Chapter 7

Right Triangles and Trigonometry

Chapter Overview and Pacing

Year-long pacing: pages T20–T21.

LESSON OBJECTIVES	PACING (days) Regular		PACING (days) Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
7-1 Geometric Mean *(pp. 342–348)* • Find the geometric mean between two numbers. • Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.	1	1	0.5	0.5
7-2 The Pythagorean Theorem and Its Converse *(pp. 349–356)* ***Preview:*** Physically explore side-length relationships in the Pythagorean Theorem. • Use the Pythagorean Theorem. • Use the converse of the Pythagorean Theorem.	2 (with 7-2 Preview)	1	1 (with 7-2 Preview)	0.5
7-3 Special Right Triangles *(pp. 357–363)* • Use properties of 45°-45°-90° triangles. • Use properties of 30°-60°-90° triangles.	2	2	1	1
7-4 Trigonometry *(pp. 364–370)* • Find trigonometric ratios using right triangles. • Solve problems using trigonometric ratios.	2	2	1	1
7-5 Angle of Elevation and Depression *(pp. 371–376)* • Solve problems involving angles of elevation. • Solve problems involving angles of depression.	1	2	0.5	1
7-6 The Law of Sines *(pp. 377–384)* • Use the Law of Sines to solve triangles. • Solve problems by using the Law of Sines. ***Follow-Up:*** Use geometry software to investigate the Law of Sines.	optional	2 (with 7-6 Follow-Up)	optional	1 (with 7-6 Follow-Up)
7-7 The Law of Cosines *(pp. 385–391)* • Use the Law of Cosines to solve triangles. • Solve problems by using the Law of Cosines. ***Follow-Up:*** Use trigonometric ratios to identify trigonometric identities.	optional	2	optional	1
Study Guide and Practice Test *(pp. 392–397)* **Standardized Test Practice** *(pp. 398–399)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**10**	**14**	**5**	**7**

An electronic version of this chapter is available on ***StudentWorks™****. This backpack solution CD-ROM allows students instant access to the Student Edition, lesson worksheet pages, and web resources.*

Timesaving Tools

TeacherWorks™

All-In-One Planner and Resource Center

See pages T5 and T21.

Chapter Resource Manager

Chapter 7 Resource Masters										
Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Prerequisite Skills Workbook	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	GeomPASS: Tutorial Plus (lessons)	Materials
351–352	353–354	355	356		11–12, 31–32		7-1	7-1		
357–358	359–360	361	362	407	11–12, 31–32	GCC 29 SC 13	7-2	7-2	13	(*Preview:* patty paper, ruler)
363–364	365–366	367	368		17–18		7-3	7-3	14	grid paper, scissors
369–370	371–372	373	374	407, 409	11–12	SC 14	7-4	7-4		paper, scissors, metric ruler, protractor
375–376	377–378	379	380				7-5	7-5	15	
381–382	383–384	385	386	408		GCC 30	7-6	7-6		
387–388	389–390	391	392	408	21–22, 25–26		7-7	7-7	16	
				393–406, 410–412						

**Key to Abbreviations:* GCC = Graphing Calculator and Computer Masters
SC = School-to-Career Masters

Chapter 7

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

In previous courses, students evaluated radical expressions and equations involving fractions. The Pythagorean Theorem is reviewed in Chapter 1. In Chapter 6, students solve proportions. Chapter 4 introduces students to the Angle Sum Theorem.

This Chapter

This chapter provides students with an introduction to trigonometry. Students learn how to use the geometric mean to solve problems involving side length. They solve problems using the Pythagorean Theorem and its converse. Trigonometric ratios are defined and then used to solve right triangle problems. Students also use the Law of Sines and the Law of Cosines to solve non-right triangles.

Future Connections

Students gain a basic knowledge of trigonometric ratios from this chapter. This knowledge will help them in higher-level math courses, including pre-calculus and calculus. Architects, surveyors, and civil engineers use trigonometric ratios in their work.

7-1 Geometric Mean

The geometric mean between two numbers is the square root of their product. For two positive numbers a and b, the geometric mean is the positive number x for which the proportion $a:x = x:b$ is true. This proportion is equivalent to $x = \sqrt{ab}$.

The geometric mean has a particular application to a right triangle. If an altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other. The measure of this altitude is the geometric mean between the measures of the two segments of the hypotenuse. Moreover, the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

7-2 The Pythagorean Theorem and Its Converse

The Pythagorean Theorem states that in a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse. The converse of the Pythagorean Theorem is useful in determining whether given measures are those of a right triangle. If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle. A Pythagorean triple is a group of three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number.

7-3 Special Right Triangles

A 45°-45°-90° triangle is the only type of isosceles right triangle. One of its special properties is that the hypotenuse is $\sqrt{2}$ times as long as a leg, so the ratio of the sides is $1:1:\sqrt{2}$. A 30°-60°-90° triangle also has special properties. The measures of the sides are x, $x\sqrt{3}$, and $2x$, giving the sides a ratio of $1:\sqrt{3}:2$. Knowing these properties can save you valuable time when you are solving problems involving special right triangles.

7-4 Trigonometry

A ratio of the lengths of the sides of a right triangle is called a trigonometric ratio. The three most common trigonometric ratios are sine, cosine, and tangent, abbreviated sin, cos, and tan. Sine of $\angle A$ is the measure of the leg opposite $\angle A$ divided by the measure of the hypotenuse. Cosine of $\angle A$ is the measure of the leg adjacent $\angle A$ divided by the measure of the hypotenuse. Tangent of $\angle A$ is the measure of the leg opposite the angle divided by the measure of the leg adjacent the angle. The value of this ratio does not depend on the size of the triangle or the measures of the sides.

Trigonometric ratios are used to find missing measures of a right triangle. You only need to know the measures of two sides or the measure of one side and one acute angle. The inverse of each trigonometric ratio yields the angle measure. The inverses are written $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

7-5 Angle of Elevation and Depression

An angle of elevation is the angle between the line of sight and the horizontal when an observer looks upward. An angle of depression is the angle between the line of sight and the horizontal when an observer looks downward. Trigonometric ratios can be used to solve problems involving angles of elevation and depression. Angles of elevation and depression to two different objects can be used to find the difference between those objects.

7-6 The Law of Sines

In trigonometry, the Law of Sines can be used to find missing measures of triangles that are not right triangles. Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. The Law of Sines can be used to solve a triangle. This means finding the measure of every side and angle. The Law of Sines can be used to solve a triangle if you know the measures of two angles and any side of a triangle, or if you know the measures of two sides and an angle opposite one of these sides of the triangle.

There is one case in which using the Law of Sines to solve a triangle will yield an ambiguous solution. If you know the measures of two sides and an angle opposite one of the sides (SSA) and the angle is acute, it is possible to find two different triangles. One triangle will be an acute triangle, and the other will be obtuse.

7-7 The Law of Cosines

The Law of Cosines allows you to solve a triangle in some situations when the Law of Sines cannot be used. This occurs when you know the measures of two sides and the included angle (SAS) or three sides (SSS). Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then the following equations are true:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Chapter 7

DAILY INTERVENTION and Assessment

Key to Abbreviations:
TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

INTERVENTION / ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 341, 348, 356, 363, 370, 376, 383 Practice Quiz 1, p. 363 Practice Quiz 2, p. 383	5-Minute Check Transparencies *Prerequisite Skills Workbook*, pp. 11–12, 17–18, 21–22, 25–26, 31–32 Quizzes, *CRM* pp. 407–408 Mid-Chapter Test, *CRM* p. 409 Study Guide and Intervention, *CRM* pp. 351–352, 357–358, 363–364, 369–370, 375–376, 381–382, 387–388	GeomPASS: Tutorial Plus, Lessons 13, 14, 15, and 16 www.geometryonline.com/self_check_quiz www.geometryonline.com/extra_examples
Mixed Review	pp. 348, 356, 363, 370, 376, 383, 390	Cumulative Review, *CRM* p. 410	
Error Analysis	Find the Error, pp. 348, 353, 380 Common Misconceptions, p. 372	Find the Error, *TWE* pp. 345, 353, 380 Unlocking Misconceptions, *TWE* p. 359 Tips for New Teachers, *TWE* p. 358	
Standardized Test Practice	pp. 348, 356, 362, 370, 372, 373, 376, 382, 390, 392, 398, 399	*TWE* pp. 398–399 Standardized Test Practice, *CRM* pp. 411–412	Standardized Test Practice CD-ROM www.geometryonline.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 348, 356, 362, 369, 376, 382, 389 Open Ended, pp. 345, 353, 360, 367, 373, 380, 387 Standardized Test, p. 399	Modeling: *TWE* pp. 356, 376 Speaking: *TWE* pp. 363, 370 Writing: *TWE* pp. 348, 383, 390 Open-Ended Assessment, *CRM* p. 405	
Chapter Assessment	Study Guide, pp. 392–396 Practice Test, p. 397	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 393–398 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 399–404 Vocabulary Test/Review, *CRM* p. 406	ExamView® Pro (see below) MindJogger Videoquizzes www.geometryonline.com/vocabulary_review www.geometryonline.com/chapter_test

Yearly ProgressPro

For more information on Yearly ProgressPro, see p. 174.

Geometry Lesson	Yearly ProgressPro Skill Lesson
7-1	Geometric Mean
7-2	The Pythagorean Theorem and Its Converse
7-3	Special Right Triangles
7-4	Trigonometry
7-5	Angles of Elevation and Depression
7-6	The Laws of Sines
7-7	The Law of Cosines

ExamView® Pro

Use the networkable **ExamView® Pro** to:

- Create **multiple versions** of tests.
- Create **modified** tests for *Inclusion* students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your test from a program bank of artwork.

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Geometry provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 341
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 345, 353, 360, 367, 373, 380, 387)
- Writing in Math questions in every lesson, pp. 348, 356, 362, 369, 376, 382, 389
- Reading Study Tip, p. 364
- WebQuest, pp. 346, 390

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 341, 392
- Study Notebook suggestions, pp. 345, 349, 353, 360, 367, 373, 380, 388
- Modeling activities, pp. 356, 376
- Speaking activities, pp. 363, 370
- Writing activities, pp. 348, 383, 390
- Differentiated Instruction (Verbal/Linguistic), p. 386
- ELL Resources, pp. 340, 346, 354, 361, 368, 374, 381, 386, 389, 392

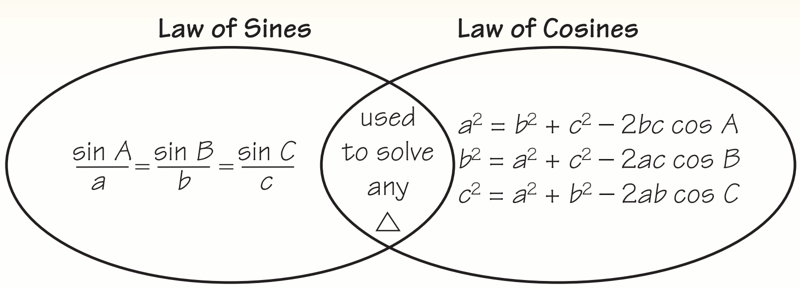
Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 7 Resource Masters,* pp. vii-viii)
- Proof Builder helps students learn and understand theorems and postulates from the chapter. (*Chapter 7 Resource Masters,* pp. ix–x)
- Reading to Learn Mathematics master for each lesson (*Chapter 7 Resource Masters*, pp. 355, 361, 367, 373, 379, 385, 391)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading Strategies for the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

PROJECT CRISS Study Skill

A Venn diagram can help students understand the commonalities and differences in two or more concepts. Show students the example below that compares and contrasts the Law of Sines and the Law of Cosines. While studying Chapter 7, have students work in groups to make Venn diagrams for special right triangles (Lesson 7-3) and for angles of elevation and depression (Lesson 7-5).



CReating Independence Through Student-Owned Strategies

Chapter 7 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
7-1	1, 3, 6, 8, 9, 10	
7-2 Preview	3, 6	
7-2	3, 6, 8, 9, 10	
7-3	3, 6, 8, 9, 10	
7-4	3, 6, 8, 9, 10	
7-5	3, 6, 8, 9, 10	
7-6	3, 6, 8, 9, 10	
7-6 Follow-Up	3, 6	
7-7	3, 6, 8, 9, 10	
7-7 Follow-Up	3, 6	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 7 Right Triangles and Trigonometry

What You'll Learn

- **Lessons 7-1, 7-2, and 7-3** Solve problems using the geometric mean, the Pythagorean Theorem, and its converse.
- **Lessons 7-4 and 7-5** Use trigonometric ratios to solve right triangle problems.
- **Lessons 7-6 and 7-7** Solve triangles using the Law of Sines and the Law of Cosines.

Key Vocabulary

- geometric mean (p. 342)
- Pythagorean triple (p. 352)
- trigonometric ratio (p. 364)
- Law of Sines (p. 377)
- Law of Cosines (p. 385)

Why It's Important

Trigonometry is used to find the measures of the sides and angles of triangles. These ratios are frequently used in real-world applications such as architecture, aviation, and surveying. *You will learn how surveyors use trigonometry in Lesson 7-6.*

Bob Daemmrich/The Image Works

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 7 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 7 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1 — Proportions

Solve each proportion. Round to the nearest hundredth, if necessary. *(For review, see Lesson 6-1.)*

1. $\frac{3}{4} = \frac{12}{a}$ **16**
2. $\frac{c}{5} = \frac{8}{3}$ **13.33**
3. $\frac{e}{20} = \frac{6}{5} = \frac{f}{10}$ **$e = 24$, $f = 12$**
4. $\frac{4}{3} = \frac{6}{y} = \frac{1}{z}$ **$y = 4.5$, $z = 0.75$**

For Lesson 7-2 — Pythagorean Theorem

Find the measure of the hypotenuse of each right triangle having legs with the given measures. Round to the nearest hundredth, if necessary. *(For review, see Lesson 1-3.)*

5. 5 and 12 **13**
6. 6 and 8 **10**
7. 15 and 15 **21.21**
8. 14 and 27 **30.41**

For Lessons 7-3 and 7-4 — Radical Expressions

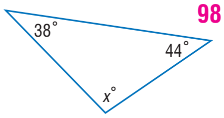
Simplify each expression. *(For review, see pages 744 and 745.)*

9. $\sqrt{8}$ **$2\sqrt{2}$**
10. $\sqrt{10^2 - 5^2}$ **$5\sqrt{3}$**
11. $\sqrt{39^2 - 36^2}$ **15**
12. $\frac{7}{\sqrt{2}}$ **$\frac{7\sqrt{2}}{2}$**

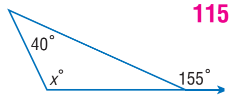
For Lessons 7-5 through 7-7 — Angle Sum Theorem

Find *x*. *(For review, see Lesson 4-2.)*

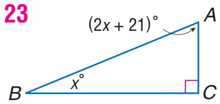
13. **98**



14. **115**



15. **23**

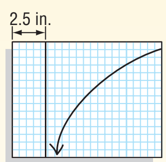


FOLDABLES™ Study Organizer

Right Triangles and Trigonometry Make this Foldable to help you organize your notes. Begin with seven sheets of grid paper.

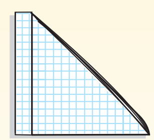
Step 1 Fold

Fold each sheet along the diagonal from the corner of one end to 2.5 inches away from the corner of the other end.



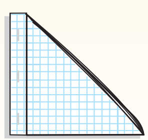
Step 2 Stack

Stack the sheets, and fold the rectangular part in half.



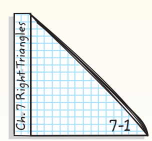
Step 3 Staple

Staple the sheets in three places.



Step 4 Label

Label each sheet with a lesson number, and the rectangular part with the chapter title.



Reading and Writing As you read and study the chapter, write notes, define terms, and solve problems in your Foldable.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 7. Page references are included for additional student help.

Additional review is provided in the *Prerequisite Skills Workbook*, pages 11–12, 17–18, 21–22, 25–26, 31–32.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
7-2	Using the Pythagorean Theorem, p. 348
7-3	Simplifying Radical Expressions, p. 356
7-4	Solving Equations That Involve Fractions, p. 363
7-5	Angles Formed by Parallel Lines and a Transversal, p. 370
7-6	Solving Proportions, p. 376
7-7	Evaluating Expressions, p. 383

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Journals and Visuals Use this Foldable journal for writing about right triangles and trigonometry. After students make their triangle-shaped journal, have them label the front of each triangle to correspond to the seven lessons in this chapter. Under the tabs of their Foldable, students take notes, define terms, solve problems, and write examples of how laws are used. On the front of each section, ask students to design a visual (graph, diagram, picture, chart) that presents the information introduced in the lesson in a concise, easy-to-study format. Encourage students to label their visuals and write captions.

7-1 Lesson Notes

1 Focus

5-Minute Check Transparency 7-1 Use as a quiz or review of Chapter 6.

Mathematical Background notes are available for this lesson on p. 340C.

How can the geometric mean be used to view paintings?

Ask students:

- What happens if you view the painting at a distance that is less than the geometric mean of the two distances described above? **You are too close to the painting and cannot see all of it at once. Your eyes must move left, right, up, and down to see all the details of the painting.**
- Name some other scenarios where you would be best positioned at a geometric mean to view something. **watching a program on television or a movie in a theatre, viewing a sports event, seeing a production on a stage, using a computer, skimming a book or magazine, etc.**

7-1 Geometric Mean

What You'll Learn

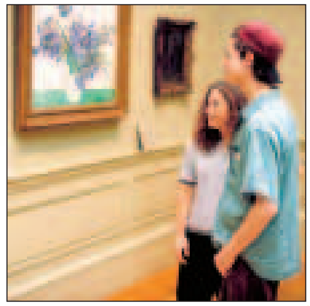
- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

Vocabulary

- geometric mean

How can the geometric mean be used to view paintings?

When you look at a painting, you should stand at a distance that allows you to see all of the details in the painting. The distance that creates the best view is the geometric mean of the distance from the top of the painting to eye level and the distance from the bottom of the painting to eye level.



GEOMETRIC MEAN The **geometric mean** between two numbers is the positive square root of their product.

Study Tip

Means and Extremes

In the equation $x^2 = ab$, the two x's in x^2 represent the *means*, and a and b represent the *extremes* of the proportion.

Key Concept — *Geometric Mean*

For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Example 1 *Geometric Mean*

Find the geometric mean between each pair of numbers.

a. 4 and 9

Let x represent the geometric mean.

$\frac{4}{x} = \frac{x}{9}$ Definition of geometric mean

$x^2 = 36$ Cross products

$x = \sqrt{36}$ Take the positive square root of each side.

$x = 6$ Simplify.

b. 6 and 15

$\frac{6}{x} = \frac{x}{15}$ Definition of geometric mean

$x^2 = 90$ Cross products

$x = \sqrt{90}$ Take the positive square root of each side.

$x = 3\sqrt{10}$ Simplify.

$x \approx 9.5$ Use a calculator.

Robert Brenner/PhotoEdit

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 351–352
- Skills Practice, p. 353
- Practice, p. 354
- Reading to Learn Mathematics, p. 355
- Enrichment, p. 356

Prerequisite Skills Workbook, pp. 11–12, 31–32

Transparencies

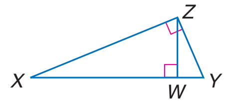
5-Minute Check Transparency 7-1

Answer Key Transparencies

Technology

Interactive Chalkboard

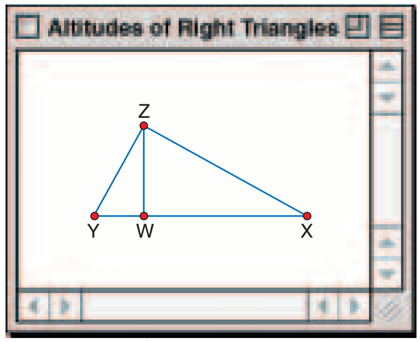
ALTITUDE OF A TRIANGLE Consider right triangle XYZ with altitude $\overline{WZ}$ drawn from the right angle Z to the hypotenuse $\overline{XY}$. A special relationship exists for the three right triangles, $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$.



Geometry Software Investigation

Right Triangles Formed by the Altitude

Use The Geometer's Sketchpad to draw a right triangle XYZ with right angle Z. Draw the altitude $\overline{ZW}$ from the right angle to the hypotenuse. Explore the relationships among the three right triangles formed.



Think and Discuss

1. Find the measures of $\angle X$, $\angle XZY$, $\angle Y$, $\angle XWZ$, $\angle XZW$, $\angle YWZ$, and $\angle YZW$. **See students' work.**
2. What is the relationship between the measures of $\angle X$ and $\angle YZW$? What is the relationship between the measures of $\angle Y$ and $\angle XZW$? **They are equal.**
3. Drag point Z to another position. Describe the relationship between the measures of $\angle X$ and $\angle YZW$ and between the measures of $\angle Y$ and $\angle XZW$. **They are equal.**
4. Make a conjecture about $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$. **They are similar.**

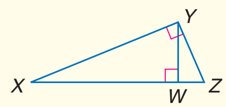
Study Tip

Altitudes of a Right Triangle
The altitude drawn to the hypotenuse originates from the right angle. The other two altitudes of a right triangle are the legs.

The results of the Geometry Software Investigation suggest the following theorem.

Theorem 7.1

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.



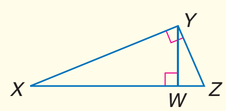
Example: $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$

You will prove this theorem in Exercise 45.

By Theorem 7.1, since $\triangle XWY \sim \triangle YWZ$, the corresponding sides are proportional. Thus, $\frac{XW}{YW} = \frac{YW}{ZW}$. Notice that $\overline{XW}$ and $\overline{ZW}$ are segments of the hypotenuse of the largest triangle.

Theorem 7.2

The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.



Example: YW is the geometric mean of XW and ZW.

You will prove this theorem in Exercise 46.

www.geometryonline.com/extra_examples

2 Teach

GEOMETRIC MEAN

In-Class Example Power Point®

1 Find the geometric mean between each pair of numbers.

a. 2 and 50 **10**

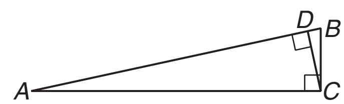
b. 25 and 7 **≈13.2**

ALTITUDE OF A TRIANGLE

In-Class Example Power Point®

Teaching Tip Remind students that they automatically discard the negative root when finding the altitude or geometric mean because these values represent lengths, and lengths cannot be negative.

2 In $\triangle ABC$, $BD = 6$ and $AD = 27$. Find CD.



$CD \approx 12.7$

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Try These exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

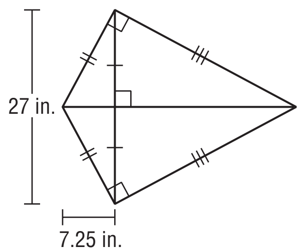
Geometry Software Investigation

Students should recognize that if they were given just one angle measure other than the right angle, they would have enough information to find all the other angles represented in the figure. They should also see that given one segment length, they could use proportions to find any of the other segment lengths in the figure as well.

In-Class Examples

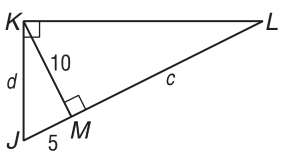
Power Point®

3 **KITES** Ms. Alspach is constructing a kite for her son. She has to arrange perpendicularly two support rods, the shorter of which is 27 inches long. If she has to place the short rod 7.25 inches from one end of the long rod in order to form two right triangles with the kite fabric, what is the length of the long rod?



≈ 32.39 in.

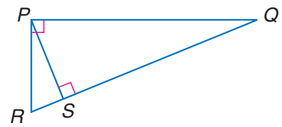
4 Find c and d in $\triangle JKL$.



$c = 20$; $d \approx 11.2$

Example 2 Altitude and Segments of the Hypotenuse

In $\triangle PQR$, $RS = 3$ and $QS = 14$. Find PS.



Let $x = PS$.

$\frac{RS}{PS} = \frac{PS}{QS}$

$\frac{3}{x} = \frac{x}{14}$ — $RS = 3$, $QS = 14$, and $PS = x$

$x^2 = 42$ — Cross products

$x = \sqrt{42}$ — Take the positive square root of each side.

$x \approx 6.5$ — Use a calculator.

PS is about 6.5.

Study Tip

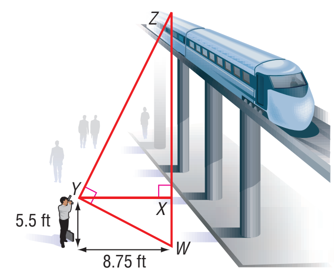
Square Roots Since these numbers represent measures, you can ignore the negative square root value.

Ratios in right triangles can be used to solve problems.

Example 3 Altitude and Length of the Hypotenuse

ARCHITECTURE **Mr. Martinez is designing a walkway that must pass over an elevated train. To find the height of the elevated train, he holds a carpenter's square at eye level and sights along the edges from the street to the top of the train. If Mr. Martinez's eye level is 5.5 feet above the street and he is 8.75 feet from the train, find the distance from the street to the top of the train. Round to the nearest tenth.**

Draw a diagram. Let $\overline{YX}$ be the altitude drawn from the right angle of $\triangle WYZ$.



$\frac{WX}{YX} = \frac{YX}{ZX}$

$\frac{5.5}{8.75} = \frac{8.75}{ZX}$ — $WX = 5.5$ and $YX = 8.75$

$5.5ZX = 76.5625$ — Cross products

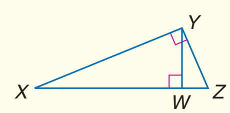
$ZX \approx 13.9$ — Divide each side by 5.5.

Mr. Martinez estimates that the elevated train is $5.5 + 13.9$ or about 19.4 feet high.

The altitude to the hypotenuse of a right triangle determines another relationship between the segments.

Theorem 7.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.



Example: $\frac{XZ}{XY} = \frac{XY}{XW}$ and $\frac{XZ}{YZ} = \frac{ZY}{WZ}$

You will prove Theorem 7.3 in Exercise 47.

DAILY INTERVENTION

Differentiated Instruction

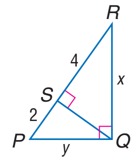
Intrapersonal Allow students to sit quietly and explore similarities and differences between Theorem 7.2 and Theorem 7.3. Encourage students to use the examples in the book or create their own to reinforce the concepts outlined in these two theorems. Ask students to think and write about why the formulas for geometric mean work for a right triangle with an altitude drawn to its hypotenuse.

344 Chapter 7 Right Triangles and Trigonometry

Example 4 Hypotenuse and Segment of Hypotenuse

Find x and y in $\triangle PQR$.

$\overline{PQ}$ and $\overline{RQ}$ are legs of right triangle PQR. Use Theorem 7.3 to write a proportion for each leg and then solve.



$\frac{PR}{PQ} = \frac{PQ}{PS}$		$\frac{PR}{RQ} = \frac{RQ}{SR}$	
$\frac{6}{y} = \frac{y}{2}$	$PS = 2$, $PQ = y$, $PR = 6$	$\frac{6}{x} = \frac{x}{4}$	$RS = 4$, $RQ = x$, $PR = 6$
$y^2 = 12$	Cross products	$x^2 = 24$	Cross products
$y = \sqrt{12}$	Take the square root.	$x = \sqrt{24}$	Take the square root.
$y = 2\sqrt{3}$	Simplify.	$x = 2\sqrt{6}$	Simplify.
$y \approx 3.5$	Use a calculator.	$x \approx 4.9$	Use a calculator.

Study Tip

Simplifying Radicals

Remember that $\sqrt{12} = \sqrt{4} \cdot \sqrt{3}$. Since $\sqrt{4} = 2$, $\sqrt{12} = 2\sqrt{3}$. For more practice simplifying radicals, see pages 744 and 745.

Check for Understanding

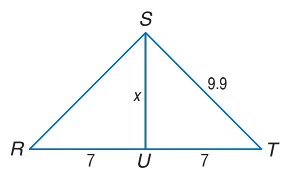
Concept Check

1. **OPEN ENDED** Find two numbers whose geometric mean is 12. **1. Sample answer: 2 and 72**

2. **Draw and label** a right triangle with an altitude drawn from the right angle. From your drawing, explain the meaning of *the hypotenuse and the segment of the hypotenuse adjacent to that leg* in Theorem 7.3. **See margin.**

3. **FIND THE ERROR** $\triangle RST$ is a right isosceles triangle. Holly and Ian are finding the measure of altitude $\overline{SU}$. **3. Ian; his proportion shows that the altitude is the geometric mean of the two segments of the hypotenuse.**

Holly	Ian
$\frac{RS}{SU} = \frac{SU}{RT}$	$\frac{RU}{SU} = \frac{SU}{UT}$
$\frac{9.9}{x} = \frac{x}{14}$	$\frac{7}{x} = \frac{x}{7}$
$x^2 = 138.6$	$x^2 = 49$
$x = \sqrt{138.6}$	$x = 7$
$x \approx 11.8$	



Who is correct? Explain your reasoning.

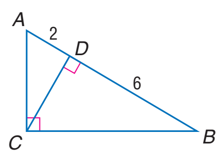
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1
8, 9	2
10–12	3, 4

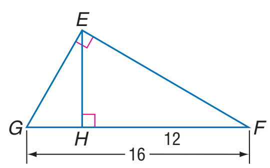
Find the geometric mean between each pair of numbers.

4. 9 and 4 **6**
5. 36 and 49 **42**
6. 6 and 8 **$4\sqrt{3} \approx 6.9$**
7. $2\sqrt{2}$ and $3\sqrt{2}$ **$2\sqrt{3} \approx 3.5$**

Find the measure of the altitude drawn to the hypotenuse.

8. 

$2\sqrt{3} \approx 3.5$

9. 

$4\sqrt{3} \approx 6.9$

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Ask students to name the hypotenuse of $\triangle RST$, and point out that Ian correctly found the geometric mean between the two segments of the *hypotenuse*, as the rule states in Theorem 7.2.

About the Exercises...

Organization by Objective

- **Geometric Mean:** 13–20
- **Altitude of a Triangle:** 21–32

Odd/Even Assignments

Exercises 13–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 40 requires the Internet or other research materials.

Assignment Guide

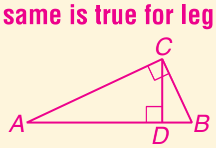
Basic: 13–31 odd, 35–43 odd, 44, 48–66

Average: 13–43 odd, 44, 48–66

Advanced: 14–44 even, 45–63 (optional: 64–66)

Answer

2. For leg $\overline{CB}$, $\overline{DB}$ is the segment of the hypotenuse that shares an endpoint. Thus, it is the adjacent segment. The same is true for leg $\overline{AC}$ and segment $\overline{AD}$.



Study Guide and Intervention, p. 351 (shown) and p. 352

Geometric Mean The **geometric mean** between two numbers is the square root of their product. For two positive numbers a and b, the geometric mean of a and b is the positive number x in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

Example **Find the geometric mean between each pair of numbers.**

a. 12 and 3

Let x represent the geometric mean.

$\frac{12}{x} = \frac{x}{3}$ Definition of geometric mean

$x^2 = 36$ Cross multiply.

$x = \sqrt{36}$ or 6 Take the square root of each side.

b. 8 and 4

Let x represent the geometric mean.

$\frac{8}{x} = \frac{x}{4}$

$x^2 = 32$

$x = \sqrt{32}$

≈ 5.7

Exercises

Find the geometric mean between each pair of numbers.

1. 4 and 4 **4**

2. 4 and 6 $\sqrt{24} \approx 4.9$

3. 6 and 9 $\sqrt{54} \approx 7.3$

4. $\frac{1}{2}$ and 2 **1**

5. $2\sqrt{3}$ and $3\sqrt{3}$ $\sqrt{18} \approx 4.2$

6. 4 and 25 **10**

7. $\sqrt{3}$ and $\sqrt{6}$ $18^{\frac{1}{4}} \approx 2.1$

8. 10 and 100 $\sqrt{1000} \approx 31.6$

9. $\frac{1}{2}$ and $\frac{1}{4}$ $\sqrt{\frac{1}{8}} \approx 0.4$

10. $\frac{2\sqrt{2}}{5}$ and $\frac{3\sqrt{2}}{5}$ $\sqrt{\frac{12}{25}} \approx 0.7$

11. 4 and 16 **8**

12. 3 and 24 $\sqrt{72} \approx 8.5$

The geometric mean and one extreme are given. Find the other extreme.

13. $\sqrt{24}$ is the geometric mean between a and b. Find b if $a = 2$. **12**

14. $\sqrt{12}$ is the geometric mean between a and b. Find b if $a = 3$. **4**

Determine whether each statement is *always*, *sometimes*, or *never* true.

15. The geometric mean of two positive numbers is greater than the average of the two numbers. **never**

16. If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1. **sometimes**

Skills Practice, p. 353 and Practice, p. 354 (shown)

Find the geometric mean between each pair of numbers to the nearest tenth.

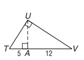
1. 8 and 12 $\sqrt{96} \approx 9.8$

2. $3\sqrt{7}$ and $6\sqrt{7}$ $\sqrt{126} \approx 11.2$

3. $\frac{4}{5}$ and 2 $\sqrt{\frac{8}{5}} \approx 1.3$

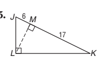
Find the measure of each altitude. State exact answers and answers to the nearest tenth.

4.



$\sqrt{60} \approx 7.7$

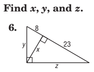
5.



$\sqrt{102} \approx 10.1$

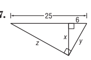
Find x, y, and z.

6.



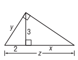
$\sqrt{184} \approx 13.6$; $\sqrt{248} \approx 15.7$; $\sqrt{713} \approx 26.7$

7.



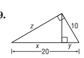
$\sqrt{114} \approx 10.7$; $\sqrt{150} \approx 12.2$; $\sqrt{475} \approx 21.8$

8.



4.5; $\sqrt{13} \approx 3.6$; 6.5

9.



15; 5; $\sqrt{300} \approx 17.3$

10. CIVIL ENGINEERING An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth. **2.88 mi**

Reading to Learn Mathematics, p. 355 ELL

Pre-Activity **How can the geometric mean be used to view paintings?**

Read the introduction to Lesson 7-1 at the top of page 342 in your textbook.

- What is a disadvantage of standing too close to a painting?
  Sample answer: You don't get a good overall view.
- What is a disadvantage of standing too far from a painting?
  Sample answer: You can't see all the details in the painting.

Reading the Lesson

1. In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.

a. Complete the following by writing an algebraic expression in each blank.

If a and b are two positive numbers, then the geometric mean between a and b is $\underline{\sqrt{ab}}$ and their arithmetic mean is $\underline{\frac{a+b}{2}}$.

b. Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean. **Sample answer: The geometric mean between two numbers is the square root of their product. The arithmetic mean of two numbers is half their sum.**

2. Let r and s be two positive numbers. In which of the following equations is z equal to the geometric mean between r and s? **A, C, D, F**

A. $\frac{s}{z} = \frac{z}{r}$ **B.** $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z = z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

3. Supply the missing words or phrases to complete the statement of each theorem.

a. The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the __**geometric mean**__ between the measures of the two segments of the __**hypotenuse**__.

b. If the altitude is drawn from the vertex of the __**right**__ angle of a right triangle to its hypotenuse, then the measure of a __**leg**__ of the triangle is the __**geometric mean**__ between the measure of the hypotenuse and the segment of the __**hypotenuse**__ adjacent to that leg.

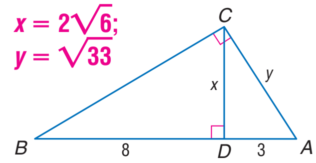
c. If the altitude is drawn from the __**vertex**__ of the right angle of a right triangle to its __**hypotenuse**__, then the two triangles formed are __**similar**__ to the given triangle and to each other.

Helping You Remember

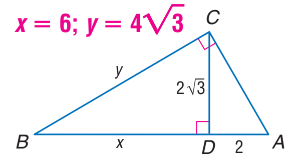
4. A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers? **Sample answer: Write a proportion in which the two means are equal. This common mean is the geometric mean between the two extremes.**

Find x and y.

