AB &= AE + EB && \text{Segment Addition Postulate} \\
AB &= 12 + 48 \text{ or } 60 && \text{Substitution and addition}
\end{aligned}
$$




Since the diameter is 60, $r = 30$.

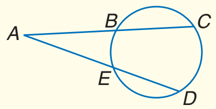
**SEGMENTS INTERSECTING OUTSIDE A CIRCLE** Nonparallel chords of a circle can be extended to form secants that intersect in the exterior of a circle. The special relationship among secant segments excludes the chord.

**Study Tip**

*Helping You Remember*
To remember this concept, the wording of Theorem 10.16 can be simplified by saying that each side of the equation is the product of the exterior part and the whole segment.

**Theorem 10.16**

If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.



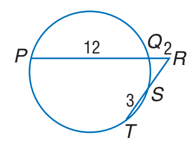

**Example:** $AB \cdot AC = AE \cdot AD$

You will prove this theorem in Exercise 30.

### Example 3 *Intersection of Two Secants*

**Find $RS$ if $PQ = 12$, $QR = 2$, and $TS = 3$.**

Let $RS = x$.




$$
\begin{aligned}
QR \cdot PR &= RS \cdot RT && \text{Secant Segment Products} \\
2 \cdot (12 + 2) &= x \cdot (x + 3) && \text{Substitution} \\
28 &= x^2 + 3x && \text{Distributive Property} \\
0 &= x^2 + 3x - 28 && \text{Subtract 28 from each side.} \\
0 &= (x + 7)(x - 4) && \text{Factor.}
\end{aligned}
$$

$x + 7 = 0 \qquad x - 4 = 0$

$x = -7 \qquad x = 4 \qquad$ Disregard negative value.



## Geometry Activity

**Materials:** compass, straightedge

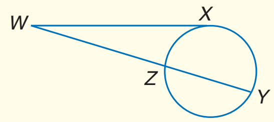
- Tell students to draw chords that are not congruent and that do not intersect at the center of the circle.
- Students can use a protractor to verify that the triangles are similar, and they can determine the scale factor by which the triangles are related.

570 Chapter 10 Circles

The same secant segment product can be used with a secant segment and a tangent. In this case, the tangent is both the exterior part and the whole segment. This is stated in Theorem 10.17.

## Theorem 10.17

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

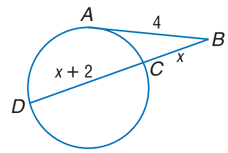



**Example:** $WX \cdot WX = WZ \cdot WY$

You will prove this theorem in Exercise 31.

## Example 4 Intersection of a Secant and a Tangent.

**Find $x$. Assume that segments that appear to be tangent are tangent.**




$(AB)^2 = BC \cdot BD$

$4^2 = x(x + x + 2)$

$16 = x(2x + 2)$

$16 = 2x^2 + 2x$

$0 = 2x^2 + 2x - 16$

$0 = x^2 + x - 8$

This expression is not factorable. Use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-8)}}{2(1)}$ $a = 1, b = 1, c = -8$

$= \frac{-1 + \sqrt{33}}{2}$ or $x = \frac{-1 - \sqrt{33}}{2}$ Disregard the negative solution.

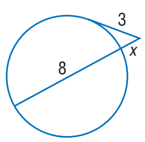
$\approx 2.37$ Use a calculator.

## Check for Understanding

*Concept Check*

1. **Show** how the products for secant segments are similar to the products for a tangent and a secant segment. **See margin.**

2. **FIND THE ERROR** Becky and Latisha are writing products to find $x$. Who is correct? Explain your reasoning.




| Becky | Latisha |
|---|---|
| $3^2 = x \cdot 8$ | $3^2 = x(x + 8)$ |
| $9 = 8x$ | $9 = x^2 + 8x$ |
| $\frac{9}{8} = x$ | $0 = x^2 + 8x - 9$ |
| | $0 = (x + 9)(x - 1)$ |
| | $x = 1$ |

2. Latisha; the length of the tangent segment squared equals the product of the exterior secant segment and the entire secant, not the interior secant segment.

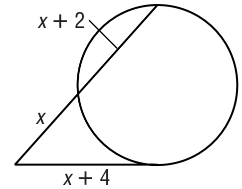
www.geometryonline.com/extra_examples



## In-Class Example

Power Point®

4 Find $x$. Assume that segments that appear to be tangent are tangent. **8**




## 3 Practice/Apply

### Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

**DAILY INTERVENTION**

**FIND THE ERROR** Point out to students that in this lesson they do not use the entire length of the interior secant segment in any of the relationships. Explain that in each case with exterior secant segments, they do use the entire length of the exterior secant segment.

### Answer

1. Sample answer: The product equation for secant segments equates the product of exterior segment measure and the whole segment measure for each secant. In the case of secant-tangent, the product involving the tangent segment becomes (measure of tangent segment)$^2$ because the exterior segment and the whole segment are the same segment.

**DAILY INTERVENTION**

### Differentiated Instruction

**Visual/Spatial** Students can use their spatial skills to remember the concepts of this lesson. Tell students to visualize the similar triangles that would be made by the chords, secants, and/or tangents in the figures to help them determine how to set up their products.

## About the Exercises…

**Organization by Objective**

- **Segments Intersecting Inside a Circle:** 8–11, 20, 27
- **Segments Intersecting Outside a Circle:** 12–19

**Odd/Even Assignments**

Exercises 8–19, 22–28 are structured so that students practice the same concepts whether they are assigned odd or even problems.

## Assignment Guide

**Basic:** 9, 13–31 odd, 32–48

**Average:** 9–31 odd, 32–48

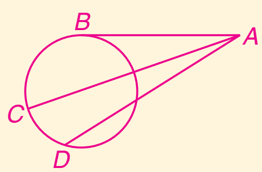
**Advanced:** 8–30 even, 32–45 (optional: 46–48)

## Concept Check

Have students create and label a three-column chart with an example of each segment relationship described in this lesson, color code the parts that are equal, and write each relationship in algebraic form.

### Answer

**3. Sample answer:**

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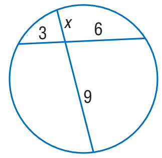
3. **OPEN ENDED** Draw a circle with two secant segments and one tangent segment that intersect at the same point. **See margin.**

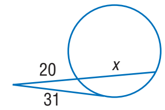
### Guided Practice

| GUIDED PRACTICE KEY | |
|---|---|
| **Exercises** | **Examples** |
| 4 | 1 |
| 5 | 4 |
| 6 | 3 |
| 7 | 2 |

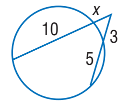
**Find $x$. Round to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.**

4. 

**2**

5. 

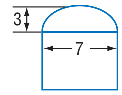
**28.1**

6. 

**2**

### Application

7. **HISTORY** The Roman Coliseum has many "entrances" in the shape of a door with an arched top. The ratio of the arch width to the arch height is 7:3. Find the ratio of the arch width to the radius of the circle that contains the arch. **≈7:3.54**



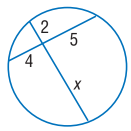

★ indicates increased difficulty

## Practice and Apply

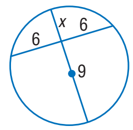
| Homework Help | |
|---|---|
| **For Exercises** | **See Examples** |
| 8–11 | 1 |
| 12–15 | 4 |
| 16–19 | 3 |
| 20, 27 | 2 |

**Extra Practice** See page 775.

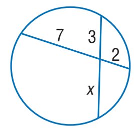
**Find $x$. Round to the nearest tenth if necessary. Assume that segments that appear to be tangent are tangent.**

8. 

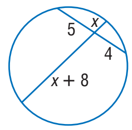
**10**

9. 

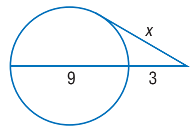
**4**

10. 

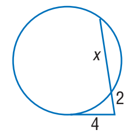
**4.7**

★ 11. 

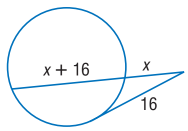
**2**

12. 

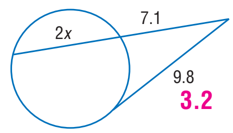
**6**

13. 

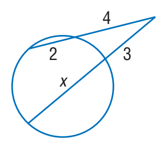
**6**

14. 

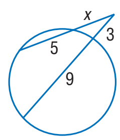
**8**

15. 

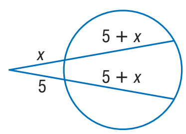
**3.2**

16. 

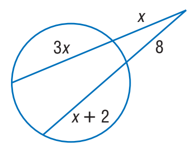
**5**

17. 

**4**

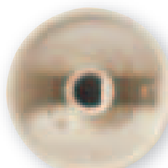
18. 

**5**

19. 

**5.6**

20. **KNOBS** If you remove a knob from a kitchen appliance, you may notice that the hole is not completely round. Suppose the flat edge is 4 millimeters long and the distance from the curved edge to the flat edge is about 4.25 millimeters. Find the radius of the circle containing the hole. **about 2.6 mm**





Doug Martin

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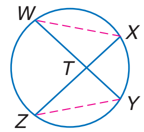
21. **PROOF** Copy and complete the proof of Theorem 10.15.

**Given:** $\overline{WY}$ and $\overline{ZX}$ intersect at $T$.

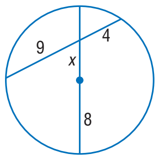
**Prove:** $WT \cdot TY = ZT \cdot TX$



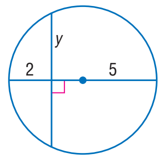

| Statements | Reasons |
|---|---|
| a. $\angle W \cong \angle Z$, $\angle X \cong \angle Y$ | a. ? **Inscribed angles that intercept the same arc are congruent.** |
| b. ? **$\triangle WXT \sim \triangle ZYT$** | b. AA Similarity |
| c. $\frac{WT}{ZT} = \frac{TX}{TY}$ | c. ? **Definition of similar triangles** |
| d. ? **$WT \cdot TY = ZT \cdot TX$** | d. Cross products |

**Find each variable. Round to the nearest tenth, if necessary.**

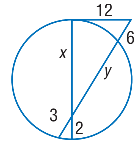
22. **5.3**



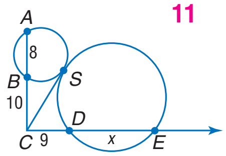

23. **4**



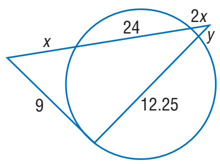

24. **15, 22.5**



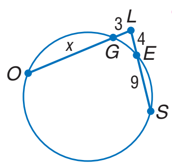

25. **11**



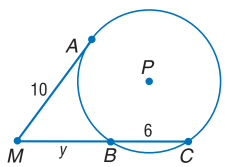

26. **$x = 3$, $y = 8.6$**




27. **14.3**




28. **7.4**



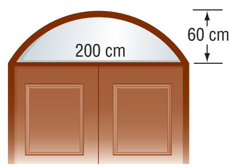

## Career Choices

**Construction Worker**

Construction workers must know how to measure and fit shapes together to make a sound building that will last for years to come. These workers also must master using machines to cut wood and metal to certain specifications that are based on geometry.

**Online Research** For more information about a career as a construction worker, visit: www.geometryonline.com/careers

29. **CONSTRUCTION** An arch over a courtroom door is 60 centimeters high and 200 centimeters wide. Find the radius of the circle containing the arc of the arch. **113.3 cm**

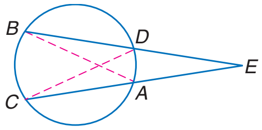



**30–31. See p. 589B.**

30. **PROOF** Write a two-column proof of Theorem 10.16.

**Given:** secants $\overline{EC}$ and $\overline{EB}$

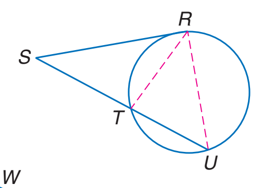
**Prove:** $EA \cdot EC = ED \cdot EB$



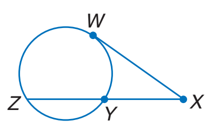

31. **PROOF** Write a two-column proof of Theorem 10.17.

**Given:** tangent $\overline{RS}$, secant $\overline{SU}$

**Prove:** $(RS)^2 = ST \cdot SU$




32. **CRITICAL THINKING** In the figure, $Y$ is the midpoint of $\overline{XZ}$. Find $WX$ in terms of $XY$. Explain your reasoning.




**$WX = \sqrt{2} \cdot XY$; see margin for explanation.**

www.geometryonline.com/self_check_quiz



David Young-Wolff/PhotoEdit

## Answer

32. $ZY = XY$

$(WX)^2 = XY \cdot XZ$

$(WX)^2 = XY(XY + ZY)$

$(WX)^2 = XY(2XY)$

$(WX)^2 = 2(XY)^2$

$WX = \sqrt{2(XY)^2}$

$WX = \sqrt{2} \cdot XY$

## Enrichment, p. 582

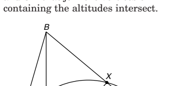
***The Nine-Point Circle***

The figure below illustrates a surprising fact about triangles and circles. Given any $\triangle ABC$, there is a circle that contains all of the following nine points:

(1) the midpoints $K$, $L$, and $M$ of the sides of $\triangle ABC$

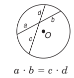
(2) the points $X$, $Y$, and $Z$, where $\overline{AX}$, $\overline{BY}$, and $\overline{CZ}$ are the altitudes of $\triangle ABC$

(3) the points $R$, $S$, and $T$ which are the midpoints of the segments $\overline{AH}$, $\overline{BH}$, and $\overline{CH}$ that join the vertices of $\triangle ABC$ to the point $H$ where the lines containing the altitudes intersect.




## Study Guide and Intervention, p. 577 (shown) and p. 578

**Segments Intersecting Inside a Circle** If two chords intersect in a circle, then the products of the measures of the chords are equal.




$a \cdot b = c \cdot d$

**Example** Find $x$.

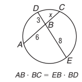
The two chords intersect inside the circle, so the products $AB \cdot BC$ and $EB \cdot BD$ are equal.

$AB \cdot BC = EB \cdot BD$

$6 \cdot x = 8 \cdot 3$ Substitution

$6x = 24$ Simplify.

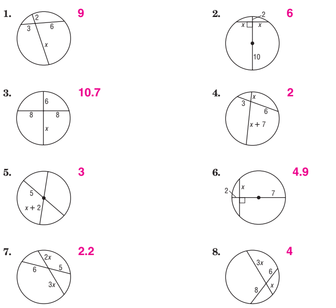
$x = 4$ Divide each side by 6.




$AB \cdot BC = EB \cdot BD$

**Exercises**

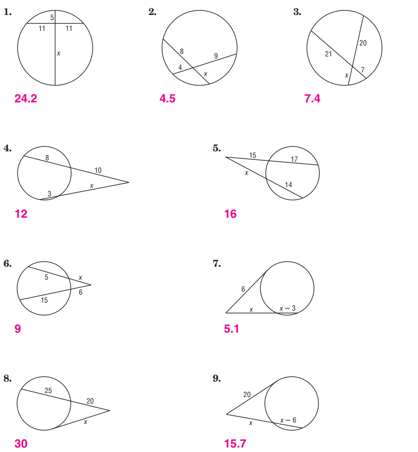
**Find $x$ to the nearest tenth.**



1. **9**
2. **6**
3. **10.7**
4. **2**
5. **3**
6. **4.9**
7. **2.2**
8. **4**

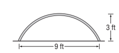
## Skills Practice, p. 579 and Practice, p. 580 (shown)

**Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.**



1. **24.2**
2. **4.5**
3. **7.4**
4. **12**
5. **16**
6. **9**
7. **5.1**
8. **30**
9. **15.7**

10. **CONSTRUCTION** An arch over an apartment entrance is 3 feet high and 9 feet wide. Find the radius of the circle containing the arc of the arch. **4.875 ft**




## Reading to Learn Mathematics, p. 581 ELL

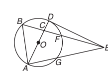
**Pre-Activity** **How are lengths of intersecting chords related?**

Read the introduction to Lesson 10-7 at the top of page 569 in your textbook.

- What kinds of angles of the circle are formed at the points of the star? **inscribed angles**
- What is the sum of the measures of the five angles of the star? **180**

**Reading the Lesson**

1. Refer to $\odot O$. Name each of the following.




a. a diameter **$\overline{AD}$**

b. a chord that is not a diameter **$\overline{AB}$, $\overline{BF}$, or $\overline{AG}$**

c. two chords that intersect in the interior of the circle **$\overline{AD}$ and $\overline{BF}$**

d. an exterior point **$E$**

e. two secant segments that intersect in the exterior of the circle **$\overline{EA}$ and $\overline{EB}$**

f. a tangent segment **$\overline{ED}$**

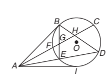
g. a right angle **$\angle ADE$**

h. an external secant segment **$\overline{EF}$ or $\overline{EG}$**

i. a secant-tangent angle with vertex on the circle **$\angle ADE$**

j. an inscribed angle **$\angle BAD$, $\angle DAG$, $\angle BAG$, or $\angle ABF$**

2. Supply the missing length to complete each equation.




a. $BH \cdot HD = FH \cdot$ **$HC$**

b. $AC \cdot AF = AD \cdot$ **$AE$**

c. $AD \cdot AE = AB \cdot$ **$AB$**

d. $AB =$ **$AI$**

e. $AF \cdot AC = ($**$AI$ or $AB$**$)^2$

f. $EG \cdot$ **$GB$** $= FG \cdot GC$

**Helping You Remember**

3. Some students find it easier to remember geometric theorems if they restate them in their own words. Restate Theorem 10.16 in a way that you find easier to remember.

**Sample answer: Suppose you draw a secant to a circle through a point $A$ outside the circle. Multiply the distances from point $A$ to the points where the secant intersects the circle. The corresponding product will be the same for any other secant through point $A$ to the same circle.**

# 4 Assess

## Open-Ended Assessment

**Modeling** Provide students with three or four cutout circles, thin masking tape, and two or three sheets of construction paper. Have students arbitrarily model segments intersecting inside and outside a circle with the tape and the cutouts. Use the sheets of construction paper for points outside the circle. Students can then use a metric ruler to measure segments and test the theorems in this lesson.

## Getting Ready for Lesson 10-8

**Prerequisite Skill** Students will learn about equations of circles in Lesson 10-8. They will apply the Distance Formula to write equations for circles and graph circles. Use Exercises 46–48 to determine your students' familiarity with the Distance Formula.

## Answer

33. Sample answer: The product of the parts of one intersecting chord equals the product of the parts of the other chord. Answers should include the following.
- $\overline{AF}, \overline{FD}, \overline{EF}, \overline{FB}$
- $AF \cdot FD = EF \cdot FB$

---

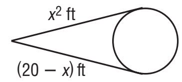
33. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**How are the lengths of intersecting chords related?**

Include the following in your answer:
- the segments formed by intersecting segments, $\overline{AD}$ and $\overline{EB}$, and
- the relationship among these segments.

*Standardized Test Practice*

34. Find two possible values for $x$ from the information in the figure. **D**




Ⓐ −4, −5 Ⓑ −4, 5
Ⓒ 4, 5 Ⓓ 4, −5

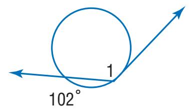
35. **ALGEBRA** Mr. Rodriguez can wash his car in 15 minutes, while his son Marcus takes twice as long to do the same job. If they work together, how long will it take them to wash the car? **C**

Ⓐ 5 min Ⓑ 7.5 min Ⓒ 10 min Ⓓ 22.5 min

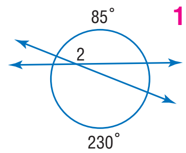
## Maintain Your Skills

*Mixed Review* **Find the measure of each numbered angle. Assume that segments that appear tangent are tangent.** *(Lesson 10-6)*

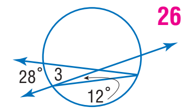
36. **129**




37. **157.5**

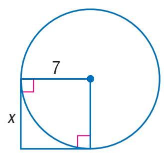



38. **26**

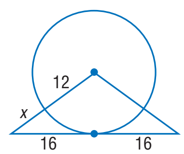



**Find $x$. Assume that segments that appear to be tangent are tangent.** *(Lesson 10-5)*

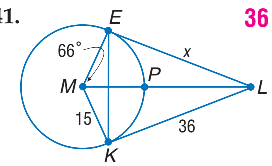
39. **7**




40. **8**

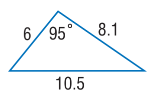



41. **36**

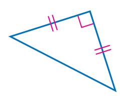



42. **INDIRECT MEASUREMENT** Joseph Blackarrow is measuring the width of a stream on his land to build a bridge over it. He picks out a rock across the stream as landmark $A$ and places a stone on his side as point $B$. Then he measures 5 feet at a right angle from $\overline{AB}$ and marks this $C$. From $C$, he sights a line to point $A$ on the other side of the stream and measures the angle to be about 67°. How far is it across the stream rounded to the nearest whole foot? *(Lesson 7-5)* **12 ft**

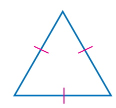
**Classify each triangle by its sides and by its angles.** *(Lesson 4-1)*

43. 


**scalene, obtuse**

44. 

**isosceles, right**

45. 

**equilateral, acute, or equiangular**

*Getting Ready for the Next Lesson* **PREREQUISITE SKILL** **Find the distance between each pair of points.**
*(To review the **Distance Formula**, see Lesson 1-3.)*

46. $C(-2, 7), D(10, 12)$ **13**
47. $E(1, 7), F(3, 4)$ **$\sqrt{13}$**
48. $G(9, -4), H(15, -2)$ **$\sqrt{40}$**



574 Chapter 10 Circles

# 10-8 Equations of Circles

## What You'll Learn

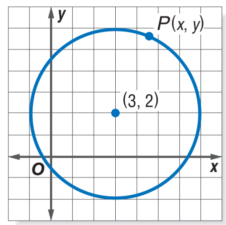
- Write the equation of a circle.
- Graph a circle on the coordinate plane.

## What kind of equations describes the ripples of a splash?

When a rock enters the water, ripples move out from the center forming concentric circles. If the rock is assigned coordinates, each ripple can be modeled by an equation of a circle.

**EQUATION OF A CIRCLE** The fact that a circle is the *locus* of points in a plane equidistant from a given point creates an equation for any circle.

Suppose the center is at (3, 2) and the radius is 4. The radius is the distance from the center. Let $P(x, y)$ be the endpoint of any radius.




$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

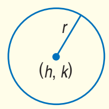
$4 = \sqrt{(x - 3)^2 + (y - 2)^2}$ $d = 4, (x_1, y_1) = (3, 2)$

$16 = (x - 3)^2 + (y - 2)^2$ Square each side.

Applying this same procedure to an unknown center $(h, k)$ and radius $r$ yields a general equation for any circle.

**Key Concept** — *Standard Equation of a Circle*

An equation for a circle with center at $(h, k)$ and radius of $r$ units is $(x - h)^2 + (y - k)^2 = r^2$.




**Study Tip**

*Equation of Circles*
Note that the equation of a circle is kept in the form shown above. The terms being squared are not expanded.

**Example 1** *Equation of a Circle*

**Write an equation for each circle.**

**a. center at (−2, 4), $d = 4$**

If $d = 4$, $r = 2$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$[x - (-2)]^2 + [y - 4]^2 = 2^2$ $(h, k) = (-2, 4), r = 2$

$(x + 2)^2 + (y - 4)^2 = 4$ Simplify.

**b. center at origin, $r = 3$**

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$(x - 0)^2 + (y - 0)^2 = 3^2$ $(h, k) = (0, 0), r = 3$

$x^2 + y^2 = 9$ Simplify.





# 10-8 Lesson Notes

## 1 Focus

**5-Minute Check Transparency 10-8** Use as a quiz or review of Lesson 10-7.

**Mathematical Background** notes are available for this lesson on p. 520D.

**What kind of equations describes the ripples of a splash?**

*Ask students:*

- In order to cause water ripples to form concentric circles, what has to happen? **Something must break the surface tension of the water, creating the force that causes the ripples, like the rock in the example.**
- If the rock is thrown with a greater force, would you see fewer circles or more circles? **more circles**

## Resource Manager

### Workbook and Reproducible Masters

***Chapter 10 Resource Masters***

- Study Guide and Intervention, pp. 583–584
- Skills Practice, p. 585
- Practice, p. 586
- Reading to Learn Mathematics, p. 587
- Enrichment, p. 588
- Assessment, p. 604

***Teaching Geometry With Manipulatives Masters,*** pp. 1, 17

### Transparencies

5-Minute Check Transparency 10-8
Answer Key Transparencies

### Technology

GeomPASS: Tutorial Plus, Lesson 19
Interactive Chalkboard
Multimedia Applications: Virtual Activities

# 2 Teach

## EQUATION OF A CIRCLE

**In-Class Examples** Power Point®

**1** Write an equation for each circle.

**a.** center at $(3, -3)$, $d = 12$
$(x - 3)^2 + (y + 3)^2 = 36$

**b.** center at $(-12, -1)$, $r = 8$
$(x + 12)^2 + (y + 1)^2 = 64$

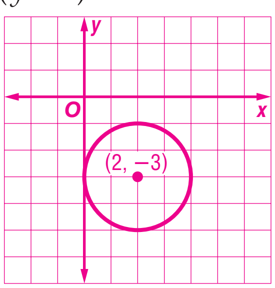
*Teaching Tip* Students should note that the two tangent lines have slopes that indicate they are perpendicular to each other. Students should also remember that a radius is the shortest distance from the tangent to the center of a circle.