10. $x = 2\sqrt{6}$; $y = \sqrt{33}$

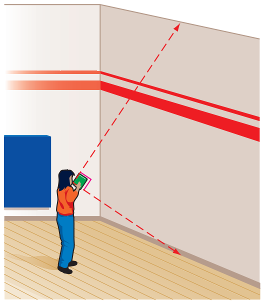


11. $x = 6$; $y = 4\sqrt{3}$



Application

12. DANCES Khaliah is making a banner for the dance committee. The banner is to be as high as the wall of the gymnasium. To find the height of the wall, Khaliah held a book up to her eyes so that the top and bottom of the wall were in line with the top edge and binding of the cover. If Khaliah's eye level is 5 feet off the ground and she is standing 12 feet from the wall, how high is the wall? **33.8 ft**



Practice and Apply

For Exercises	See Examples
13–20	1
21–26	2
27–32	3, 4

Homework Help

Extra Practice See page 766.

Find the geometric mean between each pair of numbers.

13. 5 and 6 **14.** 24 and 25 **15.** $\sqrt{45}$ and $\sqrt{80}$ **16.** $\sqrt{28}$ and $\sqrt{1372}$

17. $\frac{3}{5}$ and 1 **18.** $\frac{8\sqrt{3}}{5}$ and $\frac{6\sqrt{3}}{5}$ **19.** $\frac{2\sqrt{2}}{6}$ and $\frac{5\sqrt{2}}{6}$ **20.** $\frac{13}{7}$ and $\frac{5}{7}$

13. $\sqrt{30} \approx 5.5$

14. $10\sqrt{6} \approx 24.5$

15. $2\sqrt{15} \approx 7.7$

16. 14

17. $\frac{\sqrt{15}}{5} \approx 0.8$

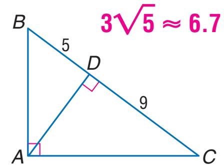
18. $\frac{12}{5} = 2.4$

19. $\frac{\sqrt{5}}{3} \approx 0.7$

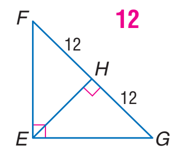
20. $\frac{\sqrt{65}}{7} \approx 1.2$

Find the measure of the altitude drawn to the hypotenuse.

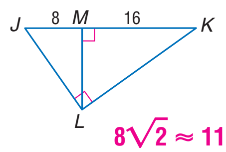
21. $3\sqrt{5} \approx 6.7$



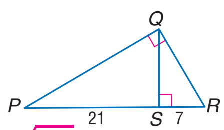
22. 12



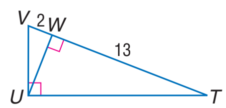
23. $8\sqrt{2} \approx 11.3$



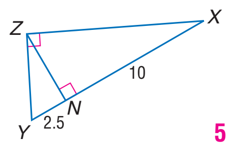
24. $\sqrt{147} \approx 12.1$



25. $\sqrt{26} \approx 5.1$

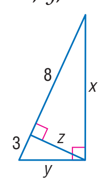


26. 5

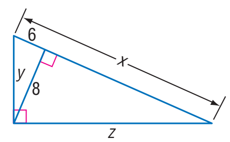


Find x, y, and z. **27–32. See margin.**

27.



28.



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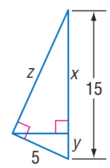
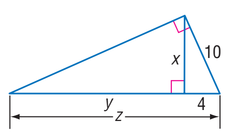
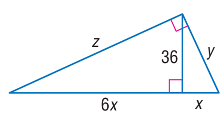


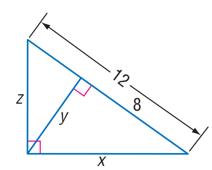
30.



31.



32.



Enrichment, p. 356

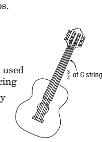
Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

C	D	E	F	G	A	B	C′
$\frac{1}{1}$	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.



The geometric mean and one extreme are given. Find the other extreme.

★ **33.** $\sqrt{17}$ is the geometric mean between a and b. Find b if $a = 7$. $\frac{17}{7}$

★ **34.** $\sqrt{12}$ is the geometric mean between x and y. Find x if $y = \sqrt{3}$. $4\sqrt{3} \approx 6.9$

Determine whether each statement is *always*, *sometimes*, or *never* true.

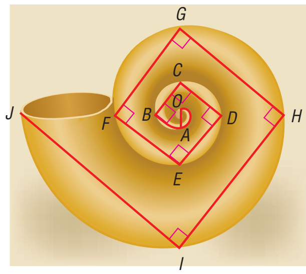
35. The geometric mean for consecutive positive integers is the average of the two numbers. **never**

36. The geometric mean for two perfect squares is a positive integer. **always**

37. The geometric mean for two positive integers is another integer. **sometimes**

38. The measure of the altitude of a triangle is the geometric mean between the measures of the segments of the side it intersects. **sometimes**

39. BIOLOGY The shape of the shell of a chambered nautilus can be modeled by a geometric mean. Consider the sequence of segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, $\overline{OE}$, $\overline{OF}$, $\overline{OG}$, $\overline{OH}$, $\overline{OI}$, and $\overline{OJ}$. The length of each of these segments is the geometric mean between the lengths of the preceding segment and the succeeding segment. Explain this relationship. (*Hint:* Consider $\triangle FGH$.) **See margin.**



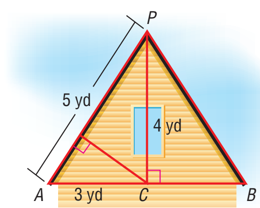
40. Sample answer: The golden ratio occurs when the geometric mean is approximately 1.62.

WebQuest

You can use geometric mean and the Quadratic Formula to discover the golden mean. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

40. RESEARCH Refer to the information at the left. Use the Internet or other resource to write a brief description of the golden ratio.

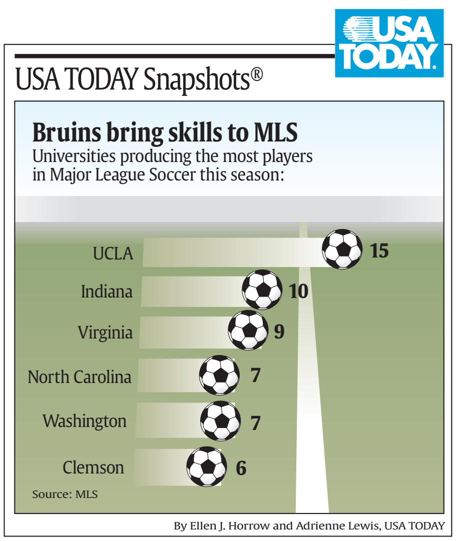
41. CONSTRUCTION In the United States, most building codes limit the steepness of the slope of a roof to $\frac{4}{3}$, as shown at the right. A builder wants to put a support brace from point C perpendicular to $\overline{AP}$. Find the length of the brace. **2.4 yd**



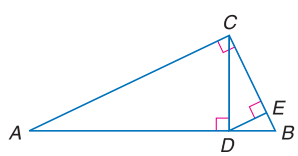
SOCCER **For Exercises 42 and 43, refer to the graphic.**

42. Find the geometric mean between the number of players from Indiana and North Carolina. $\sqrt{70} \approx 8.4$

43. Are there two schools whose geometric mean is the same as the geometric mean between UCLA and Clemson? If so, which schools? **yes; Indiana and Virginia**



44. CRITICAL THINKING Find the exact value of DE, given $AD = 12$ and $BD = 4$. $2\sqrt{3}$



www.geometryonline.com/self_check_quiz

Kaz Chiba/PhotoDisc

Answers

39. $\triangle FGH$ is a right triangle. $\overline{OG}$ is the altitude from the vertex of the right angle to the hypotenuse of that triangle. So, by Theorem 7.2, OG is the geometric mean between OF and OH, and so on.

Answers (p. 346)

27. $x = 2\sqrt{22} \approx 9.4$; $y = \sqrt{33} \approx 5.7$; $z = 2\sqrt{6} \approx 4.9$

28. $x = \frac{50}{3}$; $y = 10$; $z = \frac{40}{3}$

29. $x = \frac{40}{3}$; $y = \frac{5}{3}$; $z = 10\sqrt{2} \approx 14.1$

30. $x = 2\sqrt{21} \approx 9.2$; $y = 21$; $z = 25$

31. $x = 6\sqrt{6} \approx 14.7$; $y = 6\sqrt{42} \approx 38.9$; $z = 36\sqrt{7} \approx 95.2$

32. $x = 4\sqrt{6} \approx 9.8$; $y = 4\sqrt{2} \approx 5.7$; $z = 4\sqrt{3} \approx 6.9$

4 Assess

Open-Ended Assessment

Writing Draw and label several figures on the board, and ask students to write down the correct geometric mean proportions without working out the problem. Then have volunteers come to the board and write appropriate proportions that correspond with the figures, and allow seated students to check their work.

Getting Ready for Lesson 7-2

Prerequisite Skill Students will review the Pythagorean Theorem and learn about its converse in Lesson 7-2. Use Exercises 64–66 to determine your students' familiarity with using the Pythagorean Theorem.

Answers

48. Sample answer: The geometric mean can be used to help determine the optimum viewing distance. Answers should include the following.
    - If you are too far from a painting, you may not be able to see fine details. If you are too close, you may not be able to see the entire painting.
    - A curator can use the geometric mean to help determine how far from the painting the roping should be.

PROOF Write the specified type proof for each theorem. 45–47. See p. 399A.

45. two-column proof of Theorem 7.1
46. paragraph proof of Theorem 7.2
47. two-column proof of Theorem 7.3

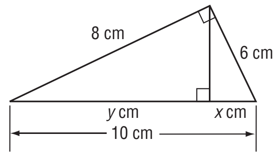
48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See margin.

    How can the geometric mean be used to view paintings?

    Include the following in your answer:
    - an explanation of what happens when you are too far or too close to a painting, and
    - an explanation of how the curator of a museum would determine where to place roping in front of paintings on display.

Standardized Test Practice

49. Find x and y. C
    (A) 4 and 6 (B) 2.5 and 7.5
    (C) 3.6 and 6.4 (D) 3 and 7



50. **ALGEBRA** Solve $5x^2 + 405 = 1125$. B
    (A) ± 15 (B) ± 12
    (C) $\pm 4\sqrt{3}$ (D) ± 4

Maintain Your Skills

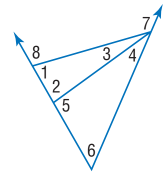
Mixed Review

51. 15, 18, 21
52. 14, 44, 134

Find the first three iterations of each expression. *(Lesson 6-6)*

51. $x + 3$, where x initially equals 12
52. $3x + 2$, where x initially equals 4
53. $x^2 - 2$, where x initially equals 3 **7, 47, 2207**
54. $2(x - 3)$, where x initially equals 1 **−4, −14, −34**
55. The measures of the sides of a triangle are 20, 24, and 30. Find the measures of the segments formed where the bisector of the smallest angle meets the opposite side. *(Lesson 6-5)* $8\frac{8}{9}$, $11\frac{1}{9}$

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition. *(Lesson 5-2)*



56. all angles with a measure less than $m\angle 8$ $\angle 6, \angle 4, \angle 2, \angle 3$
57. all angles with a measure greater than $m\angle 1$ $\angle 5, \angle 7$
58. all angles with a measure less than $m\angle 7$ $\angle 1, \angle 6$
59. all angles with a measure greater than $m\angle 6$ $\angle 2, \angle 7, \angle 8$

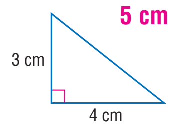
Write an equation in slope-intercept form for the line that satisfies the given conditions. *(Lesson 3-4)*

60. $m = 2$, y-intercept $= 4$ $y = 2x + 4$
61. x-intercept is 2, y-intercept $= -8$ $y = 4x - 8$
62. passes through $(2, 6)$ and $(-1, 0)$ $y = 2x + 2$
63. $m = -4$, passes through $(-2, -3)$ $y = -4x - 11$

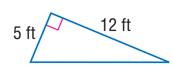
Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use the Pythagorean Theorem to find the length of the hypotenuse of each right triangle.**
*(To review **using the Pythagorean Theorem**, see Lesson 1-4.)*

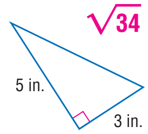
64. **5 cm**



65. **13 ft**



66. $\sqrt{34} \approx 5.8$ in.



The Pythagorean Theorem

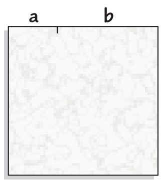
In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. Ancient cultures used the Pythagorean Theorem before it was officially named in 1909.

Use square pieces of patty paper and algebra. Then you too can discover this relationship among the measures of the sides of a right triangle.

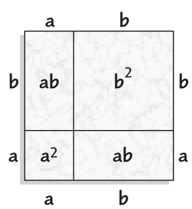
Activity

Use paper folding to develop the Pythagorean Theorem.

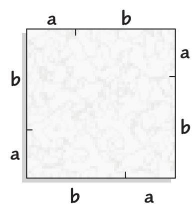
Step 1 On a piece of patty paper, make a mark along one side so that the two resulting segments are not congruent. Label one as *a* and the other as *b*.



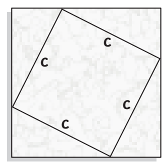
Step 2 Copy these measures on the other sides in the order shown at the right. Fold the paper to divide the square into four sections. Label the area of each section.



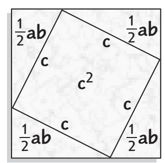
Step 3 On another sheet of patty paper, mark the same lengths *a* and *b* on the sides in the different pattern shown at the right.



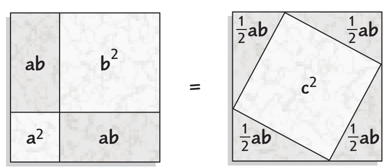
Step 4 Use your straightedge and pencil to connect the marks as shown at the right. Let *c* represent the length of each hypotenuse.



Step 5 Label the area of each section, which is $\frac{1}{2}ab$ for each triangle and c^2 for the square.



Step 6 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$.

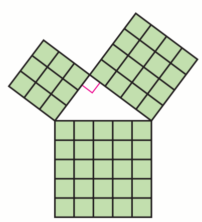


The parts that are not shaded tell us that $a^2 + b^2 = c^2$.

Model

1. Use a ruler to find actual measures for *a*, *b*, and *c*. Do these measures confirm that $a^2 + b^2 = c^2$? **yes**
2. Repeat the activity with different *a* and *b* values. What do you notice? $a^2 + b^2 = c^2$

Analyze the model



3. **Explain** why the drawing at the right is an illustration of the Pythagorean Theorem. **Sample answer: The sum of the areas of the two smaller squares is equal to the area of the largest square.**

Geometry Activity

A Preview of Lesson 7-2

Getting Started

Objective Physically explore side-length relationships in the Pythagorean Theorem.

Materials patty paper, ruler, pencil

Teach

- For Step 2, have students use a ruler and a pencil to draw the lines that the creases make.
- Advise students that the key to this activity is making sure measures *a* and *b* are exactly the same on both sheets of paper. Without using a ruler, students may use one marked edge of the first paper to mark accurate lengths on the second paper.
- After Step 6, some students may still be skeptical that the shaded areas of the two pieces of patty paper are equal. Point out that the two pieces of paper are the same size, so they have the same area. Then ask students to cut out all the shaded triangles from the second piece of paper and arrange them so that they fit over the shaded areas on the first piece of paper.

Assess

Exercises 1 and 2 provide students with actual measurements they can use to confirm the Pythagorean Theorem. In **Exercise 3**, students prove that three lengths meet the criteria for the Pythagorean Theorem.

Resource Manager

Teaching Geometry with Manipulatives
- p. 115 (student recording sheet)
- p. 17 (ruler)

Glencoe Mathematics Classroom Manipulative Kit
- rulers

Study Notebook

Ask students to summarize what they have learned about the Pythagorean Theorem.

7-2 Lesson Notes

1 Focus

5-Minute Check Transparency 7-2 Use as a quiz or review of Lesson 7-1.

Mathematical Background notes are available for this lesson on p. 340C.

How are right triangles used to build suspension bridges?

Ask students:

- How can you reword the Pythagorean Theorem by substituting the names of the parts of the Talmadge Memorial Bridge for the legs and hypotenuse of the triangle? **The sum of the squares of the measure of the height of the tower from the roadway and length of the roadway from the tower to the end of the bridge equals the square of the measure of the cable.**
- You may notice that most suspension cables have some slack in them. In this case, does the actual length of the cable satisfy the criteria of the Pythagorean Theorem? Explain. **No; a suspension cable is cut to allow for expansion and contraction due to temperature changes, and when it has slack, its length actually exceeds the length required to satisfy the Pythagorean Theorem.**

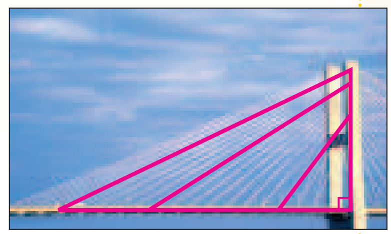
7-2 The Pythagorean Theorem and Its Converse

What You'll Learn

- Use the Pythagorean Theorem.
- Use the converse of the Pythagorean Theorem.

How are right triangles used to build suspension bridges?

The Talmadge Memorial Bridge over the Savannah River has two soaring towers of suspension cables. Note the right triangles being formed by the roadway, the perpendicular tower, and the suspension cables. The Pythagorean Theorem can be used to find measures in any right triangle.



Vocabulary

- Pythagorean triple

Study Tip

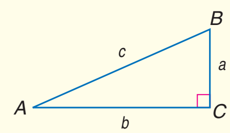
Look Back
To review **finding the hypotenuse of a right triangle,** see Lesson 1-3.

THE PYTHAGOREAN THEOREM In Lesson 1-3, you used the Pythagorean Theorem to find the distance between two points by finding the length of the hypotenuse when given the lengths of the two legs of a right triangle. You can also find the measure of any side of a right triangle given the other two measures.

Theorem 7.4

Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Symbols: $a^2 + b^2 = c^2$



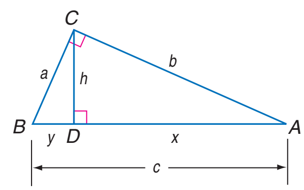
The geometric mean can be used to prove the Pythagorean Theorem.

Proof *Pythagorean Theorem*

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$

Proof:



Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$.

Two geometric means now exist.

$$\frac{c}{a} = \frac{a}{y} \quad \text{and} \quad \frac{c}{b} = \frac{b}{x}$$

$$a^2 = cy \quad \text{and} \quad b^2 = cx \quad \text{Cross products}$$

Add the equations.

$$a^2 + b^2 = cy + cx$$

$$a^2 + b^2 = c(y + x) \quad \text{Factor.}$$

$$a^2 + b^2 = c^2 \quad \text{Since } c = y + x \text{, substitute } c \text{ for } (y + x).$$

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 357–358
- Skills Practice, p. 359
- Practice, p. 360
- Reading to Learn Mathematics, p. 361
- Enrichment, p. 362
- Assessment, p. 407

Graphing Calculator and Computer Masters, p. 29

School-to-Career Masters, p. 13

Prerequisite Skills Workbook, pp. 11–12, 31–32

Teaching Geometry With Manipulatives Masters, pp. 114, 116

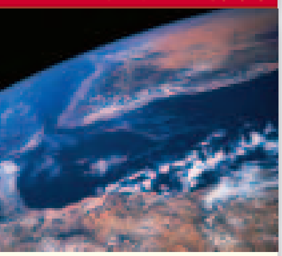
Transparencies

5-Minute Check Transparency 7-2
Answer Key Transparencies

Technology

GeomPASS: Tutorial Plus, Lesson 13
Interactive Chalkboard

More About. . .



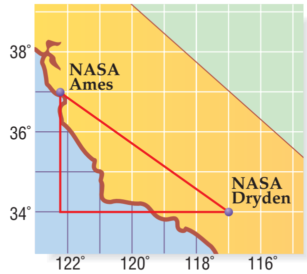
Maps

Due to the curvature of Earth, the distance between two points is often expressed as *degree distance* using latitude and longitude. This measurement closely approximates the distance on a plane.

Source: NASA

Example 1 Find the Length of the Hypotenuse

LONGITUDE AND LATITUDE **NASA Dryden is located at about 117 degrees longitude and 34 degrees latitude. NASA Ames is located at about 122 degrees longitude and 37 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth between NASA Dryden and NASA Ames.**



The change in longitude between the two locations is $|117–122|$ or 5 degrees. Let this distance be a.

The change in latitude is $|37–34|$ or 3 degrees latitude. Let this distance be b.

Use the Pythagorean Theorem to find the distance in degrees from NASA Dryden to NASA Ames, represented by c.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 3^2 = c^2$ $a = 5, b = 3$

$25 + 9 = c^2$ Simplify.

$34 = c^2$ Add.

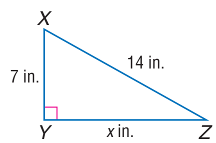
$\sqrt{34} = c$ Take the square root of each side.

$5.8 \approx c$ Use a calculator.

The degree distance between NASA Dryden and NASA Ames is about 5.8 degrees.

Example 2 Find the Length of a Leg

Find x.



$(XY)^2 + (YZ)^2 = (XZ)^2$ Pythagorean Theorem

$7^2 + x^2 = 14^2$ $XY = 7, XZ = 14$

$49 + x^2 = 196$ Simplify.

$x^2 = 147$ Subtract 49 from each side.

$x = \sqrt{147}$ Take the square root of each side.

$x = 7\sqrt{3}$ Simplify.

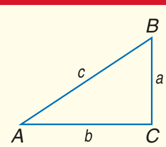
$x \approx 12.1$ Use a calculator.

CONVERSE OF THE PYTHAGOREAN THEOREM The converse of the Pythagorean Theorem can help you determine whether three measures of the sides of a triangle are those of a right triangle.

Theorem 7.5

Converse of the Pythagorean Theorem If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Symbols: If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



You will prove this theorem in Exercise 38.

www.geometryonline.com/extra_examples

StockTrek/PhotoDisc

2 Teach

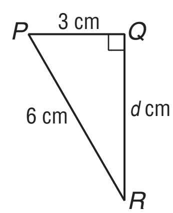
THE PYTHAGOREAN THEOREM

In-Class Examples Power Point®

1 **LONGITUDE AND LATITUDE** Carson City, Nevada, is located at about 120 degrees longitude and 39 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth degree if you were to travel directly from NASA Ames to Carson City, Nevada. **about 2.8 degrees**

Teaching Tip Students can also rewrite the Pythagorean Theorem as $b^2 = c^2 - a^2$ or $a^2 = c^2 - b^2$ when they are using it to find the measure of either leg of a right triangle.

2 Find d. $d \approx 5.2$



Building on Prior Knowledge

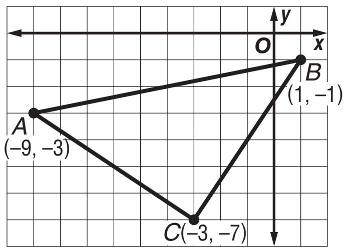
Students used the Distance Formula in Chapter 1 when they were learning how to calculate segment lengths. In this lesson, they use this formula to verify the side lengths of right triangles.

CONVERSE OF THE PYTHAGOREAN THEOREM

In-Class Examples

Power Point®

3 COORDINATE GEOMETRY
Verify that $\triangle ABC$ is a right triangle.



$AB = \sqrt{104}$; $BC = \sqrt{52}$; $AC = \sqrt{52}$; $\triangle ABC$ is a right triangle because $AC^2 + BC^2 = AB^2$.

4 Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean triple.

a. 9, 12, and 15
The segments form the sides of a right triangle, and the measures form a Pythagorean triple.

b. 21, 42, and 54
The segments do not form the sides of a right triangle, and the measures do not form a Pythagorean triple.

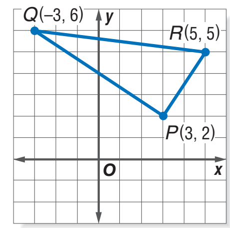
c. $4\sqrt{3}$, 4, and 8
The segments form the sides of a right triangle, but the measures do not form a Pythagorean triple.

Study Tip

Distance Formula
When using the Distance Formula, be sure to follow the order of operations carefully. Perform the operation inside the parentheses first, square each term, and then add.

Example 3 Verify a Triangle is a Right Triangle

COORDINATE GEOMETRY Verify that ΔPQR is a right triangle.



Use the Distance Formula to determine the lengths of the sides.

$$PQ = \sqrt{(-3-3)^2 + (6-2)^2} \quad x_1 = 3, y_1 = 2, x_2 = -3, y_2 = 6$$
$$= \sqrt{(-6)^2 + 4^2} \quad \text{Subtract.}$$
$$= \sqrt{52} \quad \text{Simplify.}$$

$$QR = \sqrt{[5-(-3)]^2 + (5-6)^2} \quad x_1 = -3, y_1 = 6, x_2 = 5, y_2 = 5$$
$$= \sqrt{8^2 + (-1)^2} \quad \text{Subtract.}$$
$$= \sqrt{65} \quad \text{Simplify.}$$

$$PR = \sqrt{(5-3)^2 + (5-2)^2} \quad x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 5$$
$$= \sqrt{2^2 + 3^2} \quad \text{Subtract.}$$
$$= \sqrt{13} \quad \text{Simplify.}$$

By the converse of the Pythagorean Theorem, if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

$$PQ^2 + PR^2 = QR^2 \quad \text{Converse of the Pythagorean Theorem}$$
$$\left(\sqrt{52}\right)^2 + \left(\sqrt{13}\right)^2 \stackrel{?}{=} \left(\sqrt{65}\right)^2 \quad PQ = \sqrt{52}, PR = \sqrt{13}, QR = \sqrt{65}$$
$$52 + 13 \stackrel{?}{=} 65 \quad \text{Simplify.}$$
$$65 = 65 \quad \text{Add.}$$

Since the sum of the squares of two sides equals the square of the longest side, ΔPQR is a right triangle.

A **Pythagorean triple** is three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number. One common Pythagorean triple is 3-4-5, in which the sides of a right triangle are in the ratio 3:4:5. If the measures of the sides of any right triangle are whole numbers, the measures form a Pythagorean triple.

Example 4 Pythagorean Triples

Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.

a. 8, 15, 16

Since the measure of the longest side is 16, 16 must be c, and a or b are 15 and 8.

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$
$$8^2 + 15^2 \stackrel{?}{=} 16^2 \quad a = 8, b = 15, c = 16$$
$$64 + 225 \stackrel{?}{=} 256 \quad \text{Simplify.}$$
$$289 \neq 256 \quad \text{Add.}$$

Since $289 \neq 256$, segments with these measures cannot form a right triangle. Therefore, they do not form a Pythagorean triple.

DAILY INTERVENTION — Differentiated Instruction

Visual/Spatial Explain that many artists use right triangles in their works because they are so appealing to the eye. Right triangles can serve as guidelines to draw mountains in the background or to create vanishing points and perspective. Ask students to construct one or more right triangles on a blank sheet of paper, find the lengths of the sides and then try to compose a picture using the triangle in an image or as a guide for an image. Examples could be a picture of a road vanishing in the distance or a house with right triangles as part of its roof.

b. 20, 48, and 52

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$20^2 + 48^2 \stackrel{?}{=} 52^2$ — $a = 20, b = 48, c = 52$

$400 + 2304 \stackrel{?}{=} 2704$ — Simplify.

$2704 = 2704$ — Add.

These segments form the sides of a right triangle since they satisfy the Pythagorean Theorem. The measures are whole numbers and form a Pythagorean triple.

c. $\frac{\sqrt{3}}{5}$, $\frac{\sqrt{6}}{5}$, and $\frac{3}{5}$

$a^2 + b^2 = c^2$ — Pythagorean Theorem

$\left(\frac{\sqrt{3}}{5}\right)^2 + \left(\frac{\sqrt{6}}{5}\right)^2 \stackrel{?}{=} \left(\frac{3}{5}\right)^2$ — $a = \frac{\sqrt{3}}{5}, b = \frac{\sqrt{6}}{5}, c = \frac{3}{5}$

$\frac{3}{25} + \frac{6}{25} \stackrel{?}{=} \frac{9}{25}$ — Simplify.

$\frac{9}{25} = \frac{9}{25}$ — Add.

Since $\frac{9}{25} = \frac{9}{25}$, segments with these measures form a right triangle. However, the three numbers are not whole numbers. Therefore, they do not form a Pythagorean triple.

Study Tip

Comparing Numbers
If you cannot quickly identify the greatest number, use a calculator to find decimal values for each number and compare.

Check for Understanding

Concept Check

1. **FIND THE ERROR** Maria and Colin are determining whether 5-12-13 is a Pythagorean triple.

Colin	Maria
$a^2 + b^2 = c^2$	$a^2 + b^2 = c^2$
$13^2 + 5^2 \stackrel{?}{=} 12^2$	$5^2 + 12^2 \stackrel{?}{=} 13^2$
$169 + 25 \stackrel{?}{=} 144$	$25 + 144 \stackrel{?}{=} 169$
$193 \neq 144$	$169 = 169$
no	yes

Who is correct? Explain your reasoning.

2. **Explain** why a Pythagorean triple can represent the measures of the sides of a right triangle.

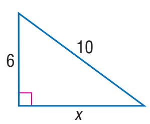
3. **OPEN ENDED** Draw a pair of similar right triangles. List the corresponding sides, the corresponding angles, and the scale factor. Are the measures of the sides of each triangle a Pythagorean triple? **See margin.**

1. Maria; Colin does not have the longest side as the value of c.

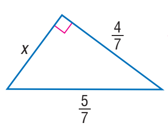
2. Since the numbers in a Pythagorean triple satisfy the equation $a^2 + b^2 = c^2$, they represent the sides of a right triangle by the converse of the Pythagorean Theorem.

Guided Practice

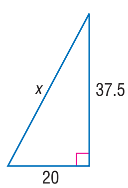
Find x.

4. 

8

5. 

3/7

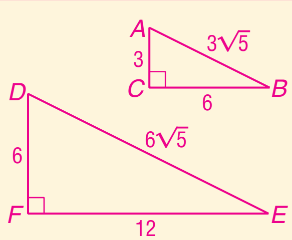
6. 

42.5

GUIDED PRACTICE KEY

Exercises	Examples
4, 5	2
6	1
7	3
8–10	4

Answers

3. 

Sample answer: $\triangle ABC \sim \triangle DEF$, $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$, $\overline{AB}$ corresponds to $\overline{DE}$, $\overline{BC}$ corresponds to $\overline{EF}$, and $\overline{AC}$ corresponds to $\overline{DF}$. The scale factor is $\frac{2}{1}$. No; the measures of the sides do not form a Pythagorean triple since $6\sqrt{5}$ and $3\sqrt{5}$ are not whole numbers.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Point out that in all right triangles, the hypotenuse is always the longest side because it is opposite the 90° angle. Students have to make sure that the value they use to check for c is the largest value given. Also, tell students that the order of a and b does not matter. Maria could have used 12 for a and 5 for b, and she still would have been correct.

About the Exercises...

Organization by Objective
- **The Pythagorean Theorem:** 12–17
- **Converse of the Pythagorean Theorem:** 18–29

Odd/Even Assignments
Exercises 12–29 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 48 and 49 require a graphing calculator.

Assignment Guide

Basic: 13–17 odd, 41, 43–70

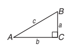
Average: 13–43 odd, 44–70

Advanced: 12–44 even, 45–60 (optional: 61–70)

Lesson 7-2 The Pythagorean Theorem and Its Converse 353

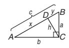
Study Guide and Intervention, p. 357 (shown) and p. 358

The Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.



$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

Example 1 **Prove the Pythagorean Theorem.**



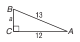
With altitude $\overline{CD}$, each leg a and b is a geometric mean between hypotenuse c and the segment of the hypotenuse adjacent to that leg.

$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get
$a^2 + b^2 = cy + cx = c(y + x) = c^2$.

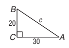
Example 2

a. Find a.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 12^2 = 13^2$	$b = 12$, $c = 13$
$a^2 + 144 = 169$	Simplify.
$a^2 = 25$	Subtract.
$a = 5$	Take the square root of each side.

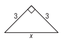
b. Find c.



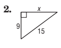
$a^2 + b^2 = c^2$	Pythagorean Theorem
$20^2 + 30^2 = c^2$	$a = 20$, $b = 30$
$400 + 900 = c^2$	Simplify.
$1300 = c^2$	Add.
$\sqrt{1300} = c$	Take the square root of each side.
$36.1 \approx c$	Use a calculator.

Exercises

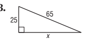
Find x.

1. 

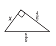
$\sqrt{18} \approx 4.2$

2. 

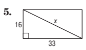
12

3. 

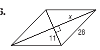
60

4. 

$\frac{3}{10}$

5. 

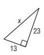
$\sqrt{1345} \approx 36.7$

6. 

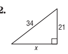
$\sqrt{663} \approx 25.7$

Skills Practice, p. 359 and Practice, p. 360 (shown)

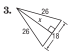
Find x.

1. 

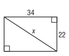
$\sqrt{698} \approx 26.4$

2. 

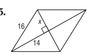
$\sqrt{715} \approx 26.7$

3. 

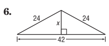
$\sqrt{595} \approx 24.4$

4. 

$\sqrt{1640} \approx 40.5$

5. 

$\sqrt{60} \approx 7.7$

6. 

$\sqrt{135} \approx 11.6$

Determine whether $\triangle GHI$ is a right triangle for the given vertices. Explain.

7. $G(2, 7)$, $H(3, 6)$, $I(-4, -1)$
yes; $GH = \sqrt{2}$, $HI = \sqrt{98}$, $IG = \sqrt{100}$, $(\sqrt{2})^2 + (\sqrt{98})^2 = (\sqrt{100})^2$

8. $G(-6, 2)$, $H(1, 12)$, $I(-2, 1)$
no; $GH = \sqrt{149}$, $HI = \sqrt{130}$, $IG = \sqrt{17}$, $(\sqrt{130})^2 + (\sqrt{17})^2 \neq (\sqrt{149})^2$

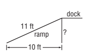
9. $G(-2, 1)$, $H(3, -1)$, $I(-4, -4)$
yes; $GH = \sqrt{29}$, $HI = \sqrt{58}$, $IG = \sqrt{29}$, $(\sqrt{29})^2 + (\sqrt{29})^2 = (\sqrt{58})^2$

10. $G(-2, 4)$, $H(4, 1)$, $I(-1, -9)$
yes; $GH = \sqrt{45}$, $HI = \sqrt{125}$, $IG = \sqrt{170}$, $(\sqrt{45})^2 + (\sqrt{125})^2 = (\sqrt{170})^2$

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

11. 9, 40, 41 yes, yes
12. 7, 28, 29 no, no
13. 24, 32, 40 yes, yes
14. $\frac{9}{5}, \frac{12}{5}, 3$ yes, no
15. $\frac{1}{3}, \frac{2\sqrt{2}}{3}, 1$ yes, no
16. $\frac{\sqrt{4}}{7}, \frac{2\sqrt{3}}{7}, \frac{4}{7}$ yes, no

17. **CONSTRUCTION** The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock? about 4.6 ft high



Reading to Learn Mathematics, p. 361 ELL

Pre-Activity **How are right triangles used to build suspension bridges?**

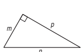
Read the introduction to Lesson 7-2 at the top of page 350 in your textbook.

Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning. Sample answer: No; their sides are not proportional. In the smaller triangle, the longer leg is more than twice the length of the shorter leg, while in the larger triangle, the longer leg is less than twice the length of the shorter leg.

Reading the Lesson

1. Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used. Sample answer: The Pythagorean Theorem is used to find the third side of a right triangle if you know the lengths of any two of the sides. The converse is used to tell whether a triangle with three given side lengths is a right triangle.

2. Refer to the figure. For this figure, which statements are true? B, E, F, G



A. $m^2 + n^2 = p^2$
B. $n^2 = m^2 + p^2$
C. $m^2 = n^2 + p^2$
D. $m^2 = p^2 - n^2$
E. $p^2 = n^2 - m^2$
F. $n^2 - p^2 = m^2$
G. $n = \sqrt{m^2 + p^2}$
H. $p = \sqrt{m^2 - n^2}$

3. Is the following statement true or false?
A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.
Sample answer: The statement is false because in a Pythagorean triple, all three numbers must be *whole* numbers.