**2** A circle with a diameter of 10 has its center in the first quadrant. The lines $y = -3$ and $x = -1$ are tangent to the circle. Write an equation of the circle.
$(x - 4)^2 + (y - 2)^2 = 25$

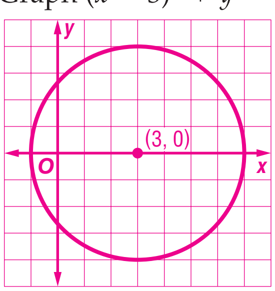
## GRAPH CIRCLES

**In-Class Example** Power Point®

**3** **a.** Graph $(x - 2)^2 + (y + 3)^2 = 4$.




**b.** Graph $(x - 3)^2 + y^2 = 16$.

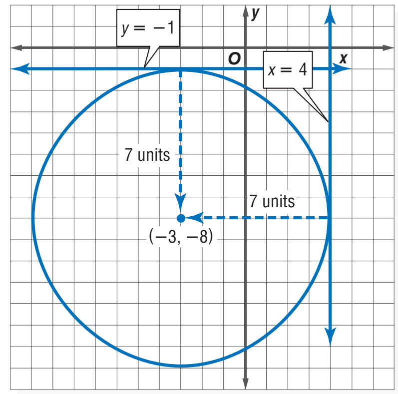



Other information about a circle can be used to find the equation of the circle.

### Example 2 *Use Characteristics of Circles*

**A circle with a diameter of 14 has its center in the third quadrant. The lines $y = -1$ and $x = 4$ are tangent to the circle. Write an equation of the circle.**

Sketch a drawing of the two tangent lines.




Since $d = 14$, $r = 7$. The line $x = 4$ is perpendicular to a radius. Since $x = 4$ is is a vertical line, the radius lies on a horizontal line. Count 7 units to the left from $x = 4$. Find the value of $h$.

$$h = 4 - 7 \text{ or } -3$$

Likewise, the radius perpendicular to the line $y = -1$ lies on a vertical line. The value of $k$ is 7 units down from $-1$.

$$k = -1 - 7 \text{ or } -8$$

The center is at $(-3, -8)$, and the radius is 7. An equation for the circle is $(x + 3)^2 + (y + 8)^2 = 49$.

**GRAPH CIRCLES** You can analyze the equation of a circle to find information that will help you graph the circle on a coordinate plane.

**Study Tip**

*Graphing Calculator*
To use the center and radius to graph a circle, select a suitable window that contains the center of the circle. For a TI-83 Plus, press ZOOM 5. Then use **9: Circle (** on the **Draw** menu. Put in the coordinates of the center and then the radius so that the screen shows "Circle (−2, 3, 4)". Then press ENTER.

### Example 3 *Graph a Circle*

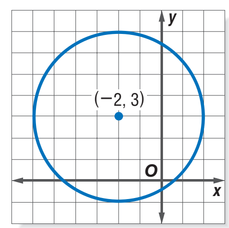
**a. Graph $(x + 2)^2 + (y - 3)^2 = 16$.**

Compare each expression in the equation to the standard form.

| | |
|---|---|
| $(x - h)^2 = (x + 2)^2$ | $(y - k)^2 = (y - 3)^2$ |
| $x - h = x + 2$ | $y - k = y - 3$ |
| $-h = 2$ | $-k = -3$ |
| $h = -2$ | $k = 3$ |

$r^2 = 16$, so $r = 4$.

The center is at $(-2, 3)$, and the radius is 4. Graph the center. Use a compass set at a width of 4 grid squares to draw the circle.

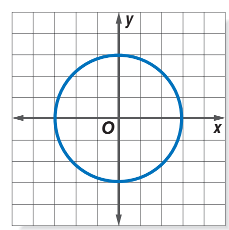



**b. Graph $x^2 + y^2 = 9$.**

Write the equation in standard form.

$(x - 0)^2 + (y - 0)^2 = 3^2$

The center is at $(0, 0)$, and the radius is 3. Draw a circle with radius 3, centered at the origin.




If you know three points on the circle, you can find the center and radius of the circle and write its equation.



**DAILY INTERVENTION** **Differentiated Instruction**

**Logical/Mathematical** Explain that students will rely heavily on their geometric knowledge and reasoning skills to solve the problems in this lesson. Allow students to explain how to explore and collaborate as they work through examples and exercises. Encourage students to recall definitions, concepts, and theorems to help explain why they use certain methods to solve problems.

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## Study Tip

**Locus**

The center of the circle is the locus of points equidistant from the three given points. This is a **compound locus** because the point satisfies more than one condition.

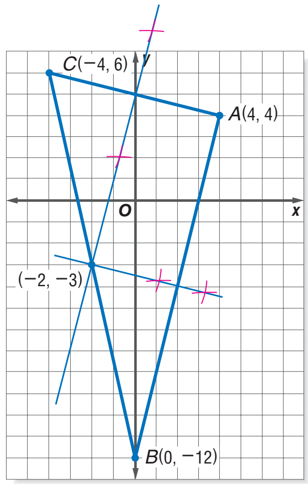
## Example 4 A Circle Through Three Points

**CELL PHONES** **Cell phones work by the transfer of phone signals from one tower to another via satellite. Cell phone companies try to locate towers so that they service multiple communities. Suppose three large metropolitan areas are modeled by the points $A(4, 4)$, $B(0, -12)$, and $C(-4, 6)$, and each unit equals 100 miles. Determine the location of a tower equidistant from all three cities, and write an equation for the circle.**

**Explore** You are given three points that lie on a circle.

**Plan** Graph $\triangle ABC$. Construct the perpendicular bisectors of two sides to locate the center, which is the location of the tower. Find the length of a radius. Use the center and radius to write an equation.

**Solve** Graph $\triangle ABC$ and construct the perpendicular bisectors of two sides. The center appears to be at $(-2, -3)$. This is the location of the tower.




Find $r$ by using the Distance Formula with the center and any of the three points.

$$r = \sqrt{[-2-4]^2 + [-3-4]^2}$$
$$= \sqrt{85}$$

Write an equation.

$$[x-(-2)]^2 + [y-(-3)]^2 = (\sqrt{85})^2$$
$$(x+2)^2 + (y+3)^2 = 85$$

**Examine** You can verify the location of the center by finding the equations of the two bisectors and solving a system of equations. You can verify the radius by finding the distance between the center and another of the three points on the circle.

## Check for Understanding

**Concept Check**

1. **OPEN ENDED** Draw an obtuse triangle on a coordinate plane and construct the circle that circumscribes it. **1–2. See margin.**
2. **Explain** how the definition of a circle leads to its equation.

**Guided Practice**

**GUIDED PRACTICE KEY**

| Exercises | Examples |
|---|---|
| 3, 4, 8 | 1 |
| 5 | 2 |
| 6 | 3 |
| 7 | 3, 4 |

**Write an equation for each circle.**

3. center at $(-3, 5)$, $r = 10$ $(x+3)^2 + (y-5)^2 = 100$
4. center at origin, $r = \sqrt{7}$ $x^2 + y^2 = 7$
5. diameter with endpoints at $(2, 7)$ and $(-6, 15)$ $(x+2)^2 + (y-11)^2 = 32$

**Graph each equation. 6–7. See margin.**

6. $(x+5)^2 + (y-2)^2 = 9$
7. $(x-3)^2 + y^2 = 16$

8. Write an equation of a circle that contains $M(-2, -2)$, $N(2, -2)$, and $Q(2, 2)$. Then graph the circle. $x^2 + y^2 = 8$; **See margin for graph.**

www.geometryonline.com/extra_examples



## In-Class Example

Power Point®

4 **ELECTRICITY** Strategically located substations are extremely important in the transmission and distribution of a power company's electric supply. Suppose three substations are modeled by the points $D(3, 6)$, $E(-1, 0)$, and $F(3, -4)$. Determine the location of a town equidistant from all three substations, and write an equation for the circle.
**(4, 1); $(x-4)^2 + (y-1)^2 = 26$**

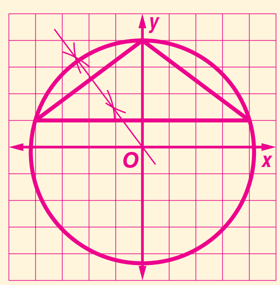
## 3 Practice/Apply

### Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include examples of how to write the equation for a circle and how to graph a circle given various information.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

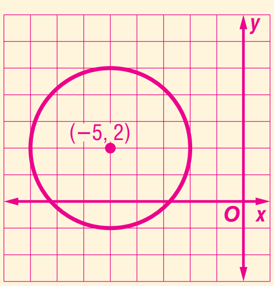
### Answers

1. Sample answer:




2. A circle is the locus of all points in a plane (coordinate plane) a given distance (the radius) from a given point (the center). The equation of a circle is written from knowing the location of the given point and the radius.

6.

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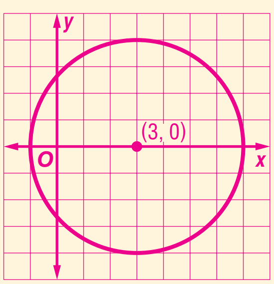


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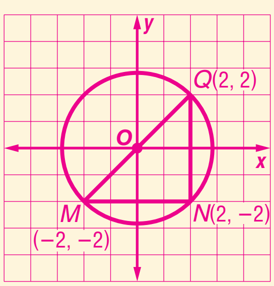


## About the Exercises...

### Organization by Objective

- **Equation of a Circle:** 10–23
- **Graph Circles:** 24–31

### Odd/Even Assignments

Exercises 10–33 are structured so that students practice the same concepts whether they are assigned odd or even problems.

## Assignment Guide

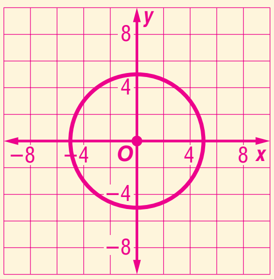
**Basic:** 11, 15–21 odd, 25–37 odd, 41, 43, 45–56

**Average:** 11–41 odd, 43, 45–56

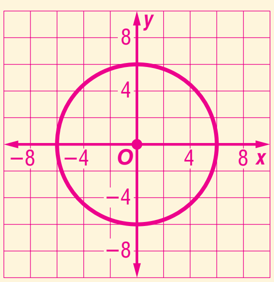
**Advanced:** 10–42 even, 43–56

### Answers

24.




25.

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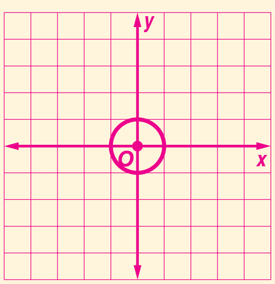


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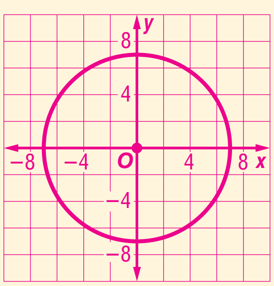
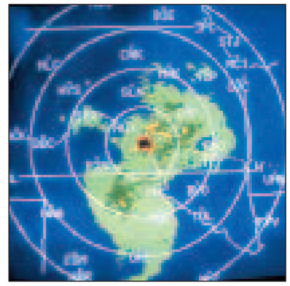



*Application*

9. **WEATHER** Meteorologists track severe storms using Doppler radar. A polar grid is used to measure distances as the storms progress. If the center of the radar screen is the origin and each ring is 10 miles farther from the center, what is the equation of the fourth ring? $x^2 + y^2 = 1600$



★ indicates increased difficulty

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 10–17 | 1 |
| 18–23 | 2 |
| 24–29 | 3 |
| 30–31 | 4 |

**Extra Practice**
See page 776.

11. $(x + 2)^2 + (y + 8)^2 = 25$
12. $(x - 1)^2 + (y + 4)^2 = 17$
13. $x^2 + y^2 = 36$
14. $(x - 5)^2 + (y - 10)^2 = 49$
15. $x^2 + (y - 5)^2 = 100$
16. $(x + 8)^2 + (y - 8)^2 = 64$
17. $(x + 3)^2 + (y + 10)^2 = 144$
18. $(x + 3)^2 + (y - 6)^2 = 9$
20. $(x + 11)^2 + (y - 2)^2 = 32$
21. $(x + 2)^2 + (y - 1)^2 = 10$

**Write an equation for each circle.**

10. center at origin, $r = 3$ $x^2 + y^2 = 9$
11. center at $(-2, -8)$, $r = 5$
12. center at $(1, -4)$, $r = \sqrt{17}$
★ 13. center at $(0, 0)$, $d = 12$
14. center at $(5, 10)$, $r = 7$
15. center at $(0, 5)$, $d = 20$
16. center at $(-8, 8)$, $d = 16$
17. center at $(-3, -10)$, $d = 24$
18. a circle with center at $(-3, 6)$ and a radius with endpoint at $(0, 6)$
19. a circle with a diameter that has endpoints at $(2, -2)$ and $(-2, 2)$ $x^2 + y^2 = 8$
20. a circle with a diameter that has endpoints at $(-7, -2)$ and $(-15, 6)$
21. a circle with center at $(-2, 1)$ and a radius with endpoint at $(1, 0)$

★ 22. a circle with $d = 12$ and a center translated 18 units left and 7 units down from the origin $(x + 18)^2 + (y + 7)^2 = 36$

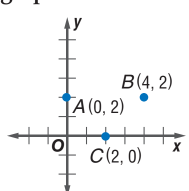
★ 23. a circle with its center in quadrant I, radius of 5 units, and tangents $x = 2$ and $y = 3$ $(x - 7)^2 + (y - 8)^2 = 25$

**Graph each equation.** 24–29. See margin.

24. $x^2 + y^2 = 25$
25. $x^2 + y^2 = 36$
26. $x^2 + y^2 - 1 = 0$
27. $x^2 + y^2 - 49 = 0$
28. $(x - 2)^2 + (y - 1)^2 = 4$
29. $(x + 1)^2 + (y + 2)^2 = 9$

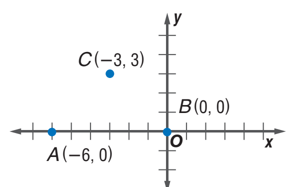
**Write an equation of the circle containing each set of points. Copy and complete the graph of the circle.**

30.