4. If x, y, and z form a Pythagorean triple and k is a positive integer, which of the following groups of numbers are also Pythagorean triples? B, D
A. $3x, 4y, 5z$ B. $3x, 3y, 3z$ C. $-3x, -3y, -3z$ D. kx, ky, kz

Helping You Remember

5. Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?
Sample answer: Draw a right triangle. Label the lengths of the two legs as *a* and *b* and the length of the hypotenuse as *c*.

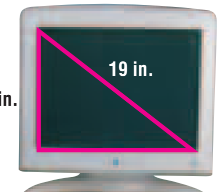
7. **COORDINATE GEOMETRY** Determine whether $\triangle JKL$ with vertices $J(-2, 2)$, $K(-1, 6)$, and $L(3, 5)$ is a right triangle. Explain.
yes; $JK = \sqrt{17}$, $KL = \sqrt{17}$, $JL = \sqrt{34}$; $(\sqrt{17})^2 + (\sqrt{17})^2 = (\sqrt{34})^2$

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

8. 15, 36, 39 yes, yes
9. $\sqrt{40}$, 20, 21 no, no
10. $\sqrt{44}$, 8, $\sqrt{108}$ yes, no

Application

11. **COMPUTERS** Computer monitors are usually measured along the diagonal of the screen. A 19-inch monitor has a diagonal that measures 19 inches. If the height of the screen is 11.5 inches, how wide is the screen? about 15.1 in.



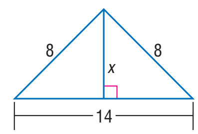
★ indicates increased difficulty

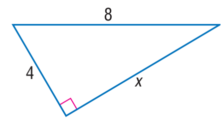
Practice and Apply

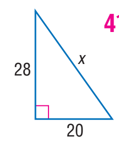
Homework Help	
For Exercises	See Examples
14, 15	1
12, 13, 16, 17	2
18–21	3
22–29	4

Extra Practice
See page 767.

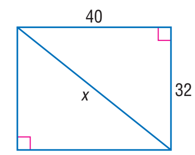
Find x. 12. $\sqrt{15} \approx 3.9$ 13. $4\sqrt{3} \approx 6.9$

12. 

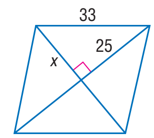
13. 

14. 

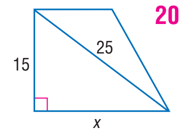
$4\sqrt{74} \approx 34.4$

15. 

$8\sqrt{41} \approx 51.2$

16. 

$4\sqrt{29} \approx 21.5$

17. 

20

COORDINATE GEOMETRY Determine whether $\triangle QRS$ is a right triangle for the given vertices. Explain. 18–21. See margin for explanations.

18. $Q(1, 0)$, $R(1, 6)$, $S(9, 0)$ yes
19. $Q(3, 2)$, $R(0, 6)$, $S(6, 6)$ no
20. $Q(-4, 6)$, $R(2, 11)$, $S(4, -1)$ no
21. $Q(-9, -2)$, $R(-4, -4)$, $S(-6, -9)$ yes

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

22. 8, 15, 17
23. 7, 24, 25
24. 20, 21, 31
25. 37, 12, 34
26. $\frac{1}{5}, \frac{1}{7}, \frac{\sqrt{74}}{35}$ yes, no
27. $\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{3}, \frac{35}{36}$ no, no
28. $\frac{3}{5}, \frac{4}{5}, 1$ yes, no
29. $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$ yes, no

For Exercises 30–35, use the table of Pythagorean triples.

a	*b*	*c*
5	12	13
10	24	? 26
15	? 36	39
? 20	48	52

30. Copy and complete the table.
31. A *primitive* Pythagorean triple is a Pythagorean triple with no common factors except 1. Name any primitive Pythagorean triples contained in the table.
32. Describe the pattern that relates these sets of Pythagorean triples.
33. These Pythagorean triples are called a *family*. Why do you think this is?
34. Are the triangles described by a family of Pythagorean triples similar? Explain.
35. For each Pythagorean triple, find two triples in the same family.
a. 8, 15, 17 b. 9, 40, 41 c. 7, 24, 25

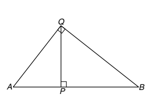
22. yes, yes
23. yes, yes
24. no, no
25. no, no
31. 5-12-13
32. Sample answer: The triples are all multiples of the triple 5-12-13.
33. Sample answer: They consist of any number of similar triangles.
34. Yes; the measures of the sides are always multiples of 5, 12, and 13.
35a. 16-30-34; 24-45-51
35b. 18-80-82; 27-120-123
35c. 14-48-50; 21-72-75

Enrichment, p. 362

Converse of a Right Triangle Theorem

You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

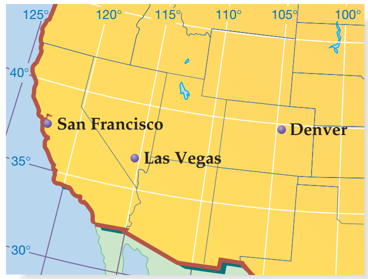
If $\triangle ABQ$ is a right triangle with right angle at Q, then QP is the geometric mean between AP and PB, where P is between A and B and $\overline{QP}$ is perpendicular to $\overline{AB}$.



1. Write the converse of the if-then form of the theorem.

GEOGRAPHY **For Exercises 36 and 37, use the following information.**

Denver is located at about 105 degrees longitude and 40 degrees latitude. San Francisco is located at about 122 degrees longitude and 38 degrees latitude. Las Vegas is located at about 115 degrees longitude and 36 degrees latitude. Using the lines of longitude and latitude, find each degree distance.

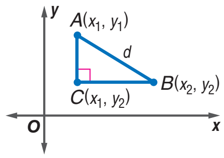


36. San Francisco to Denver **≈17.1 degrees**

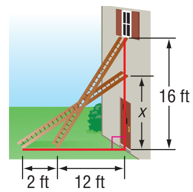
37. Las Vegas to Denver **≈10.8 degrees**

38. **PROOF** Write a paragraph proof of Theorem 7.5. **See margin.**

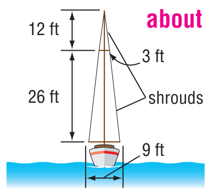
★ **39.** **PROOF** Use the Pythagorean Theorem and the figure at the right to prove the Distance Formula. **See margin.**



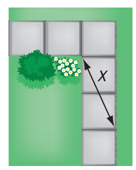
40. **PAINTING** A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?
$2\sqrt{51}$ ft ≈ 14.3 ft



41. **SAILING** The mast of a sailboat is supported by wires called *shrouds*. What is the total length of wire needed to form these shrouds? **about 76.53 ft**



★ **42.** **LANDSCAPING** Six congruent square stones are arranged in an L-shaped walkway through a garden. If $x = 15$ inches, then find the area of the L-shaped walkway. **270 in^2**



Career Choices

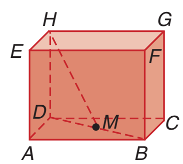
Military

All branches of the military use navigation. Some of the jobs using navigation include radar/sonar operators, boat operators, airplane navigators, and space operations officers.

Online Research For information about a career in the military, visit: www.geometryonline.com/careers

43. **NAVIGATION** A fishing trawler off the coast of Alaska was ordered by the U.S. Coast Guard to change course. They were to travel 6 miles west and then sail 12 miles south to miss a large iceberg before continuing on the original course. How many miles out of the way did the trawler travel? **about 4.6 mi**

44. **CRITICAL THINKING** The figure at the right is a rectangular prism with $AB = 8$, $BC = 6$, and $BF = 8$, and M is the midpoint of BD. Find BD and HM. How are EM, FM, and GM related to HM?
$BD = 10$, $HM = \sqrt{89} \approx 9.4$; $HM = EM = FM = GM$



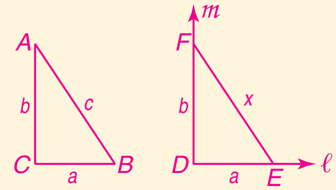
www.geometryonline.com/self_check_quiz

Phil Mislinski/Getty Images

Answers

38. Given: $\triangle ABC$ with sides of measure a, b, and c, where $c^2 = a^2 + b^2$

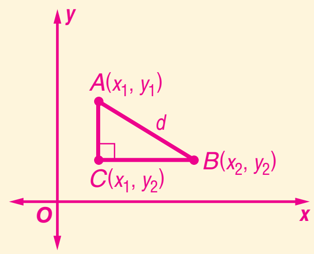
Prove: $\triangle ABC$ is a right triangle.



Proof: Draw $\overline{DE}$ on line ℓ with measure equal to a. At D, draw line $m \perp \overline{DE}$. Locate point F on m so that $DF = b$. Draw $\overline{FE}$ and call its measure x. Because $\triangle FED$ is a right triangle, $a^2 + b^2 = x^2$. But $a^2 + b^2 = c^2$, so $x^2 = c^2$ or $x = c$. Thus, $\triangle ABC \cong \triangle FED$ by SSS. This means $\angle C \cong \angle D$. Therefore, $\angle C$ must be a right angle, making $\triangle ABC$ a right triangle.

39. Given: $\triangle ABC$ with right angle at C, $AB = d$

Prove: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Proof:

Statements (Reasons)
1. $\triangle ABC$ with right angle at C, $AB = d$ (Given)
2. $(CB)^2 + (AC)^2 = (AB)^2$ (Pythagorean Theorem)
3. $\lvert x_2 - x_1 \rvert = CB$; $\lvert y_2 - y_1 \rvert = AC$ (Distance on a number line)
4. $\lvert x_2 - x_1 \rvert^2 + \lvert y_2 - y_1 \rvert^2 = d^2$ (Substitution)
5. $(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$ (Substitution)
6. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$ (Take the square root of each side.)
7. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Reflexive Property)

Answers (p. 354)

18. $QR = 6$, $RS = 10$, $QS = 8$; $6^2 + 8^2 = 10^2$

19. $QR = 5$, $RS = 6$, $QS = 5$; $5^2 + 5^2 \neq 6^2$

20. $QR = \sqrt{61}$, $RS = \sqrt{148}$, $QS = \sqrt{113}$; $(\sqrt{61})^2 + (\sqrt{113})^2 \neq (\sqrt{148})^2$

21. $QR = \sqrt{29}$, $RS = \sqrt{29}$, $QS = \sqrt{58}$; $(\sqrt{29})^2 + (\sqrt{29})^2 = (\sqrt{58})^2$

4 Assess

Open-Ended Assessment

Modeling Ask students to recall how right triangles are modeled in suspension bridges, and have them demonstrate how a right triangle could model real world objects such as sailboats, flags, houses, bookshelves, and so on.

Getting Ready for Lesson 7-3

Prerequisite Skill In Lesson 7-3, students will be working more extensively with radical expressions to find side lengths of special right triangles. Use Exercises 61–70 to determine your students' familiarity with simplifying radical expressions.

Assessment Options

Quiz (Lessons 7-1 and 7-2) is available on p. 407 of the *Chapter 7 Resource Masters*.

Answers

45. Sample answer: The road, the tower that is perpendicular to the road, and the cables form the right triangles. Answers should include the following.
    - Right triangles are formed by the bridge, the towers, and the cables.
    - The cable is the hypotenuse in each triangle.

---

45. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

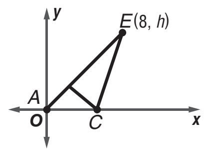
How are right triangles used to build suspension bridges?

Include the following in your answer:
- the locations of the right triangles, and
- an explanation of which parts of the right triangle are formed by the cables.

Standardized Test Practice

46. In the figure, if $AE = 10$, what is the value of h? **A**

Ⓐ 6 Ⓑ 8
Ⓒ 10 Ⓓ 12



47. ALGEBRA If $x^2 + 36 = (9 - x)^2$, then find x. **C**

Ⓐ 6 Ⓑ no solution
Ⓒ 2.5 Ⓓ 10

Graphing Calculator

PROGRAMMING **For Exercises 48 and 49, use the following information.**

The TI-83 Plus program uses a procedure for finding *Pythagorean triples* that was developed by Euclid around 320 B.C. Run the program to generate a list of Pythagorean triples.

```
PROGRAM: PYTHTRIP
:For (X, 2, 6)          :Disp B,A,C
:For (Y, 1, 5)          :Else
:If X > Y               :Disp A,B,C
:Then                   :End
:int (X² – Y² + 0.5)→A  :End
:2XY→B                  :Pause
:int (X²+Y²+0.5)→C      :Disp " "
:If A > B               :End
:Then                   :End
                        :Stop
```

48. List all the members of the 3-4-5 family that are generated by the program.

48. 3-4-5, 6-8-10, 12-16-20, 24-32-40, 27-36-45

49. A geometry student made the conjecture that if three whole numbers are a Pythagorean triple, then their product is divisible by 60. Does this conjecture hold true for each triple that is produced by the program? **yes**

Maintain Your Skills

Mixed Review

Find the geometric mean between each pair of numbers. *(Lesson 7-1)*

50. 3 and 12 **6**
51. 9 and 12 **$6\sqrt{3} \approx 10.4$**
52. 11 and 7 **$\sqrt{77} \approx 8.8$**
53. 6 and 9 **$3\sqrt{6} \approx 7.3$**
54. 2 and 7 **$\sqrt{14} \approx 3.7$**
55. 2 and 5 **$\sqrt{10} \approx 3.2$**

Find the value of each expression. Then use that value as the next x in the expression. Repeat the process and describe your observations. *(Lesson 6-6)*

56. $\sqrt{2x}$, where x initially equals 5
57. 3^x, where x initially equals 1
58. $x^{\frac{1}{2}}$, where x initially equals 4
59. $\frac{1}{x}$, where x initially equals 4

56. $\sqrt{10}$; it converges to 2.
57. 3; it approaches positive infinity.
58. 2; it converges to 1.
59. 0.25; it alternates between 0.25 and 4.

60. Determine whether the sides of a triangle could have the lengths 12, 13, and 25. Explain. *(Lesson 5-4)* **no; $12 + 13 \not> 25$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression by rationalizing the denominator.**
*(To review **simplifying radical expressions**, see pages 744 and 745.)*

61. $\frac{7}{\sqrt{3}}$ **$\frac{7\sqrt{3}}{3}$**
62. $\frac{18}{\sqrt{2}}$ **$9\sqrt{2}$**
63. $\frac{\sqrt{14}}{\sqrt{2}}$ **$\sqrt{7}$**
64. $\frac{3\sqrt{11}}{\sqrt{3}}$ **$\sqrt{33}$**
65. $\frac{24}{\sqrt{2}}$ **$12\sqrt{2}$**
66. $\frac{12}{\sqrt{3}}$ **$4\sqrt{3}$**
67. $\frac{2\sqrt{6}}{\sqrt{3}}$ **$2\sqrt{2}$**
68. $\frac{15}{\sqrt{3}}$ **$5\sqrt{3}$**
69. $\frac{2}{\sqrt{8}}$ **$\frac{\sqrt{2}}{2}$**
70. $\frac{25}{\sqrt{10}}$ **$\frac{5\sqrt{10}}{2}$**

356 Chapter 7 Right Triangles and Trigonometry

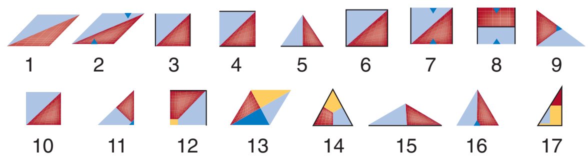
7-3 Special Right Triangles

What You'll Learn

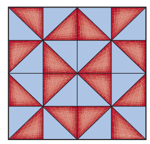
- Use properties of 45°-45°-90° triangles.
- Use properties of 30°-60°-90° triangles.

How is triangle tiling used in wallpaper design?

Triangle tiling is the process of laying copies of a single triangle next to each other to fill an area. One type of triangle tiling is *wallpaper tiling*. There are exactly 17 types of triangle tiles that can be used for wallpaper tiling.

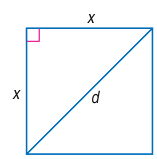


Tile 4 is made up of two 45°-45°-90° triangles that form a square. This tile is rotated to make the wallpaper design shown at the right.



PROPERTIES OF 45°-45°-90° TRIANGLES Facts about 45°-45°-90° triangles are used to solve many geometry problems. The Pythagorean Theorem allows us to discover special relationships that exist among the sides of a 45°-45°-90° triangle.

Draw a diagonal of a square. The two triangles formed are isosceles right triangles. Let x represent the measure of each side and let d represent the measure of the hypotenuse.

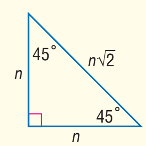


$d^2 = x^2 + x^2$	Pythagorean Theorem
$d^2 = 2x^2$	Add.
$d = \sqrt{2x^2}$	Take the positive square root of each side.
$d = \sqrt{2} \cdot \sqrt{x^2}$	Factor.
$d = x\sqrt{2}$	Simplify.

This algebraic proof verifies that the length of the hypotenuse of any 45°-45°-90° triangle is $\sqrt{2}$ times the length of its leg. The ratio of the sides is $1 : 1 : \sqrt{2}$.

Theorem 7.6

In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



7-3 Lesson Notes

1 Focus

5-Minute Check Transparency 7-3 Use as a quiz or review of Lesson 7-2.

Mathematical Background notes are available for this lesson on p. 340C.

How is triangle tiling used in wallpaper design?

Ask students:

- How many of the wallpaper tiles are made up of right triangles? Which ones are they? **12 tiles; tiles 3, 4, 5, 6, 7, 10, 11, 12, 13, 15, 16, and 17**
- What percent of wallpaper tiling depends on right triangles? **12 out of 17 or about 71%**
- If the length of the hypotenuse of one of the triangles in tile 12 is $7\sqrt{2}$ centimeters, then how long is each leg of the triangle? **7 cm**

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters
- Study Guide and Intervention, pp. 363–364
- Skills Practice, p. 365
- Practice, p. 366
- Reading to Learn Mathematics, p. 367
- Enrichment, p. 368

Prerequisite Skills Workbook, pp. 17–18
Teaching Geometry With Manipulatives Masters, p. 1

Transparencies

5-Minute Check Transparency 7-3
Answer Key Transparencies

Technology

GeomPASS: Tutorial Plus, Lesson 14
Interactive Chalkboard

2 Teach

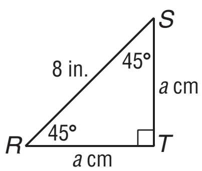
PROPERTIES OF 45°-45°-90° TRIANGLES

In-Class Examples

Power Point®

1 **WALLPAPER TILING** Note that the figure in Example 1 can be divided into four equal square quadrants so that each square contains 8 triangles. What is the area of one of these squares if the hypotenuse of each 45°-45°-90° triangle measures $7\sqrt{2}$ millimeters? **196 mm²**

2 Find *a*. $a = 4\sqrt{2}$



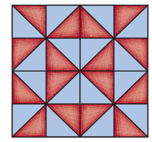
Teaching Tip Point out to students that one corner of the wallpaper square consists of four 45°-45°-90° triangles, the combination of which forms a new larger 45°-45°-90° triangle. Explain how it is interesting that tiled groups of 4, 8, and 16 of the smaller triangles form larger 45°-45°-90° triangles.

Tips for New Teachers

Intervention Some students may have difficulty keeping the properties of 45°-45°-90° and 30°-60°-90° triangles straight. Suggest that they periodically reproduce the figures in Theorem 7.6 and Theorem 7.7 to keep these relationships fresh. Advise them, however, that they have the intuitive knowledge to derive either of these two figures by using the Pythagorean Theorem with a square or an equilateral triangle.

Example 1 Find the Measure of the Hypotenuse

WALLPAPER TILING **Assume that the length of one of the legs of the 45°-45°-90° triangles in the wallpaper in the figure is 4 inches. What is the length of the diagonal of the entire wallpaper square?**

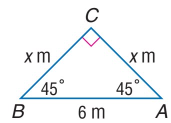


The length of each leg of the 45°-45°-90° triangle is 4 inches. The length of the hypotenuse is $\sqrt{2}$ times as long as a leg. The length of the hypotenuse of one of the triangles is $4\sqrt{2}$. There are four 45°-45°-90° triangles along the diagonal of the square. So, the length of the diagonal of the square is $4(4\sqrt{2})$ or $16\sqrt{2}$ inches.

Example 2 Find the Measure of the Legs

Find *x*.

The length of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.



$$AB = (AC)\sqrt{2}$$

$$6 = x\sqrt{2} \qquad AB = 6,\ AC = x$$

$$\frac{6}{\sqrt{2}} = x \qquad \text{Divide each side by } \sqrt{2}.$$

$$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x \qquad \text{Rationalize the denominator.}$$

$$\frac{6\sqrt{2}}{2} = x \qquad \text{Multiply.}$$

$$3\sqrt{2} = x \qquad \text{Divide.}$$

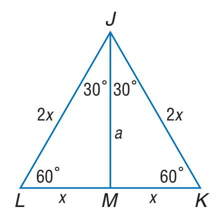
Study Tip

Rationalizing Denominators
To rationalize a denominator, multiply the fraction by 1 in the form of a radical over itself so that the product in the denominator is a rational number.

PROPERTIES OF 30°-60°-90° TRIANGLES

There is also a special relationship among the measures of the sides of a 30°-60°-90° triangle.

When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. $\overline{LM}$ and $\overline{KM}$ are congruent segments, so let $LM = x$ and $KM = x$. By the Segment Addition Postulate, $LM + KM = KL$. Thus, $KL = 2x$. Since $\triangle JKL$ is an equilateral triangle, $KL = JL = JK$. Therefore, $JL = 2x$ and $JK = 2x$.



Let *a* represent the measure of the altitude. Use the Pythagorean Theorem to find *a*.

$$(JM)^2 + (LM)^2 = (JL)^2 \qquad \text{Pythagorean Theorem}$$

$$a^2 + x^2 = (2x)^2 \qquad JM = a,\ LM = x,\ JL = 2x$$

$$a^2 + x^2 = 4x^2 \qquad \text{Simplify.}$$

$$a^2 = 3x^2 \qquad \text{Subtract } x^2 \text{ from each side.}$$

$$a = \sqrt{3x^2} \qquad \text{Take the positive square root of each side.}$$

$$a = \sqrt{3} \cdot \sqrt{x^2} \qquad \text{Factor.}$$

$$a = x\sqrt{3} \qquad \text{Simplify.}$$

So, in a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$. The ratio of the sides is $1:\sqrt{3}:2$.

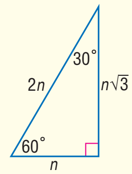
DAILY INTERVENTION — Differentiated Instruction

Logical/Mathematical Suggest that students close their books and divide a piece of paper into two columns. At the top of one column, ask them to draw a square with diagonal *d* and side *x*. At the top of the other column, ask students to draw an equilateral triangle with one altitude and tell them to label the segment that the altitude divides with two *x*'s. Then have students systematically use the Pythagorean Theorem to figure out the side relationships of the 45°-45°-90° and 30°-60°-90° triangles in these two figures.

The relationship of the side measures leads to Theorem 7.7.

Theorem 7.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.



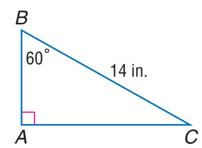
Study Tip

30°-60°-90° Triangle
The shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

Example 3 30°-60°-90° Triangles

Find AC.

$\overline{AC}$ is the longer leg, $\overline{AB}$ is the shorter leg, and $\overline{BC}$ is the hypotenuse.



$AB = \frac{1}{2}(BC)$

$= \frac{1}{2}(14)$ or 7 $\quad$ $BC = 14$

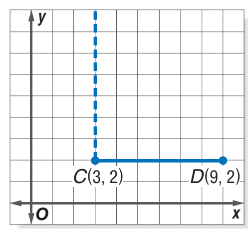
$AC = \sqrt{3}(AB)$

$= \sqrt{3}(7)$ or $7\sqrt{3}$ $\quad$ $AB = 7$

Example 4 Special Triangles in a Coordinate Plane

COORDINATE GEOMETRY **Triangle *PCD* is a 30°-60°-90° triangle with right angle *C*. $\overline{CD}$ is the longer leg with endpoints *C*(3, 2) and *D*(9, 2). Locate point *P* in Quadrant I.**

Graph *C* and *D*. $\overline{CD}$ lies on a horizontal gridline of the coordinate plane. Since $\overline{PC}$ will be perpendicular to $\overline{CD}$, it lies on a vertical gridline. Find the length of $\overline{CD}$.



$CD = |9 - 3| = 6$

$\overline{CD}$ is the longer leg. $\overline{PC}$ is the shorter leg.

So, $CD = \sqrt{3}(PC)$. Use *CD* to find *PC*.

$CD = \sqrt{3}(PC)$

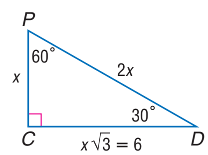
$6 = \sqrt{3}(PC)$ $\quad$ $CD = 6$

$\frac{6}{\sqrt{3}} = PC$ $\quad$ Divide each side by $\sqrt{3}$.

$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = PC$ $\quad$ Rationalize the denominator.

$\frac{6\sqrt{3}}{3} = PC$ $\quad$ Multiply.

$2\sqrt{3} = PC$ $\quad$ Simplify.



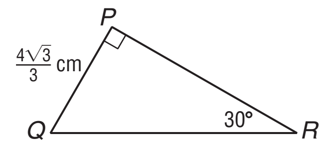
Point *P* has the same *x*-coordinate as *C*. *P* is located $2\sqrt{3}$ units above *C*. So, the coordinates of *P* are $(3, 2 + 2\sqrt{3})$ or about (3, 5.46).

www.geometryonline.com/extra_examples

PROPERTIES OF 30°-60°-90° TRIANGLES

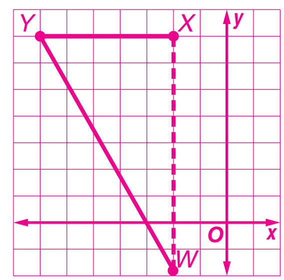
In-Class Examples Power Point®

3 Find *QR*.



$\frac{8\sqrt{3}}{3}$ **cm**

4 $\triangle WXY$ is a 30°-60°-90° triangle with right angle *X* and $\overline{WX}$ as the longer leg. Graph points *X*(−2, 7) and *Y*(−7, 7), and locate point *W* in Quadrant III.



$W(-2, 7 - 5\sqrt{3})$

DAILY INTERVENTION

Unlocking Misconceptions

30°-60°-90° Triangles Point out that a common mistake is for students to quickly assume that the longer leg of a 30°-60°-90° triangle is twice the length of the shorter leg. Demonstrate for students that this cannot be so by drawing a 30°-60°-90° triangle on the board and extending the figure into an equilateral triangle. Explain that the shorter leg of the original right triangle is exactly half the distance of one side of the equilateral triangle. Tell students that the hypotenuse is also a side of the equilateral triangle, so this has to be the side that is twice the length of the shorter leg.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Properties of 45°-45°-90° Triangles:** 12, 13, 17, 22, 25
- **Properties of 30°-60°-90° Triangles:** 14–16, 18–21, 23–24, 27–31

Odd/Even Assignments
Exercises 12–31 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–37 odd, 41–66
Average: 13–41 odd, 42–66
Advanced: 12–40 even, 41–58 (optional: 59–66)
All: Quiz 1 (1–5)

Check for Understanding

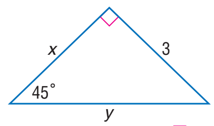
Concept Check

1. **OPEN ENDED** Draw a 45°-45°-90° triangle. Be sure to label the angles and the sides and to explain how you made the drawing. **1–2. See margin.**
2. **Draw** a 30°-60°-90° triangle with the shorter leg 2 centimeters long. Label the angles and the remaining sides.
3. **Write** an equation to find the length of a rectangle that has a diagonal twice as long as its width. **The length of the rectangle is $\sqrt{3}$ times the width; $\ell = \sqrt{3}w$.**

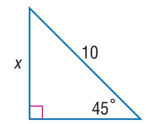
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5, 11	1, 2
6–8	3
9, 10	4

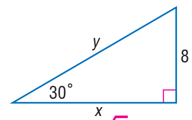
Find x and y.

4. 

$x = 3$; $y = 3\sqrt{2}$

5. 

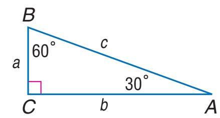
$x = 5\sqrt{2}$; $y = 5\sqrt{2}$

6. 

$x = 8\sqrt{3}$; $y = 16$

Find the missing measures.

7. If $c = 8$, find a and b. **$a = 4$; $b = 4\sqrt{3}$**
8. If $b = 18$, find a and c. **$a = 6\sqrt{3}$; $c = 12\sqrt{3}$**

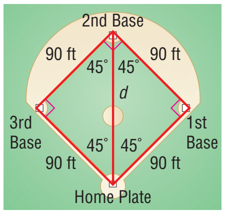


Triangle ABD is a 30°-60°-90° triangle with right angle B and with $\overline{AB}$ as the shorter leg. Graph A and B, and locate point D in Quadrant I. **9–10. See p. 399A.**

9. $A(8, 0)$, $B(8, 3)$
10. $A(6, 6)$, $B(2, 6)$

Application

11. **SOFTBALL** Find the distance from home plate to second base if the bases are 90 feet apart. **$90\sqrt{2}$ or 127.28 ft**



★ indicates increased difficulty

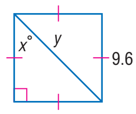
Practice and Apply

Homework Help

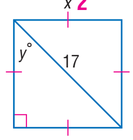
For Exercises	See Examples
12, 13, 17, 22, 25	1, 2
14–16, 18–21, 23, 24	3
27–31	4

Extra Practice
See page 767.

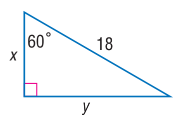
Find x and y.

12. 

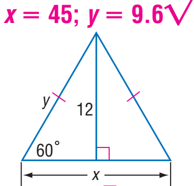
$x = 45$; $y = 9.6\sqrt{2}$

13. 

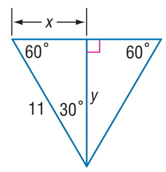
$x = \frac{17\sqrt{2}}{2}$; $y = 45$

14. 

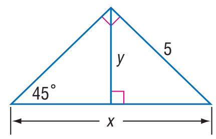
$x = 9$; $y = 9\sqrt{3}$

15. 

$x = 8\sqrt{3}$; $y = 8\sqrt{3}$

16. 

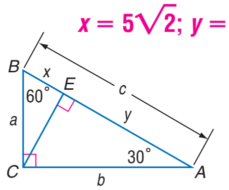
$x = 5.5$; $y = 5.5\sqrt{3}$

17. 

$x = 5\sqrt{2}$; $y = \frac{5\sqrt{2}}{2}$

For Exercises 18 and 19, use the figure at the right.

18. If $a = 10\sqrt{3}$, find CE and y. **$CE = 15$; $y = 15\sqrt{3}$**
19. If $x = 7\sqrt{3}$, find a, CE, y, and b.
$a = 14\sqrt{3}$; $CE = 21$; $y = 21\sqrt{3}$; $b = 42$



Answers

1. Sample answer: Construct two perpendicular lines. Use a ruler to measure 3 cm from the point of intersection on the one ray. Use the compass to copy the 3 cm segment. Connect the two endpoints to form a 45°-45°-90° triangle with sides of 3 cm and a hypotenuse of $3\sqrt{2}$ cm.

2. Sample answer: Draw a line using a ruler. Then use a protractor to measure a 90° angle. On one ray mark a 2 cm length, and at that endpoint use the protractor to measure a 30° angle toward the other ray. Where this ray intersects the other ray should form a 60° angle, completing the 30°-60°-90° triangle with sides 2 cm, $2\sqrt{3}$ cm, and a hypotenuse of 4 cm.

20. The length of an altitude of an equilateral triangle is 12 feet. Find the length of a side of the triangle. **$8\sqrt{3}$ ft ≈ 13.86 ft**

21. The perimeter of an equilateral triangle is 45 centimeters. Find the length of an altitude of the triangle. **$7.5\sqrt{3}$ cm ≈ 12.99 cm**

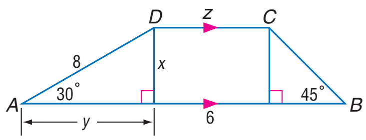
22. The length of a diagonal of a square is $22\sqrt{2}$ millimeters. Find the perimeter of the square. **88 mm**

23. The altitude of an equilateral triangle is 7.4 meters long. Find the perimeter of the triangle. **$14.8\sqrt{3}$ m ≈ 25.63 m**

24. The diagonals of a rectangle are 12 inches long and intersect at an angle of 60°. Find the perimeter of the rectangle. **$12 + 12\sqrt{3}$ or about 32.78 in.**

25. The sum of the squares of the measures of the sides of a square is 256. Find the measure of a diagonal of the square. **$8\sqrt{2} \approx 11.31$**

★ **26.** Find x, y, z, and the perimeter of $ABCD$.
$x = 4$; $y = 4\sqrt{3}$; $z = 6$;
$24 + 4\sqrt{2} + 4\sqrt{3}$ or ≈ 36.59 units



27. $\triangle PAB$ is a 45°-45°-90° triangle with right angle B. Find the coordinates of P in Quadrant I for $A(-3, 1)$ and $B(4, 1)$. **(4, 8)**

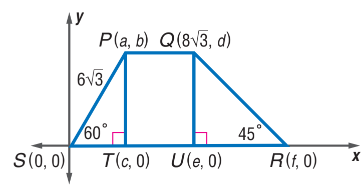
28. $\triangle PGH$ is a 45°-45°-90° triangle with $m\angle P = 90$. Find the coordinates of P in Quadrant I for $G(4, -1)$ and $H(4, 5)$. **(1, 2), (7, 2)**

29. $\triangle PCD$ is a 30°-60°-90° triangle with right angle C and $\overline{CD}$ the longer leg. Find the coordinates of P in Quadrant III for $C(-3, -6)$ and $D(-3, 7)$.

29. $\left(-3 - \frac{13\sqrt{3}}{3}, -6\right)$ or about (−10.51, −6)

★ **30.** $\triangle PCD$ is a 30°-60°-90° triangle with $m\angle C = 30$ and hypotenuse $\overline{CD}$. Find the coordinates of P for $C(2, -5)$ and $D(10, -5)$ if P lies above $\overline{CD}$. **$(8, -5 + 2\sqrt{3})$**

31. If $\overline{PQ} \parallel \overline{SR}$, use the figure to find a, b, c, and d.
$a = 3\sqrt{3}$, $b = 9$, $c = 3\sqrt{3}$, $d = 9$



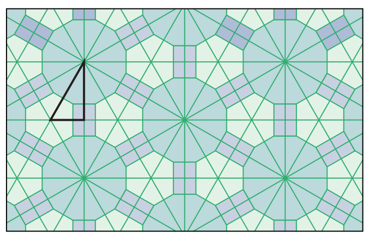
More About . . .

Triangle Tiling

Buildings in Federation Square in Melbourne, Australia, feature a tiling pattern called a pinwheel tiling. The sides of each right triangle are in the ratio $1 : 2 : \sqrt{5}$.

Source: www.federationsquare.com.au

TRIANGLE TILING **For Exercises 32–35, use the following information.**
Triangle tiling refers to the process of taking many copies of a single triangle and laying them next to each other to fill an area. For example, the pattern shown is composed of tiles like the one outlined.



32. How many 30°-60°-90° triangles are used to create the basic circular pattern? **12**

33. Which angle of the 30°-60°-90° triangle is being rotated to make the basic shape? **30° angle**

34. Explain why there are no gaps in the basic pattern.

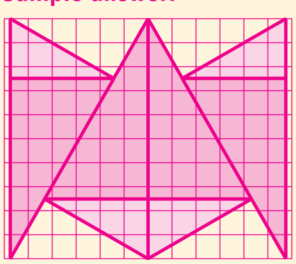
34. There are no gaps because when a 30° angle is rotated 12 times, it rotates 360°.

35. Use grid paper to cut out 30°-60°-90° triangles. Color the same pattern on each triangle. Create one basic figure that would be part of a wallpaper tiling. **See margin.**

John Gollings, courtesy Federation Square

Answer

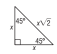
35. Sample answer:



Study Guide and Intervention, p. 363 (shown) and p. 364

Properties of 45°-45°-90° Triangles The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 **If the leg of a 45°-45°-90° right triangle is x units, show that the hypotenuse is $x\sqrt{2}$ units.**



Using the Pythagorean Theorem with $a = b = x$, then

$$c^2 = a^2 + b^2 = x^2 + x^2 = 2x^2$$
$$c = \sqrt{2x^2} = x\sqrt{2}$$

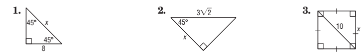
Example 2 **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

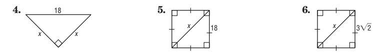
$$a = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \text{ units}$$

Exercises

Find x.



1. **$8\sqrt{2} \approx 11.3$** 2. **3** 3. **$5\sqrt{2} \approx 7.1$**



4. **$9\sqrt{2} \approx 12.7$** 5. **$18\sqrt{2} \approx 25.5$** 6. **6**

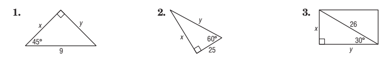
7. Find the perimeter of a square with diagonal 12 centimeters. **$24\sqrt{2} \approx 33.9$ cm**

8. Find the diagonal of a square with perimeter 20 inches. **$5\sqrt{2} \approx 7.1$ in.**

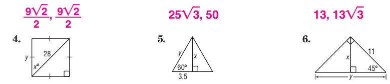
9. Find the diagonal of a square with perimeter 28 meters. **$7\sqrt{2} \approx 9.9$ m**

Skills Practice, p. 365 and Practice, p. 366 (shown)

Find x and y.

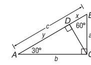


1. **$\frac{9\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}$** 2. **$25\sqrt{3}$, 50** 3. **13, $13\sqrt{3}$**



4. **45, $14\sqrt{2}$** 5. **$3.5\sqrt{3}$, 7** 6. **$\frac{11\sqrt{2}}{2}$; $11\sqrt{2}$**

For Exercises 7–9, use the figure at the right.



7. If $a = 4\sqrt{3}$, find b and c.
$b = 12$, $c = 8\sqrt{3}$

8. If $x = 3\sqrt{3}$, find a and CD.
$a = 6\sqrt{3}$, $CD = 9$

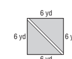
9. If $a = 4$, find CD, b, and y.
$CD = 2\sqrt{3}$, $b = 4\sqrt{3}$, $y = 6$

10. The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.
$6.5\sqrt{3}$ in. or about 11.26 in.

11. $\triangle MIP$ is a 30°-60°-90° triangle with right angle at I, and $\overline{IP}$ the longer leg. Find the coordinates of M in Quadrant I for $I(3, 3)$ and $P(12, 3)$.
$(3, 3 + 3\sqrt{3})$ or about (3, 8.19)

12. $\triangle TJK$ is a 45°-45°-90° triangle with right angle at J. Find the coordinates of T in Quadrant II for $J(-2, -3)$ and $K(3, -3)$.
(−2, 2)

13. BOTANICAL GARDENS One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?
$6\sqrt{2}$ yd or about 8.48 yd



Reading to Learn Mathematics, p. 367 ELL

Pre-Activity **How is triangle tiling used in wallpaper design?**

Read the introduction to Lesson 7-3 at the top of page 357 in your textbook.

- How can you most completely describe the larger triangle and the two smaller triangles in tile 15? **Sample answer: The larger triangle is an isosceles obtuse triangle. The two smaller triangles are congruent scalene right triangles.**
- How can you most completely describe the larger triangle and the two smaller triangles in tile 16? (Include angle measures in describing all the triangles.) **Sample answer: The larger triangle is equilateral, so each of its angle measures is 60. The two smaller triangles are congruent right triangles in which the angle measures are 30, 60, and 90.**

Reading the Lesson

1. Supply the correct number or numbers to complete each statement.
a. In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by **$\sqrt{2}$**.
b. In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by **2**.
c. In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of **60**.
d. In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by **$\sqrt{3}$**.
e. In an isosceles right triangle, each leg is opposite an angle with a measure of **45**.
f. In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by **$\sqrt{3}$**.
g. In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by **2** and multiply the result by **$\sqrt{3}$**.
h. To find the length of a side of a square, divide the length of the diagonal by **$\sqrt{2}$**.

2. Indicate whether each statement is *always*, *sometimes*, or *never* true.
a. The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem. **sometimes**
b. The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple. **never**
c. The lengths of all three sides of a 30°-60°-90° triangle are positive integers. **never**

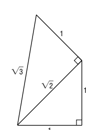
Helping You Remember

3. Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle?
Sample answer: Draw a 30°-60°-90° triangle. Label the length of the shorter leg as 1. Then the length of the hypotenuse is 2, and the length of the longer leg is $\sqrt{3}$. Just remember: 1, 2, $\sqrt{3}$.

Enrichment, p. 368

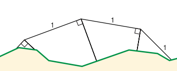
Constructing Values of Square Roots

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.



By continuing this process as shown below, you can construct a "wheel" of square roots. This wheel is called the "Wheel of Theodorus" after a Greek philosopher who lived about 400 B.C.

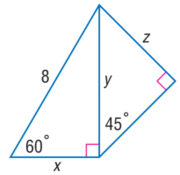
Continue constructing the wheel until you make a segment of length $\sqrt{18}$.



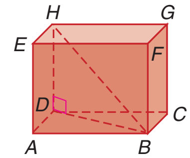
Answers

42. Sample answer: Congruent triangles of different color can be arranged to create patterns. Answers should include the following.
- 5, 9, 15, and 17; 3, 4, 6, 7, 10, 11, and 12
- Placing 45° angles next to one another forms 90° angles, which can be placed next to each other, leaving no holes.

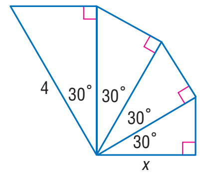
36. Find x, y, and z. $x = 4$; $y = 4\sqrt{3}$; $z = 2\sqrt{6}$



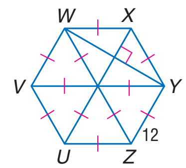
37. If $BD = 8\sqrt{3}$ and $m\angle DHB = 60$, find BH. 16



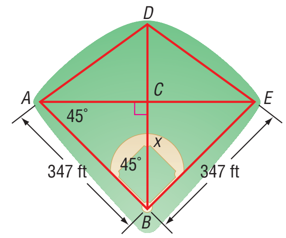
★ **38.** Each triangle in the figure is a 30°-60°-90° triangle. Find x. 2.25



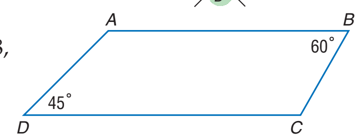
★ **39.** In regular hexagon $UVWXYZ$, each side is 12 centimeters long. Find WY. $12\sqrt{3} \approx 20.78$ cm



40. **BASEBALL** The diagram at the right shows some dimensions of Comiskey Park in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul-ball pole to the right field foul-ball pole. If the center fielder is standing at C, how far is he from home plate? $\frac{347\sqrt{2}}{2} \approx 245.4$ ft



41. **CRITICAL THINKING** Given figure $ABCD$, with $\overline{AB} \parallel \overline{DC}$, $m\angle B = 60$, $m\angle D = 45$, $BC = 8$, and $AB = 24$, find the perimeter. $52 + 4\sqrt{3} + 4\sqrt{6}$ units



42. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See margin.

How is triangle tiling used in wallpaper design?

Include the following in your answer:
- which of the numbered designs contain 30°-60°-90° triangles and which contain 45°-45°-90° triangles, and
- a reason why rotations of the basic design left no holes in the completed design.

Standardized Test Practice

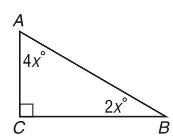
43. In the right triangle, what is AB if $BC = 6$? C

Ⓐ 12 units

Ⓑ $6\sqrt{2}$ units

Ⓒ $4\sqrt{3}$ units

Ⓓ $2\sqrt{3}$ units



44. **SHORT RESPONSE** For real numbers a and b, where $b \neq 0$, if $a \star b = \frac{a^2}{b^2}$, then $(3 \star 4)(5 \star 3) = $ ___?___. $\frac{25}{16}$

362 Chapter 7 Right Triangles and Trigonometry

Maintain Your Skills

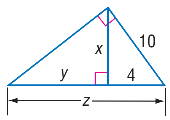
Mixed Review Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. *(Lesson 7-2)*

45. 3, 4, 5 **yes, yes** **46.** 9, 40, 41 **yes, yes** **47.** 20, 21, 31 **no, no**

48. 20, 48, 52 **yes, yes** **49.** 7, 24, 25 **yes, yes** **50.** 12, 34, 37 **no, no**

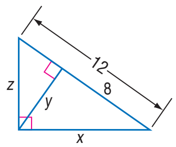
Find x, y, and z. *(Lesson 7-1)*

51.

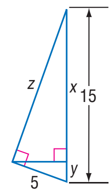


$2\sqrt{21} \approx 9.2; 21; 25$

52.

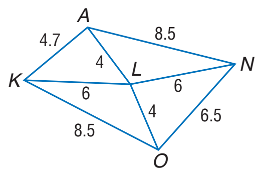


53.



52. $4\sqrt{6} \approx 9.8$; $4\sqrt{2} \approx 5.7$; $4\sqrt{3} \approx 6.9$
53. $\frac{40}{3}, \frac{5}{3}, 10\sqrt{2} \approx 14.1$

Write an inequality or equation relating each pair of angles. *(Lesson 5-5)*



54. $m\angle ALK$, $m\angle ALN$ $m\angle ALK < m\angle ALN$

55. $m\angle ALK$, $m\angle NLO$ $m\angle ALK < m\angle NLO$

56. $m\angle OLK$, $m\angle NLO$ $m\angle OLK > m\angle NLO$

57. $m\angle KLO$, $m\angle ALN$ $m\angle KLO = m\angle ALN$

58. Determine whether $\triangle JKL$ with vertices $J(-3, 2)$, $K(-1, 5)$, and $L(4, 4)$ is congruent to $\triangle RST$ with vertices $R(-6, 6)$, $S(-4, 3)$, and $T(1, 4)$. Explain. *(Lesson 4-4)*

58. $JK = \sqrt{13}$, $KL = \sqrt{26}$, $JL = \sqrt{53}$, $RS = \sqrt{13}$, $ST = \sqrt{26}$, $RT = \sqrt{53}$; $\triangle JKL \cong \triangle RST$ by SSS

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation.
*(To review **solving equations**, see pages 737 and 738.)*

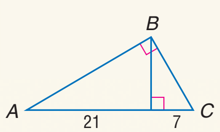
59. $5 = \frac{x}{3}$ **15** **60.** $\frac{x}{9} = 0.14$ **1.26** **61.** $0.5 = \frac{10}{k}$ **20** **62.** $0.2 = \frac{13}{g}$ **65**

63. $\frac{7}{n} = 0.25$ **28** **64.** $9 = \frac{m}{0.8}$ **7.2** **65.** $\frac{24}{x} = 0.4$ **60** **66.** $\frac{35}{y} = 0.07$ **500**

Practice Quiz 1 — *Lessons 7-1 through 7-3*

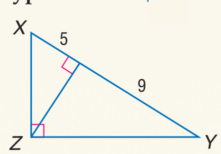
Find the measure of the altitude drawn to the hypotenuse. *(Lesson 7-1)*

1.



$7\sqrt{3} \approx 12.1$

2.

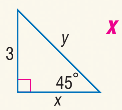


$3\sqrt{5} \approx 6.7$

3. Determine whether $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ is a right triangle. Explain. *(Lesson 7-2)* yes; $AB = \sqrt{5}$, $BC = \sqrt{50}$, $AC = \sqrt{45}$; $(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$

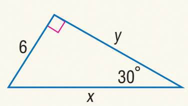
Find x and y. *(Lesson 7-3)*

4.



$x = 3$; $y = 3\sqrt{2}$

5.



$x = 12$; $y = 6\sqrt{3}$

www.geometryonline.com/self_check_quiz

4 Assess

Open-Ended Assessment

Speaking Have students name the side relationships for 45°-45°-90° triangles and 30°-60°-90° triangles. For some examples and problems in the book, ask students to call out these side relationships using the numbers and variables in the figures.

Getting Ready for Lesson 7-4

Prerequisite Skill Students will learn about trigonometric ratios in Lesson 7-4. They will use fractions to solve equations and express side lengths. Use Exercises 59–66 to determine your students' familiarity with solving equations that involve fractions.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 7-1 through 7-3. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

7-4 Lesson Notes

1 Focus

5-Minute Check Transparency 7-4 Use as a quiz or review of Lesson 7-3.

Mathematical Background notes are available for this lesson on p. 340D.

How can surveyors determine angle measures?

Ask students:

- What side of a right triangle is represented by the theodolite's line of sight? **the hypotenuse**
- The Statue of Liberty is approximately 91.5 meters tall from the base of the pedestal to the tip of the torch. If a theodolite were placed 100 meters from the base of the pedestal, about how far would the theodolite be from the top of the torch? **about 135.5 m**

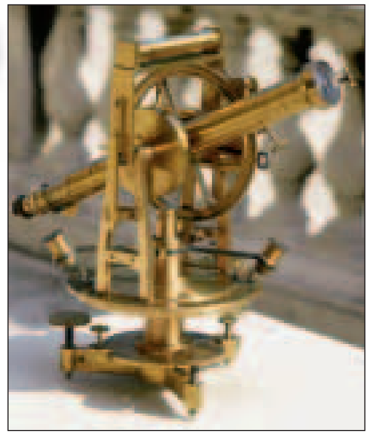
7-4 Trigonometry

What You'll Learn

- Find trigonometric ratios using right triangles.
- Solve problems using trigonometric ratios.

How can surveyors determine angle measures?

The old surveyor's telescope shown at right is called a theodolite (thee AH duh lite). It is an optical instrument used to measure angles in surveying, navigation, and meteorology. It consists of a telescope fitted with a level and mounted on a tripod so that it is free to rotate about its vertical and horizontal axes. After measuring angles, surveyors apply trigonometry to calculate distance or height.



Vocabulary

- trigonometry
- trigonometric ratio
- sine
- cosine
- tangent

TEACHING TIP

In the expression sin *A*, *A* represents the measure of $\angle A$ in degrees.

Study Tip

Reading Math

SOH-CAH-TOA is a mnemonic device for learning the ratios for sine, cosine, and tangent using the first letter of each word in the ratios.

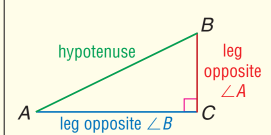
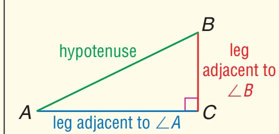
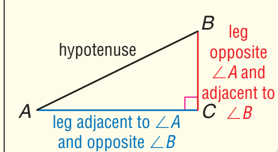
$\sin A = \frac{opp}{hyp}$

$\cos A = \frac{adj}{hyp}$

$\tan A = \frac{opp}{adj}$

TRIGONOMETRIC RATIOS The word **trigonometry** comes from two Greek terms, *trigon*, meaning triangle, and *metron*, meaning measure. The study of trigonometry involves triangle measurement. A ratio of the lengths of sides of a right triangle is called a **trigonometric ratio**. The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**.

Key Concept — *Trigonometric Ratios*

Words	Symbols	Models
sine of $\angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}$ sine of $\angle B = \frac{\text{measure of leg opposite } \angle B}{\text{measure of hypotenuse}}$	$\sin A = \frac{BC}{AB}$ $\sin B = \frac{AC}{AB}$	
cosine of $\angle A = \frac{\text{measure of leg adjacent to } \angle A}{\text{measure of hypotenuse}}$ cosine of $\angle B = \frac{\text{measure of leg adjacent to } \angle B}{\text{measure of hypotenuse}}$	$\cos A = \frac{AC}{AB}$ $\cos B = \frac{BC}{AB}$	
tangent of $\angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A}$ tangent of $\angle B = \frac{\text{measure of leg opposite } \angle B}{\text{measure of leg adjacent to } \angle B}$	$\tan A = \frac{BC}{AC}$ $\tan B = \frac{AC}{BC}$	

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 369–370
- Skills Practice, p. 371
- Practice, p. 372
- Reading to Learn Mathematics, p. 373
- Enrichment, p. 374
- Assessment, pp. 407, 409

School-to-Career Masters, p. 14
Prerequisite Skills Workbook, pp. 11–12
Teaching Geometry With Manipulatives Masters, pp. 16, 17, 18, 117, 118

Transparencies

5-Minute Check Transparency 7-4
Answer Key Transparencies

Technology

Interactive Chalkboard

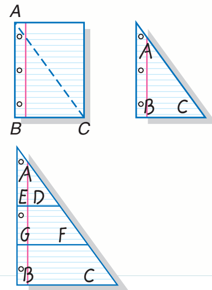
Trigonometric ratios are related to the acute angles of a right triangle, *not* the right angle.

Study Tip

Equivalent Ratios
Notice that the ratio $\frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ is the same as $\frac{\text{leg adjacent } \angle C}{\text{hypotenuse}}$. Thus, $\sin A = \cos C = \frac{a}{b}$. Likewise, $\cos A = \sin B = \frac{c}{b}$.

Geometry Activity

Trigonometric Ratios



- Fold a rectangular piece of paper along a diagonal from A to C. Then cut along the fold to form right triangle ABC. Write the name of each angle on the inside of the triangle.
- Fold the triangle so that there are two segments perpendicular to $\overline{BA}$. Label points D, E, F, and G as shown. Use a ruler to measure $\overline{AC}$, $\overline{AB}$, $\overline{BC}$, $\overline{AF}$, $\overline{AG}$, $\overline{FG}$, $\overline{AD}$, $\overline{AE}$, and $\overline{DE}$ to the nearest millimeter.

Analyze

1. What is true of $\triangle AED$, $\triangle AGF$, and $\triangle ABC$? **They are similar triangles.**
2. Copy the table. Write the ratio of the side lengths for each trigonometric ratio. Then calculate a value for each ratio to the nearest ten-thousandth.

	In $\triangle AED$	in $\triangle AGF$	In $\triangle ABC$
sin A	$\frac{DE}{AD} \approx 0.6114$	$\frac{FG}{AF} \approx 0.6114$	$\frac{BC}{AC} \approx 0.6114$
cos A	$\frac{AE}{AD} \approx 0.7913$	$\frac{AG}{AF} \approx 0.7913$	$\frac{AB}{AC} \approx 0.7913$
tan A	$\frac{DE}{AE} \approx 0.7727$	$\frac{FG}{AG} \approx 0.7727$	$\frac{BC}{AB} \approx 0.7727$

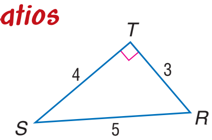
3. Study the table. Write a sentence about the patterns you observe with the trigonometric ratios.
4. What is true about $m\angle A$ in each triangle? **$m\angle A$ is the same in all triangles.**

3. Sample answer: Regardless of the side lengths, the trigonometric ratio is the same when comparing angles in similar triangles.

As the Geometry Activity suggests, the value of a trigonometric ratio depends *only* on the measure of the angle. It does not depend on the size of the triangle.

Example 1 Find Sine, Cosine, and Tangent Ratios

Find sin R, cos R, tan R, sin S, cos S, and tan S. Express each ratio as a fraction and as a decimal.



$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5}$ or 0.8

$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5}$ or 0.6

$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{ST}{RT} = \frac{4}{3}$ or $1.\overline{3}$

$\sin S = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5}$ or 0.6

$\cos S = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5}$ or 0.8

$\tan S = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{RT}{ST} = \frac{3}{4}$ or 0.75

www.geometryonline.com/extra_examples

2 Teach

TRIGONOMETRIC RATIOS

Building on Prior Knowledge

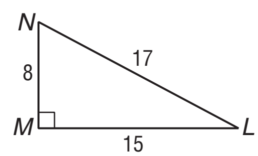
In Lesson 7-1, students learned that they could use the Distance Formula and the Pythagorean Theorem to find lengths in a coordinate plane. They revisit these methods in this lesson and combine them with trigonometry to find the angles of a triangle in a coordinate plane.

In-Class Example

Power Point®

Teaching Tip Assure the class that sine, cosine, and tangent are very useful and not as imposing as they first seem. Explain that if students regularly practice using these ratios to solve problems, they will become as familiar as squaring and taking the square root of numbers and are just as easy to key into a calculator.

1 Find sin L, cos L, tan L, sin N, cos N, and tan N. Express each ratio as a fraction and as a decimal.



$\sin L = \frac{8}{17}$ or 0.47

$\cos L = \frac{15}{17}$ or 0.88

$\tan L = \frac{8}{15}$ or 0.53

$\sin N = \frac{15}{17}$ or 0.88

$\cos N = \frac{8}{17}$ or 0.47

$\tan N = \frac{15}{8}$ or 1.88

Geometry Activity

Materials: rectangular paper, scissors, metric ruler, pencil

- When students are folding the triangle to form two segments that are parallel to $\overline{BC}$, tell them that it does not matter how close or far apart these segments are, and encourage the class to come up with a variety of different lengths to demonstrate that this activity would work for any lengths.
- Students can also begin the activity with different sizes of rectangular paper.

Teaching Tip Encourage students to familiarize themselves with using their calculators to find cosines, sines and tangents and their inverses. Allow students to practice with easy values, such as cos 0° = 1, sin 30° = 0.5, $\cos^{-1} 0.5 = 60°$, etc., so they can be sure they are using their calculators correctly.

In-Class Example

2 Find each value to the nearest ten thousandth.

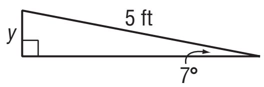
a. tan 56° **1.4826**

b. cos 89° **0.0175**

USE TRIGONOMETRIC RATIOS

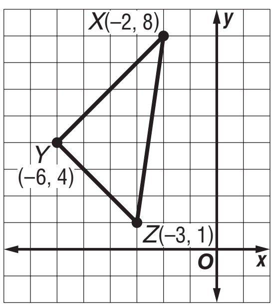
In-Class Examples

3 **EXERCISING** A fitness trainer sets the incline on a treadmill to 7°. The walking surface is 5 feet long. Approximately how many inches did the trainer raise the end of the treadmill from the floor?



≈7.3 in.

4 **COORDINATE GEOMETRY** Find $m\angle X$ in right $\triangle XYZ$ for $X(-2, 8)$, $Y(-6, 4)$, and $Z(-3, 1)$.



≈36.9

Study Tip

Graphing Calculator

Be sure your calculator is in degree mode rather than radian mode. Your calculator may require you to input the angle *before* using the trigonometric key.

Example 2 Use a Calculator to Evaluate Expressions

Use a calculator to find each value to the nearest ten thousandth.

a. cos 39°

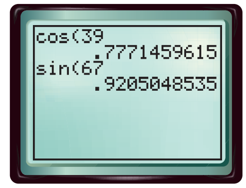
KEYSTROKES: [COS] 39 [ENTER]

cos 39° ≈ 0.7771

b. sin 67°

KEYSTROKES: [SIN] 67 [ENTER]

sin 67° ≈ 0.9205



USE TRIGONOMETRIC RATIOS You can use trigonometric ratios to find the missing measures of a right triangle if you know the measures of two sides of a triangle or the measure of one side and one acute angle.

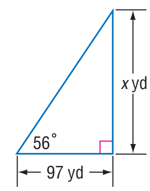
Example 3 Use Trigonometric Ratios to Find a Length

SURVEYING **Dakota is standing on the ground 97 yards from the base of a cliff. Using a theodolite, he noted that the angle formed by the ground and the line of sight to the top of the cliff was 56°. Find the height of the cliff to the nearest yard.**

Let x be the height of the cliff in yards.

$\tan 56° = \frac{x}{97}$ $\quad$ $\tan = \frac{\text{leg opposite}}{\text{leg adjacent}}$

$97 \tan 56° = x$ $\quad$ Multiply each side by 97.



Use a calculator to find x.

KEYSTROKES: 97 [TAN] 56 [ENTER] *143.8084139*

The cliff is about 144 yards high.

Study Tip

Tangent

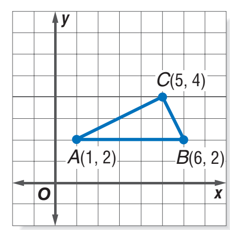
The tangent of angle *A* is the same as the slope of the line through the origin that forms an angle of measure *A* with the positive *x*-axis.

When solving equations like $3x = -27$, you use the inverse of multiplication to find x. In trigonometry, you can find the measure of the angle by using the inverse of sine, cosine, or tangent.

Given equation	To find the angle	Read as
$\sin A = x$	$A = \sin^{-1}(x)$	*A* equals *the inverse sine of x.*
$\cos A = y$	$A = \cos^{-1}(y)$	*A* equals *the inverse cosine of y.*
$\tan A = z$	$A = \tan^{-1}(z)$	*A* equals *the inverse tangent of z.*

Example 4 Use Trigonometric Ratios to Find an Angle Measure

COORDINATE GEOMETRY **Find $m\angle A$ in right triangle ABC for $A(1, 2)$, $B(6, 2)$, and $C(5, 4)$.**



Explore You know the coordinates of the vertices of a right triangle and that $\angle C$ is the right angle. You need to find the measure of one of the angles.

Plan Use the Distance Formula to find the measure of each side. Then use one of the trigonometric ratios to write an equation. Use the inverse to find $m\angle A$.

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical The easiest way for auditory learners to remember the ratios for sine, cosine and tangent is for them to chant SOH-CAH-TOA. When introducing this mnemonic device to students, have them repeat it as a class a few times in rhythm. Point out that SOH and CAH each have one syllable because the "H" is silent, so students can remember that one "silent" hypotenuse is involved for the sine and cosine ratios. TOA has two syllables and involves the two legs for the tangent ratio.

Study Tip

Calculators
The second functions of the SIN, COS, and TAN keys are usually the inverses.

Solve

$AB = \sqrt{(6-1)^2 + (2-2)^2}$ $\quad BC = \sqrt{(5-6)^2 + (4-2)^2}$

$= \sqrt{25 + 0}$ or 5 $\quad = \sqrt{1 + 4}$ or $\sqrt{5}$

$AC = \sqrt{(5-1)^2 + (4-2)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$ or $2\sqrt{5}$

Use the cosine ratio.

$\cos A = \frac{AC}{AB}$ $\quad \cos = \frac{\text{leg adjacent}}{\text{hypotenuse}}$

$\cos A = \frac{2\sqrt{5}}{5}$ $\quad AC = 2\sqrt{5}$ and $AB = 5$

$A = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right)$ $\quad$ Solve for A.

Use a calculator to find $m\angle A$.

KEYSTROKES: 2nd [COS^{-1}] 2 2nd [$\sqrt{\ }$] 5) ÷ 5 ENTER

$m\angle A \approx 26.56505118$

The measure of $\angle A$ is about 26.6.

Examine Use the sine ratio to check the answer.

$\sin A = \frac{BC}{AB}$ $\quad \sin = \frac{\text{leg opposite}}{\text{hypotenuse}}$

$\sin A = \frac{\sqrt{5}}{5}$ $\quad BC = \sqrt{5}$ and $AB = 5$

KEYSTROKES: 2nd [SIN^{-1}] 2nd [$\sqrt{\ }$] 5) ÷ 5 ENTER

$m\angle A \approx 26.56505118$

The answer is correct.

Check for Understanding

Concept Check

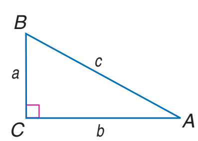
1. The triangles are similar, so the ratios remain the same.

1. **Explain** why trigonometric ratios do not depend on the size of the right triangle.
2. **OPEN ENDED** Draw a right triangle and label the measures of one acute angle and the measure of the side opposite that angle. Then solve for the remaining measures. **2–4. See margin.**
3. **Compare and contrast** the sine, cosine, and tangent ratios.
4. **Explain** the difference between $\tan A = \frac{x}{y}$ and $\tan^{-1}\left(\frac{x}{y}\right) = A$.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
5, 6	1
7–12	2
17	3
13–16	4

Use △ABC to find sin A, cos A, tan A, sin B, cos B, and tan B. Express each ratio as a fraction and as a decimal to the nearest hundredth. **5–6. See margin.**



5. $a = 14$, $b = 48$, and $c = 50$
6. $a = 8$, $b = 15$, and $c = 17$

Use a calculator to find each value. Round to the nearest ten-thousandth.

7. sin 57° **0.8387**
8. cos 60° **0.5000**
9. cos 33° **0.8387**
10. tan 30° **0.5774**
11. tan 45° **1.0000**
12. sin 85° **0.9962**

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include the "SOH-CAH-TOA" mnemonic device and an example similar to Example 1 on p. 365 for quick reference.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Trigonometric Ratios:** 18–36
- **Use Trigonometric Ratios:** 37–48, 52–54

Odd/Even Assignments
Exercises 18–54 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 22–27 require a scientific calculator.

Assignment Guide

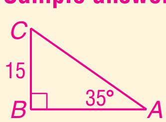
Basic: 19–49 odd, 59, 61–64, 69–82 (optional: 65–68)

Average: 19–61 odd, 62–64, 69–82 (optional: 65–68)

Advanced: 18–60 even, 61–76 (optional: 77–82)

Answers

2. Sample answer:



$m\angle B = 90$, $m\angle C = 55$, $b \approx 26.2$, $c \approx 21.4$

3. All three ratios involve two sides of a right triangle. The sine ratio is the measure of the opposite leg divided by the measure of the hypotenuse. The cosine ratio is the measure of the adjacent leg divided by the measure of the hypotenuse. The tangent ratio is the measure of the opposite leg divided by the measure of the adjacent leg.

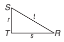
4. The tan is the ratio of the measure of the opposite leg divided by the measure of the adjacent leg for a given angle in a right triangle. The $\tan^{-1}$ is the measure of the angle with a certain tangent ratio.

5. $\frac{14}{50} = 0.28$; $\frac{48}{50} = 0.96$; $\frac{14}{48} \approx 0.29$; $\frac{48}{50} = 0.96$; $\frac{14}{50} = 0.28$; $\frac{48}{14} \approx 3.43$

6. $\frac{8}{17} \approx 0.47$; $\frac{15}{17} \approx 0.88$; $\frac{8}{15} \approx 0.53$; $\frac{15}{17} \approx 0.88$; $\frac{8}{17} \approx 0.47$; $\frac{15}{8} \approx 1.88$

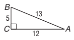
Study Guide and Intervention, p. 369 (shown) and p. 370

Trigonometric Ratios The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.



$\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t}$ $\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t}$ $\tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}$

Example **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a decimal to the nearest thousandth.**



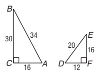
$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \approx 0.385$

$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.923$

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.417$

Exercises

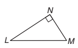
Find the indicated trigonometric ratio as a fraction and as a decimal. If necessary, round to the nearest ten-thousandth.



1. sin *A* $\frac{15}{17}$; 0.8824
2. tan *B* $\frac{8}{15}$; 0.5333
3. cos *A* $\frac{8}{17}$; 0.4706
4. cos *B* $\frac{15}{17}$; 0.8824
5. sin *D* $\frac{4}{5}$; 0.8
6. tan *E* $\frac{3}{4}$; 0.75
7. cos *E* $\frac{4}{5}$; 0.8
8. cos *D* $\frac{3}{5}$; 0.6

Skills Practice, p. 371 and Practice, p. 372 (shown)

Use $\triangle LMN$ to find sin *L*, cos *L*, tan *L*, sin *M*, cos *M*, and tan *M*. Express each ratio as a fraction and as a decimal to the nearest hundredth.

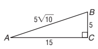


1. $\ell = 15, m = 36, n = 39$
   $\sin L = \frac{15}{39} \approx 0.38$; $\cos L = \frac{36}{39} \approx 0.92$; $\tan L = \frac{15}{36} \approx 0.42$; $\sin M = \frac{36}{39} \approx 0.92$; $\cos M = \frac{15}{39} \approx 0.38$; $\tan M = \frac{36}{15} = 2.4$
2. $\ell = 12, m = 12\sqrt{3}, n = 24$
   $\sin L = \frac{12}{24} = 0.50$; $\cos L = \frac{12\sqrt{3}}{24} \approx 0.87$; $\tan L = \frac{12}{12\sqrt{3}} \approx 0.58$; $\sin M = \frac{12\sqrt{3}}{24} \approx 0.87$; $\cos M = \frac{12}{24} = 0.50$; $\tan M = \frac{12\sqrt{3}}{12} \approx 1.73$

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 92.4 0.9991
4. tan 27.5 0.5206
5. cos 64.8 0.4258

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.

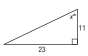


6. cos *A* $\frac{3\sqrt{10}}{10} \approx 0.9487$
7. tan *B* $\frac{3}{1} = 3.0000$
8. sin *A* $\frac{\sqrt{10}}{10} \approx 0.3162$

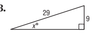
Find the measure of each acute angle to the nearest tenth of a degree.

9. sin *B* = 0.7823 51.5
10. tan *A* = 0.2356 13.3
11. cos *R* = 0.6401 50.2

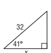
Find *x*. Round to the nearest tenth.

12. 

64.4

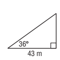
13. 

18.1

14. 

24.2

15. **GEOGRAPHY** Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter? 31 m



Reading to Learn Mathematics, p. 373 ELL

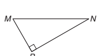
Pre-Activity **How can surveyors determine angle measures?**

Read the introduction to Lesson 7-4 at the top of page 364 in your textbook.

- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.) Sample answers: (1) To avoid collisions between ships, and (2) to prevent ships from losing their bearings and getting lost at sea.
- What does *calibrated* mean? Sample answer: marked precisely to permit accurate measurements to be made

Reading the Lesson

1. Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.



A. sin *N* $\frac{MP}{MN}$ B. cos *N* $\frac{NP}{MN}$
C. tan *N* $\frac{MP}{NP}$ D. tan *M* $\frac{NP}{MP}$
E. sin *M* $\frac{NP}{MN}$ F. cos *M* $\frac{MP}{MN}$

2. Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

a. sin 20 v	i. the degree measure of an acute angle whose cosine is 0.8
b. cos 20 ii	ii. the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle
c. $\sin^{-1}$ 0.8 vi	iii. the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8
d. $\tan^{-1}$ 0.8 iii	iv. the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle
e. tan 20 iv	v. the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle
f. $\cos^{-1}$ 0.8 i	vi. the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8

Helping You Remember

3. How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?
Sample answer: The *co* in *cosine* comes from *complement*, as in *complementary* angles. The cosine of an acute angle is equal to the sine of its complement.

Find the measure of each angle to the nearest tenth of a degree.

13. $\tan A = 1.4176$ $m\angle A \approx 54.8$
14. $\sin B = 0.6307$ $m\angle B \approx 39.1$

COORDINATE GEOMETRY **Find the measure of the angle to the nearest tenth in each right triangle *ABC*.**

15. $\angle A$ in $\triangle ABC$, for $A(6, 0)$, $B(-4, 2)$, and $C(0, 6)$ $m\angle A \approx 33.7$
16. $\angle B$ in $\triangle ABC$, for $A(3, -3)$, $B(7, 5)$, and $C(7, -3)$ $m\angle B \approx 26.6$

Application

17. **SURVEYING** Maureen is standing on horizontal ground level with the base of the CN Tower in Toronto, Ontario. The angle formed by the ground and the line segment from her position to the top of the tower is 31.2°. She knows that the height of the tower to the top of the antennae is about 1815 feet. Find her distance from the CN Tower to the nearest foot. 2997 ft

★ indicates increased difficulty

Practice and Apply

Homework Help	
For Exercises	See Examples
18–21, 28–36	1
22–27	2
43–48	3
37–42, 52–54	4

Extra Practice
See page 767.

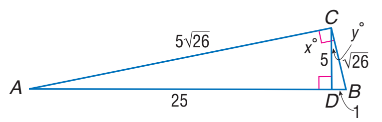
Use $\triangle PQR$ with right angle *R* to find sin *P*, cos *P*, tan *P*, sin *Q*, cos *Q*, and tan *Q*. Express each ratio as a fraction, and as a decimal to the nearest hundredth. 18–21. See margin.

18. $p = 12$, $q = 35$, and $r = 37$
19. $p = \sqrt{6}$, $q = 2\sqrt{3}$, and $r = 3\sqrt{2}$
20. $p = \frac{3}{2}$, $q = \frac{3\sqrt{3}}{2}$, and $r = 3$
21. $p = 2\sqrt{3}$, $q = \sqrt{15}$, and $r = 3\sqrt{3}$

Use a calculator to find each value. Round to the nearest ten-thousandth.