$(x - 2)^2 + (y - 2)^2 = 4$

31.




$(x + 3)^2 + y^2 = 9$

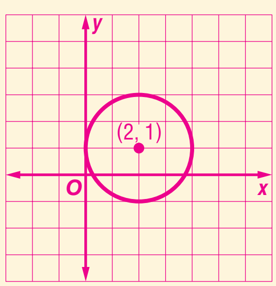
32. Find the radius of a circle with equation $(x - 2)^2 + (y - 2)^2 = r^2$ that contains the point at $(2, 5)$. 3

33. Find the radius of a circle with equation $(x - 5)^2 + (y - 3)^2 = r^2$ that contains the point at $(5, 1)$. 2

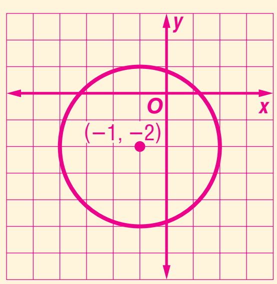
34. **COORDINATE GEOMETRY** Refer to the Examine part of Example 4. Verify the coordinates of the center by solving a system of equations that represent the perpendicular bisectors. See p. 589B.



28.

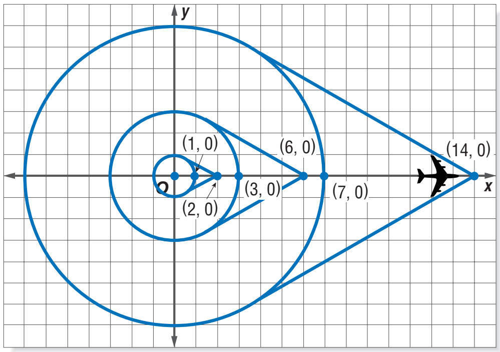



29.




578 Chapter 10 Circles

**AERODYNAMICS** **For Exercises 35–37, use the following information.**
The graph shows cross sections of spherical sound waves produced by a supersonic airplane. When the radius of the wave is 1 unit, the plane is 2 units from the origin. A wave of radius 3 occurs when the plane is 6 units from the center.




35. $x^2 + y^2 = 49$

**35.** Write the equation of the circle when the plane is 14 units from the center.

**36.** What type of circles are modeled by the cross sections? **concentric circles**

**37.** What is the radius of the circle for a plane 26 units from the center? **13**

★ **38.** The equation of a circle is $(x - 6)^2 + (y + 2)^2 = 36$. Determine whether the line $y = 2x - 2$ is a secant, a tangent, or neither of the circle. Explain. **See margin.**

★ **39.** The equation of a circle is $x^2 - 4x + y^2 + 8y = 16$. Find the center and radius of the circle. **(2, −4); $r = 6$**

**40. WEATHER** The geographic center of Tennessee is near Murfreesboro. The closest Doppler weather radar is in Nashville. If Murfreesboro is designated as the origin, then Nashville has coordinates (−58, 55), where each unit is one mile. If the radar has a radius of 80 miles, write an equation for the circle that represents the radar coverage from Nashville. $(x + 58)^2 + (y - 55)^2 = 6400$

**41. RESEARCH** Use the Internet or other materials to find the closest Doppler radar to your home. Write an equation of the circle for the radar coverage if your home is the center. **See students' work.**

**More About. . .**

**Space Travel**
The Apollo program was designed to successfully land a man on the moon. The first landing was July 20, 1969. There were a total of six landings on the moon during 1969–1972.
**Source:** www.infoplease.com

**42. SPACE TRAVEL** Apollo 8 was the first manned spacecraft to orbit the moon at an average altitude of 185 kilometers above the moon's surface. Determine an equation to model a single circular orbit of the Apollo 8 command module if the radius of the moon is 1740 kilometers. Let the center of the moon be at the origin. $x^2 + y^2 = 3{,}705{,}625$

**43. CRITICAL THINKING** Determine the coordinates of any intersection point of the graphs of each pair of equations.

**a.** $x^2 + y^2 = 9,\ y = x + 3$ **(0, 3) or (−3, 0)**

**b.** $x^2 + y^2 = 25,\ x^2 + y^2 = 9$ **none**

**c.** $(x + 3)^2 + y^2 = 9,\ (x - 3)^2 + y^2 = 9$ **(0, 0)**

**44. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

**What kind of equations describe the ripples of a splash?**

Include the following in your answer:
- the general form of the equation of a circle, and
- the equations of five ripples if each ripple is 3 inches farther from the center.

www.geometryonline.com/self_check_quiz



NASA

## Answers

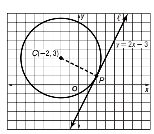
**38.** secant, because it intersects the circle at (0, −2) and (2.4, 2.8)

**44.** Sample answer: Equations of concentric circles; answers should include the following.
- $(x - h)^2 + (y - k)^2 = r^2$
- $x^2 + y^2 = 9,\ x^2 + y^2 = 36,\ x^2 + y^2 = 81,\ x^2 + y^2 = 144,\ x^2 + y^2 = 225$

## Enrichment, p. 588

*Equations of Circles and Tangents*

Recall that the circle whose radius is $r$ and whose center has coordinates $(h, k)$ is the graph of $(x - h)^2 + (y - k)^2 = r^2$. You can use this idea and what you know about circles and tangents to find an equation of the circle that has a given center and is tangent to a given line.

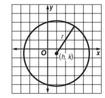



**Use the following steps to find an equation for the circle that has center $C(-2, 3)$ and is tangent to the graph $y = 2x - 3$. Refer to the figure.**

1. State the slope of the line $\ell$ that has equation $y = 2x - 3$.
**2**

## Study Guide and Intervention, p. 583 (shown) and p. 584

**Equation of a Circle** A **circle** is the locus of points in a plane equidistant from a given point. You can use this definition to write an equation of a circle.




**Standard Equation of a Circle** An equation for a circle with center at $(h, k)$ and a radius of $r$ units is $(x - h)^2 + (y - k)^2 = r^2$.

**Example** **Write an equation for a circle with center (−1, 3) and radius 6.**
Use the formula $(x - h)^2 + (y - k)^2 = r^2$ with $h = -1$, $k = 3$, and $r = 6$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle
$(x - (-1))^2 + (y - 3)^2 = 6^2$ Substitution
$(x + 1)^2 + (y - 3)^2 = 36$ Simplify.

**Exercises**

**Write an equation for each circle.**

1. center at (0, 0), $r = 8$
$x^2 + y^2 = 64$

2. center at (−2, 3), $r = 5$
$(x + 2)^2 + (y - 3)^2 = 25$

3. center at (2, −4), $r = 1$
$(x - 2)^2 + (y + 4)^2 = 1$

4. center at (−1, −4), $r = 2$
$(x + 1)^2 + (y + 4)^2 = 4$

5. center at (−2, −6), diameter = 8
$(x + 2)^2 + (y + 6)^2 = 16$

6. center at $\left(-\frac{1}{2}, \frac{1}{4}\right)$, $r = \sqrt{3}$
$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = 3$

7. center at the origin, diameter = 4
$x^2 + y^2 = 4$

8. center at $\left(1, -\frac{5}{8}\right)$, $r = \sqrt{5}$
$(x - 1)^2 + \left(y + \frac{5}{8}\right)^2 = 5$

9. Find the center and radius of a circle with equation $x^2 + y^2 = 20$.
center (0, 0); radius $2\sqrt{5}$

10. Find the center and radius of a circle with equation $(x + 4)^2 + (y + 3)^2 = 16$.
center (−4, −3); radius 4

## Skills Practice, p. 585 and Practice, p. 586 (shown)

**Write an equation for each circle.**

1. center at origin, $r = 7$
$x^2 + y^2 = 49$

2. center at (0, 0), $d = 18$
$x^2 + y^2 = 81$

3. center at (−7, 11), $r = 8$
$(x + 7)^2 + (y - 11)^2 = 64$

4. center at (12, −9), $d = 22$
$(x - 12)^2 + (y + 9)^2 = 121$

5. center at (−6, −4), $r = \sqrt{5}$
$(x + 6)^2 + (y + 4)^2 = 5$

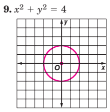
6. center at (3, 0), $d = 28$
$(x - 3)^2 + y^2 = 196$

7. a circle with center at (−5, 3) and a radius with endpoint (2, 3)
$(x + 5)^2 + (y - 3)^2 = 49$

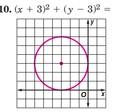
8. a circle whose diameter has endpoints (4, 6) and (−2, 6)
$(x - 1)^2 + (y - 6)^2 = 9$

**Graph each equation.**

9. $x^2 + y^2 = 4$




10. $(x + 3)^2 + (y - 3)^2 = 9$




11. **EARTHQUAKES** When an earthquake strikes, it releases seismic waves that travel in concentric circles from the epicenter of the earthquake. Seismograph stations monitor seismic activity and record the intensity and duration of earthquakes. Suppose a station determines that the epicenter of an earthquake is located about 50 kilometers from the station. If the station is located at the origin, write an equation for the circle that represents a possible epicenter of the earthquake. $x^2 + y^2 = 2500$

## Reading to Learn Mathematics, p. 587 ELL

**Pre-Activity** **What kind of equations describe the ripples of a splash?**

Read the introduction to Lesson 10-8 at the top of page 575 in your textbook.

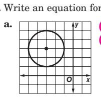
In a series of concentric circles, what is the same about all the circles, and what is different? **Sample answer: They all have the same center, but different radii.**

**Reading the Lesson**

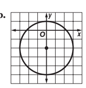
1. Identify the center and radius of each circle.
a. $(x - 2)^2 + (y - 3)^2 = 16$ **(2, 3); 4**
b. $(x + 1)^2 + (y + 5)^2 = 9$ **(−1, −5); 3**
c. $x^2 + y^2 = 49$ **(0, 0); 7**
d. $(x - 8)^2 + (y + 1)^2 = 36$ **(8, −1); 6**
e. $x^2 + (y - 10)^2 = 144$ **(0, 10); 12**
f. $(x + 3)^2 + y^2 = 5$ **(−3, 0); $\sqrt{5}$**

2. Write an equation for each circle.
a. center at origin, $r = 8$ $x^2 + y^2 = 64$
b. center at (3, 9), $r = 1$ $(x - 3)^2 + (y - 9)^2 = 1$
c. center at (−5, −6), $r = 10$ $(x + 5)^2 + (y + 6)^2 = 100$
d. center at (0, −7), $r = 7$ $x^2 + (y + 7)^2 = 49$
e. center at (12, 0), $d = 12$ $(x - 12)^2 + y^2 = 36$
f. center at (−4, 8), $d = 22$ $(x + 4)^2 + (y - 8)^2 = 121$
g. center at (4.5, −3.5), $r = 1.5$ $(x - 4.5)^2 + (y + 3.5)^2 = 2.25$
h. center at (0, 0), $r = \sqrt{13}$ $x^2 + y^2 = 13$

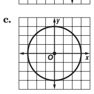
3. Write an equation for each circle.

a.


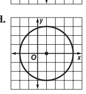
$(x + 3)^2 + (y - 3)^2 = 4$

b.


$x^2 + (y + 2)^2 = 9$

c.


$x^2 + y^2 = 9$

d.


$(x - 1)^2 + y^2 = 9$

**Helping You Remember**

4. A good way to remember a new mathematical formula or equation is to relate it to one you already know. How can you use the Distance Formula to help you remember the standard equation of a circle? **Sample answer: Use the Distance Formula to find the distance between the center $(h, k)$ and a general point $(x, y)$ on the circle. Square each side to obtain the standard equation of a circle.**

## 4 Assess

### Open-Ended Assessment

**Speaking** Allow pairs of students to quiz each other with selected questions from the Practice and Apply section. Let students take turns calling out the equations of circles, naming the centers of circles, and stating the lengths of radii.

### Assessment Options

**Quiz (Lessons 10-7 and 10-8)** is available on p. 604 of the *Chapter 10 Resource Masters*.

*Standardized Test Practice*

45. Which of the following is an equation of a circle with center at $(-2, 7)$ and a diameter of 18? **B**

(A) $x^2 + y^2 - 4x + 14y + 53 = 324$
(B) $x^2 + y^2 + 4x - 14y + 53 = 81$
(C) $x^2 + y^2 - 4x + 14y + 53 = 18$
(D) $x^2 + y^2 + 4x - 14y + 53 = 3$

46. **ALGEBRA** Jordan opened a one-gallon container of milk and poured one pint of milk into his glass. What is the fractional part of one gallon left in the container? **D**

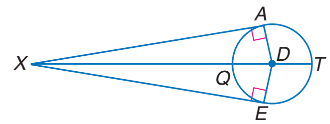
(A) $\frac{1}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{7}{8}$

## Maintain Your Skills

*Mixed Review*

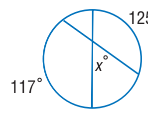
**Find each measure if $EX = 24$ and $DE = 7$.** *(Lesson 10-7)*

47. $AX$ **24**
48. $DX$ **25**
49. $QX$ **18**
50. $TX$ **32**

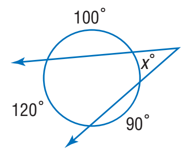



**Find $x$.** *(Lesson 10-6)*

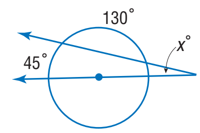
51. **59**




52. **35**




53. **20**

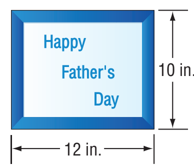



**For Exercises 54 and 55, use the following information.**
Triangle $ABC$ has vertices $A(-3, 2)$, $B(4, -1)$, and $C(0, -4)$.

54. What are the coordinates of the image after moving $\triangle ABC$ 3 units left and 4 units up? *(Lesson 9-2)* **(−6, 6), (1, 3), (−3, 0)**

55. What are the coordinates of the image of $\triangle ABC$ after a reflection in the $y$-axis? *(Lesson 9-1)* **(3, 2), (−4, −1), (0, −4)**

56. **CRAFTS** For a Father's Day present, a kindergarten class is making foam plaques. The edge of each plaque is covered with felt ribbon all the way around with 1 inch overlap. There are 25 children in the class. How much ribbon does the teacher need to buy for all 25 children to complete this craft? *(Lesson 1-6)* **1125 in. or 31.25 yd**




**WebQuest** Internet Project

**"Geocaching" Sends Folks on a Scavenger Hunt**

It's time to complete your project. Use the information and data you have gathered about designing a treasure hunt to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

www.geometryonline.com/webquest



580 Chapter 10 Circles

# Chapter 10 Study Guide and Review

## Vocabulary and Concept Check

| | | | |
|---|---|---|---|
| arc (p. 530) | circumference (p. 523) | major arc (p. 530) | radius (p. 522) |
| center (p. 522) | circumscribed (p. 537) | minor arc (p. 530) | secant (p. 561) |
| central angle (p. 529) | diameter (p. 522) | pi ($\pi$) (p. 524) | semicircle (p. 530) |
| chord (p. 522) | inscribed (p. 537) | point of tangency (p. 552) | tangent (p. 552) |
| circle (p. 522) | intercepted (p. 544) | | |

A complete list of postulates and theorems can be found on pages R1–R8.

**Exercises** Choose the letter of the term that best matches each phrase.

1. arcs of a circle that have exactly one point in common a
2. a line that intersects a circle in exactly one point j
3. an angle with a vertex that is on the circle and with sides containing chords of the circle h
4. a line that intersects a circle in exactly two points i
5. an angle with a vertex that is at the center of the circle b
6. arcs that have the same measure f
7. the distance around a circle d
8. circles that have the same radius g
9. a segment that has its endpoints on the circle c
10. circles that have different radii, but the same center e

**a.** adjacent arcs
**b.** central angle
**c.** chord
**d.** circumference
**e.** concentric circles
**f.** congruent arcs
**g.** congruent circles
**h.** inscribed angle
**i.** secant
**j.** tangent

## Lesson-by-Lesson Review

### 10-1 Circles and Circumference

See pages 522–528.

**Concept Summary**

- The diameter of a circle is twice the radius.
- The circumference $C$ of a circle with diameter $d$ or a radius of $r$ can be written in the form $C = \pi d$ or $C = 2\pi r$.

**Example** Find $r$ to the nearest hundredth if $C = 76.2$ feet.

$C = 2\pi r$ Circumference formula

$76.2 = 2\pi r$ Substitution

$\frac{76.2}{2\pi} = r$ Divide each side by $2\pi$.

$12.13 \approx r$ Use a calculator.

11. 7.5 in.; 47.12 in. 12. 12.8 m, 40.21 m
13. 10.82 yd; 21.65 yd 14. 26 cm; 163.36 cm

**Exercises** The radius, diameter, or circumference of a circle is given. Find the missing measures. Round to the nearest hundredth if necessary. *See Example 4 on page 524.*

11. $d = 15$ in., $r =$ ___?___, $C =$ ___?___
12. $r = 6.4$ m, $d =$ ___?___, $C =$ ___?___
13. $C = 68$ yd, $r =$ ___?___, $d =$ ___?___
14. $d = 52$ cm, $r =$ ___?___, $C =$ ___?___
15. $C = 138$ ft, $r =$ ___?___, $d =$ ___?___
16. $r = 11$ mm, $d =$ ___?___, $C =$ ___?___

21.96 ft; 43.93 ft 22 mm; 69.12 mm

www.geometryonline.com/vocabulary_review



## Chapter 10 Study Guide and Review

### Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 10 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 10 is available on p. 602 of the *Chapter 10 Resource Masters*.

### Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

### Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

### MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

**FOLDABLES™ Study Organizer**

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Have students look through the chapter to make sure they have included notes and examples in their Foldables for each lesson of Chapter 10.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

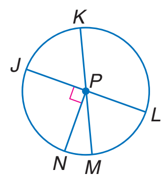
## 10-2 Angles and Arcs

See pages 529–535.

### Concept Summary

- The sum of the measures of the central angles of a circle with no interior points in common is 360.
- The measure of each arc is related to the measure of its central angle.
- The length of an arc is proportional to the length of the circumference.

**Examples** **In ⊙$P$, $m\angle MPL = 65$ and $\overline{NP} \perp \overline{PL}$.**




**1 Find $m\widehat{NM}$.**

$\widehat{NM}$ is a minor arc, so $m\widehat{NM} = m\angle NPM$.

$\angle JPN$ is a right angle and $m\angle MPL = 65$, so $m\angle NPM = 25$.

$m\widehat{NM} = 25$

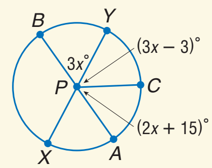
**2 Find $m\widehat{NJK}$.**

$\widehat{NJK}$ is composed of adjacent arcs, $\widehat{NJ}$ and $\widehat{JK}$. $\angle MPL \cong \angle JPK$, so $m\angle JPK = 65$.

| | |
|---|---|
| $m\widehat{NJ} = m\angle NPJ$ or 90 | $\angle NPJ$ is a right angle |
| $m\widehat{NJK} = m\widehat{NJ} + m\widehat{JK}$ | Arc Addition Postulate |
| $m\widehat{NJK} = 90 + 65$ or 155 | Substitution |

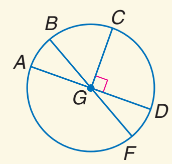
**Exercises** **Find each measure.**

*See Example 1 on page 529.*




17. $m\widehat{YC}$ **60**
18. $m\widehat{BC}$ **123**
19. $m\widehat{BX}$ **117**
20. $m\widehat{BCA}$ **180**

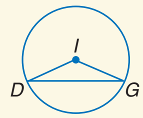
**In ⊙$G$, $m\angle AGB = 30$ and $\overline{CG} \perp \overline{GD}$. Find each measure.** *See Example 2 on page 531.*



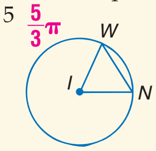

21. $m\widehat{AB}$ **30**
22. $m\widehat{BC}$ **60**
23. $m\widehat{FD}$ **30**
24. $m\widehat{CDF}$ **120**
25. $m\widehat{BCD}$ **150**
26. $m\widehat{FAB}$ **180**

**Find the length of the indicated arc in each ⊙$I$.** *See Example 4 on page 532.*

27. $\widehat{DG}$ if $m\angle DGI = 24$ and $r = 6$ **$\frac{22}{5}\pi$**




28. $\widehat{WN}$ if $\triangle IWN$ is equilateral and $WN = 5$ **$\frac{5}{3}\pi$**



## 10-3 Arcs and Chords

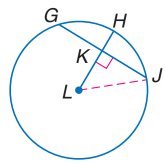
See pages 536–543.

### Concept Summary

- The endpoints of a chord are also the endpoints of an arc.
- Diameters perpendicular to chords bisect chords and intercepted arcs.

**Examples**

**Circle $L$ has a radius of 32 centimeters. $\overline{LH} \perp \overline{GJ}$, and $GJ = 40$ centimeters. Find $LK$.**




Draw radius $\overline{LJ}$. $LJ = 32$ and $\triangle LKJ$ is a right triangle.

$\overline{LH}$ bisects $\overline{GJ}$, since they are perpendicular.

$KJ = \frac{1}{2}(GJ)$ Definition of segment bisector

$= \frac{1}{2}(40)$ or 20 $GJ = 40$, and simplify.

Use the Pythagorean Theorem to find $LK$.

$(LK)^2 + (KJ)^2 = (LJ)^2$ Pythagorean Theorem

$(LK)^2 + 20^2 = 32^2$ $KJ = 20$, $LJ = 32$

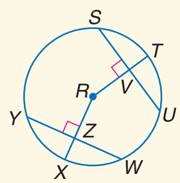
$(LK)^2 + 400 = 1024$ Simplify.

$(LK)^2 = 624$ Subtract 400 from each side.

$LK = \sqrt{624}$ Take the square root of each side.

$LK \approx 24.98$ Use a calculator.

**Exercises** **In $\odot R$, $SU = 20$, $YW = 20$, and $m\overset{\frown}{YX} = 45$. Find each measure.** *See Example 3 on page 538.*




**29.** $SV$ **10**

**30.** $WZ$ **10**

**31.** $UV$ **10**

**32.** $m\overset{\frown}{YW}$ **90**

**33.** $m\overset{\frown}{ST}$ **45**

**34.** $m\overset{\frown}{SU}$ **90**

## 10-4 Inscribed Angles

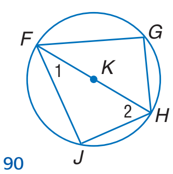
See pages 544–551.

### Concept Summary

- The measure of the inscribed angle is half the measure of its intercepted arc.
- The angles of inscribed polygons can be found by using arc measures.

**Example**

**ALGEBRA** **Triangles $FGH$ and $FHJ$ are inscribed in $\odot K$ with $\overset{\frown}{FG} \cong \overset{\frown}{FJ}$. Find $x$ if $m\angle 1 = 6x - 5$, and $m\angle 2 = 7x + 4$.**




$FJH$ is a right angle because $\overset{\frown}{FJH}$ is a semicircle.

$m\angle 1 + m\angle 2 + m\angle FJH = 180$ Angle Sum Theorem

$(6x - 5) + (7x + 4) + 90 = 180$ $m\angle 1 = 6x - 5$, $m\angle 2 = 7x + 4$, $m\angle FJH = 90$

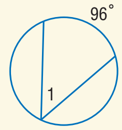
$13x + 89 = 180$ Simplify.

$x = 7$ Solve for $x$.

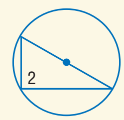


Chapter 10 Study Guide and Review 583

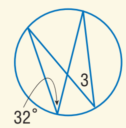
**Exercises** Find the measure of each numbered angle.
See Example 1 on page 545.