22. sin 6° 0.1045
23. tan 42.8° 0.9260
24. cos 77° 0.2250
25. sin 85.9° 0.9974
26. tan 12.7° 0.2254
27. cos 22.5° 0.9239

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



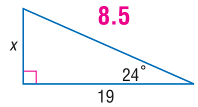
28. sin *A*
29. tan *B*
30. cos *A*
31. sin *x*°
32. cos *x*°
33. tan *A*
34. cos *B*
35. sin *y*°
36. tan *x*°

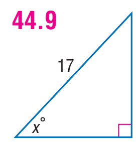
28. $\frac{\sqrt{26}}{26} \approx 0.1961$
29. $\frac{5}{1} = 5.0000$
30. $\frac{5\sqrt{26}}{26} \approx 0.9806$
31. $\frac{5\sqrt{26}}{26} \approx 0.9806$
32. $\frac{\sqrt{26}}{26} \approx 0.1961$
33. $\frac{1}{5} = 0.2000$
34. $\frac{\sqrt{26}}{26} \approx 0.1961$
35. $\frac{\sqrt{26}}{26} \approx 0.1961$
36. $\frac{5}{1} = 5.0000$

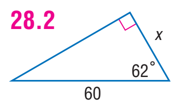
Find the measure of each angle to the nearest tenth of a degree.

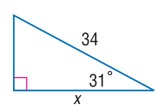
37. $\sin B = 0.7245$ 46.4
38. $\cos C = 0.2493$ 75.6
39. $\tan E = 9.4618$ 84.0
40. $\sin A = 0.4567$ 27.2
41. $\cos D = 0.1212$ 83.0
42. $\tan F = 0.4279$ 23.2

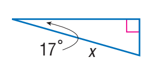
Find *x*. Round to the nearest tenth.

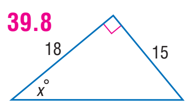
43. 8.5


44. 44.9


45. 28.2


46. 29.1


47. 22.6


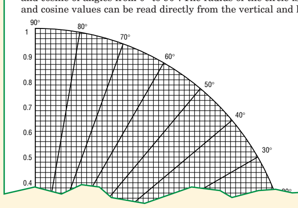
48. 39.8


David R. Frazier/Photo Researchers

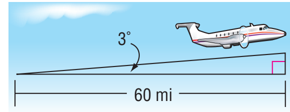
Enrichment, p. 374

Sine and Cosine of Angles

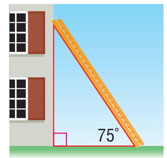
The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.



49. **AVIATION** A plane is one mile above sea level when it begins to climb at a constant angle of 3° for the next 60 ground miles. About how far above sea level is the plane after its climb? **4.1 mi**



SAFETY **For Exercises 50 and 51, use the following information.**
To guard against a fall, a ladder should make an angle of 75° or less with the ground.



50. What is the maximum height that a 20-foot ladder can reach safely? **19.32 ft**

51. How far from the building is the base of the ladder at the maximum height? **5.18 ft**

COORDINATE GEOMETRY **Find the measure of each angle to the nearest tenth in each right triangle.**

52. $\angle J$ in $\triangle JCL$ for $J(2, 2)$, $C(2, -2)$, and $L(7, -2)$ **about 51.3**

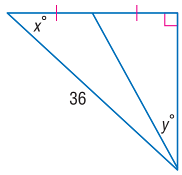
53. $\angle C$ in $\triangle BCD$ for $B(-1, -5)$, $C(-6, -5)$, and $D(-1, 2)$ **about 54.5**

54. $\angle X$ in $\triangle XYZ$ for $X(-5, 0)$, $Y(7, 0)$, and $Z(0, \sqrt{35})$ **49.8**

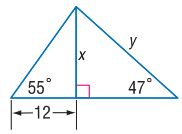
★55. Find the perimeter of $\triangle ABC$ if $m\angle A = 35$, $m\angle C = 90$, and $AB = 20$ inches. **about 47.9 in.**

Find x and y. Round to the nearest tenth.

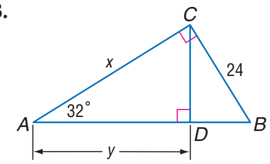
★56.



★57.

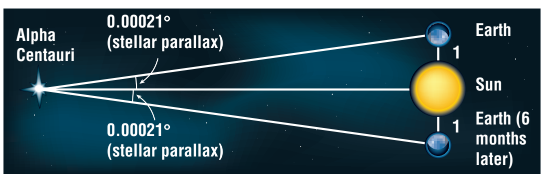


★58.



56. $x = 41.8$; $y = 29.2$
57. $x = 17.1$; $y = 23.4$
58. $x = 38.4$; $y = 32.6$
59. about 272,837 astronomical units
60. The stellar parallax would be too small.
62. See margin.

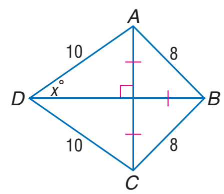
ASTRONOMY **For Exercises 59 and 60, use the following information.**
One way to find the distance between the sun and a relatively close star is to determine the angles of sight for the star exactly six months apart. Half the measure formed by these two angles of sight is called the *stellar parallax*. Distances in space are sometimes measured in *astronomical units*. An astronomical unit is equal to the average distance between Earth and the sun.



59. Find the distance between Alpha Centauri and the sun.

60. Make a conjecture as to why this method is used only for close stars.

61. **CRITICAL THINKING** Use the figure at the right to find sin $x°$. $\frac{2\sqrt{2}}{5}$



Exercise 61

62. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do surveyors determine angle measures?

Include the following in your answer:

- where theodolites are used, and
- the kind of information one obtains from a theodolite.

More About. . .

Astronomy

The stellar parallax is one of several methods of triangulation used to determine the distance of stars from the sun. Another method is trigonometric parallax, which measures the displacement of a nearby star relative to a more distant star.

Source: www.infoplease.com

Answer

62. Sample answer: Surveyors use a theodolite to measure angles to determine distances and heights. Answers should include the following.

- Theodolites are used in surveying, navigation, and meteorology. They are used to measure angles.
- The angle measures from two points, which are a fixed distance apart, to a third point.

www.geometryonline.com/self_check_quiz

Answers (p. 368)

18. $\frac{12}{37} \approx 0.32$; $\frac{35}{37} \approx 0.95$; $\frac{12}{35} \approx 0.34$; $\frac{35}{37} \approx 0.95$; $\frac{12}{37} \approx 0.32$; $\frac{35}{12} \approx 2.92$

19. $\frac{\sqrt{3}}{3} \approx 0.58$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{6}}{3} \approx 0.82$; $\frac{\sqrt{3}}{3} \approx 0.58$; $\sqrt{2} \approx 1.41$

20. $\frac{1}{2} = 0.5$; $\frac{\sqrt{3}}{2} \approx 0.87$; $\frac{\sqrt{3}}{3} \approx 0.58$; $\frac{\sqrt{3}}{2} \approx 0.87$; $\frac{1}{2} = 0.5$; $\sqrt{3} \approx 1.73$

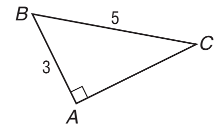
21. $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{3} \approx 0.75$; $\frac{2\sqrt{5}}{5} \approx 0.89$; $\frac{\sqrt{5}}{3} \approx 0.75$; $\frac{2}{3} \approx 0.67$; $\frac{\sqrt{5}}{2} \approx 1.12$

Standardized Test Practice

63. Find cos C. **C**

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$

(C) $\frac{4}{5}$ (D) $\frac{5}{4}$

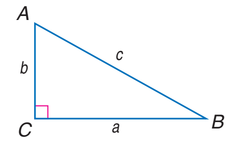


64. **ALGEBRA** If $x^2 = 15^2 + 24^2 - 15(24)$, find x. **B**

(A) 20.8 (B) 21 (C) 12 (D) 9

Extending the Lesson

Each of the basic trigonometric ratios has a reciprocal ratio. The reciprocals of the sine, cosine, and tangent are called the *cosecant*, *secant*, and the *cotangent*, respectively.



Reciprocal	Trigonometric Ratio	Abbreviation	Definition
$\frac{1}{\sin A}$	cosecant of ∠A	csc A	$\frac{\text{measure of the hypotenuse}}{\text{measure of the leg opposite } \angle A} = \frac{c}{a}$
$\frac{1}{\cos A}$	secant of ∠A	sec A	$\frac{\text{measure of the hypotenuse}}{\text{measure of the leg adjacent } \angle A} = \frac{c}{b}$
$\frac{1}{\tan A}$	cotangent of ∠A	cot A	$\frac{\text{measure of the leg adjacent } \angle A}{\text{measure of the leg opposite } \angle A} = \frac{b}{a}$

Use △*ABC* to find csc *A*, sec *A*, cot *A*, csc *B*, sec *B*, and cot *B*. Express each ratio as a fraction or as a radical in simplest form. **65–68. See margin.**

65. $a = 3, b = 4$, and $c = 5$

66. $a = 12, b = 5$, and $c = 13$

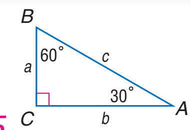
67. $a = 4, b = 4\sqrt{3}$, and $c = 8$

68. $a = 2\sqrt{2}, b = 2\sqrt{2}$, and $c = 4$

Maintain Your Skills

Mixed Review

Find each measure. *(Lesson 7-3)*



69. If $a = 4$, find b and c. $b = 4\sqrt{3}, c = 8$

70. If $b = 3$, find a and c. $a = \sqrt{3}, c = 2\sqrt{3}$

71. If $c = 5$, find a and b. $a = 2.5, b = 2.5\sqrt{3}$

Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. *(Lesson 7-2)*

72. 4, 5, 6 **no, no**

73. 5, 12, 13 **yes, yes**

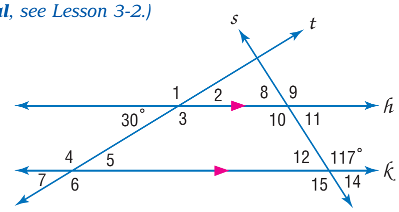
74. 9, 12, 15 **yes, yes**

75. 8, 12, 16 **no, no**

76. **TELEVISION** During a 30-minute television program, the ratio of minutes of commercials to minutes of the actual show is 4 : 11. How many minutes are spent on commercials? *(Lesson 6-1)* **8 min**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each angle measure if *h* ∥ *k*.** *(To review* ***angles formed by parallel lines and a transversal****, see Lesson 3-2.)*



77. $m\angle 15$ **117**

78. $m\angle 7$ **30**

79. $m\angle 3$ **150**

80. $m\angle 12$ **63**

81. $m\angle 11$ **63**

82. $m\angle 4$ **150**

4 Assess

Open-Ended Assessment

Speaking Draw and label the sides and angles of several right triangles on the board, and call on volunteers to name the sides they would use to find the sine, cosine, and tangent of the angles. Encourage students to use the terms *opposite*, *adjacent*, and *hypotenuse* when naming the sides.

Getting Ready for Lesson 7-5

Prerequisite Skill Students will learn about angles of elevation and depression in Lesson 7-5. They will use angle relationships and trigonometry to find distances and angles. Use Exercises 77–82 to determine your students' familiarity with determining angle relationships among angles formed by parallel lines and a transversal.

Assessment Options

Quiz (Lessons 7-3 and 7-4) is available on p. 407 of the *Chapter 7 Resource Masters*.

Mid-Chapter Test (Lessons 7-1 through 7-4) is available on p. 409 of the *Chapter 7 Resource Masters*.

Answers

65. $\csc A = \frac{5}{3}$; $\sec A = \frac{5}{4}$; $\cot A = \frac{4}{3}$; $\csc B = \frac{5}{4}$; $\sec B = \frac{5}{3}$; $\cot B = \frac{3}{4}$

66. $\csc A = \frac{13}{12}$; $\sec A = \frac{13}{5}$; $\cot A = \frac{5}{12}$; $\csc B = \frac{13}{5}$; $\sec B = \frac{13}{12}$; $\cot B = \frac{12}{5}$

67. $\csc A = 2$; $\sec A = \frac{2\sqrt{3}}{3}$; $\cot A = \sqrt{3}$; $\csc B = \frac{2\sqrt{3}}{3}$; $\sec B = 2$; $\cot B = \frac{\sqrt{3}}{3}$

68. $\csc A = \sqrt{2}$; $\sec A = \sqrt{2}$; $\cot A = 1$; $\csc B = \sqrt{2}$; $\sec B = \sqrt{2}$; $\cot B = 1$

7-5 Angles of Elevation and Depression

What You'll Learn

- Solve problems involving angles of elevation.
- Solve problems involving angles of depression.

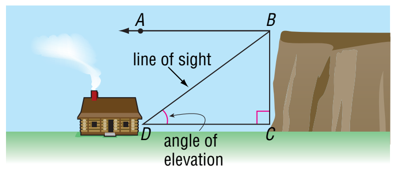
Vocabulary

- angle of elevation
- angle of depression

How do airline pilots use angles of elevation and depression?

A pilot is getting ready to take off from Mountain Valley airport. She looks up at the peak of a mountain immediately in front of her. The pilot must estimate the speed needed and the angle formed by a line along the runway and a line from the plane to the peak of the mountain to clear the mountain.

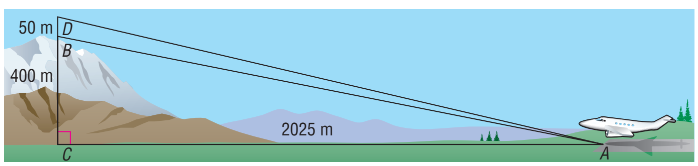
ANGLES OF ELEVATION An **angle of elevation** is the angle between the line of sight and the horizontal when an observer looks upward.



Example 1 Angle of Elevation

AVIATION **The peak of Goose Bay Mountain is 400 meters higher than the end of a local airstrip. The peak rises above a point 2025 meters from the end of the airstrip. A plane takes off from the end of the runway in the direction of the mountain at an angle that is kept constant until the peak has been cleared. If the pilot wants to clear the mountain by 50 meters, what should the angle of elevation be for the takeoff to the nearest tenth of a degree?**

Make a drawing.



Since CB is 400 meters and BD is 50 meters, CD is 450 meters. Let x represent $m\angle DAC$.

$\tan x° = \frac{CD}{AC}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$
$\tan x° = \frac{450}{2025}$	$CD = 450$, $AC = 2025$
$x = \tan^{-1}\left(\frac{450}{2025}\right)$	Solve for x.
$x \approx 12.5$	Use a calculator.

The angle of elevation for the takeoff should be more than 12.5°.

7-5 Lesson Notes

1 Focus

5-Minute Check Transparency 7-5 Use as a quiz or review of Lesson 7-4.

Mathematical Background notes are available for this lesson on p. 340D.

How do airline pilots use angles of elevation and depression?

Ask students:

- In the right triangle formed by the pilot's scenario, what triangle side does the height of the mountain represent? What does the line along the runway represent? **the side opposite the unknown angle, the side adjacent to the unknown angle**
- If the pilot is going to calculate the angle that she needs to clear the mountain given the height of the mountain and the horizontal distance from her takeoff point to the base of the mountain, what trigonometric ratio would she use? **tangent**

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 375–376
- Skills Practice, p. 377
- Practice, p. 378
- Reading to Learn Mathematics, p. 379
- Enrichment, p. 380

Teaching Geometry With Manipulatives Masters, p.18

Transparencies

5-Minute Check Transparency 7-5
Real-World Transparency 7
Answer Key Transparencies

Technology

GeomPASS: Tutorial Plus, Lesson 15
Interactive Chalkboard

2 Teach

ANGLE OF ELEVATION

In-Class Example Power Point®

1 **CIRCUS ACTS** At the circus, a person in the audience at ground level watches the high-wire routine. A 5-foot-6-inch tall acrobat is standing on a platform that is 25 feet off the ground. How far is the audience member from the base of the platform, if the angle of elevation from the audience member's line of sight to the top of the acrobat is 27°? **about 60 ft**

ANGLE OF DEPRESSION

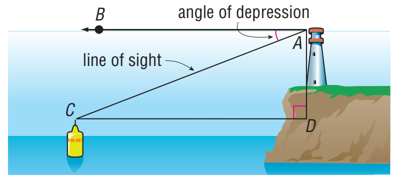
In-Class Examples Power Point®

Teaching Tip Remind students to draw upon their previous knowledge of angle relationships formed by two parallel lines and a transversal. Students may want to review these relationships.

2 A wheelchair ramp is 3 meters long and inclines at 6°. Find the height of the ramp to the nearest tenth centimeter. **about 31.4 cm**

3 Vernon is on the top deck of a cruise ship and observes two dolphins following each other directly away from the ship in a straight line. Vernon's position is 154 meters above sea level, and the angles of depression to the two dolphins are 35° and 36°. Find the distance between the two dolphins to the nearest meter. **about 8 m**

ANGLES OF DEPRESSION An **angle of depression** is the angle between the line of sight when an observer looks downward, and the horizontal.

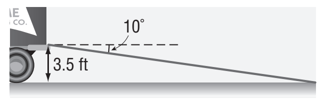


Standardized Test Practice

Example 2 Angle of Depression

Short-Response Test Item

The tailgate of a moving van is 3.5 feet above the ground. A loading ramp is attached to the rear of the van at an incline of 10°. Find the length of the ramp to the nearest tenth foot.

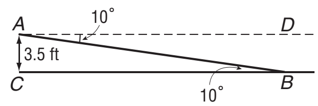


Read the Test Item

The angle of depression between the ramp and the horizontal is 10°. Use trigonometry to find the length of the ramp.

Solve the Test Item

Method 1



The ground and the horizontal level with the back of the van are parallel. Therefore, $m\angle DAB = m\angle ABC$ since they are alternate interior angles.

$$\sin 10° = \frac{3.5}{AB}$$

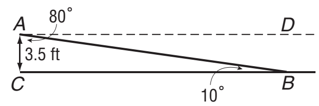
$$AB \sin 10° = 3.5$$

$$AB = \frac{3.5}{\sin 10°}$$

$$AB \approx 20.2$$

The ramp is about 20.2 feet long.

Method 2



The horizontal line from the back of the van and the segment from the ground to the back of the van are perpendicular. So, $\angle DAB$ and $\angle BAC$ are complementary angles. Therefore, $m\angle BAC = 90 - 10$ or 80.

$$\cos 80° = \frac{3.5}{AB}$$

$$AB \cos 80° = 3.5$$

$$AB = \frac{3.5}{\cos 80°}$$

$$AB \approx 20.2$$

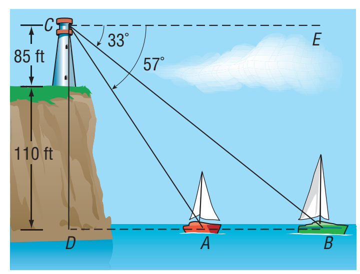
Angles of elevation or depression to two different objects can be used to find the distance between those objects.

Study Tip

Common Misconception The angle of depression is often not an angle of the triangle, but the complement to an angle of the triangle. In $\triangle DBC$, the angle of depression is $\angle BCE$, not $\angle DCB$.

Example 3 Indirect Measurement

Olivia is in a lighthouse on a cliff. She observes two sailboats due east of the lighthouse. The angles of depression to the two boats are 33° and 57°. Find the distance between the two sailboats to the nearest foot.



$\triangle CDA$ and $\triangle CDB$ are right triangles, and and $CD = 110 + 85$ or 195. The distance between the boats is AB or $BD - AD$. Use the right triangles to find these two lengths.

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Using a meterstick and a calculator, groups of students can find angles of elevation and depression for different objects in the classroom. Groups can measure one person's eye level from the floor, and the topmost height of a wall clock from the floor. The person stands 5 feet away from the clock, and the group calculates the angle of elevation from the person's line of sight to the top of the object. Repeat for items placed on the floor, and include variations like having the person stand on a chair, or placing two objects on the floor a certain distance from each other.

Because $\overline{CE}$ and $\overline{DB}$ are horizontal lines, they are parallel. Thus, $\angle ECB \cong \angle CBD$ and $\angle ECA \cong \angle CAD$ because they are alternate interior angles. This means that $m\angle CBD = 33$ and $m\angle CAD = 57$.

$\tan 33° = \frac{195}{DB}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CBD = 33$

$DB \tan 33° = 195$ — Multiply each side by DB.

$DB = \frac{195}{\tan 33°}$ — Divide each side by tan 33°.

$DB \approx 300.27$ — Use a calculator.

$\tan 57° = \frac{195}{DA}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CAD = 57$

$DA \tan 57° = 195$ — Multiply each side by DA.

$DA = \frac{195}{\tan 57°}$ — Divide each side by tan 57°.

$DA \approx 126.63$ — Use a calculator.

The distance between the boats is $DB - DA$.

$DB - DA \approx 300.27 - 126.63$ or about 174 feet.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

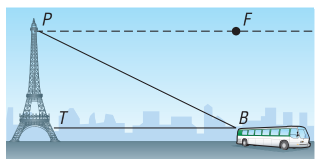
Check for Understanding

Concept Check

1. **OPEN ENDED** Find a real-life example of an angle of depression. Draw a diagram and identify the angle of depression. **1–2. See margin.**

2. **Explain** why an angle of elevation is given that name.

3. **Name** the angles of depression and elevation in the figure. **The angle of depression is $\angle FPB$ and the angle of elevation is $\angle TBP$.**



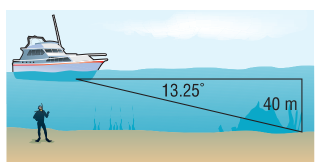
Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6	2
7	3

4. **AVIATION** A pilot is flying at 10,000 feet and wants to take the plane up to 20,000 feet over the next 50 miles. What should be his angle of elevation to the nearest tenth? (*Hint:* There are 5280 feet in a mile.) **about 2.2°**

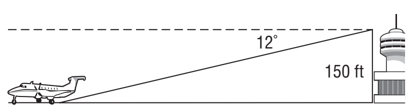
5. **SHADOWS** Find the angle of elevation of the sun when a 7.6-meter flagpole casts a 18.2-meter shadow. Round to the nearest tenth of a degree. **22.7°**

6. **SALVAGE** A salvage ship uses sonar to determine that the angle of depression to a wreck on the ocean floor is 13.25°. The depth chart shows that the ocean floor is 40 meters below the surface. How far must a diver lowered from the salvage ship walk along the ocean floor to reach the wreck? **about 169.9 m**



Standardized Test Practice

7. **SHORT RESPONSE** From the top of a 150-foot high tower, an air traffic controller observes an airplane on the runway. To the nearest foot, how far from the base of the tower is the airplane? **706 ft**



www.geometryonline.com/extra_examples

About the Exercises...

Organization by Objective
- **Angles of Elevation:** 12, 14–18
- **Angles of Depression:** 8–11, 13, 19–20

Odd/Even Assignments
Exercises 8–20 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 23 requires the Internet or other research materials.

Assignment Guide

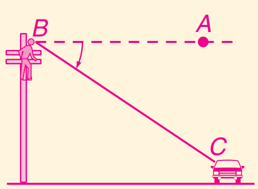
Basic: 9–23 odd, 26–48

Average: 9–25 odd, 26–48

Advanced: 8–26 even, 27–40 (optional: 41–48)

Answers

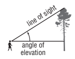
1. Sample answer: $\angle ABC$



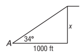
2. Sample answer: An angle of elevation is called that because the angle formed by a horizontal line and a segment joining two endpoints rises above the horizontal line.

Study Guide and Intervention, p. 375 (shown) and p. 376

Angles of Elevation Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer's line of sight and a horizontal line.



Example **The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?**



Let x = the height of the cliff.

$\tan 34° = \frac{x}{1000}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$1000(\tan 34°) = x$ — Multiply each side by 1000.

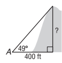
$674.5 = x$ — Use a calculator.

The height of the cliff is about 674.5 feet.

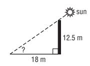
Exercises

Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.

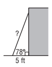
1. The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill? **460 ft**



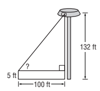
2. Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts a 18-meter-long shadow. **35°**



3. A ladder leaning against a building makes an angle of 78° with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder? **24 ft**



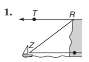
4. A person whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. That person observes an air traffic controller at the window of the 132-foot tower. What is the angle of elevation? **52°**



Skills Practice, p. 377 and Practice, p. 378 (shown)

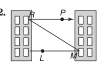
Name the angle of depression or angle of elevation in each figure.

1.



∠TRZ; ∠YZR

2.

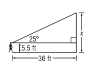


∠PRM; ∠LMR

3. **WATER TOWERS** A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree. **about 13.5°**

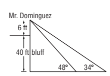
4. **CONSTRUCTION** A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot. **about 21 ft**

5. **TOWN ORDINANCES** The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot? **about 22.3 ft**



6. **GEOGRAPHY** Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan's eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter. **about 2850 m**

7. **INDIRECT MEASUREMENT** Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot? **about 27 ft**



Reading to Learn Mathematics, p. 379 — ELL

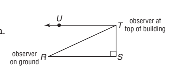
Pre-Activity **How do airline pilots use angles of elevation and depression?**

Read the introduction to Lesson 7-5 at the top of page 371 in your textbook.

What does the angle measure tell the pilot? **Sample answer: how steep her ascent must be to clear the peak**

Reading the Lesson

1. Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.



   a. What is the line of sight? **iii**
      (i) line RS (ii) line ST (iii) line RT (iv) line TU
   b. What is the angle of elevation? **ii**
      (i) ∠RST (ii) ∠SRT (iii) ∠RTS (iv) ∠UTR
   c. What is the angle of depression? **iv**
      (i) ∠RST (ii) ∠SRT (iii) ∠RTS (iv) ∠UTR
   d. How are the angle of elevation and the angle of depression related? **ii**
      (i) They are complementary.
      (ii) They are congruent.
      (iii) They are supplementary.
      (iv) The angle of elevation is larger than the angle of depression.
   e. Which postulate or theorem that you learned in Chapter 3 supports your answer for part c? **iv**
      (i) Corresponding Angles Postulate
      (ii) Alternate Exterior Angles Theorem
      (iii) Consecutive Interior Angles Theorem
      (iv) Alternate Interior Angles Theorem

2. A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement? **An angle of elevation cannot be obtuse.**

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time? **Sample answers: (1) The angle of depression and the angle of elevation are both measured between the horizontal and the line of sight. (2) The angle of depression is always congruent to the angle of elevation in the same diagram. (3) Associate the word *elevation* with the word *up* and the word *depression* with the word *down*.**

Practice and Apply

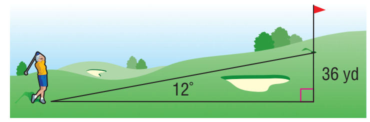
Homework Help

For Exercises	See Examples
12, 14–18	1
9–11	2
8, 13, 19, 20	3

Extra Practice
See page 768.

8. **BOATING** Two boats are observed by a parasailer 75 meters above a lake. The angles of depression are 12.5° and 7°. How far apart are the boats? **about 273 m**

9. **GOLF** A golfer is standing at the tee, looking up to the green on a hill. If the tee is 36 yards lower than the green and the angle of elevation from the tee to the hole is 12°, find the distance from the tee to the hole. **about 173.2 yd**

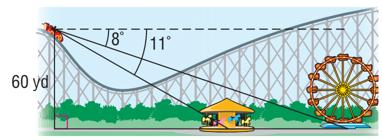


10. **AVIATION** After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight? **about 2.6°**

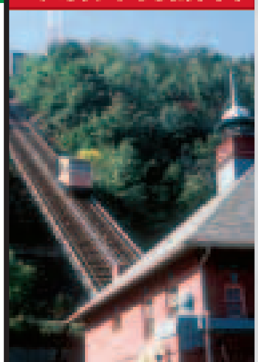
11. **SLEDDING** A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression of the run. **about 5.3°**

12. **RAILROADS** The Monongahela Incline overlooks the city of Pittsburgh, Pennsylvania. Refer to the information at the left to determine the incline of the railway. **about 35.6°**

13. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8° respectively, how far apart are the merry-go-round and the Ferris wheel? **about 118.2 yd**



More About. . .



Railroads

The Monongahela Incline is 635 feet long with a vertical rise of 369.39 feet. It was built at a cost of $50,000 and opened on May 28, 1870. It is still used by commuters to and from Pittsburgh.

Source: www.portauthority.org

CIVIL ENGINEERING **For Exercises 14 and 15, use the following information.**
The percent grade of a highway is the ratio of the vertical rise or fall over a given horizontal distance. The ratio is expressed as a percent to the nearest whole number. Suppose a highway has a vertical rise of 140 feet for every 2000 feet of horizontal distance.

14. Calculate the percent grade of the highway. **7%**
15. Find the angle of elevation that the highway makes with the horizontal. **about 4°**

16. **SKIING** A ski run has an angle of elevation of 24.4° and a vertical drop of 1100 feet. To the nearest foot, how long is the ski run? **2663 ft**

GEYSERS **For Exercises 17 and 18, use the following information.**
Kirk visits Yellowstone Park and Old Faithful on a perfect day. His eyes are 6 feet from the ground, and the geyser can reach heights ranging from 90 feet to 184 feet.

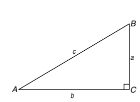
17. If Kirk stands 200 feet from the geyser and the eruption rises 175 feet in the air, what is the angle of elevation to the top of the spray to the nearest tenth? **about 40.2°**
18. In the afternoon, Kirk returns and observes the geyser's spray reach a height of 123 feet when the angle of elevation is 37°. How far from the geyser is Kirk standing to the nearest tenth of a foot? **about 155.3 ft**

★ 19. **BIRDWATCHING** Two observers are 200 feet apart, in line with a tree containing a bird's nest. The angles of elevation to the bird's nest are 30° and 60°. How far is each observer from the base of the tree? **100 ft, 300 ft**

Enrichment, p. 380

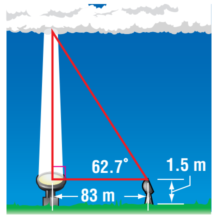
Reading Mathematics

The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**. Three other ratios are the **cosecant**, **secant**, and **cotangent**. The chart below shows abbreviations and definitions for all six ratios. Refer to the triangle at the right.



Abbreviation	Read as:	Ratio
sin A	the sine of ∠A	$\frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$
cos A	the cosine of ∠A	$\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$
tan A	the tangent of ∠A	$\frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{a}{b}$
csc A	the cosecant of ∠A	$\frac{\text{hypotenuse}}{\text{leg opposite } \angle A} = \frac{c}{a}$
sec A	the secant of ∠A	$\frac{\text{hypotenuse}}{\text{leg adjacent to } \angle A} = \frac{c}{b}$
	...ngent of ∠A	...∠A

20. **METEOROLOGY** The altitude of the base of a cloud formation is called the *ceiling*. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling? **about 162.3 m**



MEDICINE **For Exercises 21–23, use the following information.**
A doctor is using a treadmill to assess the strength of a patient's heart. At the beginning of the exam, the 48-inch long treadmill is set at an incline of 10°.

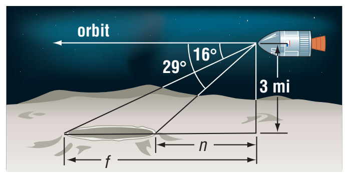
21. How far off the horizontal is the raised end of the treadmill at the beginning of the exam? **about 8.3 in.**

22. During one stage of the exam, the end of the treadmill is 10 inches above the horizontal. What is the incline of the treadmill to the nearest degree? **12°**

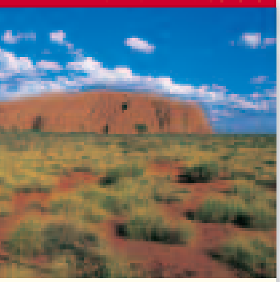
23. Suppose the exam is divided into five stages and the incline of the treadmill is increased 2° for each stage. Does the end of the treadmill rise the same distance between each stage? **no**

★ 24. **TRAVEL** Kwan-Yong uses a theodolite to measure the angle of elevation from the ground to the top of Ayers Rock to be 15.85°. He walks half a kilometer closer and measures the angle of elevation to be 25.6°. How high is Ayers Rock to the nearest meter? **about 348 m**

★ 25. **AEROSPACE** On July 20, 1969, Neil Armstrong became the first human to walk on the moon. During this mission, *Apollo 11* orbited the moon three miles above the surface. At one point in the orbit, the onboard guidance system measured the angles of depression to the far and near edges of a large crater. The angles measured 16° and 29°, respectively. Find the distance across the crater. **about 5.1 mi**



More About. . .



Travel

Ayers Rock is the largest monolith, a type of rock formation, in the world. It is approximately 3.6 kilometers long and 2 kilometers wide. The rock is believed to be the tip of a mountain, two thirds of which is underground.

Source: www.atn.com.au

Online Research **Data Update** Use the Internet to determine the angle of depression formed by someone aboard the international space station looking down to your community. Visit www.geometryonline.com/data_update to learn more.

26. **CRITICAL THINKING** Two weather observation stations are 7 miles apart. A weather balloon is located between the stations. From Station 1, the angle of elevation to the weather balloon is 33°. From Station 2, the angle of elevation to the balloon is 52°. Find the altitude of the balloon to the nearest tenth of a mile. **about 3.0 mi**

27. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How do airline pilots use angles of elevation and depression?

Include the following in your answer:
- when pilots use angles of elevation or depression, and
- the difference between angles of elevation and depression.

Answer

27. Answers should include the following.
- **Pilots use angles of elevation when they are ascending and angles of depression when descending.**
- **Angles of elevation are formed when a person looks upward and angles of depression are formed when a person looks downward.**

www.geometryonline.com/self_check_quiz

Teacher to Teacher

Susan M. Parece — Plymouth South High School, Plymouth, MA

I like to make a clinometer (also called a hypsometer) and have my students try to measure objects in the schoolyard, such as height of the building, height of the flagpole, and height of the lights. (Directions can be found at www.globe.org.uk/activities/toolkit/tollkit.pdf.)

4 Assess

Open-Ended Assessment

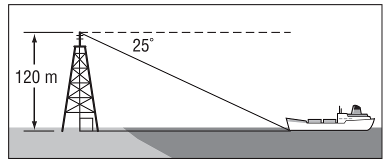
Modeling Using a ruler, have students model different angles of elevation and depression and name real-world situations that the angle depicted could be modeling.

Getting Ready for Lesson 7-6

Prerequisite Skill Students will learn about the Law of Sines in Lesson 7-6. They will use proportions to solve for side lengths and angle measures. Use Exercises 41–48 to determine your students' familiarity with solving proportions.

Standardized Test Practice

28. The top of a signal tower is 120 meters above sea level. The angle of depression from the top of tower to a passing ship is 25°. How many meters from the foot of the tower is the ship? **B**



Ⓐ 283.9 m Ⓑ 257.3 m
Ⓒ 132.4 m Ⓓ 56 m

29. **ALGEBRA** If $\frac{y}{28} = \frac{x}{16}$, then find x when $y = \frac{1}{2}$. **A**

Ⓐ $\frac{2}{7}$ Ⓑ $\frac{4}{7}$ Ⓒ $\frac{7}{4}$ Ⓓ $3\frac{1}{2}$

Maintain Your Skills

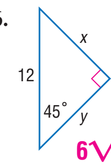
Mixed Review

Find the measure of each angle to the nearest tenth of a degree. *(Lesson 7-4)*

30. $\cos A = 0.6717$ **47.8**
31. $\sin B = 0.5127$ **30.8**
32. $\tan C = 2.1758$ **65.3**
33. $\cos D = 0.3421$ **70.0**
34. $\sin E = 0.1455$ **8.4**
35. $\tan F = 0.3541$ **19.5**

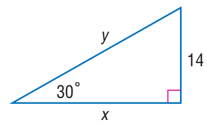
Find x and y. *(Lesson 7-3)*

36.



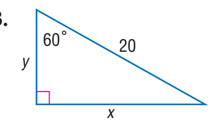
$6\sqrt{2}$; $6\sqrt{2}$

37.



$14\sqrt{3}$; 28

38.