**35.** 

**48**

**36.** 

**90**

**37.** 

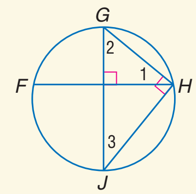
**32**

**Find the measure of each numbered angle for each situation given.**
See Example 4 on page 547.

**38.** $m\widehat{GH} = 78$ **$m\angle 1 = m\angle 3 = 39$, $m\angle 2 = 51$**

**39.** $m\angle 2 = 2x$, $m\angle 3 = x$ **$m\angle 1 = m\angle 3 = 30$, $m\angle 2 = 60$**

**40.** $m\widehat{JH} = 114$ **$m\angle 2 = 57$, $m\angle 3 = m\angle 1 = 33$**




## 10-5 Tangents

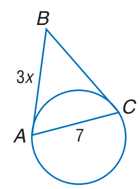
See pages 552–558.

### Concept Summary

- A line that is tangent to a circle intersects the circle in exactly one point.
- A tangent is perpendicular to a radius of a circle.
- Two segments tangent to a circle from the same exterior point are congruent.

**Example**

**ALGEBRA Given that the perimeter of $\triangle ABC = 25$, find $x$. Assume that segments that appear tangent to circles are tangent.**

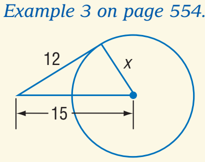



In the figure, $\overline{AB}$ and $\overline{AC}$ are drawn from the same exterior point and are tangent to $\odot Q$. So $\overline{AB} \cong \overline{AC}$.

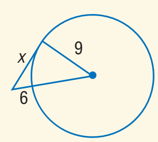
The perimeter of the triangle, $AB + BC + AC$, is 25.

| | |
|---|---|
| $AB + BC + AC = 25$ | Definition of perimeter |
| $3x + 3x + 7 = 25$ | $AB = BC = 3x$, $AC = 7$ |
| $6x + 7 = 25$ | Simplify. |
| $6x = 18$ | Subtract 7 from each side. |
| $x = 3$ | Divide each side by 6. |

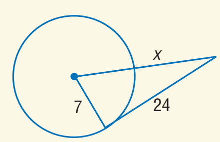
**Exercises** Find $x$. Assume that segments that appear to be tangent are tangent.
See Example 3 on page 554.

**41.** 

**9**

**42.** 

**12**

**43.** 

**18**

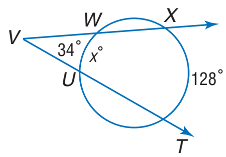
## 10-6 Secants, Tangents, and Angle Measures

See pages 561–568.

### Concept Summary

- The measure of an angle formed by two secant lines is half the positive difference of its intercepted arcs.
- The measure of an angle formed by a secant and tangent line is half its intercepted arc.

**Example** **Find $x$.**




$m\angle V = \frac{1}{2}\left(m\widehat{XT} - m\widehat{WU}\right)$

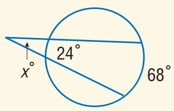
$34 = \frac{1}{2}(128 - x)$ Substitution

$-30 = -\frac{1}{2}x$ Simplify.

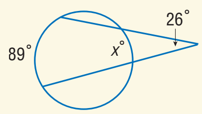
$x = 60$ Multiply each side by $-2$.

**Exercises** **Find $x$.** *See Example 3 on page 563.*

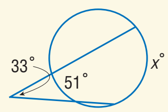
**44.** 22




**45.** 37




**46.** 117



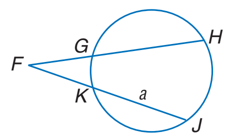

## 10-7 Special Segments in a Circle

See pages 569–574.

### Concept Summary

- The lengths of intersecting chords in a circle can be found by using the products of the measures of the segments.
- The secant segment product also applies to segments that intersect outside the circle, and to a secant segment and a tangent.

**Example** **Find $a$, if $FG = 18$, $GH = 42$, and $FK = 15$.**




Let $KJ = a$.

$FK \cdot FJ = FG \cdot FH$ Secant Segment Products

$15(a + 15) = 18(18 + 42)$ Substitution

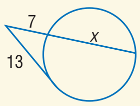
$15a + 225 = 1080$ Distributive Property

$15a = 855$ Subtract 225 from each side.

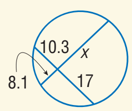
$a = 57$ Divide each side by 15.

**Exercises** **Find $x$ to the nearest tenth. Assume that segments that appear to be tangent are tangent.** *See Examples 3 and 4 on pages 570 and 571.*

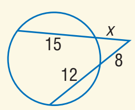
**47.** 17.1




**48.** 21.6




**49.** 7.2



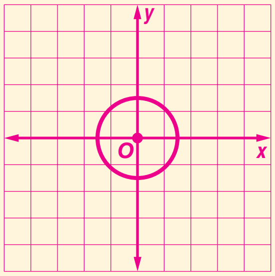



Chapter 10 Study Guide and Review 585

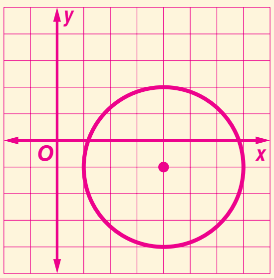
Chapter 10 For More ...
• Extra Practice, see pages 773–776.
• Mixed Problem Solving, see page 791.

## Answers

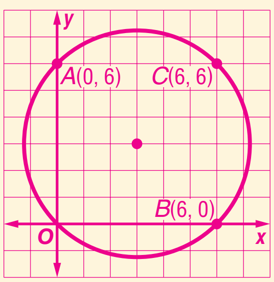
54.




55.



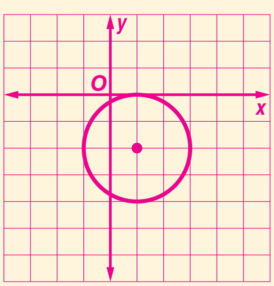

57.




## Answers (page 587)

1. **Sample answer: A chord is a segment that has its endpoints on a circle. A secant contains a chord and is a line that intersects a circle in two points. A tangent is a line that intersects a circle in exactly one point and no point of the tangent lies in the interior of the circle.**
2. **Find the midpoint of the diameter using the Midpoint Formula with the coordinates of the diameter's endpoints.**

22.






## 10-8 *Equations of Circles*

See pages 575–580.

### Concept Summary

- The coordinates of the center of a circle $(h, k)$ and its radius $r$ can be used to write an equation for the circle in the form $(x - h)^2 + (y - k)^2 = r^2$.
- A circle can be graphed on a coordinate plane by using the equation written in standard form.
- A circle can be graphed through any three noncollinear points on the coordinate plane.

### Examples

**1 Write an equation of a circle with center (−1, 4) and radius 3.**

Since the center is at $(-1, 4)$ and the radius is 3, $h = -1$, $k = 4$, and $r = 3$.

$(x - h)^2 + (y - k)^2 = r^2$ Equation of a circle

$[x - (-1)]^2 + (y - 4)^2 = 3^2$ $h = -1$, $k = 4$, and $r = 3$

$(x + 1)^2 + (y - 4)^2 = 9$ Simplify.

**2 Graph $(x - 2)^2 + (y + 3)^2 = 6.25$.**

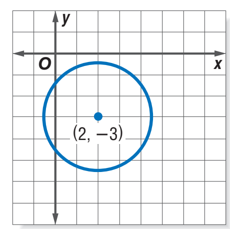
Identify the values of $h$, $k$, and $r$ by writing the equation in standard form.

$(x - 2)^2 + (y + 3)^2 = 6.25$

$(x - 2)^2 + [y - (-3)]^2 = 2.5^2$

$h = 2$, $k = -3$, and $r = 2.5$

Graph the center $(2, -3)$ and use a compass to construct a circle with radius 2.5 units.




### Exercises

**Write an equation for each circle.** *See Examples 1 and 2 on pages 575 and 576.*

50. center at $(0, 0)$, $r = \sqrt{5}$ **$x^2 + y^2 = 5$**
51. center at $(-4, 8)$, $d = 6$ **$(x + 4)^2 + (y - 8)^2 = 9$**
52. diameter with endpoints at $(0, -4)$ and $(8, -4)$ **$(x - 4)^2 + (y + 4)^2 = 16$**
53. center at $(-1, 4)$ and is tangent to $x = 1$ **$(x + 1)^2 + (y - 4)^2 = 4$**

**Graph each equation.** *See Example 3 on page 576.*

54. $x^2 + y^2 = 2.25$
55. $(x - 4)^2 + (y + 1)^2 = 9$

**54–55. See margin.**

**For Exercises 56 and 57, use the following information.**
A circle graphed on a coordinate plane contains $A(0, 6)$, $B(6, 0)$, and $C(6, 6)$.
*See Example 4 on page 577.*

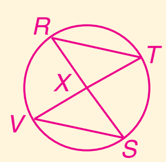
56. Write an equation of the circle. **$(x - 3)^2 + (y - 3)^2 = 18$**
57. Graph the circle. **See margin.**



**23. Sample answer:**

**Given: $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$**

**Prove: $\widehat{RT} \cong \widehat{VS}$**




**Proof:**

**Statements (Reasons)**

1. **$\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$ (Given)**
2. **$\angle RXT \cong \angle VXS$ (Vertical ∠s are ≅.)**
3. **$m\angle RXT = m\angle VXS$ (Def. of ≅ ∠s)**
4. **$m\widehat{RT} = m\angle RXT$, $m\widehat{VS} = m\angle VXS$ (Measure of arc equals measure of its central angle.)**
5. **$m\widehat{RT} = m\widehat{VS}$ (Substitution)**
6. **$\widehat{RT} \cong \widehat{VS}$ (Def. of ≅ arcs)**

Chapter 10

# Practice Test

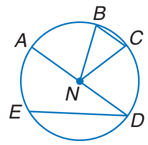
## Vocabulary and Concepts

1. **Describe** the differences among a tangent, a secant, and a chord of a circle. **1–2. See margin.**
2. **Explain** how to find the center of a circle given the coordinates of the endpoints of a diameter.

## Skills and Applications

3. Determine the radius of a circle with circumference $25\pi$ units. Round to the nearest tenth. **12.5 units**

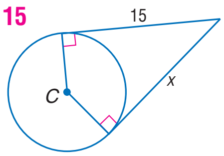
**For Questions 4–11, refer to $\odot N$.**



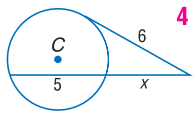

4. Name the radii of $\odot N$. $\overline{NA}$, $\overline{NB}$, $\overline{NC}$, $\overline{ND}$
5. If $AD = 24$, find $CN$. **12**
6. Is $ED > AD$? Explain. **No; diameters are the longest chords of a circle.**
7. If $AN$ is 5 meters long, find the exact circumference of $\odot N$. **$10\pi$ m**
8. If $m\angle BNC = 20$, find $m\widehat{BC}$. **20**
9. If $m\widehat{BC} = 30$ and $\widehat{AB} \cong \widehat{CD}$, find $m\widehat{AB}$. **75**
10. If $\overline{BE} \cong \overline{ED}$ and $m\widehat{ED} = 120$, find $m\widehat{BE}$. **120**
11. If $m\widehat{AE} = 75$, find $m\angle ADE$. **37.5**

**Find $x$. Assume that segments that appear to be tangent are tangent.**

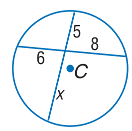
12. **15**



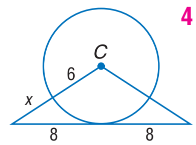

13. **4**



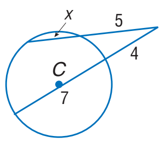

14. **9.6**



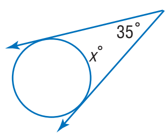

15. **4**



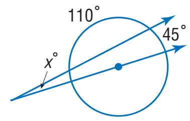

16. **3.8**



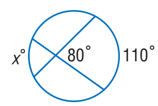

17. **145**



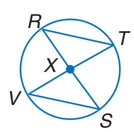

18. **10**



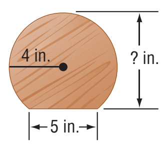

19. **50**




20. **AMUSEMENT RIDES** Suppose a Ferris wheel is 50 feet wide. Approximately how far does a rider travel in one rotation of the wheel? **157 ft**
21. Write an equation of a circle with center at $(-2, 5)$ and a diameter of 50. **$(x + 2)^2 + (y - 5)^2 = 625$**
22. Graph $(x - 1)^2 + (y + 2)^2 = 4$. **See margin.**
23. **PROOF** Write a two-column proof.
    **Given:** $\odot X$ with diameters $\overline{RS}$ and $\overline{TV}$
    **Prove:** $\widehat{RT} \cong \widehat{VS}$
    **See margin.**

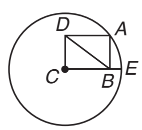



24. **CRAFTS** Takita is making bookends out of circular wood pieces as shown at the right. What is the height of the cut piece of wood? **about 7.1 in.**




25. **STANDARDIZED TEST PRACTICE** Circle $C$ has radius $r$ and $ABCD$ is a rectangle. Find $DB$. **A**

(A) $r$ (B) $r\frac{\sqrt{2}}{2}$ (C) $r\sqrt{3}$ (D) $r\frac{\sqrt{3}}{2}$

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## Portfolio Suggestion

**Introduction** After completing a chapter containing several concepts, students might benefit from going back and categorizing the concepts they found easy or challenging.

**Ask Students** Label two sheets of paper "Chapter 10—Concepts I Already Knew" and "Chapter 10—Concepts I Learned." Go back through each lesson and note the concepts in the lesson. Then categorize them on their pieces of paper. You can either write the name of the concept, explain it in your own words, or draw an example. Place these sheets in your portfolio.

Chapter 10

Practice Test

## Assessment Options

**Vocabulary Test** A vocabulary test/review for Chapter 10 can be found on p. 602 of the *Chapter 10 Resource Masters*.

**Chapter Tests** There are six Chapter 10 Tests and an Open-Ended Assessment task available in the *Chapter 10 Resource Masters*.

| Chapter 10 Tests | | | |
|---|---|---|---|
| **Form** | **Type** | **Level** | **Pages** |
| 1 | MC | basic | 589–590 |
| 2A | MC | average | 591–592 |
| 2B | MC | average | 593–594 |
| 2C | FR | average | 595–596 |
| 2D | FR | average | 597–598 |
| 3 | FR | advanced | 599–600 |

*MC = multiple-choice questions*
*FR = free-response questions*

**Open-Ended Assessment** Performance tasks for Chapter 10 can be found on p. 601 of the *Chapter 10 Resource Masters*. A sample scoring rubric for these tasks appears on p. A31.

**Unit 3 Test** A unit test/review can be found on pp. 609–610 of the *Chapter 10 Resource Masters*.

### ExamView® Pro

Use the networkable **ExamView® Pro** to:

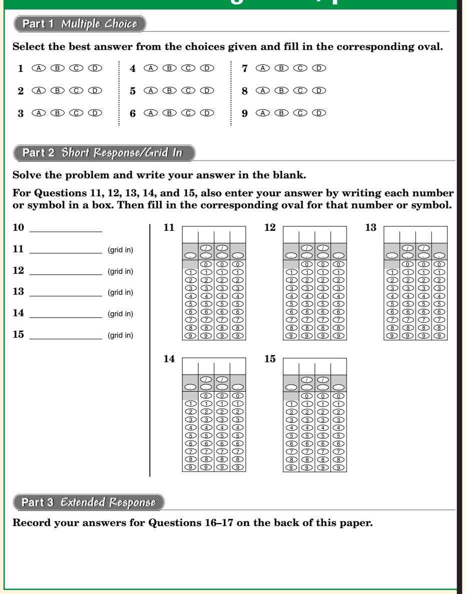
- Create **multiple versions** of tests.
- Create **modified** tests for Inclusion students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your tests from a program bank of artwork.

# Chapter 10 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 10 Resource Masters*.

**Standardized Test Practice Student Recording Sheet, p. A1**




## Additional Practice

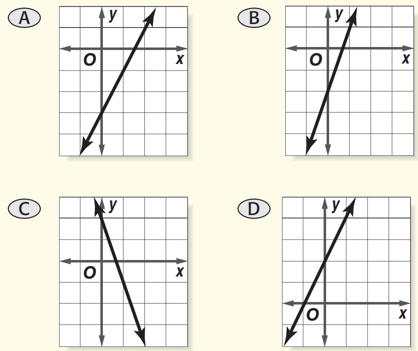
See pp. 607–608 in the *Chapter 10 Resource Masters* for additional standardized test practice.

# Chapter 10 Standardized Test Practice

## Part 1 Multiple Choice

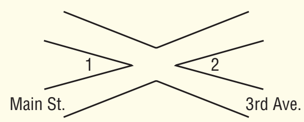
**Record your answer on the answer sheet provided by your teacher or on a sheet of paper.**

**1.** Which of the following shows the graph of $3y = 6x - 9$? (Prerequisite Skill) **A**




**2.** In Hyde Park, Main Street and Third Avenue do not meet at right angles. Use the figure below to determine the measure of $\angle 1$ if $m\angle 1 = 6x - 5$ and $m\angle 2 = 3x + 13$. (Lesson 1-5) **C**

Ⓐ 6
Ⓑ 18
Ⓒ 31
Ⓓ 36




**3.** Part of a proof is shown below. What is the reason to justify Step b? (Lesson 2-5) **A**

**Given:** $4x + \frac{4}{3} = 12$ **Prove:** $x = \frac{8}{3}$

| Statements | Reasons |
|---|---|
| a. $4x + \frac{4}{3} = 12$ | a. Given |
| b. $3\left(4x + \frac{4}{3}\right) = 3(12)$ | b. ___?___ |

Ⓐ Multiplication Property
Ⓑ Distributive Property
Ⓒ Cross products
Ⓓ none of the above

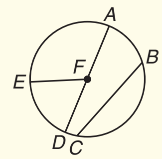
**4.** If an equilateral triangle has a perimeter of $(2x + 9)$ miles and one side of the triangle measures $(x + 2)$ miles, how long (in miles) is the side of the triangle? (Lesson 4-1) **B**

Ⓐ 3 Ⓑ 5 Ⓒ 9 Ⓓ 15

**5.** A pep team is holding up cards to spell out the school name. What symmetry does the card shown below have? (Lesson 9-1) **A**


Ⓐ only line symmetry
Ⓑ only point symmetry
Ⓒ both line and point symmetry
Ⓓ neither line nor point symmetry

**Use the figure below for Questions 6 and 7.**




**6.** In circle $F$, which are chords? (Lesson 10-1) **D**

Ⓐ $\overline{AD}$ and $\overline{EF}$
Ⓑ $\overline{AF}$ and $\overline{BC}$
Ⓒ $\overline{EF}$, $\overline{DF}$, and $\overline{AF}$
Ⓓ $\overline{AD}$ and $\overline{BC}$

**7.** In circle $F$, what is the measure of $\overset{\frown}{EA}$ if $m\angle DFE$ is 36? (Lesson 10-2) **C**

Ⓐ 54 Ⓑ 104 Ⓒ 144 Ⓓ 324

**8.** Which statement is false? (Lesson 10-3) **C**

Ⓐ Two chords that are equidistant from the center of a circle are congruent.
Ⓑ A diameter of a circle that is perpendicular to a chord bisects the chord and its arc.
Ⓒ The measure of a major arc is the measure of its central angle.
Ⓓ Minor arcs in the same circle are congruent if their corresponding chords are congruent.

**9.** Which of the segments described could be a secant of a circle? (Lesson 10-6) **D**

Ⓐ intersects exactly one point on a circle
Ⓑ has its endpoints on a circle
Ⓒ one endpoint at the center of the circle
Ⓓ intersects exactly two points on a circle



## ExamView® Pro

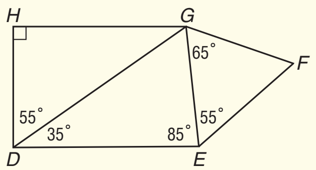
Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and state proficiency tests can be found on this CD-ROM.

**Preparing for Standardized Tests**
For test-taking strategies and more practice, see pages 795–810.

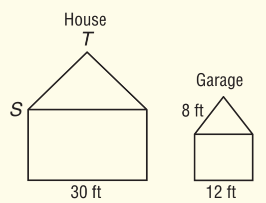
## Part 2 Short Response/Grid In

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

**10.** What is the shortest side of quadrilateral *DEFG*? (Lesson 5-3) $\overline{FG}$

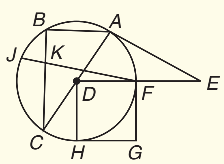



**11.** An architect designed a house and a garage that are similar in shape. How many feet long is $\overline{ST}$? (Lesson 6-2) **20**




**12.** Two triangles are drawn on a coordinate grid. One has vertices at (0, 1), (0, 7), and (6, 4). The other has vertices at (7, 7), (10, 7), and (8.5, 10). What scale factor can be used to compare the smaller triangle to the larger? (Lesson 9-5) **2**

**Use the figure below for Questions 13–15.**




**13.** Point *D* is the center of the circle. What is $m\angle ABC$? (Lesson 10-4) **90**

**14.** $\overline{AE}$ is a tangent. If $AD = 12$ and $FE = 18$, how long is $\overline{AE}$ to the nearest tenth unit? (Lesson 10-5) **27.5**

**15.** Chords $\overline{JF}$ and $\overline{BC}$ intersect at *K*. If $BK = 8$, $KC = 12$, and $KF = 16$, find *JK*. (Lesson 10-7) **6**

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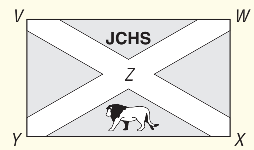
### Test-Taking Tip

**Question 4**
If a question does not provide you with a figure that represents the problem, draw one yourself. Label the figure with the given information.

## Part 3 Extended Response

**Record your answers on a sheet of paper. Show your work.**

**16.** The Johnson County High School flag is shown below. Points have been added for reference.




a. Which diagonal segments would have to be congruent for *VWXY* to be a rectangle? (Lesson 8-3) $\overline{VX}$ **and** $\overline{WY}$

b. Suppose the length of rectangle *VWXY* is 2 more than 3 times the width and the perimeter is 164 inches. What are the dimensions of the flag? (Lesson 1-6) **20 by 62**

**17.** The segment with endpoints $A(1, -2)$ and $B(1, 6)$ is the diameter of a circle.

a. Graph the points and draw the circle. (Lesson 10-1) **See margin.**

b. What is the center of the circle? (Lesson 10-1) **(1, 2)**

c. What is the length of the radius? (Lesson 10-8) **4**

d. What is the circumference of the circle? (Lesson 10-8) **$8\pi$ units**

e. What is the equation of the circle? (Lesson 10-8) $(x - 1)^2 + (y - 2)^2 = 16$



## Evaluating Extended-Response Questions

Extended-Response questions are graded by using a multilevel rubric that guides you in assessing a student's knowledge of a particular concept.

**Goal for Question 16:** Determine the relationship between segment lengths for the flag to be a parallelogram, and find the dimensions of the flag.

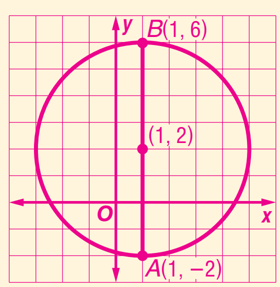
**Goal for Question 17:** Using a segment in a coordinate plane as a diameter, write an equation for a circle and find its center, radius, circumference.

**Sample Scoring Rubric:** The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.

| Score | Criteria |
|---|---|
| 4 | A correct solution that is supported by well-developed, accurate explanations |
| 3 | A generally correct solution, but may contain minor flaws in reasoning or computation |
| 2 | A partially correct interpretation and/or solution to the problem |
| 1 | A correct solution with no supporting evidence or explanation |
| 0 | An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given |

### Answer

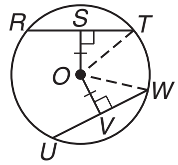
17a.