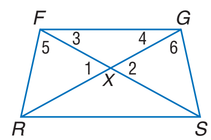
$10\sqrt{3}$; 10

39. **HOBBIES** A twin-engine airplane used for medium-range flights has a length of 78 meters and a wingspan of 90 meters. If a scale model is made with a wingspan of 36 centimeters, find its length. *(Lesson 6-2)* **31.2 cm**

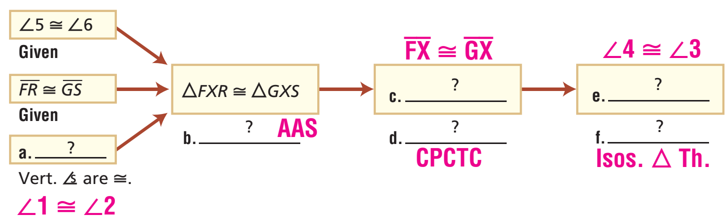
40. Copy and complete the flow proof. *(Lesson 4-6)*

Given: $\angle 5 \cong \angle 6$
$\overline{FR} \cong \overline{GS}$

Prove: $\angle 4 \cong \angle 3$



Proof:



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each proportion.**
*(To review **solving proportions**, see Lesson 6-1.)*

41. $\frac{x}{6} = \frac{35}{42}$ **5**
42. $\frac{3}{x} = \frac{5}{45}$ **27**
43. $\frac{12}{17} = \frac{24}{x}$ **34**
44. $\frac{24}{36} = \frac{x}{15}$ **10**
45. $\frac{12}{13} = \frac{48}{x}$ **52**
46. $\frac{x}{18} = \frac{5}{8}$ **11.25**
47. $\frac{28}{15} = \frac{7}{x}$ **3.75**
48. $\frac{x}{40} = \frac{3}{26}$ **$\frac{60}{13}$**

376 Chapter 7 Right Triangles and Trigonometry

7-6 The Law of Sines

What You'll Learn

- Use the Law of Sines to solve triangles.
- Solve problems by using the Law of Sines.

How are triangles used in radio astronomy?

The Very Large Array (VLA), one of the world's premier astronomical radio observatories, consists of 27 radio antennas in a Y-shaped configuration on the Plains of San Agustin in New Mexico. Astronomers use the VLA to make pictures from the radio waves emitted by astronomical objects. Construction of the antennas is supported by a variety of triangles, many of which are not right triangles.

Vocabulary

- Law of Sines
- solving a triangle

THE LAW OF SINES In trigonometry, the **Law of Sines** can be used to find missing parts of triangles that are not right triangles.

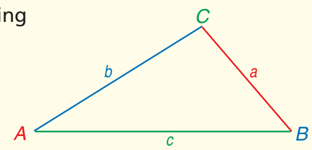
Study Tip

Obtuse Angles
There are also values for sin *A*, cos *A*, and tan *A*, when $A \geq 90°$. Values of the ratios for these angles will be found using the trigonometric functions on your calculator.

Key Concept — Law of Sines

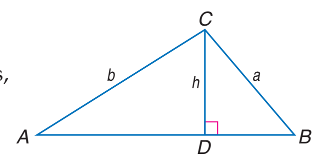
Let $\triangle ABC$ be any triangle with *a*, *b*, and *c* representing the measures of the sides opposite the angles with measures *A*, *B*, and *C*, respectively. Then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



Proof — Law of Sines

$\triangle ABC$ is a triangle with an altitude from *C* that intersects $\overline{AB}$ at *D*. Let *h* represent the measure of $\overline{CD}$. Since $\triangle ADC$ and $\triangle BDC$ are right triangles, we can find sin *A* and sin *B*.



$\sin A = \frac{h}{b}$	$\sin B = \frac{h}{a}$	Definition of sine
$b \sin A = h$	$a \sin B = h$	Cross products
$b \sin A = a \sin B$		Substitution
$\frac{\sin A}{a} = \frac{\sin B}{b}$		Divide each side by *ab*.

The proof can be completed by using a similar technique with the other altitudes to show that $\frac{\sin A}{a} = \frac{\sin C}{c}$ and $\frac{\sin B}{b} = \frac{\sin C}{c}$.

Roger Ressmeyer/CORBIS

7-6 Lesson Notes

1 Focus

5-Minute Check Transparency 7-6 Use as a quiz or review of Lesson 7-5.

Mathematical Background notes are available for this lesson on p. 340D.

How are triangles used in radio astronomy?

Ask students:

- Why is it called radio astronomy? **because they are analyzing radio signals**
- What are some examples of astronomical objects? **Sample answers: stars, planets, quasars, brown dwarfs, galaxies**

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters
- Study Guide and Intervention, pp. 381–382
- Skills Practice, p. 383
- Practice, p. 384
- Reading to Learn Mathematics, p. 385
- Enrichment, p. 386
- Assessment, p. 408

Graphing Calculator and Computer Masters, p. 30

Transparencies

5-Minute Check Transparency 7-6
Answer Key Transparencies

Technology

Interactive Chalkboard
Multimedia Applications: Virtual Activities

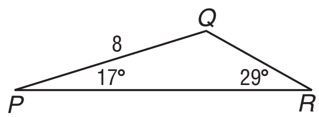
2 Teach

THE LAW OF SINES

In-Class Examples

Power Point®

1 **a.** Find p. Round to the nearest tenth.



4.8

b. Find $m\angle L$ to the nearest degree in $\triangle LMN$ if $n = 7$, $\ell = 9$, and $m\angle N = 43$.
61

Teaching Tip To save a step, students can also write $a \sin B = b \sin A$ and then substitute values to solve.

2 **a.** Solve $\triangle DEF$ if $m\angle D = 112$, $m\angle F = 8$, and $f = 2$.
$d \approx 13.2$; $e \approx 12.4$; $m\angle E = 60$

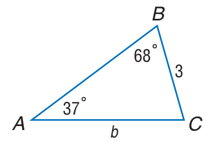
b. Solve $\triangle IJK$ if $m\angle J = 32$, $i = 30$, and $j = 16$.
$k \approx 27.3$; $m\angle I \approx 83.5$; $m\angle K \approx 64.5$

Study Tip

Rounding
If you round before the final answer, your results may differ from results in which rounding was not done until the final answer.

Example 1 Use the Law of Sines

a. Find b. Round to the nearest tenth.



Use the Law of Sines to write a proportion.

$\frac{\sin A}{a} = \frac{\sin B}{b}$	Law of Sines
$\frac{\sin 37°}{3} = \frac{\sin 68°}{b}$	$m\angle A = 37$, $a = 3$, $m\angle B = 68$
$b \sin 37° = 3 \sin 68°$	Cross products
$b = \frac{3 \sin 68°}{\sin 37°}$	Divide each side by sin 37°.
$b \approx 4.6$	Use a calculator.

b. Find $m\angle Z$ to the nearest degree in ΔXYZ if $y = 17$, $z = 14$, and $m\angle Y = 92$.

Write a proportion relating $\angle Y$, $\angle Z$, y, and z.

$\frac{\sin Y}{y} = \frac{\sin Z}{z}$	Law of Sines
$\frac{\sin 92°}{17} = \frac{\sin Z}{14}$	$m\angle Y = 92$, $y = 17$, $z = 14$
$14 \sin 92° = 17 \sin Z$	Cross products
$\frac{14 \sin 92°}{17} = \sin Z$	Divide each side by 17.
$\sin^{-1}\left(\frac{14 \sin 92°}{17}\right) = Z$	Solve for Z.
$55° \approx Z$	Use a calculator.

So, $m\angle Z \approx 55$.

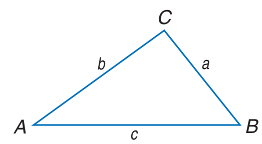
The Law of Sines can be used to solve a triangle. **Solving a triangle** means finding the measures of all of the angles and sides of a triangle.

Study Tip

Look Back
To review the **Angle Sum Theorem**, see Lesson 4-2.

Example 2 Solve Triangles

a. Solve ΔABC if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.



We know the measures of two angles of the triangle. Use the Angle Sum Theorem to find $m\angle C$.

$m\angle A + m\angle B + m\angle C = 180$	Angle Sum Theorem
$33 + 47 + m\angle C = 180$	$m\angle A = 33$, $m\angle B = 47$
$80 + m\angle C = 180$	Add.
$m\angle C = 100$	Subtract 80 from each side.

Since we know $m\angle B$ and b, use proportions involving $\frac{\sin B}{b}$.

To find a:		To find c:
$\frac{\sin B}{b} = \frac{\sin A}{a}$	Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 47°}{14} = \frac{\sin 33°}{a}$	Substitute.	$\frac{\sin 47°}{14} = \frac{\sin 100°}{c}$
$a \sin 47° = 14 \sin 33°$	Cross products	$c \sin 47° = 14 \sin 100°$
$a = \frac{14 \sin 33°}{\sin 47°}$	Divide each side by sin 47°.	$c = \frac{14 \sin 100°}{\sin 47°}$
$a \approx 10.4$	Use a calculator.	$c \approx 18.9$

Therefore, $m\angle C = 100$, $a \approx 10.4$, and $c \approx 18.9$.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Separate the class into groups of three students to work through the exercises. One group member can select the exercise. The second member sets up the equation for the Law of Sines and fills in the values. The third person uses a calculator to solve the problem. Group members can rotate tasks so each member participates in each responsibility.

378 Chapter 7 Right Triangles and Trigonometry

Study Tip

An Equivalent Proportion

The Law of Sines may also be written as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. You may wish to use this form when finding the length of a side.

b. Solve $\triangle ABC$ if $m\angle C = 98$, $b = 14$, and $c = 20$. Round angle measures to the nearest degree and side measures to the nearest tenth.

We know the measures of two sides and an angle opposite one of the sides.

$\frac{\sin B}{b} = \frac{\sin C}{c}$	Law of Sines
$\frac{\sin B}{14} = \frac{\sin 98°}{20}$	$m\angle C = 98$, $b = 14$, and $c = 20$
$20 \sin B = 14 \sin 98°$	Cross products
$\sin B = \frac{14 \sin 98°}{20}$	Divide each side by 20.
$B = \sin^{-1}\left(\frac{14 \sin 98°}{20}\right)$	Solve for B.
$B \approx 44°$	Use a calculator.
$m\angle A + m\angle B + m\angle C = 180$	Angle Sum Theorem
$m\angle A + 44 + 98 = 180$	$m\angle B = 44$ and $m\angle C = 98$
$m\angle A + 142 = 180$	Add.
$m\angle A = 38$	Subtract 142 from each side.
$\frac{\sin A}{a} = \frac{\sin C}{c}$	Law of Sines
$\frac{\sin 38°}{a} = \frac{\sin 98°}{20}$	$m\angle A = 38$, $m\angle C = 98$, and $c = 20$
$20 \sin 38° = a \sin 98°$	Cross products
$\frac{20 \sin 38°}{\sin 98°} = a$	Divide each side by sin 98°.
$12.4 \approx a$	Use a calculator.

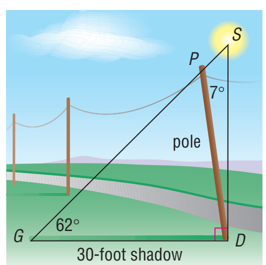
Therefore, $A \approx 38°$, $B \approx 44°$, and $a \approx 12.4$.

USE THE LAW OF SINES TO SOLVE PROBLEMS The Law of Sines is very useful in solving direct and indirect measurement applications.

Example 3 Indirect Measurement

When the angle of elevation to the sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.

Draw a diagram.



Draw $\overline{SD} \perp \overline{GD}$. Then find the $m\angle GDP$ and $m\angle GPD$.

$m\angle GDP = 90 - 7$ or 83

$m\angle GPD + 62 + 83 = 180$ or $m\angle GPD = 35$

Since you know the measures of two angles of the triangle, $m\angle GDP$ and $m\angle GPD$, and the length of a side opposite one of the angles ($\overline{GD}$ is opposite $\angle GPD$) you can use the Law of Sines to find the length of the pole.

(continued on the next page)

www.geometryonline.com/extra_examples

USE THE LAW OF SINES TO SOLVE PROBLEMS

In-Class Examples Power Point®

3 In Example 3, what happens to the length of the shadow later in the day when the angle of elevation decreases to 33°? Assume that the length and position of the telephone pole are still the same. **The length of the shadow increases to about 76.2 feet.**

Lesson 7-6 The Law of Sines 379

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include one completely worked example similar to Example 2 on pp. 378–379.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Explain that if $\angle F$ in the figure had been 90°, then Makayla and Felipe would have found the same value for d. Students should recognize that they *could* use the Law of Sines to solve right triangles, but trigonometric ratios are more efficient.

About the Exercises...

Organization by Objective
- **The Law of Sines:** 16–29
- **Use the Law of Sines to Solve Problems:** 30–38

Odd/Even Assignments
Exercises 16–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–39 odd, 40–58
Average: 17–39 odd, 40–58
Advanced: 16–40 even, 41–52 (optional: 53–58)
All: Quiz 2 (1–5)

$\frac{PD}{\sin \angle DGP} = \frac{GD}{\sin \angle GPD}$ Law of Sines

$\frac{PD}{\sin 62°} = \frac{30}{\sin 35°}$ $m\angle DGP = 62$, $m\angle GPD = 35$, and $GD = 30$

$PD \sin 35° = 30 \sin 62°$ Cross products

$PD = \frac{30 \sin 62°}{\sin 35°}$ Divide each side by sin 35°.

$PD \approx 46.2$ Use a calculator.

The telephone pole is about 46.2 feet long.

Concept Summary — *Law of Sines*

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

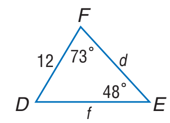
Check for Understanding

Concept Check

1. Felipe; Makayla is using the definition of the sine ratio for a right triangle, but this is not a right triangle.

1. **FIND THE ERROR** Makayla and Felipe are trying to find d in $\triangle DEF$.

Makayla	Felipe
$\sin 59° = \frac{d}{12}$	$\frac{\sin 59°}{d} = \frac{\sin 48°}{12}$



Who is correct? Explain your reasoning.

2. **OPEN ENDED** Draw an acute triangle and label the measures of two angles and the length of one side. Explain how to solve the triangle.

3. **Compare** the two cases for the Law of Sines. **2–3. See margin.**

Guided Practice

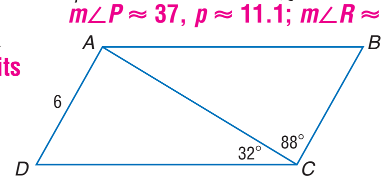
GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1
8–13	2
14	3

Find each measure using the given measures of $\triangle XYZ$. Round angle measures to the nearest degree and side measures to the nearest tenth.

4. Find y if $x = 3$, $m\angle X = 37$, and $m\angle Y = 68$. **4.6**
5. Find x if $y = 12.1$, $m\angle X = 57$, and $m\angle Z = 72$. **13.1**
6. Find $m\angle Y$ if $y = 7$, $z = 11$, and $m\angle Z = 37$. **23**
7. Find $m\angle Z$ if $y = 17$, $z = 14$, and $m\angle Y = 92$. **55**

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

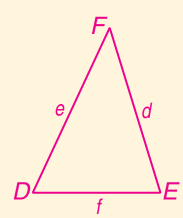
8. $m\angle R = 66$, $m\angle Q = 59$, $p = 72$
9. $p = 32$, $r = 11$, $m\angle P = 105$
10. $m\angle P = 33$, $m\angle R = 58$, $q = 22$
11. $p = 28$, $q = 22$, $m\angle P = 120$
12. $m\angle P = 50$, $m\angle Q = 65$, $p = 12$ **$m\angle R = 65$, $q \approx 14.2$, $r \approx 14.2$**
13. $q = 17.2$, $r = 9.8$, $m\angle Q = 110.7$ **$m\angle P \approx 37$, $p \approx 11.1$; $m\angle R \approx 32$**
14. Find the perimeter of parallelogram $ABCD$ to the nearest tenth. **34.6 units**



8. $m\angle P = 55$, $q \approx 75.3$, $r \approx 80.3$
9. $m\angle R \approx 19$, $m\angle Q \approx 56$, $q \approx 27.5$
10. $m\angle Q = 89$, $p \approx 12.0$, $r \approx 18.7$
11. $m\angle Q \approx 43$, $m\angle R \approx 17$, $r \approx 9.5$

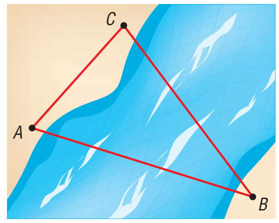
Answer

2. Sample answer: Let $m\angle D = 65$, $m\angle E = 73$, and $d = 15$. Then $\frac{\sin 65°}{15}$ is the fixed ratio or scale factor for the Law of Sines extended proportion. The length of e is found by using $\frac{\sin 65°}{15} = \frac{\sin 73°}{e}$. The $m\angle F$ is found by evaluating $180 - (m\angle D + m\angle E)$. In this problem $m\angle F = 42$. The length of f is found by using $\frac{\sin 65°}{15} = \frac{\sin 42°}{f}$.



Application

15. **SURVEYING** To find the distance between two points A and B that are on opposite sides of a river, a surveyor measures the distance to point C on the same side of the river as point A. The distance from A to C is 240 feet. He then measures the angle from A to B as 62° and measures the angle from C to B as 55°. Find the distance from A to B. **about 237.8 ft**



Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–29	2
30–38	3

Extra Practice
See page 768.

Find each measure using the given measures of $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.

16. If $m\angle L = 45$, $m\angle K = 63$, and $\ell = 22$, find k. **27.7**
17. If $k = 3.2$, $m\angle L = 52$, and $m\angle K = 70$, find ℓ. **2.7**
18. If $m = 10.5$, $k = 18.2$, and $m\angle K = 73$, find $m\angle M$. **33**
19. If $k = 10$, $m = 4.8$, and $m\angle K = 96$, find $m\angle M$. **29**
20. If $m\angle L = 88$, $m\angle K = 31$, and $m = 5.4$, find ℓ. **6.2**
21. If $m\angle M = 59$, $\ell = 8.3$, and $m = 14.8$, find $m\angle L$. **29**

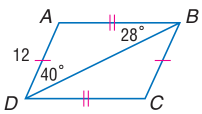
Solve each $\triangle WXY$ described below. Round measures to the nearest tenth.

22. $m\angle Y = 71, y = 7.4, m\angle X = 41$
23. $x = 10.3, y = 23.7, m\angle Y = 96$
24. $m\angle X = 25, m\angle W = 52, y = 15.6$
25. $m\angle Y = 112, x = 20, y = 56$
26. $m\angle W = 38, m\angle Y = 115, w = 8.5$
27. $m\angle W = 36, m\angle Y = 62, w = 3.1$
28. $w = 30, y = 9.5, m\angle W = 107$
29. $x = 16, w = 21, m\angle W = 88$

22. $m\angle W = 68$, $w \approx 7.3$, $x \approx 5.1$
23. $m\angle X \approx 25.6$, $m\angle W \approx 58.4$, $w \approx 20.3$
24. $m\angle Y = 103$, $w \approx 12.6$, $x \approx 6.8$
25. $m\angle X \approx 19.3$, $m\angle W \approx 48.7$, $w \approx 45.4$
26. $m\angle X = 27$, $x \approx 6.3$, $y \approx 12.5$
27. $m\angle X = 82$, $x \approx 5.2$, $y \approx 4.7$
28. $m\angle Y \approx 17.6$, $m\angle X \approx 55.4$, $x \approx 25.8$
29. $m\angle X \approx 49.6$, $m\angle Y \approx 42.4$, $y \approx 14.2$

30. An isosceles triangle has a base of 46 centimeters and a vertex angle of 44°. Find the perimeter. **about 168.8 cm**

31. Find the perimeter of quadrilateral $ABCD$ to the nearest tenth. **56.9 units**



32. **GARDENING** Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44°, find the length of fence needed to enclose the garden. **about 77.3 ft**

33. **AVIATION** Two radar stations that are 20 miles apart located a plane at the same time. The first station indicated that the position of the plane made an angle of 43° with the line between the stations. The second station indicated that it made an angle of 48° with the same line. How far is each station from the plane? **about 14.9 mi, about 13.6 mi**

34. **SURVEYING** Maria Lopez is a surveyor who must determine the distance across a section of the Rio Grande Gorge in New Mexico. Standing on one side of the ridge, she measures the angle formed by the edge of the ridge and the line of sight to a tree on the other side of the ridge. She then walks along the ridge 315 feet, passing the tree and measures the angle formed by the edge of the ridge and the new line of sight to the same tree. If the first angle is 80° and the second angle is 85°, find the distance across the gorge. **about 1194 ft**

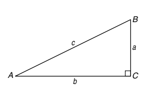
Answers (page 380)

3. In one case you need the measures of two sides and the measure of an angle opposite one of the sides. In the other case you need the measures of two angles and the measure of a side.

Enrichment, p. 386

Identities

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.



Example 1 **Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.**

$$(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2 + b^2}{c} = \frac{c^2}{c^2} = 1$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

Example 2 **Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.**

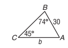
...lculator to...

Study Guide and Intervention, p. 381 (shown) and p. 382

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **In $\triangle ABC$, find b.**



$\frac{\sin C}{c} = \frac{\sin B}{b}$ Law of Sines

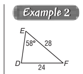
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ Divide each side by sin 45°.

$b \approx 40.8$ Use a calculator.

Example 2 **In $\triangle DEF$, find $m\angle D$.**



$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

$D \approx 81.6°$ Use a calculator.

Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a. **18.4**
2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$. **37**
3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$. **62**
4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b. **7.7**
5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a. **12.2**
6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$. **35**

Skills Practice, p. 383 and Practice, p. 384 (shown)

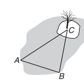
Find each measure using the given measures from $\triangle EFG$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle G = 14$, $m\angle E = 67$, and $e = 14$, find g. **3.7**
2. If $e = 12.7$, $m\angle E = 42$, and $m\angle F = 61$, find f. **16.6**
3. If $g = 14$, $f = 5.8$, and $m\angle G = 83$, find $m\angle F$. **24.3**
4. If $e = 19.1$, $m\angle G = 34$, and $m\angle E = 56$, find g. **12.9**
5. If $f = 9.6$, $g = 27.4$, and $m\angle G = 43$, find $m\angle F$. **13.8**

Solve each $\triangle STU$ described below. Round measures to the nearest tenth.

6. $m\angle T = 85, s = 4.3, t = 8.2$ **$m\angle S \approx 31.5$, $m\angle U \approx 63.5$, $u \approx 7.4$**
7. $s = 40, u = 12, m\angle S = 37$ **$m\angle T \approx 132.6$, $m\angle U \approx 10.4$, $t \approx 48.9$**
8. $m\angle U = 37, t = 2.3, m\angle T = 17$ **$m\angle S \approx 126$, $s \approx 6.4$, $u \approx 4.7$**
9. $m\angle S = 62, m\angle U = 59, s = 17.8$ **$m\angle T \approx 59$, $t \approx 17.3$, $u \approx 17.3$**
10. $t = 28.4, u = 21.7, m\angle T = 66$ **$m\angle S \approx 69.7$, $m\angle U \approx 44.3$, $s \approx 29.2$**
11. $m\angle S = 89, s = 15.3, t = 14$ **$m\angle T \approx 66.2$, $m\angle U \approx 24.8$, $u \approx 6.4$**
12. $m\angle T = 98, m\angle U = 74, u = 9.6$ **$m\angle S \approx 8$, $s \approx 1.4$, $t \approx 9.9$**
13. $t = 11.8, m\angle S = 84, m\angle T = 47$ **$m\angle U = 49$, $s \approx 16.0$, $u \approx 12.2$**

14. **INDIRECT MEASUREMENT** To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location A to location B is 85 meters. The measures of the angles at A and B are 51° and 83°, respectively. What is the distance from the edge of the lake at B to the tree on the island at C? **about 91.8 m**



Reading to Learn Mathematics, p. 385

ELL

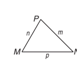
Pre-Activity **How are triangles used in radio astronomy?**

Read the introduction to Lesson 7-6 at the top of page 377 in your textbook.

Why might several antennas be better than one single antenna when studying distant objects? **Sample answer: Observing an object from only one position often does not provide enough information to calculate things such as the distance from the observer to the object.**

Reading the Lesson

1. Refer to the figure. According to the Law of Sines, which of the following are correct statements? **A, F**



A. $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$
B. $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$
C. $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$
D. $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$
E. $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$
F. $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

2. State whether each of the following statements is *true* or *false*. If the statement is false, explain why.
   a. The Law of Sines applies to all triangles. **true**
   b. The Pythagorean Theorem applies to all triangles. **False; sample answer: It only applies to right triangles.**
   c. If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides. **true**
   d. If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle. **False; sample answer: You should use the Angle Sum Theorem.**
   e. A friend tells you that in triangle RST, $m\angle R = 132$, $r = 24$ centimeters, and $s = 31$ centimeters. Can you use the Law of Sines to solve the triangle? Explain. **No; sample answer: In any triangle, the longest side is opposite the largest angle. Because a triangle can have only one obtuse angle, $\angle R$ must be the largest angle, but $s > r$, so it is impossible to have a triangle with the given measures.**

Helping You Remember

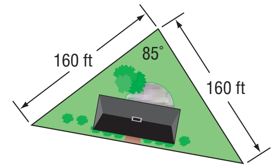
3. Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.
Sample answer: In any triangle, the ratio of the sine of an angle to the length of the opposite side is the same for all three angles.

Answers

40. Yes; in right $\triangle ABC$ $\frac{\sin A}{a} = \frac{\sin C}{c}$ where C is the right angle, $\sin A = \frac{a \sin C}{c}$. Since $m\angle C = 90$, then $\sin A = \frac{a \sin 90°}{c}$. Since the $\sin 90° = 1$, then $\sin A = \frac{a}{c}$, which is the definition of the sine ratio.

41. Sample answer: Triangles are used to determine distances in space. Answers should include the following.
  - The VLA is one of the world's premier astronomical radio observatories. It is used to make pictures from the radio waves emitted by astronomical objects.
  - Triangles are used in the construction of the antennas.

35. **REAL ESTATE** A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed? **about 536 ft**



MIRRORS **For Exercises 36 and 37, use the following information.**
Kayla, Jenna, and Paige live in a town nicknamed "Perpendicular City" because the planners and builders took great care to have all the streets oriented north-south or east-west. The three of them play a game where they signal each other using mirrors. Kayla and Jenna signal each other from a distance of 1433 meters. Jenna turns 27° to signal Paige. Kayla rotates 40° to signal Paige.

36. To the nearest tenth of a meter, how far apart are Kayla and Paige? **about 706.8 m**

37. To the nearest tenth of a meter, how far apart are Jenna and Paige? **about 1000.7 m**

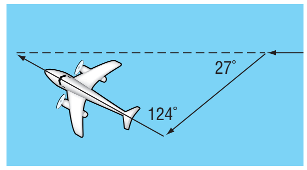
More About. . .

Aviation

From January 2001 to May 2001, Polly Vacher flew over 29,000 miles to become the first woman to fly the smallest single-engined aircraft around the world via Australia and the Pacific.

Source: www.worldwings.org

AVIATION **For Exercises 38 and 39, use the following information.**
Keisha Jefferson is flying a small plane due west. To avoid the jet stream, she must change her course. She turns the plane 27° to the south and flies 60 miles. Then she makes a turn of 124° heads back to her original course. **38. about 56.2 mi**



38. How far must she fly after the second turn to return to the original course?

39. How many miles did she add to the flight by changing course? **about 13.6 mi**

40. **CRITICAL THINKING** Does the Law of Sines apply to the acute angles of a right triangle? Explain your answer. **See margin.**

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

    How are triangles used in radio astronomy?

    Include the following in your answer:
    - a description of what the VLA is and the purpose of the VLA, and
    - the purpose of the triangles in the VLA.

Standardized Test Practice

42. **SHORT RESPONSE** In $\triangle XYZ$, if $x = 12$, $m\angle X = 48$, and $m\angle Y = 112$, solve the triangle to the nearest tenth. $m\angle Z = 20$, $y = 15.0$, **and** $z = 5.5$

43. **ALGEBRA** The table below shows the customer ratings of five restaurants in the *Metro City Guide to Restaurants*. The rating scale is from 1, the worst, to 10, the best. Which of the five restaurants has the best average rating? **A**

Restaurant	Food	Decor	Service	Value
Del Blanco's	7	9	4	7
Aquavent	8	9	4	6
Le Circus	10	8	3	5
Sushi Mambo	7	7	5	6
Metropolis Grill	9	8	7	7

(A) Metropolis Grill (B) Le Circus
(C) Aquavent (D) Del Blanco's

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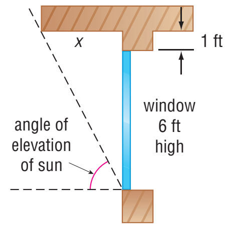
Maintain Your Skills

Mixed Review

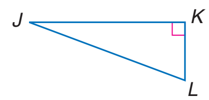
ARCHITECTURE **For Exercises 44 and 45, use the following information.**
Mr. Martinez is an architect who designs houses so that the windows receive minimum sun in the summer and maximum sun in the winter. For Columbus, Ohio, the angle of elevation of the sun at noon on the longest day is 73.5° and on the shortest day is 26.5°. Suppose a house is designed with a south-facing window that is 6 feet tall. The top of the window is to be installed 1 foot below the overhang. *(Lesson 7-5)*



44. How long should the architect make the overhang so that the window gets no direct sunlight at noon on the longest day? **about 2.07 ft**

45. Using the overhang from Exercise 44, how much of the window will get direct sunlight at noon on the shortest day? **about 5.97 ft**

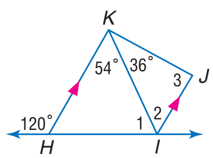
Use $\triangle JKL$ to find sin J, cos J, tan J, sin L, cos L, and tan L. Express each ratio as a fraction and as a decimal to the nearest hundredth. *(Lesson 7-4)* **46–49. See margin.**



46. $j = 8, k = 17, \ell = 15$
47. $j = 20, k = 29, \ell = 21$
48. $j = 12, k = 24, \ell = 12\sqrt{3}$
49. $j = 7\sqrt{2}, k = 14, \ell = 7\sqrt{2}$

If $\overline{KH}$ is parallel to $\overline{JI}$, find the measure of each angle. *(Lesson 4-2)*

50. $\angle 1$ **66**
51. $\angle 2$ **54**
52. $\angle 3$ **90**



Getting Ready for the Next Lesson

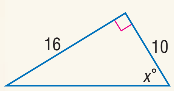
PREREQUISITE SKILL Evaluate $\frac{c^2 - a^2 - b^2}{-2ab}$ for the given values of a, b, and c.
*(To review **evaluating expressions**, see page 736.)* **53.** $\frac{13}{112}$ **54.** $\frac{61}{72}$ **55.** $-\frac{11}{80}$

53. $a = 7, b = 8, c = 10$
54. $a = 4, b = 9, c = 6$
55. $a = 5, b = 8, c = 10$
56. $a = 16, b = 4, c = 13$ $\frac{103}{128}$
57. $a = 3, b = 10, c = 9$ $\frac{7}{15}$
58. $a = 5, b = 7, c = 11$ $-\frac{47}{70}$

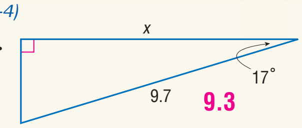
Practice Quiz 2 — *Lessons 7-4 through 7-6*

Find x to the nearest tenth. *(Lesson 7-4)*

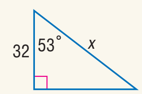
1. **58.0**



2. **9.3**

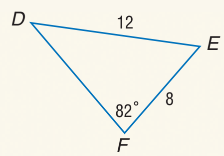


3. **53.2**



4. **COMMUNICATIONS** To secure a 500-foot radio tower against high winds, guy wires are attached to the tower 5 feet from the top. The wires form a 15° angle with the tower. Find the distance from the centerline of the tower to the anchor point of the wires. *(Lesson 7-5)* **132.6 ft**

5. Solve $\triangle DEF$. *(Lesson 7-6)*
$m\angle D \approx 41$
$m\angle E \approx 57$
$e \approx 10.2$



www.geometryonline.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Ask students to draw three triangular figures on a single sheet of paper and label and assign values to them in correspondence with the three types of triangles that can be solved using the Law of Sines (AAS, ASA and SSA). For example, one triangle would have two angle measures, one adjacent side measure and one adjacent side variable. Students can use the examples in the book if necessary. Then tell them to set up the appropriate proportions to find the unknown values.

Getting Ready for Lesson 7-7

Prerequisite Skill Students will learn about the Law of Cosines in Lesson 7-7. They will solve problems using multivariable equations. Use Exercises 53–58 to determine your students' familiarity with evaluating multivariable expressions.

Assessment Options

Practice Quiz 2 This quiz provides students with a brief review of the concepts and skills in Lessons 7-4 through 7-6. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 7-5 and 7-6) is available on p. 408 of the *Chapter 7 Resource Masters*.

Answers

46. $\frac{8}{17} \approx 0.47$; $\frac{15}{17} \approx 0.88$; $\frac{8}{15} \approx 0.53$; $\frac{15}{17} \approx 0.88$; $\frac{8}{17} \approx 0.47$; $\frac{15}{8} \approx 1.88$

47. $\frac{20}{29} \approx 0.69$; $\frac{21}{29} \approx 0.72$; $\frac{20}{21} \approx 0.95$; $\frac{21}{29} \approx 0.72$; $\frac{20}{29} \approx 0.69$; $\frac{21}{20} = 1.05$

48. $\frac{1}{2} = 0.50$; $\frac{\sqrt{3}}{2} \approx 0.87$; $\frac{\sqrt{3}}{3} \approx 0.58$; $\frac{\sqrt{3}}{2} \approx 0.87$; $\frac{1}{2} = 0.50$; $\sqrt{3} \approx 1.73$

49. $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00; $\frac{\sqrt{2}}{2} \approx 0.71$; $\frac{\sqrt{2}}{2} \approx 0.71$; 1.00

Geometry Software Investigation

A Follow-Up of Lesson 7-6

Getting Started

The Ambiguous Case of the Law of Sines As a variation, students can label the points D_1 and D_2 where the circle intersects $\overrightarrow{AD}$ and construct two radii $\overline{BD_1}$ and $\overline{BD_2}$. Then they can find and display all of the measures on the screen at one time.

Teach

- When students are comparing results, point out that the two possible measures for angle D are supplementary, and discuss why this is the case.
- Explain to students that if circle B intersects $\overrightarrow{AC}$ at just one point, then $\overrightarrow{AC}$ is tangent to circle B, and $\triangle ABD$ is a right triangle with right angle D.
- If students become concerned about the possibility of finding two solutions for a SSA problem, assure them that the problems will either be designed to yield one answer or set up in a way so students will know which solution is appropriate. Sometimes it might be appropriate for students to find both solutions.

Assess

Exercises 1–5 After working through the exercises, students will have demonstrated numerous examples of the ambiguous case of the Law of Sines and discussed the possibilities of finding one solution or no solution.

Geometry Software Investigation

A Follow-Up of Lesson 7-6

The Ambiguous Case of the Law of Sines

In Lesson 7-6, you learned that you could solve a triangle using the Law of Sines if you know the measures of two angles and any side of the triangle (AAS or ASA). You can also solve a triangle by the Law of Sines if you know the measures of two sides and an angle opposite one of the sides (SSA). When you use SSA to solve a triangle, and the given angle is acute, sometimes it is possible to find two different triangles. You can use The Geometer's Sketchpad to explore this case, called the **ambiguous case**, of the Law of Sines.

Model

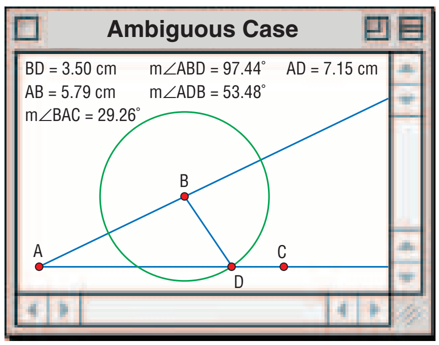
Step 1 Construct $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Construct a circle whose center is B so that it intersects $\overrightarrow{AC}$ at two points. Then, construct any radius $\overline{BD}$.