**Pages 539–543, Lesson 10-3**

**37. Given:** $\odot O$, $\overline{OS} \perp \overline{RT}$, $\overline{OV} \perp \overline{UW}$, $\overline{OS} \cong \overline{OV}$

**Prove:** $\overline{RT} \cong \overline{UW}$



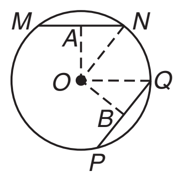

**Proof:**

**Statements (Reasons)**

1. $\overline{OT} \cong \overline{OW}$ (All radii of a $\odot$ are $\cong$.)
2. $\overline{OS} \perp \overline{RT}$, $\overline{OV} \perp \overline{UW}$, $\overline{OS} \cong \overline{OV}$ (Given)
3. $\angle OST$, $\angle OVW$ are right angles. (Def. of $\perp$ lines)
4. $\triangle STO \cong \triangle VWO$ (HL)
5. $\overline{ST} \cong \overline{VW}$ (CPCTC)
6. $ST = VW$ (Definition of $\cong$ segments)
7. $2(ST) = 2(VW)$ (Multiplication Property)
8. $\overline{OS}$ bisects $\overline{RT}$; $\overline{OV}$ bisects $\overline{UW}$. (Radius $\perp$ to a chord bisects the chord.)
9. $RT = 2(ST)$, $UW = 2(VW)$ (Def. of seg. bisector)
10. $RT = UW$ (Substitution)
11. $\overline{RT} \cong \overline{UW}$ (Definition of $\cong$ segments)

**38. Given:** $\odot O$, $\overline{MN} \cong \overline{PQ}$, $\overline{ON}$ and $\overline{OQ}$ are radii. $\overline{OA} \perp \overline{MN}$; $\overline{OB} \perp \overline{PQ}$

**Prove:** $\overline{OA} \cong \overline{OB}$




**Proof:**

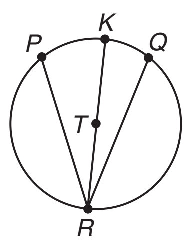
**Statements (Reasons)**

1. $\odot O$, $\overline{MN} \cong \overline{PQ}$, $\overline{ON}$ and $\overline{OQ}$ are radii, $\overline{OA} \perp \overline{MN}$, $\overline{OB} \perp \overline{PQ}$ (Given)
2. $\overline{OA}$ bisects $\overline{MN}$; $\overline{OB}$ bisects $\overline{PQ}$. ($\overline{OA}$ and $\overline{OB}$ are contained in radii. A radius $\perp$ to a chord bisects the chord.)
3. $AN = \frac{1}{2}MN$; $BQ = \frac{1}{2}PQ$ (Def. of bisector)
4. $MN = PQ$ (Def. of $\cong$ segments)
5. $\frac{1}{2}MN = \frac{1}{2}PQ$ (Mult. Prop.)
6. $AN = BQ$ (Substitution)
7. $\overline{AN} \cong \overline{BQ}$ (Def. of $\cong$ segments)
8. $\overline{ON} \cong \overline{OQ}$ (All radii of a circle are $\cong$.)
9. $\triangle AON \cong \triangle BOQ$ (HL)
10. $\overline{OA} \cong \overline{OB}$ (CPCTC)

**Pages 548–551 Lesson 10-4**

**35. Given:** $T$ lies inside $\angle PRQ$. $\overline{RK}$ is a diameter of $\odot T$.

**Prove:** $m\angle PRQ = \frac{1}{2}m\widehat{PKQ}$



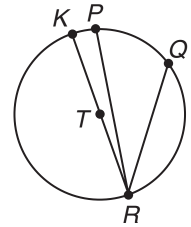

**Proof:**

**Statements (Reasons)**

1. $m\angle PRQ = m\angle PRK + m\angle KRQ$ ($\angle$ Addition Th.)
2. $m\widehat{PKQ} = m\widehat{PK} + m\widehat{KQ}$ (Arc Addition Theorem)
3. $\frac{1}{2}m\widehat{PKQ} = \frac{1}{2}m\widehat{PK} + \frac{1}{2}m\widehat{KQ}$ (Multiplication Prop.)
4. $m\angle PRK = \frac{1}{2}m\widehat{PK}$, $m\angle KRQ = \frac{1}{2}m\widehat{KQ}$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1).)
5. $\frac{1}{2}m\widehat{PKQ} = m\angle PRK + m\angle KRQ$ (Subst. (Steps 3, 4))
6. $\frac{1}{2}m\widehat{PKQ} = m\angle PRQ$ (Substitution (Steps 5, 1))

**36. Given:** $T$ lies outside $\angle PRQ$. $\overline{RK}$ is a diameter of $\odot T$.

**Prove:** $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$



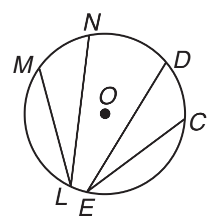

**Proof:**

**Statements (Reasons)**

1. $m\angle PRQ = m\angle KRQ - m\angle PRK$ (Angle Addition Theorem, Subtraction Property )
2. $m\widehat{PQ} = m\widehat{KQ} - m\widehat{KP}$ (Arc Addition Theorem, Subtraction Property)
3. $\frac{1}{2}m\widehat{PQ}, = \frac{1}{2}(m\widehat{KQ} - m\widehat{KP})$ (Division Property)
4. $m\angle PRK = \frac{1}{2}m\widehat{KP}$, $m\angle KRQ = \frac{1}{2}m\widehat{KQ}$ (The measure of an inscribed $\angle$ whose side is a diameter is half the measure of the intercepted arc (Case 1).)
5. $m\angle PRQ = \frac{1}{2}m\widehat{KQ} - \frac{1}{2}m\widehat{KP}$ (Subst. (Steps 1, 4))
6. $m\angle PRQ = \frac{1}{2}(m\widehat{KQ} - m\widehat{KP})$ (Distributive Property)
7. $m\angle PRQ = \frac{1}{2}m\widehat{PQ}$ (Substitution (Steps 6, 3))

**37. Given:** inscribed $\angle MLN$ and $\angle CED$, $\widehat{CD} \cong \widehat{MN}$

**Prove:** $\angle CED \cong \angle MLN$



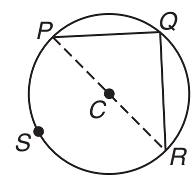

**Proof:**

**Statements (Reasons)**

1. $\angle MLN$ and $\angle CED$ are inscribed; $\widehat{CD} \cong \widehat{MN}$ (Given)
2. $m\angle MLN = \frac{1}{2}m\widehat{MN}$; $m\angle CED = \frac{1}{2}m\widehat{CD}$ (Measure of an inscribed $\angle$ = half measure of intercepted arc.)
3. $m\widehat{CD} = m\widehat{MN}$ (Def. of $\cong$ arcs)
4. $\frac{1}{2}m\widehat{CD} = \frac{1}{2}m\widehat{MN}$ (Mult. Prop.)
5. $m\angle CED = m\angle MLN$ (Substitution)
6. $\angle CED \cong \angle MLN$ (Def. of $\cong$ $\angle$s)

**38. Given:** $\widehat{PQR}$ is a semicircle.

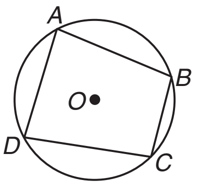
**Prove:** $\angle PQR$ is a right angle.




**Proof:** Since $\widehat{PQR}$ is a semicircle, $\widehat{PSR}$ is also a semicircle and $m\widehat{PSR} = 180$. $\angle PQR$ is an inscribed angle, and $m\angle PQR = \frac{1}{2}(m\widehat{PSR})$ or 90, making $\angle PQR$ a right angle.

**39. Given:** quadrilateral *ABCD* inscribed in $\odot O$

**Prove:** $\angle A$ and $\angle C$ are supplementary. $\angle B$ and $\angle D$ are supplementary.




**Proof:** By arc addition and the definitions of arc measure and the sum of central angles, $m\widehat{DCB} + m\widehat{DAB} = 360$. Since $m\angle C = \frac{1}{2}m\widehat{DAB}$ and $m\angle A = \frac{1}{2}m\widehat{DCB}$, $m\angle C + m\angle A = \frac{1}{2}(m\widehat{DCB} + m\widehat{DAB})$, but $m\widehat{DCB} + m\widehat{DAB} = 360$, so $m\angle C + m\angle A = \frac{1}{2}(360)$ or 180. This makes $\angle C$ and $\angle A$ supplementary. Because the sum of the measures of the interior angles of a quadrilateral is 360, $m\angle A + m\angle C + m\angle B + m\angle D = 360$. But $m\angle A + m\angle C = 180$, so $m\angle B + m\angle D = 180$, making them supplementary also.

**41.** Sides are congruent radii making it isosceles and $\angle AOC$ is a central angle for an arc of 90°, making it a right angle.

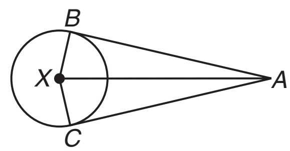
**42.** Each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

**43.** Each angle intercepts a semicircle, making them 90° angles. Each side is a chord of congruent arcs, so the chords are congruent.

### Pages 555–558, Lesson 10-5

**27. Given:** $\overline{AB}$ is tangent to $\odot X$ at *B*. $\overline{AC}$ is tangent to $\odot X$ at *C*.

**Prove:** $\overline{AB} \cong \overline{AC}$

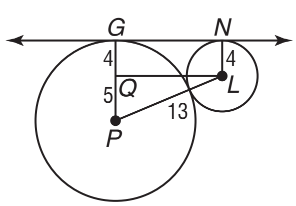



**Proof:**

**Statements (Reasons)**

1. $\overline{AB}$ is tangent to $\odot X$ at *B*, $\overline{AC}$ is tangent to $\odot X$ at *C*. (Given)
2. Draw $\overline{BX}$, $\overline{CX}$, and $\overline{AX}$. (Through any two points, there is one line.)
3. $\overline{AB} \perp \overline{BX}$, $\overline{AC} \perp \overline{CX}$ (Line tangent to a circle is $\perp$ to the radius at the pt. of tangency.)
4. $\angle ABX$ and $\angle ACX$ are right angles. (Def. of $\perp$ lines)
5. $\overline{BX} \cong \overline{CX}$ (All radii of a circle are $\cong$.)
6. $\overline{AX} \cong \overline{AX}$ (Reflexive Prop.)
7. $\triangle ABX \cong \triangle ACX$ (HL)
8. $\overline{AB} \cong \overline{AC}$ (CPCTC)

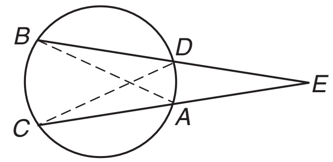
**31.** 12; Draw $\overline{PG}$, $\overline{NL}$, and $\overline{PL}$. Construct $\overline{LQ} \perp \overline{GP}$, thus *LQGN* is a rectangle. $GQ = NL = 4$, so $QP = 5$. Using the Pythagorean Theorem, $(QP)^2 + (QL)^2 = (PL)^2$. So, $QL = 12$. Since $GN = QL$, $GN = 12$.




### Pages 571–574, Lesson 10-7

**30. Given:** $\overline{EC}$ and $\overline{EB}$ are secant segments.

**Prove:** $EA \cdot EC = ED \cdot EB$



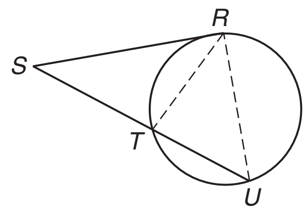

**Proof:**

**Statements (Reasons)**

1. $\overline{EC}$ and $\overline{EB}$ are secant segments. (Given)
2. $\angle DEC \cong \angle AEB$ (They name the same angle. (Reflexive Prop.))
3. $\angle ECD \cong \angle EBA$ (Inscribed $\angle$s that intercept the same arc are $\cong$.)
4. $\triangle ABE \sim \triangle DCE$ (AA Similarity)
5. $\frac{EA}{ED} = \frac{EB}{EC}$ (Definition of similar triangles)
6. $EA \cdot EC = ED \cdot EB$ (Cross Products)

**31. Given:** tangent $\overline{RS}$ and secant $\overline{US}$

**Prove:** $(RS)^2 = US \cdot TS$




**Proof:**

**Statements (Reasons)**

1. tangent $\overline{RS}$ and secant $\overline{US}$ (Given)
2. $m\angle RUT = \frac{1}{2}m\widehat{RT}$ (The measure of an inscribed angle equals half the measure of its intercepted arc.)
3. $m\angle SRT = \frac{1}{2}m\widehat{RT}$ (The measure of an angle formed by a secant and a tangent equals half the measure of its intercepted arc.)
4. $m\angle RUT = m\angle SRT$ (Substitution)
5. $\angle RUT \cong \angle SRT$ (Definition of congruent angles)
6. $\angle S \cong \angle S$ (Reflexive Prop.)
7. $\triangle SUR \sim \triangle SRT$ (AA Similarity)
8. $\frac{RS}{US} = \frac{TS}{RS}$ (Definition of similar triangles)
9. $(RS)^2 = US \cdot TS$ (Cross Products)

### Pages 577–580, Lesson 10-8

**34.** The slope of $\overline{AC}$ is $-\frac{1}{4}$, so the slope of its bisector is 4. The midpoint of $\overline{AC}$ is (0, 5). Use the slope and the midpoint to write an equation for the bisector of $\overline{AC}$: $y = 4x + 5$. The slope of $\overline{BC}$ is $-\frac{9}{2}$, so the slope of its bisector is $\frac{2}{9}$. The midpoint of $\overline{BC}$ is $(-2, -3)$. Use the slope and the midpoint to write an equation for the bisector of $\overline{BC}$: $y = \frac{2}{9}x - \frac{23}{9}$. Solving the system of equations, $y = 4x + 5$ and $y = \frac{2}{9}x - \frac{23}{9}$, yields $(-2, -3)$, which is the circumcenter. Let $(-2, -3)$ be *D*, then $DA = DB = DC = \sqrt{85}$.