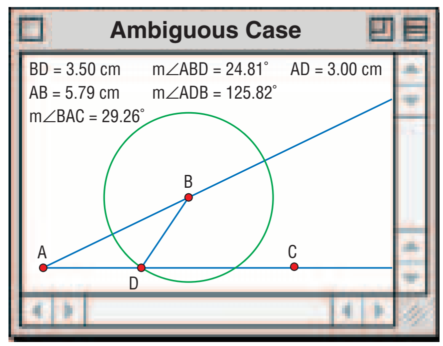
Step 2 Find the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$.

Step 3 Use the rotate tool to move D so that it lies on one of the intersection points of circle B and $\overrightarrow{AC}$. In $\triangle ABD$, find the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$.

Step 4 Using the rotate tool, move D to the other intersection point of circle B and $\overrightarrow{AC}$.

Step 5 Note the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$ in $\triangle ABD$.





Analyze

1. Which measures are the same in both triangles? **BD, AB, $m\angle A$**
2. Repeat the activity using different measures for $\angle A$, $\overline{BD}$, and $\overline{AB}$. How do the results compare to the earlier results? **Sample answer: There are two different triangles.**

Make a Conjecture **3–5. See margin.**

3. Compare your results with those of your classmates. How do the results compare?
4. What would have to be true about circle B in order for there to be one unique solution? Test your conjecture by repeating the activity.
5. Is it possible, given the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$, to have no solution? Test your conjecture and explain.

Answers

3. Sample answer: The results are the same. In each case, two triangles are possible.

4. Sample answer: Circle B intersects $\overrightarrow{AC}$ at only one point. See students' work.

5. Yes; sample answer: There is no solution if circle B does not intersect $\overrightarrow{AC}$.

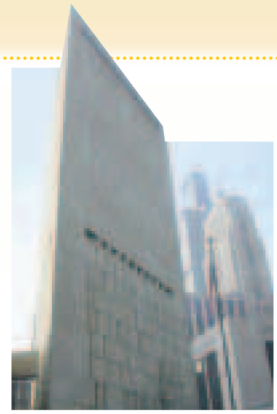
7-7 The Law of Cosines

What You'll Learn

- Use the Law of Cosines to solve triangles.
- Solve problems by using the Law of Cosines.

How are triangles used in building design?

The Chicago Metropolitan Correctional Center is a 27-story triangular federal detention center. The cells are arranged around a lounge-like common area. The architect found that a triangular floor plan allowed for the maximum number of cells to be most efficiently centered around the lounge.



Vocabulary

- Law of Cosines

THE LAW OF COSINES Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

Study Tip

Side and Angle Note that the letter of the side length on the left-hand side of each equation corresponds to the angle measure used with the cosine.

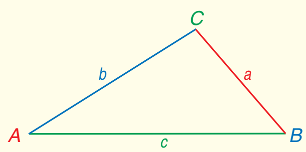
Key Concept — Law of Cosines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles A, B, and C, respectively. Then the following equations are true.

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

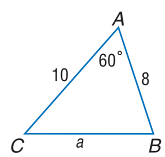


The Law of Cosines can be used to find missing measures in a triangle if you know the measures of two sides and their included angle.

Example 1 Two Sides and the Included Angle

Find a if $c = 8$, $b = 10$, and $m\angle A = 60$.

Use the Law of Cosines since the measures of two sides and the included are known.



$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 10^2 + 8^2 - 2(10)(8) \cos 60°$	$b = 10$, $c = 8$, and $m\angle A = 60$
$a^2 = 164 - 160 \cos 60°$	Simplify.
$a = \sqrt{164 - 160 \cos 60°}$	Take the square root of each side.
$a \approx 9.2$	Use a calculator.

7-7 Lesson Notes

1 Focus

5-Minute Check Transparency 7-7 Use as a quiz or review of Lesson 7-6.

Mathematical Background notes are available for this lesson on p. 340D.

How are triangles used in building design?

Ask students:

- What advantages might the triangular design offer the correction center employees? **The residents would be more confined and easier to monitor.**
- How might the triangular design of the correction facility be more cost efficient? **A triangular building may utilize less space than a rectangular one, so fewer materials would be needed, and the building could have more compact electrical and plumbing systems.**

Resource Manager

Workbook and Reproducible Masters

Chapter 7 Resource Masters

- Study Guide and Intervention, pp. 387–388
- Skills Practice, p. 389
- Practice, p. 390
- Reading to Learn Mathematics, p. 391
- Enrichment, p. 392
- Assessment, p. 408

Prerequisite Skills Workbook, pp. 21–22, 25–26

Teaching Geometry With Manipulatives Masters, p. 121

Transparencies

5-Minute Check Transparency 7-7

Answer Key Transparencies

Technology

GeomPASS: Tutorial Plus, Lesson 16

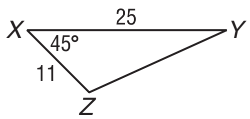
Interactive Chalkboard

2 Teach

THE LAW OF COSINES

In-Class Examples Power Point®

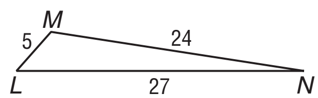
1 Find x if $y = 11$, $z = 25$, and $m\angle X = 45$.



≈ 18.9

Teaching Tip In Example 2, show students how the Law of Cosines can be rewritten to provide an alternate form: $\cos R = \dfrac{q^2 + s^2 - r^2}{2qs}$.

2 Find $m\angle L$.



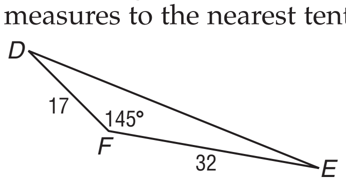
≈ 48.8

USE THE LAW OF COSINES TO SOLVE PROBLEMS

In-Class Example Power Point®

Teaching Tip In Example 3, students can store the actual value of k in their calculators. Then if they use the stored value to find L, they will get 79.19°. Advise students that answers will vary when using rounded values instead of more exact values.

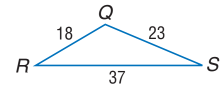
3 Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle DEF$. Then solve $\triangle DEF$. Round angle measures to the nearest degree and side measures to the nearest tenth.



Cosines, $f \approx 46.9$; $m\angle D \approx 23$; $m\angle E \approx 12$

You can also use the Law of Cosines to find the measures of angles of a triangle when you know the measures of the three sides.

Example 2 Three Sides



Find $m\angle R$.

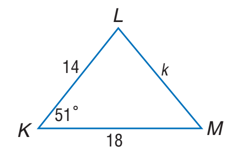
$r^2 = q^2 + s^2 - 2qs \cos R$	Law of Cosines
$23^2 = 37^2 + 18^2 - 2(37)(18) \cos R$	$r = 23$, $q = 37$, $s = 18$
$529 = 1693 - 1332 \cos R$	Simplify.
$-1164 = -1332 \cos R$	Subtract 1693 from each side.
$\dfrac{-1164}{-1332} = \cos R$	Divide each side by -1332.
$R = \cos^{-1}\left(\dfrac{1164}{1332}\right)$	Solve for R.
$R \approx 29.1°$	Use a calculator.

USE THE LAW OF COSINES TO SOLVE PROBLEMS

Most problems can be solved using more than one method. Choosing the most efficient way to solve a problem is sometimes not obvious.

When solving right triangles, you can use sine, cosine, or tangent ratios. When solving other triangles, you can use the Law of Sines or the Law of Cosines. You must decide how to solve each problem depending on the given information.

Example 3 Select a Strategy



Solve $\triangle KLM$. Round angle measure to the nearest degree and side measure to the nearest tenth.

We do not know whether $\triangle KLM$ is a right triangle, so we must use the Law of Cosines or the Law of Sines. We know the measures of two sides and the included angle. This is SAS, so use the Law of Cosines.

$k^2 = \ell^2 + m^2 - 2\ell m \cos K$	Law of Cosines
$k^2 = 18^2 + 14^2 - 2(18)(14) \cos 51°$	$\ell = 18$, $m = 14$, and $m\angle K = 51$
$k = \sqrt{18^2 + 14^2 - 2(18)(14) \cos 51°}$	Take the square root of each side.
$k \approx 14.2$	Use a calculator.

Next, we can find $m\angle L$ or $m\angle M$. If we decide to find $m\angle L$, we can use either the Law of Sines or the Law of Cosines to find this value. In this case, we will use the Law of Sines.

$\dfrac{\sin L}{\ell} = \dfrac{\sin K}{k}$	Law of Sines
$\dfrac{\sin L}{18} \approx \dfrac{\sin 51°}{14.2}$	$\ell = 18$, $k \approx 14.2$, and $m\angle K = 51$
$14.2 \sin L \approx 18 \sin 51°$	Cross products
$\sin L \approx \dfrac{18 \sin 51°}{14.2}$	Divide each side by 14.2.
$L \approx \sin^{-1}\left(\dfrac{18 \sin 51°}{14.2}\right)$	Take the inverse sine of each side.
$L \approx 80°$	Use a calculator.

Study Tip

Law of Cosines
If you use the Law of Cosines to find another measure, your answer may differ slightly from one found using the Law of Sines. This is due to rounding.

TEACHING TIP

Help students learn that when using a calculator, they can skip some steps. For example,
$L = \sin^{-1}\left(\dfrac{18 \sin 51°}{14.2}\right)$
≈ 80.1.

DAILY INTERVENTION — Differentiated Instruction — ELL

Verbal/Linguistic Have students rewrite the equations for the Law of Cosines in their own words without using variables. Then they can describe scenarios for which the Law of Cosines is the more useful. Finally, they can close their books, draw and label a triangular figure, and try to reproduce the equations for the Law of Cosines by using only their written explanations and descriptions.

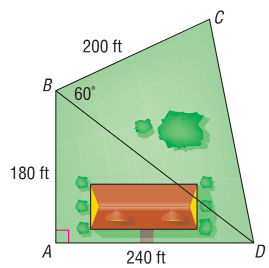
Use the Angle Sum Theorem to find $m\angle M$.

$m\angle K + m\angle L + m\angle M = 180$	Angle Sum Theorem
$51 + 80 + m\angle M \approx 180$	$m\angle K = 51$ and $m\angle L \approx 80$
$m\angle M \approx 49$	Subtract 131 from each side.

Therefore, $k \approx 14.2$, $m\angle K \approx 80$, and $m\angle M \approx 49$.

Example 4 Use Law of Cosines to Solve Problems

REAL ESTATE **Ms. Jenkins is buying some property that is shaped like quadrilateral *ABCD*. Find the perimeter of the property.**



Use the Pythagorean Theorem to find BD in $\triangle ABD$.

$(AB)^2 + (AD)^2 = (BD)^2$	Pythagorean Theorem
$180^2 + 240^2 = (BD)^2$	$AB = 180$, $AD = 240$
$90{,}000 = (BD)^2$	Simplify.
$300 = BD$	Take the square root of each side.

Next, use the Law of Cosines to find CD in $\triangle BCD$.

$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD)\cos\angle CBD$	Law of Cosines
$(CD)^2 = 200^2 + 300^2 - 2(200)(300)\cos 60°$	$BC = 200$, $BD = 300$, $m\angle CBD = 60$
$(CD)^2 = 130{,}000 - 120{,}000\cos 60°$	Simplify.
$CD = \sqrt{130{,}000 - 120{,}000\cos 60°}$	Take the square root of each side.
$CD \approx 264.6$	Use a calculator.

The perimeter of the property is $180 + 200 + 264.6 + 240$ or about 884.6 feet.

Check for Understanding

Concept Check

1. **OPEN ENDED** Draw and label one acute and one obtuse triangle, illustrating when you can use the Law of Cosines to find the missing measures.
2. **Explain** when you should use the Law of Sines or the Law of Cosines to solve a triangle. **1–2. See margin.**
3. **Find a counterexample** for the following statement.
The Law of Cosines can be used to find the length of a missing side in any triangle.

3. If two angles and one side are given, then the Law of Cosines cannot be used.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–5	1
6–7, 10	2
8–9	3

In $\triangle BCD$, given the following measures, find the measure of the missing side.

4. $c = \sqrt{2}$, $d = 5$, $m\angle B = 45$ **4.1**
5. $b = 107$, $c = 94$, $m\angle D = 105$ **159.7**

In $\triangle RST$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

6. $r = 33$, $s = 65$, $t = 56$; $m\angle S$ **90**
7. $r = 2.2$, $s = 1.3$, $t = 1.6$; $m\angle R$ **98**

Solve each triangle using the given information. Round angle measure to the nearest degree and side measure to the nearest tenth.

8. $\triangle XYZ$: $x = 5$, $y = 10$, $z = 13$
9. $\triangle KLM$: $k = 20$, $m = 24$, $m\angle L = 47$ **$\ell \approx 17.9$; $m\angle K \approx 55$; $m\angle M \approx 78$**

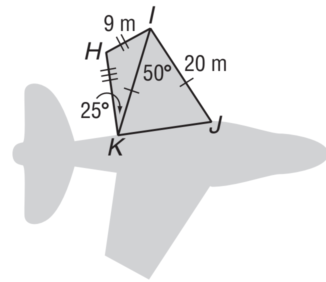
8. $m\angle X \approx 20$; $m\angle Y \approx 43$; $m\angle Z \approx 117$

www.geometryonline.com/extra_examples

In-Class Example

Power Point®

4 **AIRCRAFT** From the diagram of the plane shown, determine the approximate exterior perimeter of each wing. Round to the nearest tenth meter.



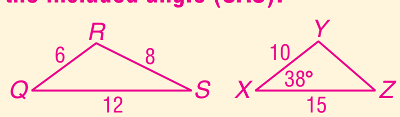
67.1 m

Concept Check

Have students compose a chart in which they include all of the types of triangles they could solve using the Pythagorean Theorem, trigonometric ratios, Law of Sines, and Law of Cosines. They can also include a column listing the formulas and/or ratios and a column for the limitations of each method. Students should recognize that the Pythagorean Theorem is the most limited but the most efficient, and the Law of Cosines is the most diverse but very involved and time-consuming. They may want to form a habit of checking for the most efficient way to solve a problem, keeping all these methods in mind.

Answers

1. Sample answer: Use the Law of Cosines when you have all three sides given (SSS) or two sides and the included angle (SAS).



2. If you have all three sides (SSS) or two sides and the included angle (SAS) given, then use the Law of Cosines. If two angles and one side (ASA or AAS) or two sides with angle opposite one of the sides (SSA) are given, then use the Law of Sines.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
- include an example each for using the Law of Cosines with SAS and with SSS.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Solve Triangles:** 11–18, 38
- **Use the Law of Cosines to Solve Problems:** 22–37, 39–41

Odd/Even Assignments

Exercises 11–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

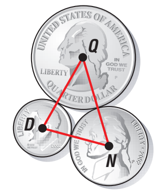
Basic: 11–37 odd, 41, 43–59

Average: 11–43 odd, 44–59

Advanced: 12–44 even, 46–56 (optional: 57–59)

Application

10. **CRAFTS** Jamie, age 25, is creating a logo for herself and two cousins, ages 10 and 5. She is using a quarter (25 cents), a dime (10 cents), and a nickel (5 cents) to represent their ages. She will hold the coins together by soldering a triangular piece of silver wire so that the three vertices of the triangle lie at the centers of the three circular coins. The diameter of the quarter is 24 millimeters, the diameter of the nickel is 22 millimeters, and the diameter of a dime is 10 millimeters. Find the measures of the three angles in the triangle. **$m\angle Q = 44.1$; $m\angle D = 88.3$; $m\angle N = 47.6$**



★ indicates increased difficulty

Practice and Apply

Homework Help	
For Exercises	**See Examples**
11–14	1
15–18, 38	2
22–37, 39–41	3

Extra Practice
See page 768.

In $\triangle TUV$, given the following measures, find the measure of the missing side.

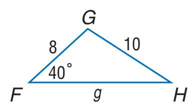
11. $t = 9.1$, $v = 8.3$, $m\angle U = 32$ **$u \approx 4.9$**
12. $t = 11$, $u = 17$, $m\angle V = 78$ **$v \approx 18.2$**
13. $u = 11$, $v = 17$, $m\angle T = 105$ **$t \approx 22.5$**
14. $v = 11$, $u = 17$, $m\angle T = 59$ **$t \approx 14.7$**

In $\triangle EFG$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

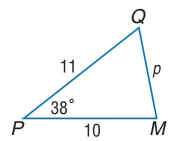
15. $e = 9.1$, $f = 8.3$, $g = 16.7$; $m\angle F$ **16**
16. $e = 14$, $f = 19$, $g = 32$; $m\angle E$ **12**
17. $e = 325$, $f = 198$, $g = 208$; $m\angle F$ **36**
18. $e = 21.9$, $f = 18.9$, $g = 10$; $m\angle G$ **27**

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth. **19–25. See margin.**

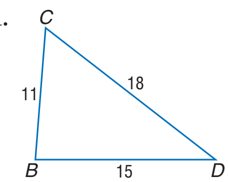
19.



20.

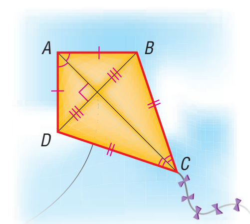


21.



22. $\triangle ABC$: $m\angle A = 42$, $m\angle C = 77$, $c = 6$
23. $\triangle ABC$: $a = 10.3$, $b = 9.5$, $m\angle C = 37$
24. $\triangle ABC$: $a = 15$, $b = 19$, $c = 28$
25. $\triangle ABC$: $m\angle A = 53$, $m\angle C = 28$, $c = 14.9$

26. **KITES** Beth is building a kite like the one at the right. If $\overline{AB}$ is 5 feet long, $\overline{BC}$ is 8 feet long, and $\overline{BD}$ is $7\frac{2}{3}$ feet long, find the measure of the angle between the short sides and the angle between the long sides to the nearest degree. **100; 57**



Solve each $\triangle LMN$ described below. Round measures to the nearest tenth.

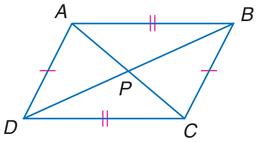
27–38. See margin.

27. $m = 44$, $\ell = 54$, $m\angle L = 23$
28. $m = 18$, $\ell = 24$, $n = 30$
29. $m = 19$, $n = 28$, $m\angle L = 49$
30. $m\angle M = 46$, $m\angle L = 55$, $n = 16$
31. $m = 256$, $\ell = 423$, $n = 288$
32. $m\angle M = 55$, $\ell = 6.3$, $n = 6.7$
33. $m\angle M = 27$, $\ell = 5$, $n = 10$
34. $n = 17$, $m = 20$, $\ell = 14$
35. $\ell = 14$, $n = 21$, $m\angle M = 60$
36. $\ell = 14$, $m = 15$, $n = 16$
37. $m\angle L = 51$, $\ell = 40$, $n = 35$
38. $\ell = 10$, $m = 11$, $n = 12$

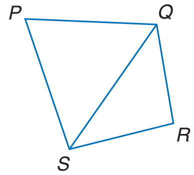
Answers

19. $m\angle H \approx 31$; $m\angle G \approx 109$; $g \approx 14.7$
20. $p \approx 6.9$; $m\angle M \approx 79$; $m\angle Q \approx 63$
21. $m\angle B \approx 86$; $m\angle C \approx 56$; $m\angle D \approx 38$
22. $m\angle B = 61$; $b \approx 5.4$; $a \approx 4.1$
23. $c \approx 6.3$; $m\angle A \approx 80$; $m\angle B \approx 63$
24. $m\angle A \approx 30$; $m\angle B \approx 39$; $m\angle C \approx 111$
25. $m\angle B = 99$; $b \approx 31.3$; $a \approx 25.3$
27. $m\angle M \approx 18.6$; $m\angle N \approx 138.4$; $n \approx 91.8$
28. $m\angle L \approx 53.2$; $m\angle M \approx 36.9$; $m\angle N \approx 89.9$
29. $\ell \approx 21.1$; $m\angle M \approx 42.8$; $m\angle N \approx 88.2$
30. $m\angle N = 79$; $\ell \approx 13.4$; $m \approx 11.7$
31. $m\angle L \approx 101.9$; $m\angle M \approx 36.3$; $m\angle N \approx 41.8$
32. $m \approx 6.0$; $m\angle L \approx 59.3$; $m\angle N \approx 65.7$
33. $m \approx 6.0$; $m\angle L \approx 22.2$; $m\angle N \approx 130.8$
34. $m\angle L \approx 43.5$; $m\angle M \approx 79.5$; $m\angle N \approx 57.0$
35. $m \approx 18.5$; $m\angle L \approx 40.9$; $m\angle N \approx 79.1$
36. $m\angle L \approx 53.6$; $m\angle M \approx 59.6$; $m\angle N \approx 66.8$
37. $m\angle N \approx 42.8$; $m\angle M \approx 86.2$; $m \approx 51.4$
38. $m\angle L \approx 51.3$; $m\angle M \approx 59.1$; $m\angle N \approx 69.6$

★ **39.** In quadrilateral $ABCD$, $AC = 188$, $BD = 214$, $m\angle BPC = 70$, and P is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the perimeter of quadrilateral $ABCD$. **561.2 units**



★ **40.** In quadrilateral $PQRS$, $PQ = 721$, $QR = 547$, $RS = 593$, $PS = 756$, and $m\angle P = 58$. Find QS, $m\angle PQS$, and $m\angle R$. **$QS \approx 716.7$; $m\angle PQS \approx 63.5$; $m\angle R \approx 77.8$**



41. BUILDINGS Refer to the information at the left. Find the measures of the angles of the triangular building to the nearest tenth. **59.8, 63.4, 56.8**

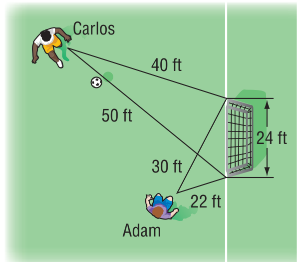
More About . . .

Buildings

The Swissôtel in Chicago, Illinois, is built in the shape of a triangular prism. The lengths of the sides of the triangle are 180 feet, 186 feet, and 174 feet.

Source: Swissôtel

42. SOCCER Carlos and Adam are playing soccer. Carlos is standing 40 feet from one post of the goal and 50 feet from the other post. Adam is standing 30 feet from one post of the goal and 22 feet from the other post. If the goal is 24 feet wide, which player has a greater angle to make a shot on goal? **Adam; 52.3°**

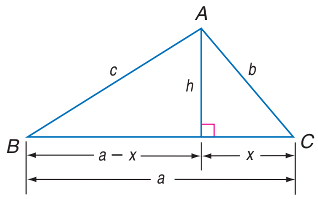


43. PROOF Justify each statement for the derivation of the Law of Cosines.

Given: $\overline{AD}$ is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$

Proof:



Statement	Reasons
a. $c^2 = (a - x)^2 + h^2$	a. __?__ **Pythagorean Theorem**
b. $c^2 = a^2 - 2ax + x^2 + h^2$	b. __?__ **Substitution**
c. $x^2 + h^2 = b^2$	c. __?__ **Pythagorean Theorem**
d. $c^2 = a^2 - 2ax + b^2$	d. __?__ **Substitution**
e. $\cos C = \frac{x}{b}$	e. __?__ **Def. of cosine**
f. $b \cos C = x$	f. __?__ **Cross products**
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	g. __?__ **Substitution**
h. $c^2 = a^2 + b^2 - 2ab \cos C$	h. __?__ **Commutative Property**

44. CRITICAL THINKING Graph $A(-6, -8)$, $B(10, -4)$, $C(6, 8)$, and $D(5, 11)$ on a coordinate plane. Find the measure of interior angle ABC and the measure of exterior angle DCA. **$m\angle ABC \approx 85.6$; $m\angle DCA \approx 124.7$**

45. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How are triangles used in building construction?

Include the following in your answer:

- why the building was triangular instead of rectangular, and
- why the Law of Sines could not be used to solve the triangle.

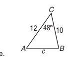
www.geometryonline.com/self_check_quiz

Study Guide and Intervention, p. 387 (shown) and p. 388

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **In $\triangle ABC$, find c.**



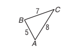
$c^2 = a^2 + b^2 - 2ab \cos C$ — Law of Cosines

$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$ — $a = 12$, $b = 10$, $m\angle C = 48$

$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$ — Take the square root of each side.

$c \approx 9.1$ — Use a calculator.

Example 2 **In $\triangle ABC$, find $m\angle A$.**



$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$ — $a = 7$, $b = 5$, $c = 8$

$49 = 25 + 64 - 80 \cos A$ — Multiply.

$-40 = -80 \cos A$ — Subtract 89 from each side.

$\frac{1}{2} = \cos A$ — Divide each side by −80.

$\cos^{-1}\frac{1}{2} = A$ — Use the inverse cosine.

$60° = A$ — Use a calculator.

Exercises

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62$, find a. **13.5**
2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$. **51**
3. If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$. **42**
4. If $a = 20$, $c = 25$, and $m\angle B = 82$, find b. **29.8**
5. If $b = 18$, $c = 28$, and $m\angle A = 59$, find a. **24.3**
6. If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$. **51**

Skills Practice, p. 389 and Practice, p. 390 (shown)

In $\triangle JKL$, given the following measures, find the measure of the missing side.

1. $j = 1.3$, $k = 10$, $m\angle L = 77$ **$\ell \approx 9.8$**
2. $j = 9.6$, $\ell = 1.7$, $m\angle K = 43$ **$k \approx 8.4$**
3. $j = 11$, $k = 7$, $m\angle L = 63$ **$\ell \approx 10.0$**
4. $k = 4.7$, $\ell = 5.2$, $m\angle J = 112$ **$j \approx 8.2$**

In $\triangle MNQ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

5. $m = 17$, $n = 23$, $q = 25$; $m\angle Q$ **75.7**
6. $m = 24$, $n = 28$, $q = 34$; $m\angle M$ **44.2**
7. $m = 12.9$, $n = 18$, $q = 20.5$; $m\angle N$ **60.2**
8. $m = 23$, $n = 30.1$, $q = 42$; $m\angle Q$ **103.7**

Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle ABC$. Then sole each triangle. Round angle measures to the nearest degree and side measure to the nearest tenth.

9. $a = 13$, $b = 18$, $c = 19$ **Cosines; $m\angle A \approx 41$; $m\angle B \approx 65$; $m\angle C \approx 74$**
10. $a = 6$, $b = 19$, $m\angle C = 38$ **Cosines; $m\angle A \approx 15$; $m\angle B \approx 127$; $c \approx 14.7$**
11. $a = 17$, $b = 22$, $m\angle B = 49$ **Sines; $m\angle A \approx 36$; $m\angle C \approx 95$; $c \approx 29.0$**
12. $a = 15.5$, $b = 18$, $m\angle C = 72$ **Cosines; $m\angle A \approx 48$; $m\angle B \approx 60$; $c \approx 19.8$**

Solve each $\triangle FGH$ described below. Round measures to the nearest tenth.

13. $m\angle F = 54$, $f = 12.5$, $g = 11$ **$m\angle G \approx 45.4$, $m\angle H \approx 80.6$, $h \approx 15.2$**
14. $f = 20$, $g = 23$, $m\angle H = 47$ **$m\angle F \approx 57.4$, $m\angle G \approx 75.6$, $h \approx 17.4$**
15. $f = 15.8$, $g = 11$, $h = 14$ **$m\angle F \approx 77.4$, $m\angle G \approx 42.8$, $m\angle H \approx 59.8$**
16. $f = 36$, $h = 30$, $m\angle G = 54$ **$m\angle F \approx 73.1$, $m\angle H \approx 52.9$, $g \approx 30.4$**

17. **REAL ESTATE** The Esposito family purchased a triangular plot of land on which they plan to build a barn and corral. The lengths of the sides of the plot are 320 feet, 286 feet, and 305 feet. What are the measures of the angles formed on each side of the property? **65.5, 54.4, 60.1**

Reading to Learn Mathematics, p. 391 ELL

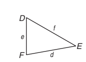
Pre-Activity **How are triangles used in building design?**

Read the introduction to Lesson 7-7 at the top of page 385 in your textbook.

What could be a disadvantage of a triangular room? **Sample answer: Furniture will not fit in the corners.**

Reading the Lesson

1. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$? **B, E, H**



A. $d^2 = e^2 + f^2 - ef \cos D$
B. $e^2 = d^2 + f^2 - 2df \cos E$
C. $d^2 = e^2 + f^2 + 2ef \cos D$
D. $f^2 = d^2 + e^2 - 2ef \cos F$
E. $f^2 = d^2 + e^2 - 2de \cos F$
F. $d^2 = e^2 + f^2$
G. $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$
H. $d = \sqrt{e^2 + f^2 - 2ef \cos D}$

2. Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)
   a. SSS **Law of Cosines**
   b. ASA **Law of Sines**
   c. AAS **Law of Sines**
   d. SAS **Law of Cosines**
   e. SSA **Law of Sines**

3. Indicate whether each statement is *true* or *false*. If the statement is false, explain why.
   a. The Law of Cosines applies to right triangles. **true**
   b. The Pythagorean Theorem applies to acute triangles. **False; sample answer: It only applies to right triangles.**
   c. The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle. **False; sample answer: It is used when you are given the measures of two sides and the included angle.**
   d. The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters. **False; sample answer: 5 + 8 < 15, so, by the Triangle Inequality Theorem, no such triangle exists.**

Helping You Remember

4. A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be? **cos 90 = 0, so in a right triangle, where the included angle is the right angle, the Law of Cosines becomes the Pythagorean Theorem.**

Answers

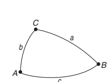
45. Sample answer: Triangles are used to build supports, walls, and foundations. Answers should include the following.

- **The triangular building was more efficient with the cells around the edge.**
- **The Law of Sines requires two angles and a side or two sides and an angle opposite one of those sides.**

Enrichment, p. 392

Spherical Triangles

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.



The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Standardized Test Practice

46. For $\triangle DEF$, find d to the nearest tenth if $e = 12$, $f = 15$, and $m\angle D = 75$. **B**

Ⓐ 18.9 Ⓑ 16.6 Ⓒ 15.4 Ⓓ 9.8

47. **ALGEBRA** Ms. LaHue earns a monthly base salary of \$1280 plus a commission of 12.5% of her total monthly sales. At the end of one month, Ms. LaHue earned \$4455. What were her total sales for the month? **C**

Ⓐ \$3175 Ⓑ \$10,240 Ⓒ \$25,400 Ⓓ \$35,640

Maintain Your Skills

Mixed Review

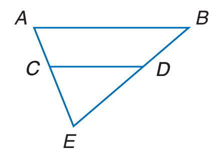
Find each measure using the given measures from $\triangle XYZ$. Round angle measure to the nearest degree and side measure to the nearest tenth. *(Lesson 7-6)*

48. If $y = 4.7$, $m\angle X = 22$, and $m\angle Y = 49$, find x. **2.3**

49. If $y = 10$, $x = 14$, and $m\angle X = 50$, find $m\angle Y$. **33**

50. **SURVEYING** A surveyor is 100 meters from a building and finds that the angle of elevation to the top of the building is 23°. If the surveyor's eye level is 1.55 meters above the ground, find the height of the building. *(Lesson 7-5)* **about 44.0 m**

For Exercises 51–54, determine whether $\overline{AB} \parallel \overline{CD}$. *(Lesson 6-4)*



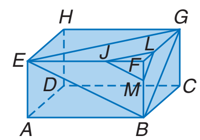
51. $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$ **yes**

52. $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$ **yes**

53. $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$ **no**

54. $AB = 5.4$, $BE = 18$, $CD = 3$, and $DE = 10$ **yes**

Use the figure at the right to write a paragraph proof. *(Lesson 6-3)*



55. **Given:** $\triangle JFM \sim \triangle EFB$
$\triangle LFM \sim \triangle GFB$
Prove: $\triangle JFL \sim \triangle EFG$ **See margin.**

56. **Given:** $\overline{JM} \parallel \overline{EB}$
$\overline{LM} \parallel \overline{GB}$
Prove: $\overline{JL} \parallel \overline{EG}$ **See margin.**

COORDINATE GEOMETRY **The vertices of $\triangle XYZ$ are $X(8, 0)$, $Y(-4, 8)$, and $Z(0, 12)$. Find the coordinates of the points of concurrency of $\triangle XYZ$ to the nearest tenth.** *(Lesson 5-1)*

57. orthocenter **(−1.6, 9.6)** 58. centroid **(1.3, 6.7)** 59. circumcenter **(2.8, 5.2)**

WebQuest Internet Project

Who is Behind This Geometry Idea Anyway?

It's time to complete your project. Use the information and data you have gathered about your research topic, two mathematicians, and a geometry problem to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

www.geometryonline.com/webquest

4 Assess

Open-Ended Assessment

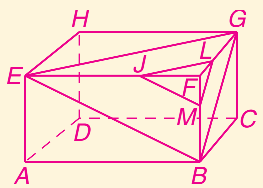
Writing Select some exercises to reproduce on the board for the class. Have students write the equations necessary to solve for different angles and sides and fill in the values without solving the problems. Volunteers can write their equations on the board.

Assessment Options

Quiz (Lessons 7-7) is available on p. 408 of the *Chapter 7 Resource Masters*.

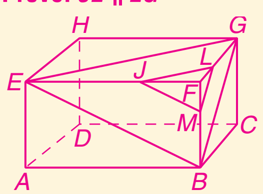
Answers

55. Given: $\triangle JFM \sim \triangle EFB$
$\triangle LFM \sim \triangle GFB$
Prove: $\triangle JFL \sim \triangle EFG$



Proof: Since $\triangle JFM \sim \triangle EFB$ and $\triangle LFM \sim \triangle GFB$, then by definition of similar triangles, $\frac{JF}{EF} = \frac{MF}{BF}$ and $\frac{MF}{BF} = \frac{LF}{GF}$. By the Transitive Property of Equality, $\frac{JF}{EF} = \frac{LF}{GF}$. $\angle F \cong \angle F$ by the Reflexive Property of Congruence. Then, by SAS Similarity, $\triangle JFL \sim \triangle EFG$.

56. Given: $\overline{JM} \parallel \overline{EB}$
$\overline{LM} \parallel \overline{GB}$
Prove: $\overline{JL} \parallel \overline{EG}$



Proof: Since $\overline{JM} \parallel \overline{EB}$ and $\overline{LM} \parallel \overline{GB}$, then $\angle MJF \cong \angle BEF$ and $\angle FML \cong \angle FBG$ because if two parallel lines are cut by a transversal, corresponding angles are congruent. $\angle EFB \cong \angle EFB$ and $\angle BFG \cong \angle BFG$ by the Reflexive Property of Congruence. Then $\triangle EFB \sim \triangle JFM$ and $\triangle FBG \sim \triangle FML$ by AA Similarity. Then $\frac{JF}{EF} = \frac{MF}{BF}$, $\frac{MF}{BF} = \frac{LF}{GF}$, by the definition of similar triangles. $\frac{JF}{EF} = \frac{LF}{GF}$ by the Transitive Property of Equality, and $\angle EFG \cong \angle EFG$ by the Reflexive Property of Congruence. Thus, $\triangle JFL \sim \triangle EFG$ by SAS Similarity and $\angle FJL \cong \angle FEG$ by the definition of similar triangles. $\overline{JL} \parallel \overline{EG}$ because if two lines are cut by a tranversal so that the corresponding angles are congruent, then the lines are parallel.

390 Chapter 7 Right Triangles and Trigonometry

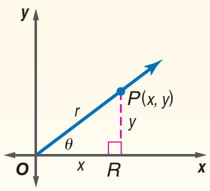
Geometry Activity

A Follow-Up of Lesson 7-7

Trigonometric Identities

In algebra, the equation $2(x + 2) = 2x + 4$ is called an *identity* because the equation is true for all values of x. There are equations involving trigonometric ratios that are true for all values of the angle measure. These are called **trigonometric identities**.

In the figure, $P(x, y)$ is in Quadrant I. The Greek letter theta (pronounced THAY tuh) θ, represents the measure of the angle formed by the x-axis and $\overline{OP}$. Triangle POR is a right triangle. Let r represent the length of the hypotenuse. Then the following are true.



$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\quad\sec\theta = \frac{r}{x}$ $\quad\cot\theta = \frac{x}{y}$

Notice that $\frac{1}{\sin\theta} = \frac{1}{\frac{y}{r}}$ ⇨ $\frac{1}{\frac{y}{r}} = 1 \div \frac{y}{r}$ ⇨ $1 \div \frac{y}{r} = 1 \cdot \frac{r}{y}$ or $\frac{r}{y}$ ⇨ $\frac{r}{y} = \csc\theta$.

So, $\frac{1}{\sin\theta} = \csc\theta$. This is known as one of the **reciprocal identities**.

Activity

Verify that $\cos^2\theta + \sin^2\theta = 1$.

The expression $\cos^2\theta$ means $(\cos\theta)^2$. To verify an identity, work on only one side of the equation and use what you know to show how that side is equivalent to the other side.

$\cos^2\theta + \sin^2\theta \stackrel{?}{=} 1$	Original equation
$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 \stackrel{?}{=} 1$	Substitute.
$\frac{x^2}{r^2} + \frac{y^2}{r^2} \stackrel{?}{=} 1$	Simplify.
$\frac{x^2 + y^2}{r^2} \stackrel{?}{=} 1$	Combine fractions with like denominators.
$\frac{r^2}{r^2} \stackrel{?}{=} 1$	Pythagorean Theorem: $x^2 + y^2 = r^2$
$1 = 1$ ✓	Simplify.

Since $1 = 1$, $\cos^2\theta + \sin^2\theta = 1$.

Analyze **1–2. See margin.**

1. The identity $\cos^2\theta + \sin^2\theta = 1$ is known as a **Pythagorean identity**. Why do you think the word *Pythagorean* is used to name this?
2. Find two more reciprocal identities involving $\frac{1}{\cos\theta}$ and $\frac{1}{\tan\theta}$.

Verify each identity. **3–6. See p. 399A.**

3. $\frac{\sin\theta}{\cos\theta} = \tan\theta$
4. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
5. $\tan^2\theta + 1 = \sec^2\theta$
6. $\cot^2\theta + 1 = \csc^2\theta$

Geometry Activity

A Follow-Up of Lesson 7-7

Getting Started

Trigonometric Identities

Explain to students that this investigation provides them with the tools necessary for deriving trigonometric identities, which they will later use to solve problems and prove theorems.

Teach

- Have students write an equation for the Law of Cosines and substitute the symbols in the figure, using θ as the angle.
- Ask students to set up a proportion with the Law of Sines, using θ and 90° for the angles and then solving for $\sin\theta$.
- Encourage students to become comfortable with verifying the identities, and advise them that these skills will be useful when they are faced with more complex problems and proofs.

Assess

Exercises 1–6 allow students to systematically practice and become more comfortable with finding equivalent trigonometric expressions and verifying trigonometric identities. Some students may need to strengthen their algebra skills before working through these exercises.

Answers

1. Sample answer: It is of the form $a^2 + b^2 = c^2$, where $c = 1$.
2. $\frac{1}{\cos\theta} = \sec\theta$; $\frac{1}{\tan\theta} = \cot\theta$

Chapter 7 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 7 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 7 is available on p. 406 of the *Chapter 7 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 7 Study Guide and Review

Vocabulary and Concept Check

ambiguous case (p. 384)	geometric mean (p. 342)	Pythagorean triple (p. 352)	tangent (p. 364)
angle of depression (p. 372)	Law of Cosines (p. 385)	reciprocal identities (p. 391)	trigonometric identity (p. 391)
angle of elevation (p. 371)	Law of Sines (p. 377)	sine (p. 364)	trigonometric ratio (p. 364)
cosine (p. 364)	Pythagorean identity (p. 391)	solving a triangle (p. 378)	trigonometry (p. 364)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **State whether each statement is *true* or *false*. If false, replace the underlined word or words to make a true sentence.**

1. The Law of Sines can be applied if you know the measures of two sides and an angle <u>opposite</u> one of these sides of the triangle. **true**
2. The tangent of $\angle A$ is the measure of the leg <u>adjacent</u> to $\angle A$ divided by the measure of the leg <u>opposite</u> $\angle A$. **false; opposite; adjacent**
3. In <u>any</u> triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse. **false; a right**
4. An angle of <u>elevation</u> is the angle between the line of sight and the horizontal when an observer looks upward. **true**
5. The geometric mean between two numbers is the positive square root of their <u>product</u>. **true**
6. In a <u>30°-60°-90°</u> triangle, two of the sides will have the same length. **false; 45°-45°-90°**
7. Looking at a city while flying in a plane is an example that uses angle of <u>elevation</u>. **false; depression**

Lesson-by-Lesson Review

7-1 Geometric Mean

See pages 342–348.

Concept Summary

- The geometric mean of two numbers is the square root of their product.
- You can use the geometric mean to find the altitude of a right triangle.

Examples

1 Find the geometric mean between 10 and 30.

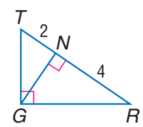
$\frac{10}{x} = \frac{x}{30}$ Definition of geometric mean

$x^2 = 300$ Cross products

$x = \sqrt{300}$ or $10\sqrt{3}$ Take the square root of each side.

2 Find NG in $\triangle TGR$.

The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.



$\frac{TN}{GN} = \frac{GN}{RN}$ Definition of geometric mean

$\frac{2}{GN} = \frac{GN}{4}$ $TN = 2$, $RN = 4$

$8 = (GN)^2$ Cross products

$\sqrt{8}$ or $2\sqrt{2} = GN$ Take the square root of each side.

www.geometryonline.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

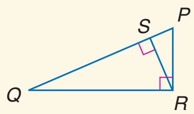
Have students look through the chapter to make sure they have included notes and examples in their Foldables for each lesson of Chapter 7.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Exercises **Find the geometric mean between each pair of numbers.**
See Example 1 on page 342.

8. 4 and 16 **8** **9.** 4 and 81 **18** **10.** 20 and 35 $10\sqrt{7} \approx 26.5$ **11.** 18 and 44 $6\sqrt{22} \approx 28.1$

12. In $\triangle PQR$, $PS = 8$, and $QS = 14$. Find RS. *See Example 2 on page 344.*
$4\sqrt{7} \approx 10.6$



7-2 The Pythagorean Theorem and Its Converse

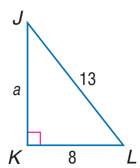
See pages 350–356.

Concept Summary

- The Pythagorean Theorem can be used to find the measures of the sides of a right triangle.
- If the measures of the sides of a triangle form a Pythagorean triple, then the triangle is a right triangle.

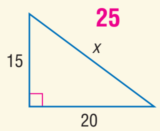
Example **Find *k*.**

$a^2 + (LK)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + 8^2 = 13^2$	$LK = 8$ and $JL = 13$
$a^2 + 64 = 169$	Simplify.
$a^2 = 105$	Subtract 64 from each side.
$a = \sqrt{105}$	Take the square root of each side.
$a \approx 10.2$	Use a calculator.

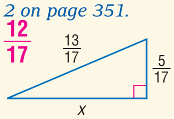


Exercises **Find *x*.** *See Example 2 on page 351.*

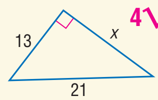
13. **25**



14. $\frac{12}{17}$



15. $4\sqrt{17} \approx 16.5$



7-3 Special Right Triangles

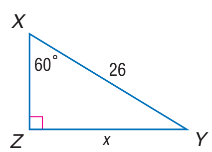
See pages 357–363.

Concept Summary

- The measure of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of the legs of the triangle. The measures of the sides are x, x, and $x\sqrt{2}$.
- In a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$.

Examples **1** **Find *x*.**

The measure of the shorter leg $\overline{XZ}$ of $\triangle XYZ$ is half the measure of the hypotenuse $\overline{XY}$. Therefore, $XZ = \frac{1}{2}(26)$ or 13. The measure of the longer leg is $\sqrt{3}$ times the measure of the shorter leg. So, $x = 13\sqrt{3}$.



Chapter 7 Study Guide and Review 393

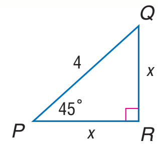
Answers

21. $\frac{3}{5} = 0.60; \frac{4}{5} = 0.80; \frac{3}{4} = 0.75;$
$\frac{4}{5} = 0.80; \frac{3}{5} = 0.60; \frac{4}{3} \approx 1.33$

22. $\frac{7}{25} = 0.28; \frac{24}{25} = 0.96; \frac{7}{24} \approx 0.29;$
$\frac{24}{25} = 0.96; \frac{7}{25} = 0.28; \frac{24}{7} \approx 3.43$

2 **Find x.**

The measure of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.



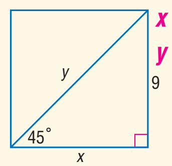
$x\sqrt{2} = 4$

$x = \frac{4}{\sqrt{2}}$

$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ or $2\sqrt{2}$

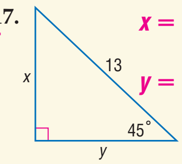
Exercises **Find x and y.** *See Examples 1 and 3 on pages 358 and 359.*

16.



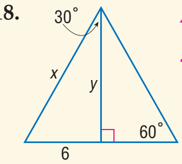
$x = 9$; $y = 9\sqrt{2}$

17.



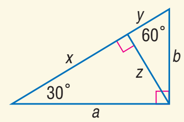
$x = \frac{13\sqrt{2}}{2}$; $y = \frac{13\sqrt{2}}{2}$

18.



$x = 12$; $y = 6\sqrt{3}$

For Exercises 19 and 20, use the figure at the right.
See Example 3 on page 359.



19. If $y = 18$, find z and a. $z = 18\sqrt{3}$, $a = 36\sqrt{3}$

20. If $x = 14$, find a, z, b, and y.

$a = \frac{28\sqrt{3}}{3}$, $z = \frac{14\sqrt{3}}{3}$, $b = \frac{28}{3}$, $y = \frac{14}{3}$

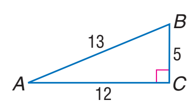
7-4 Trigonometry

See pages 364–370.

Concept Summary

- Trigonometric ratios can be used to find measures in right triangles.

Example **Find sin A, cos A, and tan A. Express each ratio as a fraction and as a decimal.**

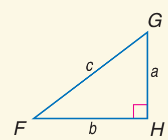


$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13}$ or about 0.38

$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13}$ or about 0.92

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12}$ or about 0.42

Exercises **Use $\triangle FGH$ to find sin F, cos F, tan F, sin G, cos G, and tan G. Express each ratio as a fraction and as a decimal to the nearest hundredth.** *See Example 1 on page 365.*



21. $a = 9$, $b = 12$, $c = 15$ **22.** $a = 7$, $b = 24$, $c = 25$

21–22. See margin.

Find the measure of each angle to the nearest tenth of a degree.
See Example 4 on pages 366 and 367.

23. $\sin P = 0.4522$ 26.9 **24.** $\cos Q = 0.1673$ 80.4 **25.** $\tan R = 0.9324$ 43.0

7-5 Angles of Elevation and Depression

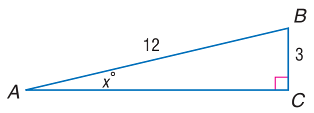
See pages 371–376.

Concept Summary

- Trigonometry can be used to solve problems related to angles of elevation and depression.

Example **A store has a ramp near its front entrance. The ramp measures 12 feet, and has a height of 3 feet. What is the angle of elevation?**

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x° = \frac{BC}{AB}$ — $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin x° = \frac{3}{12}$ — $BC = 3$ and $AB = 12$

$x = \sin^{-1}\left(\frac{3}{12}\right)$ — Find the inverse.

$x \approx 14.5$ — Use a calculator.

The angle of elevation for the ramp is about 14.5°.

Exercises **Determine the angles of elevation or depression in each situation.**

See Examples 1 and 2 on pages 371 and 372.

26. An airplane must clear a 60-foot pole at the end of a runway 500 yards long. **≈2.3°**

27. An escalator descends 100 feet for each horizontal distance of 240 feet. **≈22.6°**

28. A hot-air balloon descends 50 feet for every 1000 feet traveled horizontally. **≈2.9°**

29. **DAYLIGHT** At a certain time of the day, the angle of elevation of the sun is 44°. Find the length of a shadow cast by a building that is 30 yards high. **≈31.1 yd**

30. **RAILROADS** A railroad track rises 30 feet for every 400 feet of track. What is the measure of the angle of elevation of the track? **≈4.3°**

7-6 The Law of Sines

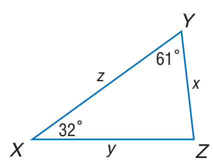
See pages 377–383.

Concept Summary

- To find the measures of a triangle by using the Law of Sines, you must either know the measures of two angles and any side (AAS or ASA), or two sides and an angle opposite one of these sides (SSA) of the triangle.
- To solve a triangle means to find the measures of all sides and angles.

Example **Solve $\triangle XYZ$ if $m\angle X = 32$, $m\angle Y = 61$, and $y = 15$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

Find the measure of $\angle Z$.



$m\angle X + m\angle Y + m\angle Z = 180$ — Angle Sum Theorem

$32 + 61 + m\angle Z = 180$ — $m\angle X = 32$ and $m\angle Y = 61$

$93 + m\angle Z = 180$ — Add.

$m\angle Z = 87$ — Subtract 93 from each side.

(continued on the next page)

Chapter 7 Study Guide and Review 395

Chapter 7
- Extra Practice, see pages 766-768.
- Mixed Problem Solving, see page 788.

Answers

33. $m\angle B \approx 41$, $m\angle C \approx 75$, $c \approx 16.1$
34. $m\angle B = 58$, $b \approx 11.1$, $a \approx 10.7$
35. $m\angle B \approx 61$, $m\angle C \approx 90$, $c \approx 9.9$
36. $m\angle C = 87$, $a \approx 10.0$, $c \approx 20.6$
39. $a \approx 17.0$, $m\angle B \approx 43$, $m\angle C \approx 73$
40. $m\angle B \approx 38$, $m\angle A \approx 89$, $a \approx 8.4$

Since we know $m\angle Y$ and y, use proportions involving sin Y and y.

To find x:		**To find z:**
$\frac{\sin Y}{y} = \frac{\sin X}{x}$	Law of Sines	$\frac{\sin Y}{y} = \frac{\sin Z}{z}$
$\frac{\sin 61°}{15} = \frac{\sin 32°}{x}$	Substitute.	$\frac{\sin 61°}{15} = \frac{\sin 87°}{z}$
$x \sin 61° = 15 \sin 32°$	Cross products	$z \sin 61° = 15 \sin 87°$
$x = \frac{15 \sin 32°}{\sin 61°}$	Divide.	$z = \frac{15 \sin 87°}{\sin 61°}$
$x \approx 9.1$	Use a calculator.	$z \approx 17.1$

Exercises **Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.**
See Example 1 on page 378.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$. **21.3**
32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$. **47**

Solve each $\triangle ABC$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth. *See Example 2 on pages 378 and 379.*

33. $a = 15$, $b = 11$, $m\angle A = 64$
34. $c = 12$, $m\angle C = 67$, $m\angle A = 55$
35. $m\angle A = 29$, $a = 4.8$, $b = 8.7$
36. $m\angle A = 29$, $m\angle B = 64$, $b = 18.5$

33–36. See margin.

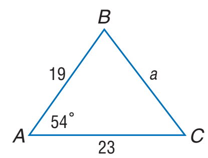
7-7 The Law of Cosines

See pages 385–390.

Concept Summary

- The Law of Cosines can be used to solve triangles when you know the measures of two sides and the included angle (SAS) or the measures of the three sides (SSS).

Example **Find a if $b = 23$, $c = 19$, and $m\angle A = 54$.**



Since the measures of two sides and the included angle are known, use the Law of Cosines.

$a^2 = b + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$	$b = 23$, $c = 19$, and $m\angle A = 54$
$a = \sqrt{23^2 + 19^2 - 2(23)(19) \cos 54°}$	Take the square root of each side.
$a \approx 19.4$	Use a calculator.

Exercises **In $\triangle XYZ$, given the following measures, find the measure of the missing side.** *See Example 1 on page 385.*

37. $x = 7.6$, $y = 5.4$, $m\angle Z = 51$ **$z \approx 5.9$**
38. $x = 21$, $m\angle Y = 73$, $z = 16$ **$y \approx 22.4$**

Solve each triangle using the given information. Round angle measure to the nearest degree and side measure to the nearest tenth.
See Example 3 on pages 386 and 387. **39–40. See margin.**

39. $c = 18$, $b = 13$, $m\angle A = 64$
40. $b = 5.2$, $m\angle C = 53$, $c = 6.7$

Answers (page 397)

2. Yes; two perfect squares can be written as $a \cdot a$ and $b \cdot b$. Multiplied together, we have $a \cdot a \cdot b \cdot b$. Taking the square root, we have ab, which is rational.

20. $m\angle A \approx 59$, $m\angle B \approx 76$, $c \approx 12.4$
21. $m\angle A \approx 56$, $m\angle C \approx 76$, $c \approx 14.2$
22. $m\angle A \approx 51$, $m\angle B \approx 72$, $m\angle C \approx 57$

Chapter 7 Practice Test

Vocabulary and Concepts

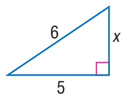
1. **State** the Law of Cosines for $\triangle ABC$ used to find $m\angle C$. $c^2 = a^2 + b^2 - 2ab \cos C$
2. **Determine** whether the geometric mean of two perfect squares will always be rational. Explain.
3. **Give** an example of side measures of a 30°-60°-90° triangle. **Sample answer: 2, $2\sqrt{3}$, 4** **2. See margin.**

Skills and Applications

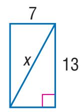
Find the geometric mean between each pair of numbers.

4. 7 and 63 **21**
5. 6 and 24 **12**
6. 10 and 50 **$10\sqrt{5}$**

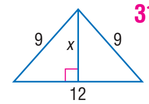
Find the missing measures.

7. 

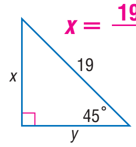
$\sqrt{11} \approx 3.32$

8. 

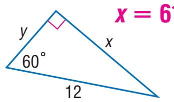
$\sqrt{218} \approx 14.8$

9. 

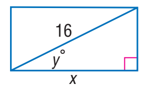
$3\sqrt{5} \approx 6.7$

10. 

$x = \frac{19\sqrt{2}}{2}$; $y = \frac{19\sqrt{2}}{2}$

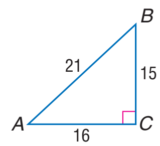
11. 

$x = 6\sqrt{3}$, $y = 6$

12. 

$x = 8\sqrt{3}$, $y = 30$

Use the figure to find each trigonometric ratio. Express answers as a fraction.



13. $\cos B$ **$\frac{5}{7}$**
14. $\tan A$ **$\frac{15}{16}$**
15. $\sin A$ **$\frac{5}{7}$**

Find each measure using the given measures from $\triangle FGH$. Round to the nearest tenth.

16. Find g if $m\angle F = 59$, $f = 13$, and $m\angle G = 71$. **14.3**
17. Find $m\angle H$ if $m\angle F = 52$, $f = 10$, and $h = 12.5$. **80.1**
18. Find f if $g = 15$, $h = 13$, and $m\angle F = 48$. **11.5**
19. Find h if $f = 13.7$, $g = 16.8$, and $m\angle H = 71$. **17.9**

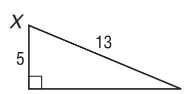
Solve each triangle. Round each angle measure to the nearest degree and each side measure to the nearest tenth. **20–22. See margin.**

20. $a = 15$, $b = 17$, $m\angle C = 45$
21. $a = 12.2$, $b = 10.9$, $m\angle B = 48$
22. $a = 19$, $b = 23.2$, $c = 21$

23. **TRAVEL** From an airplane, Janara looked down to see a city. If she looked down at an angle of 9° and the airplane was half a mile above the ground, what was the horizontal distance to the city? **3.2 mi**

24. **CIVIL ENGINEERING** A section of freeway has a steady incline of 10°. If the horizontal distance from the beginning of the incline to the end is 5 miles, how high does the incline reach? **0.9 mi**

25. **STANDARDIZED TEST PRACTICE** Find $\tan X$. **D**



(A) $\frac{5}{12}$ (B) $\frac{12}{13}$ (C) $\frac{17}{12}$ (D) $\frac{12}{5}$

www.geometryonline.com/chapter_test

Portfolio Suggestion

Introduction After working through a chapter of very detailed material, it can help to determine strengths and weaknesses.

Ask Students List all the methods and formulas for solving triangles that you learned about in this chapter. Select a favorite method to write about and explain why you are most comfortable with this particular method. Write about your least favorite method and include possible ideas and activities to help you master it. Place this in your portfolio.

Chapter 7 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 7 can be found on p. 406 of the *Chapter 7 Resource Masters*.

Chapter Tests There are six Chapter 7 Tests and an Open-Ended Assessment task available in the *Chapter 7 Resource Masters*.

Chapter 7 Tests

Form	Type	Level	Pages
1	MC	basic	393–394
2A	MC	average	395–396
2B	MC	average	397–398
2C	FR	average	399–400
2D	FR	average	401–402
3	FR	advanced	403–404

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment Performance tasks for Chapter 7 can be found on p. 405 of the *Chapter 7 Resource Masters*. A sample scoring rubric for these tasks appears on p. A28.

Unit 2 Test A unit test/review can be found on pp. 413–414 of the *Chapter 7 Resource Masters*.

First Semester Test A test for Chapters 1–7 can be found on pp. 415–416 of the *Chapter 7 Resource Masters*.

ExamView® Pro

Use the networkable **ExamView® Pro** to:

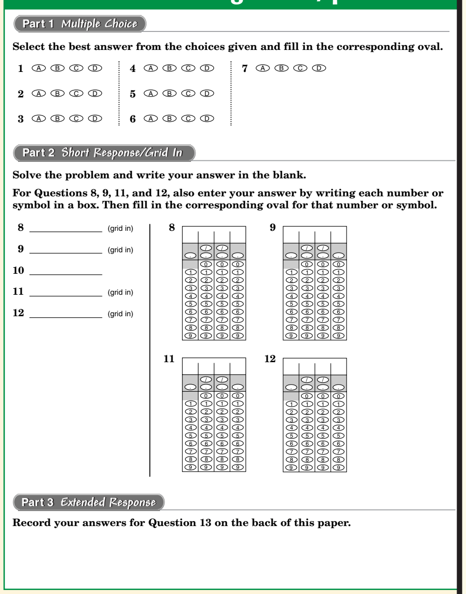
- Create **multiple versions** of tests.
- Create **modified** tests for Inclusion students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your tests from a program bank of artwork.

Chapter 7 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 7 Resource Masters*.

Standardized Test Practice
Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 8, 9, 11, and 12, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

8 ______ (grid in)
9 ______ (grid in)
10 ______
11 ______ (grid in)
12 ______ (grid in)

Part 3 Extended Response

Record your answers for Question 13 on the back of this paper.

Additional Practice

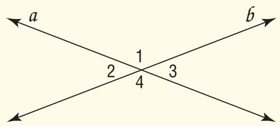
See pp. 411–412 in the *Chapter 7 Resource Masters* for additional standardized test practice.

Chapter 7 Standardized Test Practice

Part 1 Multiple Choice

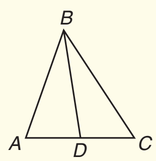
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If ∠4 and ∠3 are supplementary, which reason could you use as the first step in proving that ∠1 and ∠2 are supplementary? (Lesson 2-7) **C**



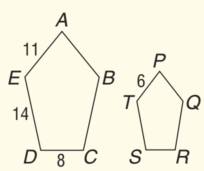
Ⓐ Definition of similar angles
Ⓑ Definition of perpendicular lines
Ⓒ Definition of a vertical angle
Ⓓ Division Property

2. In △*ABC*, $\overline{BD}$ is a median. If $AD = 3x + 5$ and $CD = 5x - 1$, find *AC*. (Lesson 5-1) **D**



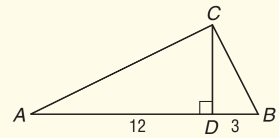
Ⓐ 3 Ⓑ 11
Ⓒ 14 Ⓓ 28

3. If pentagons *ABCDE* and *PQRST* are similar, find *SR*. (Lesson 6-2) **B**



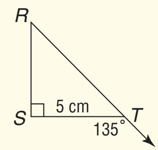
Ⓐ $14\frac{2}{3}$ Ⓑ $4\frac{4}{11}$
Ⓒ 3 Ⓓ $1\frac{5}{6}$

4. In △*ABC*, $\overline{CD}$ is an altitude and $m\angle ACB = 90°$. If $AD = 12$ and $BD = 3$, find *AC* to the nearest tenth. (Lesson 7-1) **C**



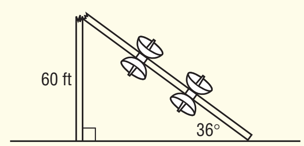
Ⓐ 6.5 Ⓑ 9.0 Ⓒ 13.4 Ⓓ 15.0

5. What is the length of $\overline{RT}$? (Lesson 7-3) **B**



Ⓐ 5 cm Ⓑ $5\sqrt{2}$ cm
Ⓒ $5\sqrt{3}$ cm Ⓓ 10 cm

6. An earthquake damaged a communication tower. As a result, the top of the tower broke off at a point 60 feet above the base. If the fallen portion of the tower made a 36° angle with the ground, what was the approximate height of the original tower? (Lesson 7-4) **D**



Ⓐ 35 ft Ⓑ 95 ft
Ⓒ 102 ft Ⓓ 162 ft

7. Miraku drew a map showing Regina's house, Steve's apartment, and Trina's workplace. The three locations formed △*RST*, where $m\angle R = 34$, $r = 14$, and $s = 21$. Which could be $m\angle S$? (Lesson 7-6) **C**

Ⓐ 15 Ⓑ 22 Ⓒ 57 Ⓓ 84

ExamView® Pro

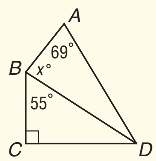
Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and state proficiency tests can be found on this CD-ROM.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 795–810.

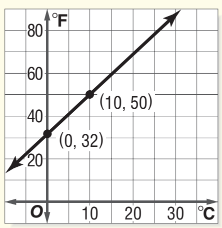
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Find $m\angle ABC$ if $m\angle CDA = 61$. (Lesson 1-6) **140**



For Questions 9 and 10, refer to the graph.

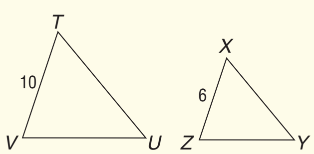


9. At the International Science Fair, a Canadian student recorded temperatures in degrees Celsius. A student from the United States recorded the same temperatures in degrees Fahrenheit. They used their data to plot a graph of Celsius versus Fahrenheit. What is the slope of their graph? (Lesson 3-3) **9/5**

10. Students used the equation of the line for the temperature graph of Celsius versus Fahrenheit to convert degrees Celsius to degrees Fahrenheit. If the line goes through points (0, 32) and (10, 50), what equation can the students use to convert degrees Celsius to degrees Fahrenheit? (Lesson 3-4)

$$y = \frac{9}{5}x + 32$$

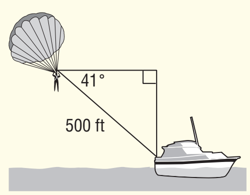
11. $\triangle TUV$ and $\triangle XYZ$ are similar. Calculate the ratio $\frac{YZ}{UV}$. (Lesson 6-3) **3/5**



Test-Taking Tip

Questions 6, 7, and 12
If a standardized test question involves trigonometric functions, draw a diagram that represents the problem and use a calculator (if allowed) or the table of trigonometric relationships provided with the test to help you find the answer.

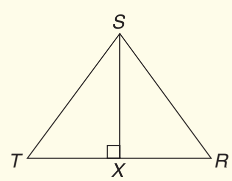
12. Dee is parasailing at the ocean. The angle of depression from her line of sight to the boat is 41°. If the cable attaching Dee to the boat is 500 feet long, how many feet is Dee above the water? (Lesson 7-5) **328**



Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

13. Toby, Rani, and Sasha are practicing for a double Dutch rope-jumping tournament. Toby and Rani are standing at points T and R and are turning the ropes. Sasha is standing at S, equidistant from both Toby and Rani. Sasha will jump into the middle of the turning rope to point X. Prove that when Sasha jumps into the rope, she will be at the midpoint between Toby and Rani. (Lessons 4-5 and 4-6) **See margin.**



www.geometryonline.com/standardized_test

Evaluating Extended Response Questions

Extended Response questions are graded by using a multilevel rubric that guides you in assessing a student's knowledge of a particular concept.

Goal: Prove that a point is a midpoint.

Sample Scoring Rubric: The following rubric is a sample scoring device. You may wish to add more detail to this sample to meet your individual scoring needs.

Score	Criteria
4	A correct solution that is supported by well-developed, accurate explanations
3	A generally correct solution, but may contain minor flaws in reasoning or computation
2	A partially correct interpretation and/or solution to the problem
1	A correct solution with no supporting evidence or explanation
0	An incorrect solution indicating no mathematical understanding of the concept or task, or no solution is given

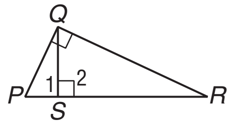
Answers

13. Since Sasha is equidistant from Toby and Rani, $\overline{ST}$ and $\overline{SR}$ are congruent and $\triangle STR$ is an isosceles triangle. According to the Isosceles Triangle Theorem, $\angle T$ and $\angle R$ are also congruent. $\overline{SX}$ is perpendicular to $\overline{TR}$, so $\angle SXT$ and $\angle SXR$ are both right angles and congruent. Two corresponding angles and the corresponding nonincluded sides are congruent (AAS Theorem), so $\triangle STX$ and $\triangle SRX$ are congruent triangles. Since these triangles are congruent, the corresponding sides $\overline{TX}$ and $\overline{RX}$ are congruent and have equal length; therefore when Sasha is jumping at Point X she will be at the midpoint between Toby and Rani.

Chapter 7 Standardized Test Practice 399

Pages 345–348, Lesson 7-1

45. Given: $\angle PQR$ is a right angle.
$\overline{QS}$ is an altitude of $\triangle PQR$.



Prove: $\triangle PSQ \sim \triangle PQR$
$\triangle PQR \sim \triangle QSR$
$\triangle PSQ \sim \triangle QSR$

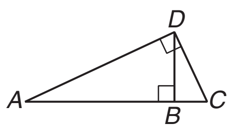
Proof:

Statements (Reasons)

1. $\angle PQR$ is a right angle; $\overline{QS}$ is an altitude of $\triangle PQR$. (Given)
2. $\overline{QS} \perp \overline{RP}$ (Definition of altitude)
3. $\angle 1$ and $\angle 2$ are right angles. (Definition of $\perp$ lines)
4. $\angle 1 \cong \angle PQR$; $\angle 2 \cong \angle PQR$ (All right $\measuredangle$s are $\cong$.)
5. $\angle P \cong \angle P$; $\angle R \cong \angle R$ (Congruence of $\measuredangle$s is reflexive.)
6. $\triangle PSQ \sim \triangle PQR$; $\triangle PQR \sim \triangle QSR$ (AA Similarity; Statements 4 and 5)
7. $\triangle PSQ \sim \triangle QSR$ (Similarity of triangles is transitive.)

46. Given: $\angle ADC$ is a right angle.
$\overline{DB}$ is an altitude of $\triangle ADC$.

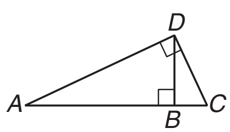
Prove: $\frac{AB}{DB} = \frac{DB}{CB}$



Proof: It is given that $\angle ADC$ is a right angle and $\overline{DB}$ is an altitude of $\triangle ADC$. $\triangle ADC$ is a right triangle by the definition of a right triangle. Therefore, $\triangle ADB \sim \triangle DCB$, because if the altitude is drawn from the vertex of the right angle to the hypotenuse of a right triangle, then the two triangles formed are similar to the given triangle and to each other. So $\frac{AB}{DB} = \frac{DB}{CB}$ by definition of similar polygons.

47. Given: $\angle ADC$ is a right angle.
$\overline{DB}$ is an altitude of $\triangle ADC$.

Prove: $\frac{AB}{AD} = \frac{AD}{AC}$; $\frac{BC}{DC} = \frac{DC}{AC}$



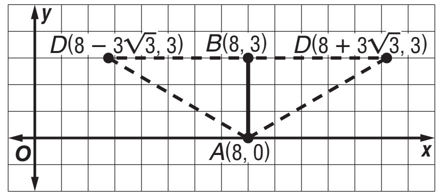
Proof:

Statements (Reasons)

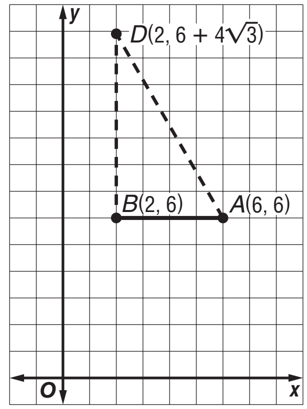
1. $\angle ADC$ is a right angle; $\overline{DB}$ is an altitude of $\triangle ADC$. (Given)
2. $\triangle ADC$ is a right triangle. (Definition of right triangle)
3. $\triangle ABD \sim \triangle ADC$; $\triangle DBC \sim \triangle ADC$ (If the altitude is drawn from the vertex of the rt. $\angle$ to the hypotenuse of a rt. $\triangle$, then the 2 $\triangle$s formed are similar to the given $\triangle$ and to each other.)
4. $\frac{AB}{AD} = \frac{AD}{AC}$; $\frac{BC}{DC} = \frac{DC}{AC}$ (Definition of similar polygons)

Pages 360–363, Lesson 7-3

9.



10.



Page 391, Geometry Activity

3.

$\frac{\sin\theta}{\cos\theta} \stackrel{?}{=} \tan\theta$	Original equation
$\frac{\frac{y}{r}}{\frac{x}{r}} \stackrel{?}{=} \frac{y}{x}$	$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\tan\theta = \frac{y}{x}$
$\frac{y}{r} \cdot \frac{r}{x} \stackrel{?}{=} \frac{y}{x}$	Multiply by the reciprocal of $\frac{x}{r}$.
$\frac{y}{x} = \frac{y}{x}$ ✓	Multiply.

4.

$\frac{\cos\theta}{\sin\theta} \stackrel{?}{=} \cot\theta$	Original equation
$\frac{\frac{x}{r}}{\frac{y}{r}} \stackrel{?}{=} \frac{x}{y}$	$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, $\cot\theta = \frac{x}{y}$
$\frac{x}{r} \cdot \frac{r}{y} \stackrel{?}{=} \frac{x}{y}$	Multiply by the reciprocal of $\frac{y}{x}$.
$\frac{x}{y} = \frac{x}{y}$ ✓	Multiply.

5.

$\tan^2\theta + 1 \stackrel{?}{=} \sec^2\theta$	Original equation
$\left(\frac{y}{x}\right)^2 + 1 \stackrel{?}{=} \left(\frac{r}{x}\right)^2$	$\tan\theta = \frac{y}{x}$, $\sec\theta = \frac{r}{x}$
$\frac{y^2}{x^2} + 1 \stackrel{?}{=} \frac{r^2}{x^2}$	Evaluate exponents.
$x^2\left(\frac{y^2}{x^2} + 1\right) \stackrel{?}{=} x^2 \cdot \frac{r^2}{x^2}$	Multiply each side by x^2.
$y^2 + x^2 \stackrel{?}{=} r^2$	Simplify.
$r^2 = r^2$ ✓	Substitution; $y^2 + x^2 = r^2$

6.

$\cot^2\theta + 1 \stackrel{?}{=} \csc^2\theta$	Original equation
$\left(\frac{x}{y}\right)^2 + 1 \stackrel{?}{=} \left(\frac{r}{y}\right)^2$	$\cot\theta = \frac{x}{y}$, $\sec\theta = \frac{r}{y}$
$\frac{x^2}{y^2} + 1 \stackrel{?}{=} \frac{r^2}{y^2}$	Evaluate exponents.
$y^2\left(\frac{x^2}{y^2} + 1\right) \stackrel{?}{=} y^2 \cdot \frac{r^2}{y^2}$	Multiply each side by y^2.
$x^2 + y^2 \stackrel{?}{=} r^2$	Simplify.
$r^2 = r^2$ ✓	Substitution; $x^2 + y^2 = r^2$

Notes